

STUDY ON TRANSVERSE MULTI-BUNCH INSTABILITY IN ELETTRA 2.0

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Abstract

One of the main characteristics of the future light sources like Elettra 2.0 is the small vacuum chamber cross section. In fact, the resistive-wall (RW) impedance due to the small vacuum chambers cross section enhances transverse coupled-bunch instabilities. In this study, the effect of the RW in the multi-bunch case is investigated versus chromaticity. The threshold currents in the presence of broad-band and RW impedances are estimated for the Elettra 2.0 storage ring at different values of chromaticity using macroparticle tracking and frequency domain semi-analytical calculations. In particular, it is found that, above a certain chromaticity, the threshold current is determined by the radial head-tail modes. In view of mitigating these instabilities, the effectiveness of the transverse bunch-by-bunch feedback system as well as bunch-lengthening harmonic cavities is also useful.

INTRODUCTION

Elettra is the first European machine synchrotron light source of the third generation for soft X-rays (originally) that came into operation in 1993. However, after many years of successfully serving the user community with excellent results, Elettra will undertake a major upgrade towards what it is called the “ultimate” light source, to maintain its leadership for its energy range of synchrotron research, by enabling new science and the development of new technologies to the general benefit. The main characteristic of this new generation of storage ring-based X-ray sources is a substantial increase in the brilliance and coherence fraction of the source as compared to today’s X-ray beams [1]. With the aim of achieving this, the Elettra 2.0 storage ring lattice includes strongly focusing quadrupoles. These quadrupoles should reach high enough gradients and for this reason, their poles must be closed to the beam. As a result, we have taken the equivalent vertical radius of the beam pipe as 7.5 mm leading to coupled-bunch instabilities that must be damped in order to achieve the high design beam current of 400 mA.

As one can see from Fig.1, one way to damp this instability is to by increasing the chromaticity, in this case, because of the chromatic frequency i.e. $\omega_\xi = \frac{\xi\omega_0}{\alpha_c}$ the beam doesn’t see the peak of resistive wall impedance and the threshold current will be increased.

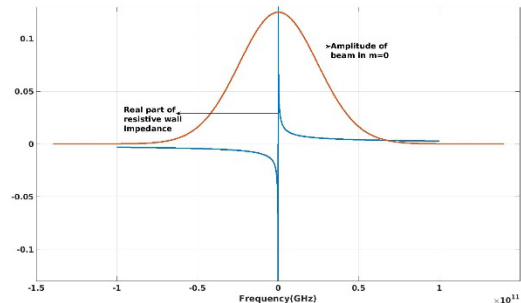


Figure 1: The beam will see the peak of impedance in the center, by increasing the chromaticity the beam will not see the peak of RW impedance.

In this paper, we investigate the threshold current in the frequency and time domain by considering the resistive wall instability. In the frequency domain, we applied the *rwmbi* code which used Laclare’s eigenvalue for finding the threshold current, and in the time domain, the multi-particle tracking simulation in *Elegant* is applied. Also, it should be mentioned that in this study, only the vertical resistive wall impedance for the copper vacuum chamber with the equivalent radius of 7.5 mm is considered.

FREQUENCY DOMAIN

The complex impedance-driven tune shift for each head-tail mode can be assessed using Laclare’s eigenvalue [2]. This method can be achieved from the transverse motion equation of a single particle or from the linearized Vlasov equation [3]. In both situations, the resulting matrix equation can be written as

$$\Delta\omega_m \sigma_m(\omega_{mq}) = \frac{\beta I}{2T_0 e} \sum_p j Z_\perp(\omega_{mp}) A_{pq}^m \sigma_m(\omega_{mp}) \quad (1)$$

where e is the unit charge, T_0 is the revolution time, Z_\perp is the transverse impedance, E is the beam energy, I is the current, β is the average transverse beta function, σ_m is the bunch spectrum and $\Delta\omega_m$ is the complex betatron frequency shift for mode m which is calculated as the eigenvalue with the largest positive imaginary component while the σ_m is the eigenvector. This calculation is done in *rwmbi* [4] code

The result for mode $m=0$ is interesting, it shows that by increasing the chromaticity the threshold is increased as one expected, but this value is decreased for the chromaticity above 3. The result for mode 0 to 3 is shown in Fig.2.

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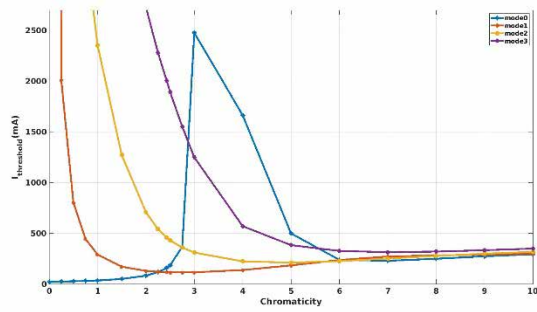


Figure 2: The Threshold current for different mode for the case with a short bunch in a single rf system, considering the RW impedance.

The spectra of the bunch (Fig.3) can explain the reason for decreasing in the threshold current in mode $m=0$.

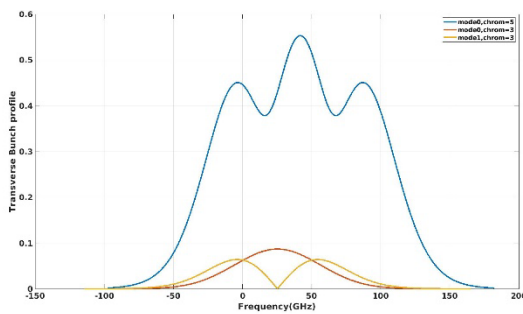


Figure 3: The bunch spectra for mode 0 and 1 for chromaticity 3 and 5.

In mode zero, we can see that by increasing the chromaticity and because of the chromatic frequency, the beam spectra started to shift from the centre and as a result, the threshold current increased, but as one can see, the spectra of the beam become double in chromaticity 5. So, one the peaks will see the RW impedance and the threshold current will decrease.

For other modes, the threshold starts from a very high value. Because the beam spectra are in the centre and it doesn't see the RW, however, by increasing the chromaticity the beam will see the impedance of RW and the current will decrease gradually. From Fig.4, one can see that the threshold converges to the same value for the chromaticity higher than 5.

The lengthened Gaussian bunch is the other situation that is investigated in the frequency domain. The lengthened bunch is achieved by decreasing the RF voltage from 2MV to 0.5 MV, and the bunch reached the lengthened value of

22(ps).

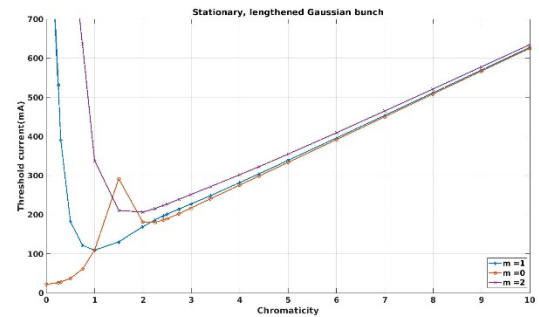


Figure 4: The Threshold current for different modes, considering the RW impedance for the case with a stationary, lengthened Gaussian bunch.

By lengthening in this way without increasing the momentum compaction factor or the energy spread, the radial Gaussian distribution of the bunch is maintained and it doesn't need a significant synchrotron tune spread. The $rwmbi$ result for this situation is pictured Fig. 4.

It should be mentioned that in the reality we don't use this method for lengthening the bunch. The 3HC is used in Elettra 2.0 for bunch lengthening which in this case the bunch length is reached to 18 (ps).

From Fig.5, it can be seen that the decreasing behavior of the threshold current for $m=0$ is seen again which can be explained in the same way by looking at the spectra of the lengthened Gaussian bunch.

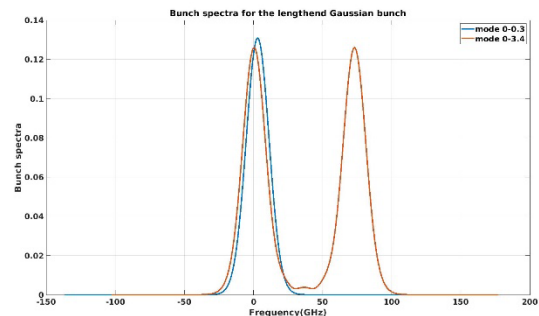


Figure 5: The Transverse bunch spectra for the lengthened Gaussian bunch case for mode 0 for chromaticity 0.3 and 3.4.

TIME DOMAIN

In the time domain, we used tracking with the Elegant code [5] to find the threshold current in different values of vertical chromaticity.

The wake of the copper vacuum chamber can be calculated by considering the length of the beam pipe, which in our case is the circumference of the vacuum chamber i.e. 259.2 m, the effective resistivity of the material in the vacuum chamber, which in our case is copper and it is equal to 1.7×10^{-8} ohm.m, and the radius of vacuum chamber which we consider 7.5 mm as we mentioned earlier. By considering these values and the time domain of 1 msec with 20,000 data points, the wakefield of the RW can be calculated by Panofsky–Wenzel theorem [6]:

$$W_{RW}^{Trans}(\tau) = \frac{8Z_0c}{\pi a^4} \left(\frac{1}{12} \left[-e^{-\tau/\tau_0} \cos\left(\frac{\sqrt{3}\tau}{\tau_0}\right) + \sqrt{3}e^{-\frac{\tau}{\tau_0}} \sin\left(\frac{\sqrt{3}\tau}{\tau_0}\right) \right] \right. \\ \left. + \frac{8Z_0c}{\pi a^4} \left(\frac{-\sqrt{2}}{\pi} \int dx \frac{-e^{-\frac{x^2\tau}{\tau_0}}}{x^6 + 8} \right) \right) \quad (2)$$

and for $\tau \gg \tau_0$ can be approximated

$$W_{RW}^{Trans}(\tau) \approx \frac{1}{\pi a^3} \sqrt{\frac{Z_0c}{\sigma\pi\tau^{1/2}}} \quad (3)$$

where Z_0 is the impedance of the free space, c being the speed of light, $\tau_0 = s_0/c$ and s_0 is the characteristic distance which is

$$s_0 = \left(\frac{2a^2}{Z_0\sigma} \right)^{1/3} \quad (4)$$

where a is the pipe radius and σ is the conductivity. So, the transverse wake potential can be pictured as shown in Fig. 6 [7]:

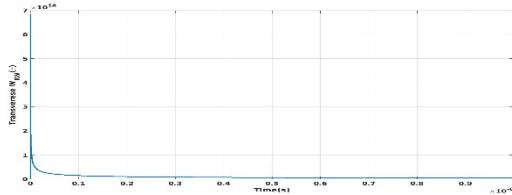


Figure 6: The Transverse wake of the copper resistive wall.

To define the threshold current in this domain, two methods can be applied [8,9]. First, look at the beam directly. In this method, for each chromaticity, the current was increased and the tracking is done until the beam become unstable. So, the current in which the beam becomes unstable is the threshold current for that chromaticity.

This method is time-consuming and one should be sure if the number of tracking is enough or not.

The second method is based on the instability growth at a high enough beam current (e.g., the nominal current like 400 mA for Elettra 2.0) for each chromaticity. In this method, the center of mass amplitude grows exponentially in turns (Fig. 7). The instability growth rate can be determined by using the linear fit of the natural logarithm of the amplitude based on the time. If the growth rate is k , the threshold for each chromaticity can be calculated as:

$$I_{th} = \frac{I_{machine}}{k\tau_y} \quad (5)$$

where τ_y is the vertical damping time. This method is much faster than the first one.

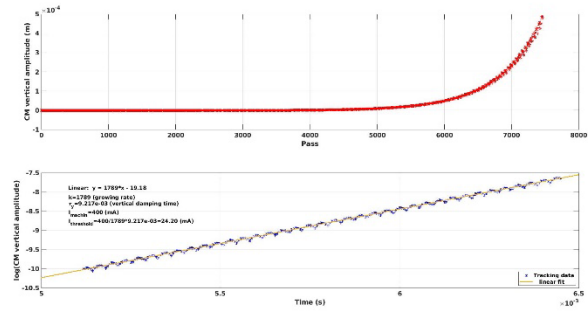


Figure 7: calculation of growth rate in zero chromaticity. In this case, the threshold is 24.20 mA.

With this method, the threshold in the time domain can be calculated for different vertical chromaticities from 0 to 5. The result is shown in Fig. 8. It can be seen that the result in time domain is near to zero mode in frequency domain up to vertical chromaticity equal to 2. Then, it acts similar to mode one in the frequency domain.

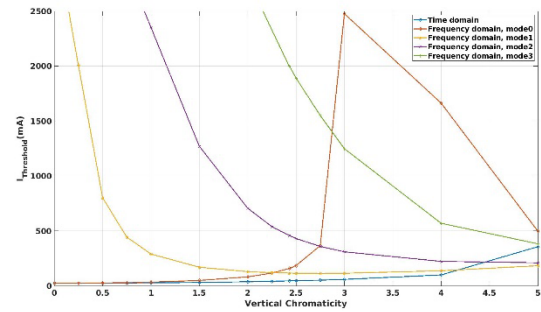


Figure 8: The blue line is the threshold current in the time domain.

CONCLUSION

In this paper, the effect of resistive wall instability is investigated and the threshold in the frequency domain and in the time domain is investigated. The comparison between the time domain and the frequency domain shows that the threshold current in the time domain is near zero modes in the frequency domain up to vertical chromaticity equal to 2. Then, it acts similarly to mode one in the frequency domain.

Also, the average growth rate in the time domain in vertical chromaticity 0-3, indicates the growth rate is equal to 0.83 (ms) which can be handled perfectly with the feedback system of Elettra 2.0.

However, the effect of 3HC should be considered too. This effect will be studied in the future.

REFERENCE

- [1] Elettra 2.0 Technical Design Report: <https://www.elettra.eu/lightsources/elettra/elettra-2-0.html>
- [2] Cullinan, F.J., Nagaoka, R., Skripka, G. and Tavares, P.F., 2016. “Transverse coupled-bunch instability thresholds in the presence of a harmonic-cavity-flattened rf potential.” *Physical Review Accelerators and Beams*, vol. 19, no 12, doi:10.1103/PhysRevAccelBeams.19.124401
- [3] J.L. Laclare, “Transverse instabilities”, in *Proc. CAS’85*, vol. 2, p. 306, CERN, Geneva, 1987..
- [4] R. Nagaoka, “rwmbi: A frequency domain solution of transverse coupled-bunch instability driven by resistive-wall and broad-band impedance (unpublished).”
- [5] M. Borland, “ELEGANT: A flexible SDDS-compliant code for accelerator simulation,” Office of Scientific and Technical Information (OSTI), Aug. 2000. doi:10.2172/761286
- [6] G. Skripka, R. Nagaoka, M. Klein, F. Cullinan, and P. F. Tavares, “Simultaneous computation of intrabunch and inter-bunch collective beam motions in storage rings”, *Nucl. Instrum. Methods Phys. Res., Sect. A*, vol. 806, pp. 221–230, 2016. doi:10.1016/j.nima.2015.10.029
- [7] A. Gamelin, W. Foosang, and R. Nagaoka, “mbtrack2, a Collective Effect Library in Python”, in *Proc. IPAC’21*, Campinas, Brazil, May 2021, pp. 282-285. doi:10.18429/JACoW-IPAC2021-MOPAB070
- [8] E. Karantzoulis, private communication.
- [9] R. Nagaoka, private communication.