



Restoring Lorentz symmetries in parity-even Einstein–Cartan domain walls conformal to Bianchi I metric embedded in weak gravity Weitzenböck vacuum manifold

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Abstract It is well known that a weak gravity limit can be associated with a restoration of Lorentz covariance. In this paper we present a physical example of a parity-even Einstein–Cartan domain wall (ECDW) embedded in a Weitzenböck zero connection vacuum manifold in the weak torsion gravity limit. Off the wall, the manifold is almost Minkowskian, and the teleparallel equivalent of general relativity is restored (TEGR). ECDW non-dynamical torsion gravity implies that the regions off the wall would naturally be Minkowskian. Thus, our generalization here is used for a parity-even ECDW in Weitzenböck geometry to allow for coupling the wall to the weak gravitational waves in teleparallel gravity. Recently, Li et al. reported giving up Weitzenböck geometry, and they also argued that local Lorentz invariance (LLI) does not affect experiments. But here we argue that in these lab experiments, weak Weitzenböck gravity is used and the specific ECDW embedded in the Weitzenböck manifold vacuum can be considered equivalent to GR (TEGR). Since conformal Bianchi type I metrics are in general not Bianchi I, we do not address here the use of the proper Weitzenböck frame instead of the orthonormal frame which introduces the local covariance violation.

1 Introduction

A weak gravity limit is known in some contexts to be associated with Lorentz covariant restoration [1], even if the full theory might exhibit violations of Lorentz symmetry at stronger gravity fields. This is relevant mainly to modified gravity theories [2], where Lorentz symmetry might be broken, and an approximated Lorentz symmetry is valid at

low energies and gravitational fields but broken at high energies or strong gravitational fields. Within the frame of modified teleparallel gravity [3,4] (book), Wu [5] showed that at teleparallel inflation, torsion-generated primordial fluctuations arise without Lorentz symmetry. He argued that arbitrary generalizations to the teleparallel equivalent of Einstein's gravity (TEGR) lose local Lorentz invariance (LLI) to reparameterize the orthonormal coordinate, giving rise to asymmetric field equations. He also showed that although some asymmetry is removed in a spatially homogeneous Bianchi type I anisotropic metric in cosmic microwave background (CMB) in this $f(T)$ gravity, where T is the product of torsion tensor contraction, can influence primordial fluctuations. Early on, we showed that [6] dynamo theory, which we also dealt with in this paper, could have LV in cosmological non-teleparallelism. In the present paper, we show that all of these reasonings may be applied to a specific example of an Einstein–Cartan domain wall (ECDW) conformal with respect to the Bianchi type I metric, where a $(2 + 1)$ -dimensional metric section is responsible for the expansion of the ECDW along this 2D direction. Instead of embedding this DW into a Minkowskian manifold, we embedded it into a Weitzenböck manifold, which would not be teleparallel equivalent to GR. The way out of this ambiguity would be to apply the weak gravity Weitzenböck manifold to allow the embedded manifold of DW to be almost Minkowskian. It is important to note, however, that Li et al. [7] recently considered giving up Weitzenböck gauge-fixing of a vanishing connection. They stressed that the Weitzenböck choice in principle, despite Lorentz violation induced in the orthonormal frame, does not invalidate or contradict experiments. Now, with regard to the physical implications of this hidden ECDW LV example, we will investigate gravitational waves (GWs), recently studied by Abbott et al. in the LIGO [7] experiment,

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which led to the discovery of GWs 100 years after Einstein [8] predicted them as a solution of his gravitational theory of GR [9]. In recent years, LIGO data have been used to check the compatibility of the data with the well-known teleparallel gravity [10, 11], and to compare the strain of the arms of the detector with strain associated with torsion waves of a spin-1 and spin-0 propagating nature [12]. In this teleparallelism theory, we compare the LIGO data with the torsion at the surface of the Earth estimated by Nitsch [13], on the order of $10^{-24} s^{-1}$. In recent years, the use of domain walls has been reported [14]. In previous works [15, 16], we have investigated invariants like the Pontrjagin and Nieh-Yan torsional invariant teleparallel equivalent of GR and its role in physics. Additionally, researchers have investigated the generation of primordial magnetic fields from massive photons and Debye screening in the presence of Cartan torsion [17]. In another gravity alternative theory, called Einstein–Cartan (EC) gravity [18], kink solutions with the potential $\sim \phi^4$ are obtained here in the framework of its Maxwell extension and DWs. An interesting result here is that dynamo action is damped by domain wall torsion. Furthermore, torsion and metric chirality are shown to play a fundamental role in the enhancement or damping of dynamo in the plasma.

In the present paper, we have mixed two important physical phenomena, namely GWs and DWs, in the presence of torsion. In the case of GW teleparallelism, one shows that the divergence of contortion is expressed in terms of the stress tensor of viscous flows, while in the DW case, it is shown that torsion also acts as a damping force on inflationary de Sitter, which is the $(2 + 1) - D$ spacetime off the planar EC DW topological defect [19]. Here we assume that, by analogy with the GR, one has both sides of the thin approximation of the domain wall as teleparallel vacua. This is of course analogous to GR DWs, since in both cases their respective curvatures vanish: in GR, the Riemann curvature vanishes, whereas in the teleparallel gravity (TG) it is the Riemann–Cartan manifold full curvature tensor that vanishes. Of course, this Riemann–Cartan curvature contains the general Cartan connection with Riemann–Christoffel symbols depending upon the metric tensor and the Cartan contortion. The matching conditions on the metric and torsion investigated by Arkowieicz et al. [20] are applied to the thick wall metric given by Guerrero et al. [14] in the case of self-gravitating domain walls, where they also investigated the thin-wall limit when the thickness of DW goes to zero. This wall thickness is compared to the thickness computed in the dilatonic domain wall with torsion in two cases: the first when the wall is non-magnetic [21], and the second when the wall is magnetized. In the latter case, the domain wall stress tensor is added not only to the Kalb–Ramond torsion [22] type tensor but also to the Maxwell stress-electromagnetic field one. Moreover, dynamo action [23] is also tested along this domain wall. The comoving magnetic field is used in the

dynamo equation as it is used in the case of chiral plasma instability [19]. Recent studies have investigated the wall dynamos in Einstein–Cartan gravity [24] as well as the role of torsion in the chiral chemical potential [25] and the action of chiral torsion on the dynamos. We show that, depending upon the handedness of torsion, dynamo action can be enhanced or can decay in the plasma. In the teleparallel domain wall case, we find that the Riemann–Cartan vanishing curvature constraint leads to a torsion source for chiral GWs, which is given by the divergence of contortion, which plays a role of the stress viscosity tensor. In these cases, dynamo and GW actions may be damped via torsion. An important aspect of this paper is the dynamo action damping driven by torsion which appears before bouncing of the universe. Actually, after the bounce, the magnetic field lines are compressed by this bouncing effect, called gravitational dynamo, which is actually how galactic clouds are formed.

Investigating the role of torsionful teleparallel DWs in this paper is interesting, because it is well known that, in general, magnetic fields are left as imprints after domain walls collapse. Here we find an exact solution of the Einstein–Cartan-scalar domain walls and the analogous EC-scalar-Maxwell equations where both are analyzed in terms of the thickness of the DWs. Moreover, we determine how the magnetic fields behave as we pass the thin-wall limit. In Sect. 3, dilatonic thick domain walls with torsion and Maxwell fields and junction conditions are given. In Sect. 4 we consider the problem of inflating domain walls and the damping of the dynamo action, whereas the GW viscous stress is obtained in teleparallelism in terms of the divergence of contortion sources. Section 6 is an appendix which addresses the even-parity symmetry of ECDWs and also the parity-even teleparallel gravity surrounding the wall. Section 7 is left for conclusions.

2 Matching conditions in the Einstein–Cartan-scalar DDW solution for gravitational dynamo

In this section, we review the dilatonic domain wall (DDW) in EC gravity and compare it with the thick domain wall metric where the DW is non-magnetized, so in this case no dynamo action is provided. Moreover, we investigate the discontinuities in the torsion axial vector in its spatial part and the junction conditions on a boundary as discussed by Bressange [22]. Let us start now by considering the general metric of the thick domain wall solution as given by Guerrero et al. in the form

$$ds^2 = \left(\cosh \frac{c_0 z}{\delta} \right)^{-2\delta} [dt^2 - dz^2 - e^{2c_0 t} (dx^2 + dy^2)], \quad (1)$$

where δ represents the thickness of the wall (not to be confused with the delta Dirac distribution $\delta(z)$). Comparison

of this geometric line element with that found in DDW with torsion was previously obtained as the conformal metric [23]

$$ds^2 = A(z)[dt^2 - dz^2 - e^{2S_0t}(dx^2 + dy^2)], \tag{2}$$

where S_0 is the zero component of the torsion axial vector. Comparing the two metric solutions, we find that that $c_0 = S_0$, and $A(z)$ of the thick DW is given by

$$A(z) = \left(\cosh \frac{c_0 z}{\delta}\right)^{-2\delta}. \tag{3}$$

Thus, from the EC field equation, the metric (2) solution is

$$A(z) = e^{S_0 z}. \tag{4}$$

From expressions (3) and (4) we obtain

$$\left(\cosh \frac{c_0 z}{\delta}\right) = (e^{S_0 z})^{-\frac{1}{2\delta}}. \tag{5}$$

Straightforward algebra then allows us to obtain the following expression:

$$\left(\cosh \frac{c_0 z}{\delta}\right) = e^{\frac{2S_0 z}{\delta}}. \tag{6}$$

Let us now imagine that we are quite close to the wall, so in this frame, $z \ll 1$, one can approximate both functions in the last expression as

$$\left(1 - \frac{S_0 z}{2\delta}\right) = \left(1 + \frac{c_0 z}{\delta}\right), \tag{7}$$

which leads to the relation $c_0 = \frac{S_0}{2}$. This provides us with the determination between the c_0 and de Sitter-like coefficient of expansion of the wall and the torsion axial vector 0-component. Thus, if we choose the torsion handedness as positive, this relation implies that c_0 is negative and the time evolution of the wall is to contract and gravitationally collapse. In the magnetic wall example in the next section, where the magnetic field lines are tied to this wall contraction in time, the magnetic field lines are compressed, forcing the gravitational dynamo to compress clouds, which by analogy can be explained in the DDW contraction. The metric which represents the EC-scalar phenomena can be geometrically expressed as

$$ds^2 = \left[\cosh \left(\frac{-S_0 z}{2\delta}\right)\right]^{-2\delta} [dt^2 - dz^2 - e^{-2S_0 t}(dx^2 + dy^2)]. \tag{8}$$

Note that the expansion or contraction of the $(x - y)$ -plane where the thick domain wall is located depends upon the handedness or the sign or helicity of the torsion 0-component of the axial vector, in the case of massless torsion. Since even-parity DWs cannot change handedness, we conclude that either the magnetic fields would decay or the dynamo could be stable around an unstable domain wall. In the next section, we show how this situation is modified with the introduction of a magnetic field in the stress-Maxwell energy. The previous section addressed the problem of chiral GWs in

teleparallelism, where the divergence of the contortion tensor acts as a source for the chiral GWs equivalently to viscous and damping stress tensors. We note that in the last metric, if the torsion 0-component S_0 vanishes, the metric reduces to a flat metric off the wall. Moreover, at $z = 0$, it reduces to the Riemann-flat Minkowski metric

3 Magnetic dynamo stability from parity-even ECDWS surrounded by teleparallel vacua walls

Before analyzing the matching conditions in the analytical solution of Einstein–Cartan–Maxwell-scalar (ECMS) field equations on a DDW background, we review how to introduce the stress matter, the electromagnetic and Kalb–Ramond-like stress-torsion tensor. This can be accomplished as follows: The ECMS gravity field equation reads

$$G^{\text{Riem}}_{ij} = \kappa T_{ij}, \tag{9}$$

where the term on the LHS of this equation is the Einstein Riemannian tensor, and the total energy-stress tensor T_{ij} , where $(i, j = 0, 1, 2, 3)$, is the sum of these three tensors

$$T_{ij}^{\text{scalar}} = \partial_i \Phi \partial_j \Phi - g_{ij} \left(\frac{1}{2} \partial_k \Phi \partial^k \Phi - V(\Phi)\right). \tag{10}$$

Besides the stress tensor, there is the torsional one, given by

$$T_i^{k\text{torsion}} = S_{iml} S^{kml} - \frac{1}{2} \delta_i^k S_z^2, \tag{11}$$

whereas the additional Maxwell electromagnetic stress-energy tensor

$$T_i^{k\text{torsion}} = F_{im} F^{km} - \frac{1}{2} \delta_i^k F^2, \tag{12}$$

where $F_{ij} = A_{j,i} - A_{i,j}$ is the electromagnetic Maxwell field tensor components, and A_j is the electromagnetic four-potential vector. We also define $F^2 = F_{ij} F^{ij} = E^2 - B^2$ as the electromagnetic scalar. Here we assume, as usual, that the electromagnetic field tensor does not couple minimally with spin-torsion density. Moreover, the component of axial vector torsion is defined along the z -coordinate and orthogonal to the wall which is located at the $(x - y)$ -plane. In these expressions, we consider the following functional forms: the scalar field $\Phi = \Phi(z)$, and the spin-torsion tensor is $S_z^2 = S_z^2(z)$, representing the spin-torsion energy density. The magnetic field is time-dependent in the sense that $B = B(t, z)$. One notes that the DDW is static in the sense that the Higgs-like potential does not depend on time. In what follows, we will use the dynamical character of the magnetic field to obtain the dynamo equation for the magnetic field. Let us now build the effective energy density and effective pressure density as

$$T_t^t = T_x^x = T_y^y = e^\lambda \Phi^{1^2} + V(\Phi) - 2S_z^2 e^{-3\lambda} + (E^2 + B^2) = \rho_{\text{eff}}, \tag{13}$$

where the prime over the scalar field indicates the first derivative on the z -coordinate. Moreover, we will assume that E^2 may be neglected. Now, the last component of the fully Einstein–Cartan–Maxwell stress-energy tensor is

$$T_z^z = -\frac{1}{2} \left(e^{-\lambda} \Phi^1 + V(\Phi) + (E^2 + B^2) - 3S_z^2 e^{-3\lambda} \right) = -p_{\text{eff}}. \tag{14}$$

The scalar field coupled to torsion in the wall is given by

$$\Phi^{11} + \Phi^1 \left(\psi^1 - S_z(z) \right) = -\frac{dV}{d\Phi} e^\lambda. \tag{15}$$

To avoid coupling of the torsion component with the scalar field, one may perform a cutoff such that

$$\psi^1 = S_z(z). \tag{16}$$

Now let us consider the DW Goetz metric [25] given by

$$ds^2 = A(z)[dt^2 - dz^2 - b(t)(dx^2 + dy^2)]. \tag{17}$$

Thus, one can observe that the time dependence of the metric component g_{tt} has been eliminated for convenience. The EC gravity field equation for the magnetic domain wall metric is

$$G_t^t - G_x^x = 0, \tag{18}$$

which results in

$$\ddot{b}b - \dot{b}^2 = 0. \tag{19}$$

A simple solution of this equation is

$$b(t) = b_0 e^{c_0 t}. \tag{20}$$

Therefore, since this metric function is the metric coefficient of the section $x - y$ of the wall, one may say that the wall inflates. Clearly, however, this happens if c_0 is positive, whereas if the metric handedness is negative or $c_0 < 0$, the domain wall gravitationally contracts, this time with the presence of magnetic field lines and torsion, which supports our idea that this DDW metric takes into account the magnetic field amplification by gravitational contraction. Once magnetic fields are present, we will see in the next section that there is a magnetic dynamo as well. Therefore, the remaining EC field equations in the Goetz [24] metric are

$$-\frac{A^{11}}{A^2} + \frac{3A^{1^2}}{2A^3} - \frac{c^2}{2A} = \frac{\Phi^{1^2}}{A} - B^2 \tag{21}$$

and

$$-\frac{A^{1^2}}{A^2} + \frac{c^2}{A} = V(\Phi) - \frac{2S_z^2}{A^3} - B^2. \tag{22}$$

Let us assume that the ansatz $A(z) = e^{S_z z}$. This ansatz is compatible with the flat spacetime Minkowski metric off the wall. This happens because there the torsion-spin density S_z

vanishes, and off the domain wall one obtains a flat metric Minkowski spacetime. Thus, all the above components of the total stress-energy density must vanish. In the effective matter density, its vanishing yields the equation

$$\Phi^{1^2} + V(\Phi) + B^2 = 0, \tag{23}$$

where we have used $\lambda = 0$ to obtain a flat metric off the wall. If the magnetic field is to be confined along the wall, then we assume the expression

$$B^2 = B_0^2 \delta(z), \tag{24}$$

and expression (16) reduces to

$$\Phi^{1^2} + V(\Phi) = 0 \tag{25}$$

off the DW. Checking of the solution matching conditions also leads to $\psi^1 = S_z \delta(z)$. Thus, the axial torsion pseudo-vector and magnetic field are both confined to the wall. Note that torsion is not confined to the wall but is orthogonal to the wall. Now let us start solving the EC equations as a certain Brans–Dicke (BD) EC gravity, by expressing the $A(z)$ metric factor as

$$A(z) \approx (1 - S_z z), \tag{26}$$

which implies that

$$[A(z)]^{-1} \approx (1 + S_z z). \tag{27}$$

This approximation is physically reasonable since we are close to the DW, so the z -coordinate is small and torsion may be weak in the present universe. Proceeding analogously to the case of A^{-3} and substituting these results into the EC DW equations, one obtains

$$\Phi^{1^2} = B^2(z) - 2c^2 + \frac{3}{2} S_z^2. \tag{28}$$

By assuming that the magnetic field energy density is confined to the wall, we may easily find the solution of this equation off the DW by dropping the B^2 term, which yields

$$\Phi(z) \approx S_z z. \tag{29}$$

The DW potential $V(\Phi)$ for the equations may be expressed as

$$V(\Phi) = B^2(z) + S_z^2 e^{-S_z z} + 6S_z^3 e^{-3S_z z}. \tag{30}$$

From the relation between the potential and torsion one may rewrite this expression as

$$V(\Phi) = B^2(z) + S_z^2 e^{-\Phi} + 6S_z^3 e^{-3\Phi}. \tag{31}$$

This expression characterizes the dilatonic nature of the domain wall. In the next section, we will derive the magnetic dynamo equation and solve for the magnetic field to obtain the magnetic field $B(z)$ and complete this solution.

This expression tells us that the axial torsion constant component S_z is orthogonal to the wall, as in dynamo quantum chromodynamics (QCD) walls investigated previously [28,29]. Note that the magnetic field energy density also must be time-independent; otherwise the potential Φ would have to be time-dependent, which was not the initial hypothesis. Despite the fact that the magnetic field on the DDW does not depend upon cosmic time, we will see in the next section that it is still possible to obtain a dynamo equation, since the $x - y$ section of the wall inflates due to the presence of a de Sitter-like ansatz $b(t) = e^{ct}$ for positive c . This is what we refer to as the chiral metric. When c is negative, the DW section would collapse and the magnetic field would be left as an imprint of a primordial magnetic field. Let us now derive the dynamo equation in the background of a DDW taking into account that the manifolds on both sides of the wall are vacuum teleparallel. Here we define a new type of metric at the $z = 0$ wall section which evolves simply to a $(2 + 1) - D$ one which is given by

$$ds^2_{(2+1)} = dt^2 - a^2(t)(dx^2 + dy^2). \tag{32}$$

From this metric one may write the magnetic field in comoving coordinates as

$$B^{DW}_c = g^{22}B(t). \tag{33}$$

Thus, the magnetic field of the comoving coordinates B_c obeys the dynamo equation [25]

$$\partial_t B^{DW}_c - \eta \nabla^2 B^{DW}_c - \mu_5 B^{DW}_c = 0, \tag{34}$$

where the last term on the LHS is the chiral current, and μ_5 is the chiral chemical potential. By making use of the fact that, as shown previously [30], the chiral chemical potential is proportional to the torsion 0-component, one may express the following dynamo equation in the form

$$\left[2a\dot{a}b + a^2\dot{b} \right] B + a^2b\dot{B} + a^2b[\eta k^2 B - S_0 B] = 0, \tag{35}$$

where we make use of expression (33). This yields

$$\left[2\frac{\dot{a}}{a} + \frac{\dot{b}}{b} \right] B + \dot{B} + [\eta k^2 - S_0]B = 0, \tag{36}$$

where η is the ohmic resistivity and k is the wave vector. Note that in the dynamo equation here, we drop the convective term which involves plasma velocity and the magnetic field. As usual, the universe is considered a very good conductor and the ohmic resistivity may also be neglected, and taking the usual DW metric form $b(t) = b_0 e^{c_0 t}$, one obtains the final form of the dynamo equation as

$$\left[2\frac{\dot{a}}{a} + c_0 - S_0 \right] B + \dot{B} = 0, \tag{37}$$

which results in a very simple solution

$$B(t) = B_0 a^{-2} \exp \left[-\frac{1}{2}(S_0 - c_0)t \right], \tag{38}$$

which means physically that the chiral dynamo in a DW with chiral torsion may be dependent on the chiral metric. If both c_0 and S_0 are positive and $S_0 \geq c_0$, then in this torsion-dominated era of the universe, there is a decay in the magnetic field. Nevertheless, if this phase of the universe is dominated by the chiral metric, the magnetic field grows in time exponentially, which can overcome the decay, although in the de Sitter case, $a = a \exp[\Lambda t]$, and therefore for Λ , a positive constant, one has

$$B(t) = B_0 \exp \left[-\frac{1}{2}(2[\Lambda + S_0] - c_0)t \right]. \tag{39}$$

Therefore, to obtain a dynamo effect, one would have to impose the following constraint on de Sitter inflation:

$$c_0 \geq 2[\Lambda + S_0]. \tag{40}$$

Thus, in the case where the chiral metric dominates over torsion and inflation decays, one is still able to obtain a dynamo effect, even in the absence of convection. Of course, one may argue that the presence of the Hubble factor already indicates that the universe is expanding and that this is already a convective sort of term in the dynamo equation above. Furthermore, since we are considering a de Sitter cosmology, the factor Λ represents the presence of convection. It is interesting to notice that the DW must inflate as a torsion defect faster than the universe inflation, and this might cause instability in the wall. However, from the last expression, this can be avoided by changing the torsion handedness. So, in this case, de Sitter inflation can be faster than DW expansion, which is cosmologically safe. Unfortunately, handedness cannot be changed in this parity conservation wall; therefore, the instability of the wall cannot be reversed. To obtain a flat Minkowski metric from the Goetz metric of the previous section, one must have the metric function $A(z)$ equal to the unit off the walls, as was done previously by HoSeong La [31]. But this is exactly given by the ansatz above, since when $z = 0$ at the wall, the exponent vanishes. So now the other metric function must be equal to the unit off the wall of $b(t)$, which is clearly given by $c = 0$. When $c^2 \approx 0$, this means physically that the DW is slowly decaying. Moreover, for the spacetime to be flat, the RHS of the Einstein–Cartan–Maxwell equation has to vanish as well, since matter, spin, and Higgs content in stress tensors must vanish, which results in the expression

$$\frac{1}{2}\Phi^1 + V(\Phi) = 0. \tag{41}$$

As a consequence, in the absence of a magnetic field and torsion axial pseudo-vector orthogonal to the wall, one obtains

$$-\frac{1}{2}\Phi^1 = V(\Phi). \tag{42}$$

Equivalently, the λ exponent must vanish to obtain a flat metric off the wall. To achieve this task, torsion must be

$$S_z = S^0_z z \delta(z), \tag{43}$$

and the magnetic field considered as parallel to the wall does not possess a component-z orthogonal to the wall. Therefore, from Eq. (21), with vanishing c , since the metric is flat at $b = 1$ off the wall, one obtains

$$\Phi(z) = \sqrt{\frac{3}{2}} S^0_z z. \tag{44}$$

Now let us obtain the dynamo equation [32] by simply considering the expression

$$\partial_t [2g^{22} B_2(z)] - 2\eta \nabla_x^2 [g^{22} B_x(z)] = 0, \tag{45}$$

where we have considered that from the Goetz metric $g^{22} = g^{33}$, and to simplify computations and by symmetry we assume that $B_x = B_y$. Therefore, although the DDW's magnetic field along the wall is stationary, inflating the DDW may induce a dynamo equation. In the next section, taking into account the discontinuity in the magnetic field, we will demonstrate that the component of the coronal mass ejection (CME) magnetic field orthogonal to the wall vanishes. By considering that the metric component g^{AC} for $(A, C = 2, 3)$ is given by

$$g^{22} = -b^{-1}(t)(1 + S_z z) = -e^{-ct} b^{-1}(t), \tag{46}$$

a simple calculus yields the final dynamo equation for

$$\partial_t [2g^{22} B_2(z)] - 2\eta \nabla_x^2 [g^{22} B_x(z)] = 0. \tag{47}$$

Now let us consider that the potential Φ is confined at the wall. Thus the expression for this potential is

$$\Phi(z) = S^0_z \delta(z). \tag{48}$$

Therefore, from this expression, the second derivative will also be proportional to the Dirac delta distribution. Substituting this value into the dynamo equation above, one obtains the equation for $B(z)$ as

$$\partial_z^2 B(z) + \omega^2 B(z) = 0, \tag{49}$$

where $\omega = i\sqrt{\frac{c}{\eta}}$, where η is the ohmic resistivity. This yields the solution for $B(z)$ as follows:

$$B(z) = B_0 \sinh\left(\sqrt{\frac{c}{\eta}} z\right). \tag{50}$$

Note that this solution is valid only if there are magnetic field components off the wall. To confine the magnetic field amplification at the wall, in the case of a thin wall, this implies that

we substitute the values of Higgs potential into the dynamo equation, and this results in a similar solution, this time with

$$B(z) = B_0 \delta(z), \tag{51}$$

where now there is no magnetic field parallel to the thin wall, off the wall. In this paper, we assume here that the photons are massless. Finally, the whole magnetic field amplification is given by

$$B(t, z) = e^{ct} B(z) = B_0 e^{ct} \sinh\left(\sqrt{\frac{c}{\eta}} z\right) \delta(z). \tag{52}$$

Therefore, if the handedness of the metric c factor is positive or right-handed, there is an amplification of the magnetic field confined in the dilatonic domain wall, which is one of our main results. Next, we will compute the thickness of the wall in various cases. From the above expression for the magnetic energy in terms of torsion, one may take the expression

$$d = \frac{B(z)^2}{S_z}, \tag{53}$$

where d is the thickness of the wall. Note that if $d = 0$, as a thin wall, one obtains either that S_z is infinity or, what is more physically reasonable, that the magnetic field vanishes. Thus, the interesting conclusion is that the thin domain wall cannot accommodate the magnetic field and only the thick DW can. Recently, Tuchin [33] investigated the impact of the domain walls on the chiral magnetic effect in hot QCD matter by making use of junction conditions. Here we will extend his approach to torsionful spacetime domain walls to show why we have chosen the magnetic fields parallel to the DDW. Note that if the magnetic fields are parallel to the wall, they trigger electric currents along the wall. Before deriving the chiral dynamo equation along the wall, let us investigate how the torsion axial vector and magnetic fields behave along the wall. Let us first address the idea behind the discontinuity of the axial torsion vector $S = (S_0, \mathbf{S})$. In the next section, we will take into account the matching conditions for the metric with torsion and a magnetic field and dynamo equation discontinuities. It is important to note that, as we showed previously, the 0-component of the axial torsion vector might mimic the chiral chemical potential μ_5 [34–36]. Let us now investigate the torsion components of the Goetz domain wall in theTEGR, where the Cartan connection vanishes. Therefore, from the Goetz DDW metric, we can obtain

$$e^0 = A^{\frac{1}{2}} dt \quad e^1 = A^{\frac{1}{2}}(z) dz, \tag{54}$$

and finally for the index $(A = 2, 3 = x, y)$, we obtain

$$e^A = A^{\frac{1}{2}} e^{\frac{c_0 t}{2}} dx^A. \tag{55}$$

By the Cartan first structure equation one obtains the components of torsion two-forms as

$$T^0 = de^0 = \frac{1}{2}A^{-\frac{1}{2}}(\partial_z A)dz \wedge dt \tag{56}$$

$$T^1 = de^1 = 0 \tag{57}$$

$$T^A = de^A = e^{\frac{c_0 t}{2}} \left[A^{-\frac{1}{2}}(\partial_z A)dz \wedge dx^A + \frac{c_0}{2}A(z)dt \wedge dt \right]. \tag{58}$$

Then, from these expressions for the torsion two-form, one obtains the following components of the torsion tensor

$$: T^0_{zt} = \frac{1}{2}A^{-\frac{1}{2}}(\partial_z A) \tag{59}$$

$$T^A_{zA} = \frac{1}{2}A^{-\frac{1}{2}}(\partial_z A)e^{\frac{c_0 t}{2}} \tag{60}$$

and

$$T^A_{tA} = \frac{c_0}{2}Ae^{-\frac{c_0 t}{2}}. \tag{61}$$

From teleparallelism, one may compute the Riemann curvature components as

$$R_{zAzA} = -\partial_z T_{AzA} = e^{c_0 t} \frac{1}{2} \left[\frac{\partial_z^2 A}{A^{\frac{1}{2}}} - \frac{1}{2} \left(\frac{(\partial_z A)^2}{A} \right) \right]. \tag{62}$$

Thus, in the case of thin walls, if one considers that the wall is a Riemann-flat vacuum on both sides of the symmetric wall as given from the last expression by causing the Riemann curvature component to vanish, this implies that

$$\left[\frac{\partial_z^2 A}{A^{\frac{1}{2}}} - \frac{1}{2} \left(\frac{(\partial_z A)^2}{A} \right) \right] = 0, \tag{63}$$

which is a second-order ordinary differential equation in $A(z)$ that yields the solution

$$A(z) = A_0 e^{S_0 z}. \tag{64}$$

But note that a metric that is conformally flat in vacuum is flat, so despite the torsion contribution, the Goetz solution of a symmetric thin domain wall embedded into the Minkowski Riemann-flat space still splits the spacetime into two Minkowski spacetimes. Of course, everything in this thin wall including the magnetic field, torsion, and scalar fields is confined on the wall and proportional to a Dirac delta distribution.

4 DDW dynamo equation and chiral GWs with contortion source

In this section we will see that the chiral GWs in the teleparallel spacetime have a divergence torsion source which is proportional to the pressure viscous tensor which tends to damp

the gravitational waves in the plasma. Moreover, we show that, in teleparallelism, the dynamo equation is damped in an appropriate compression of the magnetic fields in the universe. Let us now investigate what happens when we consider gravity wave metric perturbations in a teleparallel spacetime background. We start by considering the metric

$$ds^2 GW = a^2(\tau)[-d\tau^2 + (\delta_{ab} + 2h_{ab})dx^a dx^b]. \tag{65}$$

In this line element, $(a, b = 1, 2, 3)$ and h_{ab} are the linear tensor perturbations giving rise to the GWs. Here, the parameter τ is the conformal time. Now let us compute the contortion component

$$K_{\tau ab} = \partial_\tau g_{ab} = 2a^2 \left[\partial_\tau h_{ab} + \frac{\partial_\tau a}{a}(\delta_{ab} + 2h_{ab}) \right]. \tag{66}$$

Taking the divergence of the contortion tensor K in terms of the conformal time, one has

$$\partial_\tau K_{\tau ab} = 2a^2 [\partial_\tau^2 h_{ab} + 2H \partial_\tau h_{ab} + (\partial_\tau H)(\delta_{ab} + 2h_{ab}) + Z(a, h, H)], \tag{67}$$

where

$$Z(a, h, H) = 2a[\partial_\tau a[\partial_\tau h_{ab} + H(\delta_{ab} + 2h_{ab})]]. \tag{68}$$

This expression can be further simplified if we take H as constant, by considering the chiral GWs as

$$\partial_\tau K_{\tau ab} = 2a^2 [\partial_\tau^2 h_{ab} + 3H \partial_\tau h_{ab} + 2Hh_{ab}] 2a^2 [\Pi_{ab} + H\delta_{ab}], \tag{69}$$

where Π_{ab} is the viscous tensor acting as the source for the chiral GWs. Therefore, here the source of the GWs in teleparallelism is given by the divergence of the contortion tensor.

5 Conclusions

In this work we discuss and present several features of dynamo theory, in the presence of domain walls in EC and teleparallel spacetimes. The torsion over thick DDW planar section inflation is discussed. The importance of discussing such a kind of wall in the ECM framework stems from the fact that this is a lower-energy study of possible supergravity generalized models, where the torsion gravitino, for example, is present [41]. Fermion confinement on the five-dimensional branes, which resembles some aspects treated in this paper, was investigated in a previous paper. Junction conditions investigated by Bressange and [42, 43] in the context of Einstein–Cartan gravity could be further used here in future works. In this paper we have shown that in the thin wall limit, several physical features of the wall including torsion and magnetic fields can be suppressed from the metric off the wall, and we are left with a Riemann flat symmetric vacuum. Moreover, we show that by taking into account metric and

torsion chirality separately, one obtains dynamo amplification dependent on the metric and torsion handedness. We also show that when the wall collapses, one may obtain a gravitational dynamo for the primordial magnetic field which is also compressed as the wall collapses. This model can substitute for the classical magnetic dynamo mechanism. The gravitational dynamo is used in astronomy to describe the gravitational contraction of pre-galactic clouds. The use of teleparallel geometry here (see the appendix) is shown to be a very efficient mathematical tool, discovered by Einstein himself in 1928 [42, 43], to simplify computations, with implication for the cosmology of these DDWs. Last but not least, chiral GWs in cosmology with contortion were analyzed in detail, and we found that the divergence of contortion in conformal time can be considered as a source for these chiral gravitational waves. A preprint of a recent paper on the role of DWs in dark matter and dark energy is under construction. Even-parity torsion and Higgs in EC DWs also have parity-even torsion in the TEGR surroundings of the DW (see appendix above). Parity-odd Nieh-Yan teleparallel walls can be found because of the torsion. In that case, torsion would be a pseudovector, being the gradient of an axion [44, 45]. Rao et al. [46] recently investigated a model of teleparallel gravity with parity-odd interactions with scalar fields. It is important to point it out, however that their model is not a TEGR and may be parity-violating, contrary to the TEGR parity conservation we discussed here. In an earlier work, we presented a dynamo action where a parity-violating torsion [47] was present with a Holst curvature RC term which induced the parity violation itself. Now we may conclude that the presence of Nieh-Yan and Holst terms induces parity-violating torsion of odd-parity time.

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6 Appendix A: Teleparallel equivalent to GR “vacuum” surrounding an ECDW with even-parity symmetry EC domain wall

Here we will discuss the symmetries of the EC domain wall model we used in the text. Moreover, since the EC wall matches the teleparallel equivalent of GR (TEGR), we also discuss symmetries in this formulation of teleparallelism. First, one realizes that since TEGR is equivalent to GR, the Minkowski vacuum of GR domain walls, from vanishing with symmetries off the EC wall, is analogous to GR, but instead of having the vanishing of the Riemann curvature, we have the vanishing of the RC curvature, which matches the EC DW solution discussed in the previous section. Concerning the parity symmetry Φ given by $\Phi \rightarrow -\Phi$, the Higgs-like field potential is parity-invariant, or $V(\Phi) = V(-\Phi)$. Since we are in EC gravity DW, we might also give attention to the symmetries in the energy-momentum tensor of torsion, since the torsion pseudo-vector that is the only vector we have in the spin-density tensor that the Cartan torsion appears as

$$S_i = \epsilon^{ijkl} S_{jkl} \quad (70)$$

and in the spin-torsion energy-stress density torsion appears squared in the axial vector, we may say that the torsion potential energy stress is parity-even, or $T^{torsion}_{kl}(-S) = T^{torsion}_{kl}(S)$, as it is the same parity as the Higgs-like field Φ . The symmetry of the EC domain wall is analogous, and the torsion and $\Phi(z) = \Phi(-z)$. The same happens from $S(z) = S(-z)$. Therefore, parity-even symmetry is the main symmetry of the ECDW. Since the ECDW splits the space-time into two TEGRs, we will also investigate parity symmetry in these two TEGR types of “vacuum,” fulfilling the matching conditions. There is one way of determining the parity symmetry of the connection and torsion, which is the use of Cartan's differential calculus, where the tetrad basis can be expressed in terms of the basis one-form as $\omega^i = e^i_a dx^a$, (a,b=0,1,2,3). Then, from this basis we will find the first and second Cartan structure equations, respectively, for the torsion two-form $T^i = T^i_{jk} \omega^j \wedge \omega^k$, where \wedge is the exterior product of differential forms. Here, T^i_{jk} is the torsion tensor. Thus, the first structure equation is

$$T^i = d\omega^i + \omega^i_k \wedge \omega^k \quad (71)$$

$$R^i_j = d\omega^i_j + \omega^i_k \wedge \omega^k. \quad (72)$$

Therefore, since the R^i_k is the Riemann–Cartan manifold, the TEGR teleparallelism is that the last curvature three-form vanishes. The simplest way to effect this, which we adopt

here, is by the vanishing of spin Cartan connection. Therefore, the first and second Cartan structure equations become

$$R^i_j = 0, \tag{73}$$

and for the torsion two-form,

$$T^i = d\omega^i, \tag{74}$$

where obviously the Riemann curvature does not vanish, only the RC curvature. Therefore, is easy to check the parity symmetry in theseTEGRs as $T(-x) = -d(-\omega)(x) = d\omega x = T(x)$, where T is the torsion in the invariant coordinate language. Therefore, the ECDW torsion is parity-even as well. Parity odd-torsion was obtained from experiments by Hojman [38, 39], whereas Dyer et al. [40] showed that in Poincaré gauge theory of gravity, quadratic metric-affine torsion can be parity-even and parity-odd. From the last expression, we obtain $dT = d^2\omega = 0$ due to the Poincaré lemma for the exterior derivative.

7 Appendix B: Einstein–Cartan DWS embedded in Weitzenböck “vacuum” with vanishing connection differential forms

In this appendix we show that our ECDW embedded in the Weitzenböck teleparallel connection which vanishes as ω_b^a as in the previous appendix. In this appendix we show that the components of contortion off the ECDW vanish as we move far away the wall. They actually decay at infinite. To show this we assume that the differential Weitzenböck connection can be split into the form

$$\omega^a = \tilde{\omega}_b^a - K^a_{bc}\omega^c, \tag{75}$$

where ω represents the Riemannian connection form and K^a_{bc} represents the contortion. Note that when the Weitzenböck gauge $\omega = 0$, which is equivalent to the vanishing of the Riemann–Cartan curvature, one obtains

$$\tilde{\omega}_b^a = K^a_{bc}\omega^c. \tag{76}$$

Then, this is equivalent to the metric-contortion equation

$$K_{ijk} = \partial_i g_{jk} - \partial_j g_{ik}, \tag{77}$$

where now the metric g_{ij} will be given by the ECDW line element

$$ds^2 = A(z)[(dt^2 - dz^2) - e^{-\lambda t}(dx^2 + dy^2)]. \tag{78}$$

This conformal metric has components

$$g_{00} = -g_{11} = A(z), \tag{79}$$

and the remaining components are

$$g_{22} = g_{33} = -A(z)e^{-\lambda t}. \tag{80}$$

We note that at infinite time, this ECDW is unstable as it should be in the observations. Now, from these last two expressions, we may compute the component of the contortion tensor. Therefore, the contortion components of the ECDW in the Weitzenböck vacuum are

$$K_{100} = \partial_1 g_{00} = \partial_z A(z). \tag{81}$$

From the ECDW embedded in the Weitzenböck vacuum we have

$$A(z) = A_0 e^{-S_0 z} \approx (1 - S_0 z), \tag{82}$$

where in the last equality we approximate for regions very close to the wall ($z \approx 0$). Note that at this location, the conformal factor reduces to unity, which reduces the metric of the thin DW similar to the Minkowski manifold spacetime, where the wall is a delta singularity. Of course, in general, this equivalence to GR domain walls does not happen, and we may have a Lorentz violation. This first component is expressed in terms of torsion as $K_{100} = -S_0 e^{-S_0 z}$, where λ here is a positive constant. One observes that when we move far away from the wall $z \rightarrow \infty$, this contortion component vanishes.

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