Physics Letters B 755 (2016) 380–386

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Nuclear structure corrections to the Lamb shift in μ^3He$^+$ and μ^3H

N. Nevo Dinur [a,*], C. Ji [b,c,d], S. Bacca [b,e], N. Barnea [a]

[a] Racah Institute of Physics, The Hebrew University, Jerusalem 9190401, Israel
[b] TRIUMF, 4004 Wesbrook Mall, Vancouver, BC V6T 2A3, Canada
[c] ECT*, Villa Tambosi, 38123 Villazzano (Trento), Italy
[d] INFN-TIFPA, Trento Institute for Fundamental Physics and Applications, Trento, Italy
[e] Department of Physics and Astronomy, University of Manitoba, Winnipeg, MB R3T 2N2, Canada

ARTICLE INFO

Article history:
Received 6 January 2016
Accepted 11 February 2016
Available online 18 February 2016
Editor: W. Haxton

Keywords:
Charge radius
Muonic atom
Nuclear polarizability
Two-photon exchange

ABSTRACT

Measuring the 2S–2P Lamb shift in a hydrogen-like muonic atom allows one to extract its nuclear charge radius with a high precision that is limited by the uncertainty in the nuclear structure corrections. The charge radius of the proton thus extracted was found to be 7σ away from the CODATA value, in what has become the yet unsolved "proton radius puzzle". Further experiments currently aim at the isotopes of hydrogen and helium: the precise extraction of their radii may provide a hint at the solution of the puzzle. We present the first *ab initio* calculation of nuclear structure corrections, including the nuclear polarization correction, to the 2S–2P transition in μ^3He$^+$ and μ^3H, and assess solid theoretical error bars. Our predictions reduce the uncertainty in the nuclear structure corrections to the level of a few percent and will be instrumental to the on-going μ^3He$^+$ experiment. We also support the mirror μ^3H system as a candidate for further probing of the nucleon polarizabilities and shedding more light on the puzzle.

1. Introduction

The root-mean-square (RMS) charge radius of the proton $r_p \equiv \sqrt{\langle r_p^2 \rangle}$ was recently determined with unprecedented precision from laser spectroscopy measurements of 2S–2P transitions in muonic hydrogen μH, where the electron is replaced by a muon [1,2]. The extracted r_p differs by 7σ from the CODATA value [3], which is based in turn on many measurements involving electron–proton interactions. This discrepancy between the 'muonic' and 'electronic' proton radii ($r_p(\mu^-)$ and $r_p(e^-)$, respectively) is known as the "proton radius puzzle", and has attracted much attention (see, *e.g.*, Ref. [4] for an extensive review and Ref. [5] for a brief summary of current results and ongoing experimental effort). In an attempt to solve the puzzle, extractions of $r_p(e^-)$ from the ample electron–proton (*ep*) scattering data have been reanalyzed by, *e.g.*, Refs. [6–9], while several planned experiments aim to remeasure *ep* scattering in new kinematic regions relevant for the puzzle [10,11]. r_p extracted from electronic hydrogen is also being reexamined, both theoretically [12] and experimentally [13–15], as well as the Rydberg constant [15,16], which is relevant for several radius extraction methods. A few of the theoretical attempts to account for the discrepancy between $r_p(e^-)$ and $r_p(\mu^-)$ include new interactions that violate lepton universality [17–19] and novel proton structures [20–24]. Yet the puzzle has not been solved. Answers may be provided (see, *e.g.* Refs. [25,26]) by a planned experiment at PSI [27] to scatter electrons, muons, and their antiparticles off the proton using the same experimental setup.

Alternatively, it will be insightful to study whether the puzzle also exists in other light nuclei, and whether it depends on the atomic mass A, charge number Z, or the number of neutrons N. In particular, the CREMA collaboration plans to extract high-precision charge radii from Lamb shift measurements that were recently performed in several hydrogen-like muonic systems [5,28], namely, μD, μ^3He$^+$, and μ^4He$^+$. These measurements may unveil a dependence of the discrepancy on the isospin of the measured nucleus and, in particular, probe whether the neutron exhibits a similar effect as the puzzling proton. To obtain some control over these issues, it is advisable that nuclei with different N/Z ratios will be mapped out. It is the purpose of this Letter to perform an *ab initio* calculation of nuclear structure corrections (including nuclear po-

* Corresponding author.
E-mail addresses: nir.nevo@mail.huji.ac.il (N. Nevo Dinur), ji@ectstar.eu (C. Ji), bacca@triumf.ca (S. Bacca), nir@phys.huji.ac.il (N. Barnea).

http://dx.doi.org/10.1016/j.physletb.2016.02.023
0370-2693/

larization), with solid error estimates, for the μ^3He^+ system and for its nuclear mirror, μ^3H.

The Lamb shift [29] is the 2S–2P energy difference[1] consisting of

$$\Delta E \equiv \delta_{\rm QED} + \delta_{\rm FS}(R_c) + \delta_{\rm TPE} , \tag{1}$$

where, in decreasing order of magnitude, the three terms include: quantum electro-dynamics (QED) contributions from vacuum polarization, lepton self-energy, and relativistic recoil in $\delta_{\rm QED}$; finite-nucleus-size contributions in $\delta_{\rm FS}(R_c)$, where $R_c \equiv \sqrt{\langle R_c^2\rangle}$ is the nuclear RMS charge radius; and contributions from two-photon exchange (TPE) between the lepton and the nucleus in $\delta_{\rm TPE}$. The last term can be divided into the elastic Zemach term and the inelastic polarization term, *i.e.*, $\delta_{\rm TPE} = \delta_{\rm Zem} + \delta_{\rm pol}$. Additionally, each of these terms is separated into contributions from nuclear (δ^A) and nucleonic (δ^N) degrees of freedom, $\delta_{\rm TPE} = \delta^A_{\rm Zem} + \delta^N_{\rm Zem} + \delta^A_{\rm pol} + \delta^N_{\rm pol}$.

In light muonic atoms, $\delta_{\rm QED} \approx 10^2$–$10^3$ meV and is estimated from theory with a precision better than 10^{-3} meV [31–35]. At leading order $\delta_{\rm FS}(R_c) = \frac{m_r^3(Z\alpha)^4}{12} R_c^2$, with m_r the reduced mass of the muon–nucleus system, while higher-order contributions are at the sub-percentage level [31]. The limiting factor for the attainable precision of R_c extracted from Eq. (1) is by far the uncertainty in $\delta_{\rm TPE}$. This was confirmed in two recent papers that reviewed the theory in μD [33], and in μ^4He and μ^3He [34]. Ref. [33] covers all the theoretical contributions to the Lamb shift in μD, including a summary of recent efforts by several groups [36–40] to accurately obtain $\delta_{\rm TPE}$ in μD and reliably estimate its uncertainty, which comes out an order of magnitude larger than the uncertainties in the other terms. Ref. [34] details all the contributions for the two helium isotopes. Many terms are recalculated, not including the polarization correction $\delta_{\rm pol}$. For μ^4He^+, *ab initio* nuclear calculations were recently applied in Ref. [30], improving on decades-old estimates of $\delta_{\rm pol}$. For three-body nuclei, the only calculations of $\delta_{\rm pol}$ are outdated; based on old and simplistic nuclear models, their results are either inaccurate [41] or imprecise [42], reinforcing the need for modern, accurate, *ab initio* calculations for the three-body nuclei.

2. Calculation details

2.1. Nuclear structure corrections

The nuclear Zemach term $\delta^A_{\rm Zem}$ enters Eq. (1) as the elastic nuclear-structure contribution to $\delta^A_{\rm TPE}$.[2] This term is of order $(Z\alpha)^5$ and is defined as

$$\delta^A_{\rm Zem} = -\frac{m_r^4(Z\alpha)^5}{24}\langle r^3\rangle_{(2)} , \tag{2}$$

where $\langle r^3\rangle_{(2)}$ is the 3rd nuclear Zemach moment.[3] Friar & Payne showed [46] that the first-order corrections in $\delta^A_{\rm pol}$ contain a part that cancels $\delta^A_{\rm Zem}$ exactly. Calculation of this part can thus be avoided, providing only the sum $\delta^A_{\rm TPE} = \delta^A_{\rm pol} + \delta^A_{\rm Zem}$, as was done in Refs. [36,37] for μD. However, following Refs. [30,38,47], we calculate explicitly all the parts of $\delta^A_{\rm pol}$, including the Zemach term, as detailed below. This is done in order to: (a) allow comparison with other values in the literature, and (b) provide theoretical support for the alternative way of extracting R_c from Eq. (1) where the Zemach term is phenomenologically parameterized as [31]

$$\delta^A_{\rm Zem} = \mathcal{C} \times R_c^3 . \tag{3}$$

As in Refs. [30,38], the energy correction due to nuclear polarization is obtained as a sum of contributions

$$\delta^A_{\rm pol} = \left[\delta^{(0)}_{D1} + \delta^{(0)}_{L} + \delta^{(0)}_{T} + \delta^{(0)}_{C} + \delta^{(0)}_{M}\right] + \left[\delta^{(1)}_{R3} + \delta^{(1)}_{Z3}\right] + \left[\delta^{(2)}_{R^2} + \delta^{(2)}_{Q} + \delta^{(2)}_{D1D3}\right] + \left[\delta^{(1)}_{NS} + \delta^{(2)}_{NS}\right]. \tag{4}$$

Detailed formulas pertaining to most of the terms in Eq. (4) are found in [30] and are not repeated here. The largest contribution comes from the leading term, $\delta^{(0)}_{D1}$, related to the electric dipole. To this we add relativistic longitudinal and transverse corrections $\delta^{(0)}_L$ and $\delta^{(0)}_T$, respectively, as well as Coulomb distortion corrections $\delta^{(0)}_C$. Here we follow Ref. [38] and include in $\delta^{(0)}_C$ only the logarithmically enhanced term from the next order in $Z\alpha$. We generalize the treatment in Ref. [38] of the magnetic term $\delta^{(0)}_M$ by using the impulse approximation operator that includes the orbital angular momentum [48]. First-order corrections $\delta^{(1)}_{R3}$ and $\delta^{(1)}_{Z3}$ are related to a proton–proton correlation term and to the 3rd nuclear Zemach moment, respectively. Finally, at the next order we have the monopole $\delta^{(2)}_{R^2}$, quadrupole $\delta^{(2)}_Q$, and interference $\delta^{(2)}_{D1D3}$ terms. All the above terms are calculated using point nucleons. Finite-nucleon-size (NS) corrections appear in Eq. (4) as $\delta^{(1)}_{NS} = \delta^{(1)}_{R1} + \delta^{(1)}_{Z1}$ and $\delta^{(2)}_{NS}$, which we elaborate on below.

2.2. Nucleon-size corrections

The TPE in the point-nucleon limit is expressed as the interaction of photons with the structureless charged protons, while the neutrons are ignored. In this limit, the point-proton density operator is

$$\hat\rho_p(\boldsymbol{R}) \equiv \frac{1}{Z}\sum_{a=1}^{A}\delta(\boldsymbol{R}-\boldsymbol{R}_a)\frac{1+\tau^3_a}{2} , \tag{5}$$

where τ^3_a is the isospin projection operator. When the finite nucleon sizes are considered, $\hat\rho_p(\boldsymbol{R})$ must be convoluted with the proton's internal charge distribution, and a similar convolution is applied to the point-neutron density operator

$$\hat\rho_n(\boldsymbol{R}) \equiv \frac{1}{N}\sum_{a=1}^{A}\delta(\boldsymbol{R}-\boldsymbol{R}_a)\frac{1-\tau^3_a}{2} . \tag{6}$$

Following Refs. [30,47], we apply a low-momentum expansion for the nucleon form factors, parameterized here by their mean square charge radii, $r^2_{n/p} \equiv \langle r^2_{n/p}\rangle$. We adopt $r_n^2 = -0.1161(22)$ fm^2 [49]. For the proton, we may use either $r_p(e^-) = 0.8775(51)$ fm [3] or $r_p(\mu^-) = 0.84087(39)$ fm [2]. In fact, until the "proton radius puzzle" is resolved (or when R_c and other properties of the nuclei under consideration are measured using muons), we should use $r_p(e^-)$ for comparison with the literature, which is based on data obtained with electrons, and $r_p(\mu^-)$ for predictions in muonic systems.

The leading NS correction $\delta^{(1)}_{NS}$ is the sum of nucleon–nucleon correlations in $\delta^{(1)}_{R1}$ and Zemach-like terms in $\delta^{(1)}_{Z1}$. The former is expressed as

$$\delta^{(1)}_{R1} = -\frac{m_r^4(Z\alpha)^5}{6}\iint d\boldsymbol{R}d\boldsymbol{R}'|\boldsymbol{R}-\boldsymbol{R}'|\Big[r_p^2\langle 0|\hat\rho_p^\dagger(\boldsymbol{R})\hat\rho_p(\boldsymbol{R}')|0\rangle + \frac{N}{Z}r_n^2\langle 0|\hat\rho_n^\dagger(\boldsymbol{R})\hat\rho_p(\boldsymbol{R}')|0\rangle\Big] , \tag{7}$$

[1] We use the convention of Ref. [30] for which ΔE is negative in muonic hydrogen-like atoms.

[2] $\delta^A_{\rm Zem}$ was derived by Friar [43] as the first-order $Z\alpha$ correction to $\delta_{\rm FS}(R_c)$ and is sometimes called "the Friar term", *e.g.*, in Refs. [33,44].

[3] We refer only to charge-charge Zemach moments; for more details see, *e.g.*, Ref. [45].

which includes proton–proton (pp) and neutron–proton (np) correlations. It is an NS correction to the point-nucleon contribution $\delta^{(1)}_{R3}$ of Eq. (4) (the latter is denoted $\delta^{(1)}_{R3pp}$ in Ref. [30]). For the calculation of Zemach-like terms using point-nucleons we define

$$\langle r^k_{ij}\rangle_{(2)} \equiv \iint d\boldsymbol{R} d\boldsymbol{R}' \left|\boldsymbol{R}-\boldsymbol{R}'\right|^k \langle 0|\hat{\rho}^\dagger_i(\boldsymbol{R})|0\rangle\langle 0|\hat{\rho}_j(\boldsymbol{R}')|0\rangle , \quad (8)$$

with i, j denoting either p or n. The 3rd nuclear Zemach moment is thus calculated as

$$\langle r^3\rangle_{(2)} \approx \langle r^3_{pp}\rangle_{(2)} + 4\left[r^2_p\langle r^1_{pp}\rangle_{(2)} + \frac{N}{Z} r^2_n \langle r^1_{np}\rangle_{(2)}\right] , \quad (9)$$

where the first term is the point-nucleon limit and the second is the (approximated) NS correction. Accordingly, the point-nucleon Zemach term $\delta^{(1)}_{Z3}$ and its NS correction $\delta^{(1)}_{Z1}$ are obtained by inserting Eq. (9) into Eq. (2) and flipping the sign, *i.e.*, $\delta^A_{\rm Zem} \approx -(\delta^{(1)}_{Z3} + \delta^{(1)}_{Z1})$.

The sub-leading NS correction $\delta^{(2)}_{NS}$ is evaluated through a sum rule of the dipole response[4]

$$\delta^{(2)}_{NS} = -\frac{8\pi}{27} m^5_r (Z\alpha)^5 \left[r^2_p - \frac{N}{Z} r^2_n\right] \int\limits_{\omega_{\rm th}}^{\infty} d\omega \sqrt{\frac{\omega}{2m_r}} S_{D_1}(\omega) . \quad (10)$$

Lastly, the nucleonic TPE correction $\delta^N_{\rm TPE}$ also enters Eq. (1). We defer the treatment of this hadronic contribution to a dedicated section below.

2.3. Methods

Most of the terms in Eq. (4) can be written as sum rules of several nuclear responses with various energy-dependent weight functions [30,38]. They were evaluated using the newly developed Lanczos sum rule method [50]. Ground-state observables of ^{3}He and ^{3}H, as well as Lanczos coefficients, were obtained using the effective interaction hyperspherical harmonics (EIHH) method [51,52]. As only ingredients we employed in the nuclear Hamiltonian either one of the following state-of-the-art nuclear potentials: (i) the phenomenological AV18/UIX potential with two-nucleon [53] plus three-nucleon [54] forces; and (ii) the chiral effective field theory χEFT potential with two-nucleon [55] plus three-nucleon [56] forces.

It is of utmost importance to have realistic uncertainty estimates for our nuclear TPE predictions. These terms are the least well known in Eq. (1), and their uncertainties determine the attainable precision of R_c extracted from Lamb shift measurements. We considered many sources of uncertainty, namely: numerical; nuclear model; isospin symmetry breaking; higher-order nucleon-size corrections; missing relativistic and Coulomb-distortion corrections; higher multipoles, terms of higher-order in $Z\alpha$; and the effect of meson–exchange currents on the magnetic contribution. Their individual and cumulative effect on $\delta^A_{\rm pol}$, $\delta^A_{\rm Zem}$, and $\delta^A_{\rm TPE}$ have been estimated and applied to the results given below. More details about these uncertainty estimates are given in the Supplementary Materials [57].

[4] The sign before r^2_n in Eq. (10) is corrected from Refs. [30,38] and agrees with Ref. [40].

Table 1
Various ^{3}He and ^{3}H observables (see text for details) calculated with the AV18/UIX and χEFT potentials, compared to corresponding calculations in the literature and to experimental values. Our ground-state wave functions do not include the $T = 3/2$ channel. Our numerical uncertainties are not shown since they are smaller than one in the last decimal place. References labels correspond to: $^{a/b}$ Ref. [59] without/with inclusion of the $T = 3/2$ channel, respectively; ^{c} Ref. [58] (which includes the $T = 3/2$ channel); ^{d} Ref. [61]; ^{e} Ref. [62]; ^{f} Ref. [63]; ^{g} Ref. [64]; ^{h} Ref. [65]; ^{i} Ref. [60]; ^{j} Ref. [42]; ^{k} Ref. [66]; ^{l} Ref. [67]; ^{m} Ref. [68]; ^{n} Ref. [69].

^{3}He	BE [MeV]	$R_c(e^-)$ [fm]	$\hat{\mu}_{gs}$ [μ_N]	α_E [fm^3]
AV18/UIX	7.740	1.968	−1.73	0.149
Lit.	7.740(1)^a	–	−1.764^e	0.153(15)^g
	7.748(1)^b	–	−1.749^f	0.145^h
χEFT	7.735	1.988	−1.76	0.153
Lit.	7.750^c	–	–	0.149(5)^i
Exp.	7.71804^d	1.973(14)^m	−2.127^d	0.130(13)^j
				0.250(40)^k

^{3}H	BE [MeV]	$R_c(e^-)$ [fm]	$\hat{\mu}_{gs}$ [μ_N]	α_E [fm^3]
AV18/UIX	8.473	1.755	2.59	0.137
Lit.	8.472(1)^a	–	2.575^e	0.139(4)^l
	8.478(1)^b	–	2.569^f	
χEFT	8.478	1.777	2.63	0.139
Lit.	8.474^c	–	–	0.139(2)^i
Exp.	8.48180^d	1.759(36)^n	2.979^d	

3. Results

3.1. Benchmarks

We first compare a few observables we have calculated for the ^{3}He and ^{3}H nuclei with corresponding theoretical and experimental values available in the literature. In Table 1 we present the ground-state binding energy BE, charge radius R_c, and magnetic moment $\hat{\mu}_{gs}$, as well as the electric dipole polarizability α_E. In general, good agreement is found with other calculations.

Our results do not include isospin-symmetry breaking (ISB), except for the Coulomb interaction between protons in ^{3}He. Calculations by other groups shown in Table 1 usually do not include ISB effects; notable exceptions are Ref. [58], which includes the $T = 3/2$ isospin channel in the ground-state wave function, and Ref. [59] that provides results either including or excluding it. One observes that including ISB alters BE by a few keV. In addition, the ^{3}He BE, not used in the calibration of the Hamiltonians, is overestimated at a sub-percentage level, and this is slightly *worsened* when ISB is included. As discussed in Ref. [60], changes in BE shift the threshold of sum rules, affecting mostly sum rules with inverse energy dependence, such as α_E discussed below. For the other observables in Table 1, the estimated uncertainty stemming from ISB is $\lesssim 1\%$.

Charge radii R_c shown in Table 1 are obtained from the calculated point-proton-distribution RMS radius R_p as [70,71]

$$R^2_c = R^2_p + r^2_p + \frac{N}{Z} r^2_n + \frac{3}{4m^2_p} , \quad (11)$$

where we omit contributions from the spin-orbit radius (negligible for s-shell nuclei) and meson–exchange currents. The last term in Eq. (11) is the Darwin–Foldy term, where m_p is the proton mass, taken from Refs. [3,49]. In Table 1, we show only R_c values obtained using $r_p(e^-)$ and experimental values obtained only with electrons. As a direct result of Eq. (11), using $r_p(\mu^-)$ would decrease R_c by 0.016 (0.018) fm for ^{3}He (^{3}H). We note that the uncertainty, currently governed by nuclear-model dependence, is slightly larger than the effect of varying r_p. It should also be noted that our R_p values agree with the hyperspherical harmonics calculations of the Pisa group [58] for both nuclear potentials, suggesting a small ISB effect, while the Monte-Carlo calculations of Ref. [63] show less agreement. Considering that radii were not included in the calibration of the nuclear Hamiltonians, it would be interesting to further investigate their sensitivity to the theoretical apparatus. In particular, work is in progress to include meson–exchange currents [72]. Currently, for ^{3}He the AV18/UIX charge radius agrees with the experimental value slightly better than the χEFT result, while for ^{3}H the experimental error bar is larger than the nuclear-model dependence, and calls for a more precise measurement.

Concerning the magnetic moments, our results are comparable with the other impulse approximation calculations presented in Table 1, which deviate from experiment due to the absence of meson–exchange currents. However, we do not include meson–exchange currents in $\delta^A_{\rm TPE}$, since the contribution of the magnetic term $\delta^{(0)}_M$ is small enough to make these corrections negligible.

The electric dipole polarizability α_E is an inverse-energy-weighted sum rule of the dipole response and is therefore closely related to $\delta^A_{\rm pol}$. Our results are in agreement with previous calculations, especially the recent Ref. [60]. As in [30], α_E is found to be nuclear-model dependent. We provide first results for the unmeasured α_E of ^{3}H with the AV18/UIX potential, which lies within the uncertainty estimates of [60].

3.2. Zemach terms

We now turn to the Zemach terms, first listing available values in the literature. In Refs. [31,32] Borie calculated $\delta^A_{\rm Zem}$, following Friar [43], using a Gaussian distribution that fits the nuclear-charge-radius obtained from electron experiments. The result[5,6] was $\delta^A_{\rm Zem}(^3{\rm He}) = -10.258(305)$ meV. Recently, Krutov et al. [34] repeated this calculation and obtained $\delta^A_{\rm Zem}(^3{\rm He}) = -10.28(10)$ meV. Alternatively, inserting the 3rd nuclear Zemach moment recently extracted from $e-{}^3$He scattering data [68] into Eq. (2) gives $\delta^A_{\rm Zem}(^3{\rm He}) = -10.87(27)$ meV. As explained above, all of these results should be compared with our calculation that uses $r_p(e^-)$ as input and yields $\delta^A_{\rm Zem}(^3{\rm He})\left[r_p(e^-)\right] = -10.71(19)(16)$ meV, where the first uncertainty results from nuclear-model dependence and the second includes all other sources. Our result is in agreement with these references (based on comments made in Refs. [34,68], we assume that the error-bars in Ref. [34] are not exhaustive). However, for the muonic systems considered here we use $r_p(\mu^-)$, which gives

$$\delta^A_{\rm Zem}\left(^3{\rm He}\right)\left[r_p(\mu^-)\right] = -10.49(19)(16)\ {\rm meV}. \tag{12}$$

We note that with the given error-bars this result is also in agreement with Refs. [32,34,68].

The use of Eq. (3) is adopted from Refs. [31,32], where[7] $\mathcal{C}(^3{\rm He}) = -1.35(4)$ meV fm^{-3}. The results of Ref. [68] can also be used to extract $\mathcal{C}(^3{\rm He}) = \delta^A_{\rm Zem}/R_c^3 = -1.42(4)$ meV fm^{-3} from the $e-{}^3$He scattering data. Our calculations of $\delta^A_{\rm Zem}$ and R_c with either value of r_p give $\mathcal{C}(^3{\rm He})\left[r_p(e^-)\right] = -1.383(05)(20)$ meV fm^{-3} and $\mathcal{C}(^3{\rm He})\left[r_p(\mu^-)\right] = -1.388(05)(21)$ meV fm^{-3}, which both agree with Refs. [31,32,68]. Evidently, the nuclear-model dependence is diminished for this value, since it is proportional to the geometrical ratio $\langle r^3\rangle_{(2)}/R_c^3$. Similarly to R_c discussed above, the difference between $\delta^A_{\rm Zem}$ results obtained with the two nuclear potentials stems from the different point-proton distributions, and this largely cancels out in $\mathcal{C}$, reducing its total relative uncertainty compared to $\delta^A_{\rm Zem}$.

Repeating the above procedures we obtain predictions for μ^3H

$$\delta^A_{\rm Zem}\left(^3{\rm H}\right)\left[r_p(\mu^-)\right] = -0.227(5)(3)\ {\rm meV}, \tag{13}$$

and

$$\mathcal{C}\left(^3{\rm H}\right)\left[r_p(\mu^-)\right] = -0.0425(2)(6)\ {\rm meV\,fm^{-3}}. \tag{14}$$

For future comparisons, using $r_p(e^-)$ shifts $\delta^A_{\rm Zem}(^3{\rm H})$ by -6 µeV and $\mathcal{C}(^3{\rm H})$ by $+0.2$ µeV fm^{-3}.

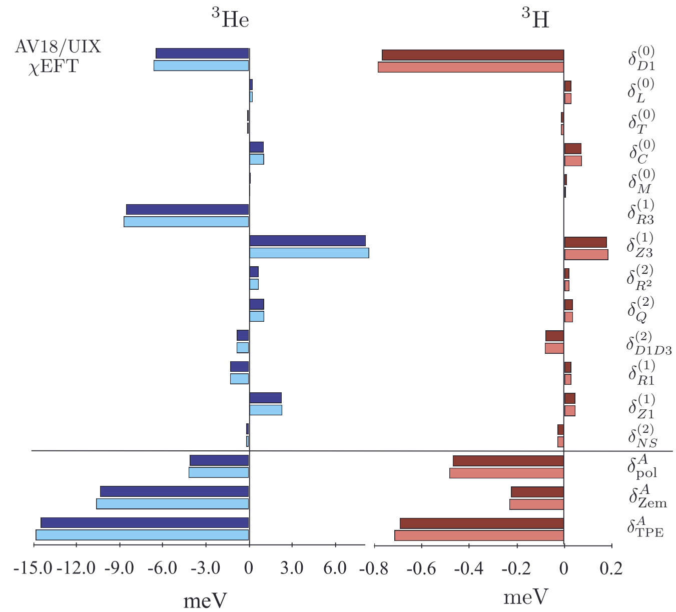

Fig. 1. Graphic representation of the various contributions to the nuclear structure and polarization corrections to the 2S–2P Lamb shift in the muonic hydrogen-like systems of ^{3}He and ^{3}H, calculated with the AV18/UIX and χEFT nuclear potentials. Notice the different scales used for the two systems.

3.3. Nuclear polarization corrections

Next, the nuclear polarization correction to the Lamb shift — $\delta^A_{\rm pol}$ — is obtained by summing up the terms in Eq. (4). Their values for $\mu^3{\rm He}^+$ and μ^3H, calculated with the two nuclear potentials, are shown[8] in Fig. 1. Here, the NS corrections are obtained using only $r_p(\mu^-)$. Taking the mean value of the two nuclear potentials we obtain

$$\delta^A_{\rm pol}\left(\mu^3{\rm He}^+\right) = -4.16(06)(16)\ {\rm meV}$$
$$\delta^A_{\rm pol}\left(\mu^3{\rm H}\right) = -0.476(10)(13)\ {\rm meV}\,, \tag{15}$$

where we retain the use of first and second brackets for uncertainties from nuclear-model dependence and from all other sources, respectively. Our result for $\mu^3{\rm He}^+$ agrees with Rinker's -4.9 ± 1.0 meV obtained fourty years ago [42]. The μ^3H case was rarely studied. We note, however, that a comparison with the simplistic calculation of Ref. [41] reveals a similar ratio of $\sim$9 between $\delta^A_{\rm pol}$ of $\mu^3{\rm He}^+$ and of μ^3H, both in Ref. [41] and in our work.

5 Ref. [32] is the arXiv version of Ref. [31], which has been updated since publication; in particular, $\delta^A_{\rm Zem}(^3{\rm He})$ was increased by $\sim$20% with respect to the published version.

6 The result is given using our sign convention.

7 See footnote 6.

8 The numerical values are detailed in the Supplementary Materials [57].

For completeness, we add Eqs. (12) and (13) to Eq. (15) to obtain the total nuclear-structure TPE corrections that enter Eq. (1)

$$\delta^A_{\rm TPE}\left(\mu^3{\rm He}^+\right) = -14.64(25)(27)\ {\rm meV}$$
$$\delta^A_{\rm TPE}\left(\mu^3{\rm H}\right) = -0.703(16)(11)\ {\rm meV}\ . \qquad (16)$$

4. Hadronic TPE

The last ingredient in $\delta_{\rm TPE}$ is the contribution from two-photon exchange with the internal degrees of freedom of the nucleons that make up the nucleus, *i.e.*, $\delta^N_{\rm TPE} = \delta^N_{\rm Zem} + \delta^N_{\rm pol}$. Since it is dictated by the hadronic scale, about 10 times higher than the nuclear interaction, this contribution can be approximated as the sum of TPE effects from each of the individual nucleons. The various terms that contribute to $\delta^N_{\rm TPE}$ are estimated based on previous studies of μH, as recently done for μD in Ref. [33]. In recent years, two-photon exchange corrections to the Lamb-shift in μH have been calculated by several groups using various methods, *e.g.*, Refs. [73–75]. As suggested by Birse and McGovern [76], below we use values for $\delta_{\rm Zem}$ and $\delta_{\rm pol}$ of μH obtained from Refs. [21,77]. These values are in agreement with the calculations given above.

4.1. Nucleon Zemach terms

As Friar showed in Ref. [37], the intrinsic Zemach term of each proton contributes to $\delta_{\rm TPE}$ of the nucleus as an additional NS correction, not accounted for in the NS corrections detailed above.[9] We denote this term $\delta^N_{\rm Zem}$ and find its contribution to be proportional to the analogous term in μH by

$$\delta^N_{\rm Zem}(\mu A) = \left(\frac{Z m_r(\mu A)}{m_r(\mu{\rm H})}\right)^4 \times \delta_{\rm Zem}(\mu{\rm H})\ . \qquad (17)$$

We take $\delta_{\rm Zem}(\mu{\rm H}) = 0.0247(13)$ meV[10] and obtain

$$\delta^N_{\rm Zem}\left(\mu^3{\rm He}^+\right) = -0.487(26)\ {\rm meV}$$
$$\delta^N_{\rm Zem}\left(\mu^3{\rm H}\right) = -0.0305(16)\ {\rm meV}\ . \qquad (18)$$

4.2. Nucleon polarization corrections

In Ref. [39], $\delta^N_{\rm pol}$ of μD was extracted from electron scattering data. Here, we resort to estimating $\delta^N_{\rm pol}$ by relating it to the proton polarization correction in μH via [36,40,80]

$$\delta^N_{\rm pol}(\mu A) = (N+Z)\left[Z m_r(\mu A)/m_r(\mu{\rm H})\right]^3 \delta_{\rm pol}(\mu{\rm H})\ , \qquad (19)$$

assuming that the neutron polarization contribution is the same as that of the proton. Here we use $\delta_{\rm pol}(\mu{\rm H}) = 9.3(1.1)$ µeV.[11] Based on current knowledge of the nucleon polarizabilities [81], we assign an additional 20% uncertainty to the neutron polarization contribution. Another possible error in $\delta^N_{\rm pol}$ arises from neglecting medium effects and nucleon–nucleon interferences in Eq. (19). These effects can be estimated by comparing the calculated $\delta^N_{\rm pol}(\mu{\rm D})$ with the result evaluated in Ref. [39] from scattering data. This yields a ∼29% correction. Until this correction is calculated rigorously in other light muonic atoms, we estimate it to be of a similar size, multiplied by $A/2$, making it the dominant source of uncertainty in our $\delta^N_{\rm TPE}$. Eventually, we obtain

$$\delta^N_{\rm pol}\left(\mu^3{\rm He}^+\right) = -0.275(123)\ {\rm meV}$$
$$\delta^N_{\rm pol}\left(\mu^3{\rm H}\right) = -0.034(16)\ {\rm meV}\ . \qquad (20)$$

4.3. Total nucleon contributions

Summing up the results in Eqs. (18) and (20) we obtain the total contribution from the nucleon degrees of freedom

$$\delta^N_{\rm TPE}\left(\mu^3{\rm He}^+\right) = -0.762(125)\ {\rm meV}$$
$$\delta^N_{\rm TPE}\left(\mu^3{\rm H}\right) = -0.065(16)\ {\rm meV}\ . \qquad (21)$$

In μ^3He$^+$, $\delta^N_{\rm TPE}$ is ∼5% of $\delta^A_{\rm TPE}$, *i.e.*, about twice the overall uncertainty in $\delta^A_{\rm TPE}$. For μ^3H we obtained that $\delta^N_{\rm TPE}$ is ∼9% of $\delta^A_{\rm TPE}$, which is more than three times the uncertainty in the latter. Therefore, our precision in predicting $\delta^A_{\rm TPE}$ can be important not only for the determination of R_c from muonic Lamb shift measurements, but also for probing $\delta^N_{\rm TPE}$, if these measurements reveal discrepancies with electronic experiments that may indicate exotic contributions to $\delta^N_{\rm TPE}$. A study of the Lamb shift in μ^3H will be especially sensitive to the nucleon polarizabilities, since their relative contribution is much larger in this case.

5. Summary

We have performed the first *ab initio* calculation of $\delta^A_{\rm Zem}$ and $\delta^A_{\rm pol}$ for both μ^3He$^+$ and μ^3H, using state-of-the-art nuclear potentials. Many possible sources of uncertainty have been considered, yet the resulting uncertainties of a few percents are much lower than in previous estimates of $\delta^A_{\rm pol}$ and $\delta^A_{\rm TPE}$, which relied on imprecise data and simplistic models. In addition, our $\delta^A_{\rm Zem}$ calculations agree with previous estimates and with recent analysis of e^{-3}He scattering, and provide predictions towards ^{3}H measurements. They were also adapted for muonic systems by incorporating $r_p(\mu^-)$ — the proton radius measured with muons.

Ultimately, our results will allow two alternative ways of extracting a much more precise R_c from a recent measurement [5,28,82] of the Lamb shift in μ^3He$^+$, and from an analogous measurement we encourage to conduct in μ^3H. The precision of the charge radii of ^{3}He and ^{3}H could be thus improved by factors of ∼5 and ∼50, respectively, which could have interesting implications for nuclear physics.

Finally, we estimate the hadronic contribution $\delta^N_{\rm TPE}$ in these systems, and find it to be larger than our uncertainty estimates in $\delta^A_{\rm TPE}$. Therefore, this combined theoretical and experimental effort may not only shed some light on the "proton radius puzzle", but could also probe the elusive nucleon polarizabilities tightly connected to it.

Acknowledgements

NND would like to thank Bezalel Bazak for suggesting the case of muonic triton, to express a special thanks to the Mainz Institute for Theoretical Physics (MITP) for its hospitality and support, and acknowledge constructive discussions with Aldo Antognini, Mike Birse, Michael Distler, Mikhail Gorchtein, Savely Karshenboim, Franz Kottmann, Randolf Pohl, and Ingo Sick. This work was supported in parts by the Natural Sciences and Engineering Research

---

9 In our notations this term appears as an NS correction to $\delta^{(1)}_{R3}$.

10 We use the same value as in [33]. Here, $\delta_{\rm Zem}(\mu{\rm H})$ stands for the elastic + non-pole parts of $\delta_{\rm TPE}(\mu{\rm H})$, and not for the non-relativistic limit that is related to the proton's 3rd Zemach moment (see Refs. [78,79]).

11 $\delta_{\rm pol}(\mu{\rm H}) = \delta^p_{\rm inelastic} + \delta^p_{\rm subtraction}$. For the former we follow Ref. [39] and adopt 13.5 µeV, which is an average of three values from Ref. [77], and for the latter we use −4.2(1.0) µeV from Ref. [21].

Council (NSERC), the National Research Council Canada, the Israel Science Foundation (Grant number 954/09), and the Pazy foundation.

Appendix A. Supplementary material

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.physletb.2016.02.023.

References

[1] R. Pohl, et al., The size of the proton, Nature 466 (2010) 213–216, http://dx.doi.org/10.1038/nature09250.
[2] A. Antognini, et al., Proton structure from the measurement of 2S–2P transition frequencies of muonic hydrogen, Science 339 (2013) 417–420, http://dx.doi.org/10.1126/science.1230016.
[3] P.J. Mohr, B.N. Taylor, D.B. Newell, Codata recommended values of the fundamental physical constants: 2010, Rev. Mod. Phys. 84 (2012) 1527, doi:10.1103/RevModPhys.84.1527.
[4] R. Pohl, R. Gilman, G.A. Miller, K. Pachucki, Muonic hydrogen and the proton radius puzzle, Annu. Rev. Nucl. Part. Sci. 63 (2013) 175–204, http://www.annualreviews.org/doi/abs/10.1146/annurev-nucl-102212-170627.
[5] R. Pohl, The Lamb shift in muonic hydrogen and the proton radius puzzle, Hyperfine Interact. 227 (2014) 23–28, doi:10.1007/s10751-014-1011-1.
[6] R.J. Hill, G. Paz, Model-independent extraction of the proton charge radius from electron scattering, Phys. Rev. D 82 (2010) 113005, doi:10.1103/PhysRevD.82.113005.
[7] I. Sick, D. Trautmann, Proton root-mean-square radii and electron scattering, Phys. Rev. C 89 (2014) 012201, doi:10.1103/PhysRevC.89.012201.
[8] E. Kraus, K. Mesick, A. White, R. Gilman, S. Strauch, Taylor series expansion fits and the proton radius puzzle, Phys. Rev. C 90 (2014) 045206, doi:10.1103/PhysRevC.90.045206, arXiv:1405.4735.
[9] I. Lorenz, U.-G. Meißner, H.-W. Hammer, Y.-B. Dong, Theoretical constraints and systematic effects in the determination of the proton form factors, Phys. Rev. D 91 (2015) 014023, doi:10.1103/PhysRevD.91.014023.
[10] A. Gasparin, et al., High precision measurement of the proton charge radius, Jefferson Laboratory Experiment 12-11-106 (unpublished), http://www.jlab.org/exp_prog/proposals/12/C12-11-106.pdf, 2012.
[11] M. Mihovilovic, H. Merkel, A1-Collaboration, Initial state radiation experiment at MAMI, AIP Conf. Proc. 1563 (2013) 187–190, doi:10.1063/1.4829406.
[12] S.G. Karshenboim, Accuracy of the optical determination of the proton charge radius, Phys. Rev. A 91 (2015) 012515, http://dx.doi.org/10.1103/PhysRevA.91.012515.
[13] A. Vutha, N. Bezginov, I. Ferchichi, M. George, V. Isaac, C. Storry, A. Weatherbee, M. Weel, E. Hessels, Progress towards a new microwave measurement of the hydrogen $n = 2$ Lamb shift: a measurement of the proton charge radius, Bull. Am. Phys. Soc. 57 (2012) D1.00138, http://meetings.aps.org/link/BAPS.2012.DAMOP.D1.138.
[14] A. Beyer, J. Alnis, K. Khabarova, A. Matveev, C.G. Parthey, D.C. Yost, R. Pohl, T. Udem, T.W. Haensch, N. Kolachevsky, Precision spectroscopy of the 2S–4P transition in atomic hydrogen on a cryogenic beam of optically excited 2S atoms, Ann. Phys. 525 (2013) 671–679, http://dx.doi.org/10.1002/andp.201300075.
[15] E. Peters, D.C. Yost, A. Matveev, T.W. Hänsch, T. Udem, Frequency-comb spectroscopy of the hydrogen 1S–3S and 1S–3D transitions, Ann. Phys. 525 (2013) L29–L34, http://dx.doi.org/10.1002/andp.201300062.
[16] J.N. Tan, S.M. Brewer, N.D. Guise, Experimental efforts at NIST towards one-electron ions in circular Rydberg states, Phys. Scr. 2011 (2011) 014009, http://dx.doi.org/10.1088/0031-8949/2011/T144/014009.
[17] B. Batell, D. McKeen, M. Pospelov, New parity-violating muonic forces and the proton charge radius, Phys. Rev. Lett. 107 (2011) 011803, http://dx.doi.org/10.1103/PhysRevLett.107.011803.
[18] D. Tucker-Smith, I. Yavin, Muonic hydrogen and MeV forces, Phys. Rev. D 83 (2011) 101702, http://dx.doi.org/10.1103/PhysRevD.83.101702.
[19] C.E. Carlson, B.C. Rislow, Constraints to new physics models for the proton charge radius puzzle from the decay $k^+ \to \mu^+ + \nu + e^- + e^+$, Phys. Rev. D 89 (2014) 035003, http://dx.doi.org/10.1103/PhysRevD.89.035003.
[20] R.J. Hill, G. Paz, Model independent analysis of proton structure for hydrogenic bound states, Phys. Rev. Lett. 107 (2011) 160402, http://dx.doi.org/10.1103/PhysRevLett.107.160402.
[21] M.C. Birse, J.A. McGovern, Proton polarisability contribution to the Lamb shift in muonic hydrogen at fourth order in chiral perturbation theory, Eur. Phys. J. A 48 (2012) 120, http://dx.doi.org/10.1140/epja/i2012-12120-8.
[22] G.A. Miller, Proton polarizability contribution: muonic hydrogen Lamb shift and elastic scattering, Phys. Lett. B 718 (2013) 1078–1082, http://dx.doi.org/10.1016/j.physletb.2012.11.016.
[23] U.D. Jentschura, Light sea fermions in electron–proton and muon–proton interactions, Phys. Rev. A 88 (2013) 062514, http://dx.doi.org/10.1103/PhysRevA.88.062514.
[24] F. Hagelstein, V. Pascalutsa, Breakdown of the expansion of finite-size corrections to the hydrogen Lamb shift in moments of charge distribution, Phys. Rev. A 91 (2015) 040502, http://dx.doi.org/10.1103/PhysRevA.91.040502.
[25] G.A. Miller, A.W. Thomas, J.D. Carroll, J. Rafelski, Toward a resolution of the proton size puzzle, Phys. Rev. A 84 (2011) 020101, doi:10.1103/PhysRevA.84.020101.
[26] O. Tomalak, M. Vanderhaeghen, Two-photon exchange corrections in elastic muon–proton scattering, Phys. Rev. D 90 (2014) 013006, doi:10.1103/PhysRevD.90.013006.
[27] R. Gilman, Studying the proton "radius" puzzle with μp elastic scattering, AIP Conf. Proc. 1563 (2013) 167–170, doi:10.1063/1.4829401.
[28] A. Antognini, et al., Illuminating the proton radius conundrum: the μHe^+ Lamb shift, Can. J. Phys. 89 (2011) 47–57, doi:10.1139/P10-113.
[29] W.E. Lamb, R.C. Retherford, Fine structure of the hydrogen atom by a microwave method, Phys. Rev. 72 (1947) 241–243, doi:10.1103/PhysRev.72.241.
[30] C. Ji, N. Nevo Dinur, S. Bacca, N. Barnea, Nuclear polarization corrections to the μ^4He^+ Lamb shift, Phys. Rev. Lett. 111 (2013) 143402, doi:10.1103/PhysRevLett.111.143402.
[31] E. Borie, Lamb shift in light muonic atoms — revisited, Ann. Phys. 327 (2012) 733–763, doi:10.1016/j.aop.2011.11.017.
[32] E. Borie, Lamb shift in light muonic atoms – revisited, arXiv:1103.1772 [physics.atom-ph], http://arxiv.org/abs/1103.1772v7.
[33] J.J. Krauth, M. Diepold, B. Franke, A. Antognini, F. Kottmann, R. Pohl, Theory of the $n = 2$ levels in muonic deuterium, arXiv:1506.01298 [physics.atom-ph], http://arxiv.org/abs/1506.01298.
[34] A. Krutov, A. Martynenko, G. Martynenko, R. Faustov, Theory of the Lamb shift in muonic helium ions, J. Exp. Theor. Phys. 120 (2015) 73–90, http://dx.doi.org/10.1134/S1063776115010033.
[35] S.G. Karshenboim, E.Y. Korzinin, V.A. Shelyuto, V.G. Ivanov, Theory of Lamb shift in muonic hydrogen, J. Phys. Chem. Ref. Data 44 (2015) 031202, doi:10.1063/1.4921197.
[36] K. Pachucki, Nuclear structure corrections in muonic deuterium, Phys. Rev. Lett. 106 (2011) 193007, http://dx.doi.org/10.1103/PhysRevLett.106.193007.
[37] J.L. Friar, Nuclear polarization corrections to μ-d atoms in zero-range approximation, Phys. Rev. C 88 (2013) 034003, http://dx.doi.org/10.1103/PhysRevC.88.034003.
[38] O.J. Hernandez, C. Ji, S. Bacca, N. Nevo Dinur, N. Barnea, Improved estimates of the nuclear structure corrections in μD, Phys. Lett. B 736 (2014) 344–349, http://dx.doi.org/10.1016/j.physletb.2014.07.039.
[39] C.E. Carlson, M. Gorchtein, M. Vanderhaeghen, Nuclear-structure contribution to the Lamb shift in muonic deuterium, Phys. Rev. A 89 (2014) 022504, http://dx.doi.org/10.1103/PhysRevA.89.022504.
[40] K. Pachucki, A. Wienczek, Nuclear structure effects in light muonic atoms, Phys. Rev. A 91 (2015) 040503, http://dx.doi.org/10.1103/PhysRevA.91.040503.
[41] C. Joachain, Polarisation nucléaire dans les atomes mésiques de tritium, d'hélium 3 et d'hélium 4, Nucl. Phys. 25 (1961) 317–327, http://dx.doi.org/10.1016/0029-5582(61)90162-6.
[42] G.A. Rinker, Nuclear polarization in muonic helium, Phys. Rev. A 14 (1976) 18–29, http://dx.doi.org/10.1103/PhysRevA.14.18.
[43] J.L. Friar, Nuclear finite-size effects in light muonic atoms, Ann. Phys. 122 (1979) 151–196, http://dx.doi.org/10.1016/0003-4916(79)90300-2.
[44] S.G. Karshenboim, E.Y. Korzinin, V.A. Shelyuto, V.G. Ivanov, Recoil correction to the proton finite-size contribution to the Lamb shift in muonic hydrogen, Phys. Rev. D 91 (2015) 073003, http://dx.doi.org/10.1103/PhysRevD.91.073003.
[45] M.O. Distler, J.C. Bernauer, T. Walcher, The RMS charge radius of the proton and Zemach moments, Phys. Lett. B 696 (2011) 343–347, http://dx.doi.org/10.1016/j.physletb.2010.12.067.
[46] J.L. Friar, G.L. Payne, Higher-order nuclear-size corrections in atomic hydrogen, Phys. Rev. A 56 (1997) 5173–5175, http://dx.doi.org/10.1103/PhysRevA.56.5173.
[47] C. Ji, N. Nevo Dinur, S. Bacca, N. Barnea, Nuclear polarization effects in muonic atoms, Few-Body Syst. 55 (2014) 917–921, http://dx.doi.org/10.1007/s00601-014-0809-3.
[48] P. Ring, P. Schuck, The Nuclear Many-Body Problem, Springer-Verlag, Berlin, Heidelberg, 1980.
[49] J. Beringer, et al., Particle Data Group, Review of particle physics, Phys. Rev. D 86 (2012) 010001, http://dx.doi.org/10.1103/PhysRevD.86.010001.
[50] N. Nevo Dinur, N. Barnea, C. Ji, S. Bacca, Efficient method for evaluating energy-dependent sum rules, Phys. Rev. C 89 (2014) 064317, http://dx.doi.org/10.1103/PhysRevC.89.064317.
[51] N. Barnea, W. Leidemann, G. Orlandini, State dependent effective interaction for the hyperspherical formalism, Phys. Rev. C 61 (2000) 054001, http://dx.doi.org/10.1103/PhysRevC.61.054001.
[52] N. Barnea, W. Leidemann, G. Orlandini, State-dependent effective interaction for the hyperspherical formalism with noncentral forces, Nucl. Phys. A 693 (2001) 565–578, http://dx.doi.org/10.1016/S0375-9474(01)00794-1.
[53] R.B. Wiringa, V.G.J. Stoks, R. Schiavilla, Accurate nucleon–nucleon potential with charge-independence breaking, Phys. Rev. C 51 (1995) 38, http://dx.doi.org/10.1103/PhysRevC.51.38.

[54] B.S. Pudliner, V.R. Pandharipande, J. Carlson, R.B. Wiringa, Quantum Monte Carlo calculations of $A \leq 6$ nuclei, Phys. Rev. Lett. 74 (1995) 4396–4399, http://dx.doi.org/10.1103/PhysRevLett.74.4396.
[55] D.R. Entem, R. Machleidt, Accurate charge-dependent nucleon–nucleon potential at fourth order of chiral perturbation theory, Phys. Rev. C 68 (2003) 041001, http://dx.doi.org/10.1103/PhysRevC.68.041001.
[56] P. Navrátil, Local three-nucleon interaction from chiral effective field theory, Few-Body Syst. 41 (2007) 117–140, http://dx.doi.org/10.1007/s00601-007-0193-3.
[57] Additional details are available as supplementary materials online.
[58] A. Kievsky, S. Rosati, M. Viviani, L.E. Marcucci, L. Girlanda, A high-precision variational approach to three- and four-nucleon bound and zero-energy scattering states, J. Phys. G, Nucl. Part. Phys. 35 (2008) 063101, http://stacks.iop.org/0954-3899/35/i=6/a=063101.
[59] A. Nogga, A. Kievsky, H. Kamada, W. Glöckle, L.E. Marcucci, S. Rosati, M. Viviani, Three-nucleon bound states using realistic potential models, Phys. Rev. C 67 (2003) 034004, http://dx.doi.org/10.1103/PhysRevC.67.034004.
[60] I. Stetcu, S. Quaglioni, J.L. Friar, A.C. Hayes, P. Navrátil, Electric dipole polarizabilities of hydrogen and helium isotopes, Phys. Rev. C 79 (2009) 064001, http://dx.doi.org/10.1103/PhysRevC.79.064001.
[61] J. Purcell, J. Kelley, E. Kwan, C. Sheu, H. Weller, Energy levels of light nuclei, Nucl. Phys. A 848 (2010) 1–74, http://dx.doi.org/10.1016/j.nuclphysa.2010.08.012.
[62] L.E. Marcucci, M. Pervin, S.C. Pieper, R. Schiavilla, R.B. Wiringa, Quantum Monte Carlo calculations of magnetic moments and $M1$ transitions in $A \leqslant 7$ nuclei including meson–exchange currents, Phys. Rev. C 78 (2008) 065501, http://dx.doi.org/10.1103/PhysRevC.78.065501.
[63] S. Pastore, S.C. Pieper, R. Schiavilla, R.B. Wiringa, Quantum Monte Carlo calculations of electromagnetic moments and transitions in $A \leq 9$ nuclei with meson-exchange currents derived from chiral effective field theory, Phys. Rev. C 87 (2013) 035503, http://dx.doi.org/10.1103/PhysRevC.87.035503.
[64] K. Pachucki, A.M. Moro, Nuclear polarizability of helium isotopes in atomic transitions, Phys. Rev. A 75 (2007) 032521, http://dx.doi.org/10.1103/PhysRevA.75.032521.
[65] W. Leidemann, Electromagnetic breakup of nuclei with $A = 3$–6, in: R. Krivec, M. Rosina, B. Golli, S. Širca (Eds.), Few-Body Problems in Physics '02, in: Few-Body Syst., vol. 14, Springer, Vienna, 2003, pp. 313–318.
[66] F. Goeckner, L.O. Lamm, L.D. Knutson, Measurement of the electric polarizability of ^{3}He, Phys. Rev. C 43 (1991) 66–72, http://dx.doi.org/10.1103/PhysRevC.43.66.
[67] V.D. Efros, W. Leidemann, G. Orlandini, Photodisintegration of the three-nucleon systems and their polarizabilities, Phys. Lett. B 408 (1997) 1–6, http://dx.doi.org/10.1016/S0370-2693(97)00772-7.
[68] I. Sick, Zemach moments of ^{3}He and ^{4}He, Phys. Rev. C 90 (2014) 064002, http://dx.doi.org/10.1103/PhysRevC.90.064002.
[69] I. Angeli, K.P. Marinova, Table of experimental nuclear ground state charge radii: an update, At. Data Nucl. Data Tables 99 (2013) 69–95, http://dx.doi.org/10.1016/j.adt.2011.12.006.
[70] J.L. Friar, J. Martorell, D.W.L. Sprung, Nuclear sizes and the isotope shift, Phys. Rev. A 56 (1997) 4579–4586, http://dx.doi.org/10.1103/PhysRevA.56.4579.
[71] A. Ong, J.C. Berengut, V.V. Flambaum, Effect of spin-orbit nuclear charge density corrections due to the anomalous magnetic moment on halonuclei, Phys. Rev. C 82 (2010) 014320, http://dx.doi.org/10.1103/PhysRevC.82.014320.
[72] S. Pastore, C. Ji, N. Nevo Dinur, S. Bacca, M. Piarulli, R.B. Wiringa, N. Barnea, in preparation.
[73] M. Gorchtein, F.J. Llanes-Estrada, A.P. Szczepaniak, Muonic-hydrogen Lamb shift: dispersing the nucleon–excitation uncertainty with a finite-energy sum rule, Phys. Rev. A 87 (2013) 052501, http://dx.doi.org/10.1103/PhysRevA.87.052501.
[74] J.M. Alarcón, V. Lensky, V. Pascalutsa, Chiral perturbation theory of muonic-hydrogen Lamb shift: polarizability contribution, Eur. Phys. J. C 74 (2014) 2852, http://dx.doi.org/10.1140/epjc/s10052-014-2852-0.
[75] C. Peset, A. Pineda, The two-photon exchange contribution to muonic hydrogen from chiral perturbation theory, Nucl. Phys. B 887 (2014) 69–111, http://dx.doi.org/10.1016/j.nuclphysb.2014.07.027.
[76] M.C. Birse, J.A. McGovern, 2015, private communication.
[77] C.E. Carlson, M. Vanderhaeghen, Higher-order proton structure corrections to the Lamb shift in muonic hydrogen, Phys. Rev. A 84 (2011) 020102, http://dx.doi.org/10.1103/PhysRevA.84.020102.
[78] A. Antognini, et al., Proton structure from the measurement of 2S–2P transition frequencies of muonic hydrogen, Science 339 (2013) 417–420, http://dx.doi.org/10.1126/science.1230016, supplementary materials.
[79] A. Antognini, F. Kottmann, F. Biraben, P. Indelicato, F. Nez, R. Pohl, Theory of the 2S–2P Lamb shift and 2S hyperfine splitting in muonic hydrogen, Ann. Phys. 331 (2013) 127–145, http://dx.doi.org/10.1016/j.aop.2012.12.003.
[80] G.A. Miller, Non-perturbative lepton sea fermions in the nucleon and the proton radius puzzle, arXiv:1501.01036 [nucl-th].
[81] L.S. Myers, J.R.M. Annand, J. Brudvik, G. Feldman, K.G. Fissum, H.W. Grießhammer, K. Hansen, S.S. Henshaw, L. Isaksson, R. Jebali, M.A. Kovash, M. Lundin, J.A. McGovern, D.G. Middleton, A.M. Nathan, D.R. Phillips, B. Schröder, S.C. Stave, Measurement of Compton scattering from the deuteron and an improved extraction of the neutron electromagnetic polarizabilities, Phys. Rev. Lett. 113 (2014) 262506, http://dx.doi.org/10.1103/PhysRevLett.113.262506.
[82] F. Kottmann, 2014, private communication.