ON THE ASYMPTOTIC BEHAVIOR OF CROSS SECTIONS AT HIGH ENERGIES

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Processes of transformation of two particles into three or four particles in the high energy region are examined in the pole approximation. In the energy range in which the total elastic scattering cross section is independent of energy, the cross section for the transformation of two particles into three does not decrease with increasing energy. The cross section for the transformation of two particles into four exhibits a logarithmic growth with energy. This result indicates that at very high energies the elastic scattering cross section tends to zero.

1. Data available at present on the collision of high energy particles make plausible the assumption that with growing energy the total effective cross section approaches a constant limit. This limit (which is of the order of $1/\mu^2$, where $1/\mu$ is the π-meson Compton wave length) is reached at energies of the order of several BeV. Along with the total cross section the elastic diffraction scattering cross section also approaches a constant limit. As far as various inelastic processes are concerned (production of showers with a given number of particles) it seems natural to assume that the cross section for each of them should approach zero with growing energy since the possible number of produced particles should increase infinitely with increasing energy, and the constant total cross section should be divided among an ever-growing number of competing processes.

It may be, however, that this simple picture is not correct. We shall give some approximate calculations of the effective cross sections for inelastic processes. These calculations lead to a very different asymptotic behavior of the inelastic cross sections. The approximations we make cannot be motivated rigorously and hence the results may not be fully convincing. Nevertheless, they show that at high energies a much more complex situation may exist.

2. Consider the transformation of two particles into three, $p_1+q_1 \rightarrow p_2+q_2+K_2$ for example, the formation of a π-meson in the collision between a meson and nucleon or nucleon and nucleon. The amplitude for this process, A, can be written in the following form

$$A = g\bar{u}(p_2)\gamma_5 u(p_1)\frac{1}{t-\mu^2}A_1 + A' \qquad (1)$$

$$(t=(p_1-p_2)^2)$$

Here the first term is a pole term corresponding to the diagram in Fig. 1, A' is the remaining part of the amplitude, g is the nucleon$-\pi$-meson coupling constant, and A_1 is the scattering amplitude of the

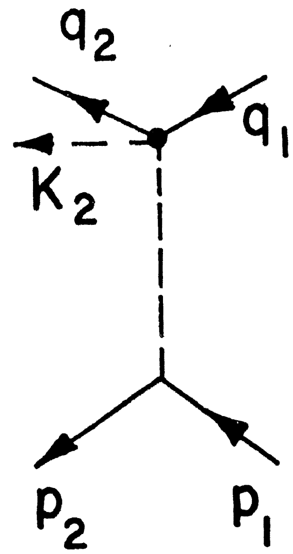

Fig. 1 Feynman diagram for single meson production.

π-meson by particle q_1 (or by a π-meson or nucleon). We shall calculate the cross section for the process under consideration in the case of small momentum transfers (of the order of μ) and retain only the pole term [1].

It seems natural to assume that since A' is determined by the singularities at $t>4\mu^2$ the neglected terms cannot fully compensate for the contribution of the pole term to the cross section. Thus at small values of t the contribution of the pole term should yield a correct order of magnitude of the cross section. A more strict approach to this problem could consist in investigating the amplitude of the process at large values of the orbital angular momentum $l>p/\mu$ (where p is the momentum of the incident particles in the c.m.s.). In such cases the dominating interaction would seem to be associated with the exchange of one meson (which corresponds to the pole term). In other words partial cross sections with large values of l will be determined only by the pole diagram considered by us [2]. It should be stressed that the partial cross sections thus obtained can be expressed in terms of the real elastic scattering cross sections determined by amplitude A_1, and are essentially positive quantities. If one takes the sum of the partial cross sections from $l\sim p/\mu$ to $l\to\infty$, one finds that the part of the cross section thus obtained is of the same order of magnitude as that obtained by calculating the cross section only with the aid of the pole term of the amplitude, and integrating over t up to $\sim -\mu^2$. It may be noted that our analysis is not identical with the Chew and Low "pole" approach [1]. The pole analysis literally yields a negative value for the cross section and is connected with extrapolation of the cross section to the non-physical region $t=\mu^2$. We, on the other hand, do not leave the physical region $t<0$. However, since we depart from the mass shell of the virtual meson by $<2\mu^2$, and the mass singularities in the meson Green's functions are located at values exceeding $9\mu^2$, it appears that the neglected terms will not affect the order of magnitude of the effect.

Retaining only the first term in (1), we arrive at the following expression for the differential cross section

$$d\sigma=\frac{f^2}{\pi}\cdot\frac{2M^2}{\mu^2}\,\frac{\left[s_1^2-2(m^2+\mu^2)s_1+(m^2-\mu^2)^2\right]^{\frac{1}{2}}}{s^2-2(M^2+m^2)s+(M^2-m^2)^2}\sigma_1(s_1)ds_1 \tag{2}$$

where

$$s=(p_1+q_1)^2,\, s_1=(q_2+k_2)^2,\, f^2=\frac{g^2}{4\pi}\left(\frac{\mu}{2M}\right)^2=0.08$$

M is the nucleon mass, μ the π-meson mass and σ_1 the cross section for elastic scattering of the meson on particle $q_1(q_1^2=m^2)$.

Small momentum transfer $|t|\lesssim\mu^2$ is possible at high colliding particle energies ($s\gg M^2$) if s_1 lies in the interval

$$M^2<s_1<s\frac{\mu^2}{M^2}\,. \tag{3}$$

Indeed, t can be expressed through s, s_1 and the angle θ between $\mathbf{p}_1$ and $\mathbf{p}_2$ in the c.m. system:

$$\begin{aligned}t=m^2+s_1-2&\left[\frac{1}{4}s+\frac{1}{2}(s_1-M^2)+\frac{1}{4s}(s_1-M^2)^2\right]^{\frac{1}{2}}\times\\ &\times\left[\frac{1}{4}s+\frac{1}{2}(m^2-M^2)+\frac{1}{4s}(m^2-M^2)^2\right]^{\frac{1}{2}}+2\cos\theta\times\\ &\times\left[\frac{1}{4}s-\frac{1}{2}(s_1+M^2)+\frac{1}{4s}(s_1-M^2)^2\right]^{\frac{1}{2}}\times\\ &\times\left[\frac{1}{4}s-\frac{1}{2}(M^2+m^2)+\frac{1}{4s}(M^2-m^2)^2\right]^{\frac{1}{2}}\end{aligned} \tag{4}$$

For $s\gg M^2$ the condition of smallness of t ($|t|\lesssim\mu^2$) can be fulfilled only for $s_1\ll s$. If, moreover, $s_1\gg M^2$ the foregoing expression for t assumes, after expansion into a series, the following form

$$t=-M^2\frac{s_1}{s}-\frac{1}{4}s\theta^2 \tag{5}$$

from which (3) can be obtained. Formula (3) defines the region in which our pole approximation can be assumed valid.

If $s_1\gg M^2$ the differential cross section (2) can be written in the form

$$d\sigma=\frac{f^2}{\pi}\cdot\frac{2M^2}{\mu^2}\sigma_1(s_1)\frac{s_1ds_1}{s^2}. \tag{6}$$

Assuming σ_1 constant and integrating over the region defined by inequality (3) we obtain the cross section of the process under consideration

$$\sigma = \frac{f^2}{\pi}\frac{\mu^2}{M^2}\sigma_1 \,. \tag{7}$$

A similar result was obtained by Chew and Low but their coefficient was larger than that in (5) by a factor $(M^2/\mu^2)^2$. This is due to the fact that they integrated over s_1 to values $\sim s$ which corresponds to momentum transfers $|t|\sim M^2$. In our opinion this extension of the region is in need of additional support since the rejected (non-polar) terms in the amplitude may play a big role.

We thus see that although numerically the cross section for shower production is not large, it does not diminish with growth of the energy and comprises a finite fraction of the elastic scattering cross section.

3. We shall now consider the energy spectrum of the produced π-meson and its angular distribution. To do this we note that the process $n+n\rightarrow n+n+\pi$ has been reduced by us to elastic π-meson-nucleon scattering. A characteristic feature of elastic scattering at high energies of the π-N system is that most of the elastic scattering events involve small scattering angles to which there correspond momentum transfers $t'=(q_1-q_2)^2$ of the order or smaller than μ^2:

$$|t'| = |(q_1-q_2)|^2 \lesssim \mu^2$$

On the other hand when Eq. (2) is integrated over s_1, large values of s_1 are especially important.

We shall now ascertain the consequences to which the conditions

$$|(q_1-q_2)^2| \lesssim \mu^2; \; s_1 = (q_2+K_2)^2 \gg m^2 \tag{8}$$

lead. In the laboratory system in which $p_1 = 0$ we have

$$t' = 2m^2 - 2q_1q_2 = 2m^2 - 2(E_1E_2 - \mathbf{q}_1\cdot\mathbf{q}_2) \approx$$
$$\approx -\frac{m^2}{E_1E_2}(E_1-E_2)^2 - \theta_{12}^2E_1E_2 \tag{9}$$

In deducing this relation the angle θ_{12} between $\mathbf{q}_1$ and $\mathbf{q}_2$ was considered small, since otherwise the condition of smallness of t' would be violated.

From (8) and (9) it follows that

$$\theta_{12}^2E_1E_2 \lesssim \mu^2; \quad \frac{m^2(E_1-E_2)^2}{E_1E_2} \lesssim \mu^2. \tag{10}$$

Due to the smallness of the recoil nucleon energy ($|t|\sim\mu^2$), $E_1 - E_2$ is practically equal to the energy of the produced π-meson (ω_2).

The second inequality in (10) therefore goes over to the following

$$\frac{m^2\omega_2^2}{E_1(E_1-\omega_2)} \lesssim \mu^2. \tag{11}$$

In effect this means that ω_2 is restricted by the following upper limit:

$$\omega_2 < \frac{\mu}{m}E_1$$

(this result is also apparent from (3)).

Let us now consider the second condition in (8),

$$s_1 = m^2+\mu^2+2q_2K_2 = m^2+\mu^2+2(E_2\omega_2 - \mathbf{q}_2\mathbf{K}_2) \gg m^2 \tag{12}$$

In view of the inequality $|t| = |(p_1-p_2)^2| \lesssim \mu^2$ one finds that in the laboratory system $|p^2| \lesssim \mu$ (since $(p_{10}-p_{20})^2 \lesssim \left(\frac{\mu}{2\mu}\right)^2 \ll \mu^2$). This is the momentum of the intermediate meson. Thus the transverse momentum of the " incident " intermediate meson is of the order or less than μ. By virtue of the properties of elastic scattering the transverse momentum of the " scattered," that is, the produced meson, K_2 should be of the same order of magnitude (for details see below). Thus the K_2 meson practically moves in the direction of q_1 if its energy is great compared with μ. Taking this into account we get

$$s_1 \approx \mu^2+m^2+2\left[E_2\omega_2 - \sqrt{E_2^2-m^2}\sqrt{\omega_2^2-\mu^2}\right.$$
$$\left.\times\left(1-\frac{1}{2}\theta_{K_2q_2}^2\right)\right] \approx m^2+\mu^2\frac{E_1}{\omega_1}+E_2\omega_2\theta_{K_2q_2}^2 \,.$$

Here $\theta_{K_2q_2}$ is the angle between $\mathbf{K}_2$ and $\mathbf{q}_2$. Introducing the two-dimensional angles $\underline{\theta}_{q_1K_2}$ and $\underline{\theta}_{12}$

$$\mathbf{q}_1\mathbf{q}_2 = |q_1||q_2|(1-\tfrac{1}{2}\theta_{12}^2); \; \mathbf{K}_2\mathbf{q}_1 = |K_2|\cdot|q_1|(1-\tfrac{1}{2}\theta_{K_2q_1}^2)$$

we see that due to the first inequality in (10) $\theta_{12} \lesssim \frac{\mu}{\sqrt{E_1E_2}} \approx \frac{\mu}{E_1}$, whereas $\underline{\theta}_{K_2q_1}|K_2|$, being the transverse momentum of the produced meson, satisfies the condition $|K_2|\theta_{K_2q_2} \lesssim \mu$. The angle $\underline{\theta}_{K_2q_2}$ can be determined through angles in the usual fashion

$$\underline{\theta}_{K_2q_2} = \underline{\theta}_{K_2q_1} + \underline{\theta}_{12}\,.$$

Returning to (10), we rewrite it as follows

$$s_1 \approx m^2 + \mu^2\frac{E_1}{\omega_2} + E_1\omega_2(\theta^2_{K_2q_1} + \theta^2_{12} + 2\underline{\theta}_{K_2q_1}\underline{\theta}_{12})\,,$$

$$\theta_{K_2q_1} \lesssim \frac{\mu}{\omega_2};\ \theta_{12} \lesssim \frac{\mu}{E_1};\ \omega_2 < \frac{\mu}{m}E_1\,. \qquad (12a)$$

The only possibility for s to be large is that $\omega_2 \ll E_1$ since the largest contribution from terms containing the angles is made by $E_1\omega_2\theta^2_{K_2q_1} \lesssim \frac{\mu^2 E_1}{\omega_2}$, which in order of magnitude is the same as the term μ^2E_1/ω_2. Thus we have $s_1 \sim \mu^2E_1/\omega_2$. From here and also from formula (4) we obtain the energy distribution of the produced π-mesons in the region of a constant cross section $\sigma_1(s_1) =$ const.:

$$s_1 ds_1 \sim \frac{d\omega_2}{\omega_2^3} \qquad (13)$$

Summing up we arrive at the following conclusions concerning the part of the process under consideration $n+n \to n+n+\pi$:

1. The transverse momenta of all particles at the end of the process are of the order of μ.

2. The energy of the produced π-meson is small compared with that of the incident particle. The energy distribution is given by the formula (13).

It is evident that these properties do not depend on the nature of the incident particle and in equal degree refer to the processes $\pi+n \to \pi+\pi+n$, $K+n \to K+\pi+n$, $Y+n \to Y+\pi+n$.

In principle, a possible way for an experimental investigation of the foregoing processes would consist in singling out those cases in which the energy and angular distribution of the produced particles correspond to elastic scattering of the incident particle by a particle possessing a mass of the order of the π-meson mass.

4. Consider now the transformation of two particles into four $p_1+q_1 \to p_2+K_1+q_2+K_2$. Under the same assumptions as those made above in our analysis

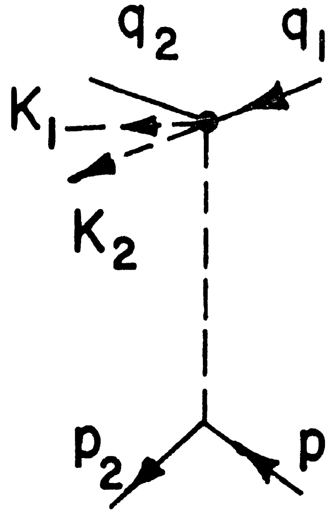

Fig. 2 Feynman diagram for double meson production.

of the transformation of two particles into three, we can single out a certain part of the cross section which is determined by the pole term of the amplitude. There are two types of " pole " processes. First, there is the " single-jet " process depicted in the diagram in Fig. 2. The upper block in this diagram is determined by the amplitude for inelastic transformation of two particles into three. Secondly, there is the " two-jet " process represented by the diagram in Fig. 3. In the pole approximation the amplitude of the two-jet process has the following form

$$A = \frac{A_1A_2}{t-\mu^2}\,, \qquad (14)$$

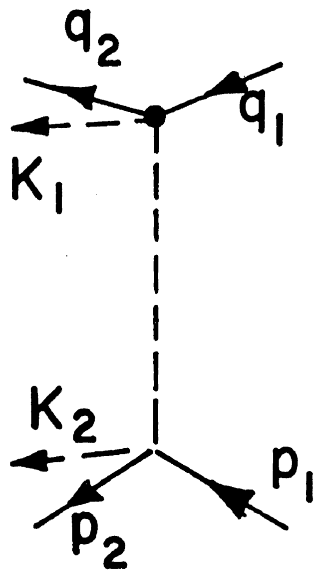

Fig. 3 Feynman diagram for double meson production.

where A_1 and A_2 are amplitudes for the corresponding elastic scattering processes. If $t=(p_1-p_2-K_2)^2$ is small ($|t|<\mu^2$), we may consider, as above, that the cross section is defined by amplitude (14). We thus obtain the following expression for the differential cross section of the process

$$d\sigma = \frac{1}{16\pi^3}\frac{[s_1^2-2(m^2+\mu^2)s_1+(m^2-\mu^2)^2]^{\frac{1}{2}}}{s^2-2(M^2+m^2)s+(M^2-m^2)^2}\times$$
$$\times[s_2^2-2(M^2+\mu^2)s_2+(M^2-\mu^2)^2]^{\frac{1}{2}}\times$$
$$\times\sigma_1(s_1)\sigma_2(s_2)ds_1ds_2\int\frac{dt}{(t-\mu^2)^2}\,, \quad (15)$$

where $s_1=(q_2+K_1)^2$ and $s_2=(p_2+K_2)^2$. As before, we may extend the integration over t to the region $|t|<\mu^2$. In this case the values of s_1 and s_2 must be restricted to the region

$$s_1s_2<\mu^2 s\,. \quad (16)$$

Formula (16) can also be easily deduced in the same way as was (3). To do this it is sufficient when calculating t to replace p_2 by p_2+K_1; that is, to replace M^2 by s_2 in formulas (4) and (5). For $s_1\gg m_2$ and $s_2\gg M^2$ expression (15) assumes the form

$$d\sigma=\frac{1}{16\pi^3}\cdot\frac{1}{\mu^2}\sigma_1(s_1)\sigma_2(s_2)\frac{s_1s_2ds_1ds_2}{s^2}\,. \quad (17)$$

Consider now the angular and energy distribution of the produced particles. We shall characterize our process by the following invariants ;

$$t_1=(q_1-q_2)^2; \qquad s_1=(K_1+q_2)^2$$
$$t_2=(p_1-p_2)^2; \qquad s_2=(K_2+p_2)^2$$
$$t=(q_1-q_2-K_1)^2.$$

Due to the properties of the matrix elements A_1 and A_2 we have

$$|t_1|\lesssim\mu^2;\ |t_2|\lesssim\mu^2$$

The condition $|t_2|\lesssim\mu^2$ yields (in the co-ordinate system in which $\mathbf{p}_1=0$) :

$$|t_2|=2p_1p_2-2M^2=$$
$$=2m(\sqrt{m^2+\mathbf{p}_2^2}-M^2)\approx\mathbf{p}_2^2\lesssim\mu^2,\ |p_2|\lesssim\mu\,. \quad (18)$$

For t_1 we obtain an expression similar to (9) :

$$|t_1|\approx m^2\frac{(E_1-E_2)^2}{E_1E_2}+E_1E_2\theta^2_{q_1q_2}\lesssim\mu^2;\ E_1E_2\theta^2_{q_1q_2}\lesssim\mu^2;$$
$$\frac{(E_1-E_2)^2}{E_1E_2}m^2\cong\frac{(\omega_1+\omega_2)^2m^2}{E_1(E_1-\omega_1-\omega_2)}\lesssim\mu^2;\ \omega_1+\omega_2<\frac{\mu}{m}E_1\,. \quad (19)$$

We see that E_2 is close to E_1 and therefore (18) gives for the transverse momentum $\mathbf{q}_{2\perp}$($\mathbf{q}_{2\perp}$ is perpendicular to $\mathbf{q}_1$) the following estimate

$$q_{2\perp}\approx E\theta_{q_1q_2}\lesssim\mu\,. \quad (20)$$

The pole approximation we are using imposes the following restriction on t

$$|t|\lesssim\mu^2.$$

Introducing the vector $R=K_1+q_2$, $R^2=s_1$ we obtain

$$t=(q_1-R)^2=m^2+s_1-2q_1R=m^2+s_1-2E_1R_0+$$
$$+2\underline{q_1}\underline{R}=-\Delta^2-(\sqrt{E_1^2-m^2}-\sqrt{E_1^2-s_1})^2-$$
$$-[\phi(E_1,E_1)-\phi(E_1,R_0)]\,, \quad (21)$$

where

$$\Delta^2=\sqrt{E_1^2-m^2}[\sqrt{R_0^2-s_1}-\sqrt{R_0^2-s_1-R_\perp^2}]>0\,,$$
$$\phi(E_1,R_0)=\sqrt{E_1^2-m^2}\sqrt{R_0^2-s_1}-E_1R_0\,.$$

The derivative of ϕ with respect to R_0 is

$$\frac{\partial\phi}{\partial R_0}=\frac{R_0\sqrt{E_1^2-m^2}}{\sqrt{R_0^2-s_1}}-E_1>0\,,$$

since $s_1>m$ and $R_0<E_1$.

Thus for $R_0<E_1$, $\phi(E_1,E_1)$ is greater than $\phi(E_1,R_0)$. The signs of all three terms in (21) are therefore the same. From the condition $|t|\lesssim\mu^2$ we can determine the restriction on Δ^2 and therefore also on $R_\perp$:

$$\Delta^2\lesssim\mu^2;\ R_\perp^2\lesssim\mu^2;\ |\mathbf{K}_{1\perp}+\mathbf{q}_{2\perp}|\lesssim\mu\,. \quad (22)$$

Together with (19) this gives

$$|K_{1\perp}|\sim\omega\theta_{K_1q_1}\lesssim\mu\,. \quad (23)$$

The virtual meson K_1 thus possesses a transverse momentum of the order of μ. Finally, it follows (on basis of (18) and (22)) that the transverse momentum of the K_2 meson is also of the order of μ:

$$K_{2\perp} = \omega_2 \theta_{K_2 q_1} \lesssim \mu \, . \qquad (24)$$

Formulas (20), (23) and (24) give the effective angular region in which the particles are emitted. The transverse momentum of *each* particle is of the order of μ. All particles (except the recoil nucleon p_2) move at small angles with respect to the incident particles. The energy distribution of the produced K_1 and K_2 mesons, integrated over their angles, is determined by s_1 and s_2 (σ_1 and σ_2 being constant):

$$s_1 s_2 ds_1 ds_2 \qquad (25)$$

The relation between s_2 and the frequency ω_2 is

$$s_2 = (p_2 + \mathbf{K}_2)^2 = M^2 + \mu^2 + 2E_2\omega_2 - \mathbf{p}_2\mathbf{K}_2 \, .$$

Since according to (18) $|\mathbf{p}_2| \lesssim \mu \ll E_2 \approx m$, this reduces to the simple form:

$$s_2 \approx 2M\omega_2$$

(providing $s_2 \gg M^2$). In analogy with (12*a*), $s_1 = (K_1 + q_1)^2$ can be expressed as

$$s_1 \sim \frac{\mu^2 E_1}{\omega_1} \sim \frac{\mu^2 s}{2m\omega_1} \, .$$

The condition $s_1 s_2 < \mu^2 s$ yields $\omega_2 < \omega_1$.

On basis of (25) we obtain the energy distribution of the produced mesons:

$$\omega_2 d\omega_2 \frac{d\omega_1}{\omega_1^3}$$

$$\mu < \omega_2 < \omega_1 \, ; \quad \mu < \omega_1 < \frac{\mu}{m} E_1$$

(the latter condition is a consequence of (16)).

In calculating (17) we did not take into account the exchange (with respect to Fig. 3) diagram in which the places of meson lines K_1 and K_2 are interchanged. The results obtained above show that one may neglect the interference terms which are due to the presence of pole terms corresponding to the diagram in Fig. 3 and the corresponding pole terms in the diagram where K_1 and K_2 are interchanged.

Indeed, the quantities s_1 and s_2 (which are of the order $\mu^2 E_1/\omega_1$ and $2m\omega_2$) enter amplitude A, which corresponds to the diagram in Fig. 3. Correspondingly s_1' and s_2' (which are of the order $\mu^2 E_1/\omega_2$ and $2m\omega_1$) enter the amplitude $A^{(c)}$ corresponding to the exchange diagram. At high energies each elastic scattering matrix element entering A and $A^{(c)}$ is proportional to its parameter s_1 (the amplitude for elastic scattering of π-mesons on nucleons has the form $s_1 f(t_1)$ and respectively $s_2 f(t_2)$ in the most important diffraction region with $\sigma = \text{const.}$). Therefore A is proportional to $s_1 s_2 \sim \omega_2/\omega_1$ and $A^{(c)}$ is proportional to $s_1' s_2' \sim \omega_1/\omega_2$. Since $\omega_1 \neq \omega_2$, neglection of the interference terms cannot change the order of magnitude of the cross section.

By integrating (17) over region (16), assuming σ_1 and σ_2 constant, we obtain for the cross section

$$\sigma = \frac{\alpha}{16\pi^3} \mu^2 \sigma_1 \sigma_2 \ln \frac{s}{\beta M^2} \, , \qquad (26)$$

where α and β are coefficients of the order of unity which appear as a result of uncertainty in the upper limit of region (16).

Formula (26) illustrates the internal inconsistency of the conception that at high energies the total and elastic cross sections are constant. The assumption of a constant elastic scattering cross section and constant total cross section has led to a logarithmic growth of the cross section for production of two mesons. The conclusions that the assumption regarding the constancy of the elastic scattering cross section is untenable has independently been drawn by V. N. Gribov [3] in a different manner.

5. In a similar way a rough treatment can be made of arbitrary multiple production of particles. The amplitudes of such processes contain pole diagrams of the type shown in Fig. 4, in which an arbitrary number of particles are projected from each vertex. The amplitude

$$A = \frac{A_1 A_2}{t - \mu^2}$$

in which A_1 and A_2 are the amplitudes for inelastic processes occurring in the collision between the intermediate meson and particles q_1 and p_1 corresponds

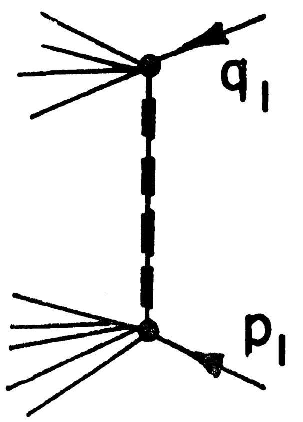

Fig. 4 Feynman diagram for multi-meson production.

to the diagram in Fig. 4. Summing the cross sections corresponding to various inelastic processes we obtain the expression

$$d\sigma(s)\sim\sigma(s_1)\sigma(s_2)s_1s_2\frac{ds_1ds_2}{s^2}, \tag{27}$$

where σ is the total cross section. The same internal contradiction as that in (17) is inherent in formula (27). Integration performed under the assumption that the cross section is constant leads to proportionality of the cross section to $\ln s$. It should be noted however, that formula (27) cannot be regarded as being as definitely established as (17), since besides the diagrams in Fig. 4 a large number of exchange diagrams must be taken into account, and it is difficult to assess the corresponding interference terms and calculate the coefficient in formula (27). An analysis of " multi-jet " processes of the type depicted in Fig. 5 lead to similar inconsistencies, $\sigma\sim(\ln s)^N$, where N is the number of "jets."

If one rejects the assumption that the cross section is constant, one can regard (17) and (27) as integral equations which the cross sections $\sigma(s)$ must obey. They are satisfied if the cross section decreases with increasing energy. A very slow decrease (only somewhat faster than $(\ln s)^{-1}$) should be sufficient. V. N. Gribov arrived at the same conclusion [3]. It may be mentioned that the showers which arise under these conditions possess a specific distribution in s_1 and s_2; most of them have a mass ($\sqrt{s_1}$ or $\sqrt{s_2}$) which is always small and independent of s whereas the other mass is of the order of s. This type of distribution is very different from that predicted by the Fermi-Landau statistical-thermodynamic theory in which each mass is (on the average) proportional to $s^{\frac{1}{4}}$.

6. It is not exactly clear just what conclusions should be drawn from the results presented above. Possibly they merely signify that the approximation made by us is not valid, i.e., we have been wrong in retaining the pole term neglecting the others. However, at present we do not see any reasons for believing this to be true. On the other hand if our approximation is reasonable and it may be considered that nuclear interactions at distances exceeding $1/\mu$ exponentially decrease at high energies as well, we are forced to accept the conclusion that the total and elastic cross sections tend to zero with infinite growth of the energy.

Experimental data available at present seem to indicate that the cross sections are constant within a broad energy interval. This result however does not contradict the conclusion that the cross sections decrease asymptotically. The point here is that the expressions (7) obtained by us contain a small numerical factor. Therefore it may be possible that the effects connected with the processes considered above become noticeable only at ultra-high energies. As mentioned above we were unable to determine the coefficient in (27). The authors must admit that they are surprised at the fact that purely numerical coefficients can lead to the appearance of new asymptotic regions. In order to understand the physics

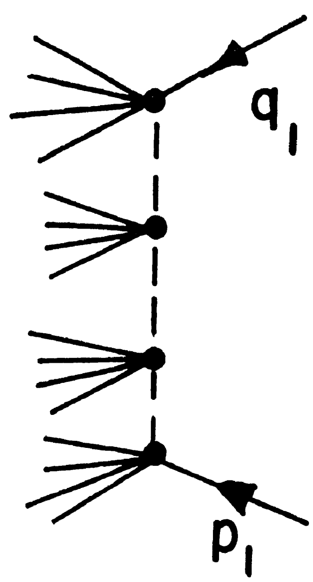

Fig. 5 Feynman diagram for multi-meson production.

of the drop of the cross sections at high energies it would be extremely desirable to visualize (e.g. by diagrams) the mechanism responsible for this phenomenon, provided, of course, that it exists in reality.

In conclusion we wish to express our appreciation to V. N. Gribov, L. D. Landau, L. B. Okun and I. M. Shmushkevich for interesting discussions connected with this work.

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ON THE ASYMPTOTIC BEHAVIOR OF SCATTERING AMPLITUDES AT HIGH ENERGIES

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The asymptotic behavior of scattering amplitudes is investigated at high energies. It is shown that at high energies the usual diffraction pattern of scattering contradicts unitarity conditions and analytic properties of the scattering amplitude formulated with the help of Mandelstam's representations. The most natural behavior in terms of these conditions is a decrease of the cross section faster than $(\ln E)^{-1}$.

1. INTRODUCTION

Asymptotic behavior of scattering amplitudes in quantum field theory has been investigated in a number of works [1-3]. In all cases, however, only very weak restrictions on possible asymptotic behavior have been obtained. At present, mainly owing to Mandelstam's work [4], in studying asymptotic behavior it has become possible to make a more extensive use of the dispersion relations for the momentum transfer, and of unitarity conditions. Since we can, so far, operate only with two-body states under unitarity conditions, evidently we cannot expect a complete solution of the problem. Nevertheless, some limited information can be obtained as will be shown below.

The description of elastic scattering at high energies is based on the so-called diffraction picture. According to this picture, particles with an impact parameter ρ smaller than a certain R (of the order of $1/\mu$; μ is the meson mass) strongly interact with the scatterer and are emitted from the elastic channel, while particles with an essentially larger impact parameter would not be scattered. This results in a diffractional scattering which is characterized by two main features : the total cross-section (σ_T) and the differential cross section of the elastic scattering in a unit interval of the square of the momentum transfer $d\sigma/dt$ are energy-independent ($-t$ is the square of the momentum transfer).