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CORRECTING HIGH-FIELD CLOSED-ORBIT ERRORS IN THE MAIN RING

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Coasting beam can be obtained in the main ring in the presence of large quadrupole-placement and dipole-roll errors by adjustment of the dc steering magnets. When the ring magnets are ramped, however, the errors are manifested in an increasing closed-orbit error indicated by the beam-position detectors. These high-field closed-orbit errors can be corrected by transverse movement of the quadrupoles. Several schemes have been used or suggested for selecting a set of quadrupoles to be moved and calculating the required displacements.[1,2,3] Which method is best depends strongly on the number of quadrupoles and sensors and the betatron oscillation number ν as well as the weight attached to the following criteria:

1. the acceptable number of moves,
2. the acceptable magnitude of a correction,
3. the acceptable residual closed-orbit distortion,
4. the ease of use,
5. the simplicity of the calculation.

Schemes which aim to correct dominant harmonics of the closed-orbit error tend to minimize the size of the moves,[1,2] however, a large number of moves must usually be made. The use of local orbit bumps to cancel errors piecemeal leads to larger corrections and more moves than necessary for some error distributions.

The basis of the technique which is presented here for application to main-ring beam-position data is the least squares minimization of rms orbit error by a repositioning of a selected set of quadrupoles; vertical and radial directions are assumed to be uncoupled. Two closely related selection algorithms have been tried with nearly equal success on test cases. Both algorithms select an n-tuplet of the 240 quadrupoles by augmenting the optimum (n-1)-tuplet of the preceding iteration by selecting one of the 241-n remaining quadrupoles. The selection criterion for the nth quadrupole used at the CERN ISR[2] is to try all n-fold fits to the error remaining from the (n-1)st iteration and select the best. This approach is limited by computing demands to $n \lesssim 10$.[4] A faster variant is to fit the remaining error with a displacement of each of the 241-n quads singly and then do a single n-fold fit with the best single quad and the previous choices taken together. This rule permits the selection of $n \lesssim 100$ quadrupoles. For small n the slower scheme is always at least as good; the faster scheme is less sensitive to local orbit bumps. When n is sufficiently large local bumps will be eliminated by the second scheme also, but one might have found a smaller set using the first scheme.

Both of the above selection schemes are only plausible rules for selecting a subset of quadrupoles for minimization of the rms closed-orbit error. Neither or both may succeed brilliantly for a given set of data. Therefore, both techniques have been embedded in an interactive computer program which allows one to inject insight or intuition in various ways. Generally, the interactive features will be most useful in judging the adequacy of solutions found by the selection algorithms. However, because the entire approach is based on minimization of the rms error rather than the maximum error, intelligent participation in the process can lead to a more economical reduction of maximum error in some cases.

If the center of the j-th quadrupole does not lie on the closed orbit there is a dipole component in the magnetic field

$$(\Delta B)_j = \delta_j \left(\frac{\partial B}{\partial x}\right)_j$$

where δ_j is the distance of the quadrupole center from the equilibrium orbit, B is the component of the magnetic field normal to the plane of the displacement and the displacement is in the $\pm x$-direction. The closed orbit elsewhere can be written as

$$x_i = a_{ij}\delta_j$$

where x_i is the closed orbit measured from the unperturbed equilibrium orbit. The a_{ij} can be derived analytically if $(\Delta B)_j$ is treated as a δ-function field error:[5]

$$a_{ij} = -\frac{k_j\sqrt{B_i\beta_j}}{2}\,(\cos\mu_{ij}\cot\pi\nu+\sin\mu_{ij})$$

where μ_{ij} is the phase advance from the displaced quad to the point i where the closed-orbit distortion is measured, ν is the betatron oscillation number, β_i and β_j are the betatron amplitudes at the measurement point and the displaced quad location respectively, and k_j is the quadrupole strength $\frac{\ell_j}{B\rho}\left(\frac{\partial B}{\partial x}\right)_j$. The a_{ij} calculated from this expression differ from thick lens results calculated by SYNCH[6] by about .5% for main-ring parameters. If one wanted to improve on the a_{ij}, for application in the booster, for example, one need calculate and store only the a_{jj} getting

$$a_{ij} = a_{jj}\sqrt{\beta_i/\beta_j}(\cos\mu_{ij}+\tan\pi\nu\sin\mu_{ij}).$$

The beam sensor by which x_i is measured is assumed to be rigidly attached to the ith quadrupole. For this reason the expressions for a_{ij} above are modified by a term $-\delta_{ij}$ to account for the displacement of the jth sensor.

Because uniform translation of all quadrupoles has no effect on the closed orbit it is clear that one cannot solve directly for Δ in the system of equations

$$X = A\Delta$$

where X is the column vector of the x_i, A is the matrix of the a_{ij}, and Δ is the column vector of the unknown displacements δ_j. However, a least squares fit in terms of selected locations j can be made:

$$A^T X = A^T A\hat{\Delta}$$

where $\hat{\Delta}$ is the vector of least squares fitted δ_j. The manner in which the j-values are selected has already been discussed.

The facilities provided by the current computer code are outlined below. Certain of these features, developed in intermediate stages of the evolution of the code, have been retained because they may be useful in interactive exploration for verification of solutions and do not complicate the routine use of the

1. Input of closed-orbit data. Data from any operative beam sensors may be used. The sensitivity of the solution to sensor noise and number of operative sensors can be estimated by adjustable amounts of additional noise and random sensor failure introduced by the code. The closed orbit corresponding to a given set of quadrupole displacements can be calculated as test data.
2. Tabulation and plotting of closed orbit. Tabulation and printer plots of the closed orbit can be made before, during and after the completed correction.
3. The sum of all quadrupole displacements calculated interactively and by the selection algorithms may be listed in order around the ring and in size order.
4. An arbitrary set of quadrupole displacements may be introduced as an ad hoc correction.
5. An arbitrary set of quadrupoles can be selected for a least squares reduction of the orbit.
6. The range over which squared error is calculated can be restricted to a portion of the ring. The output of all least squares options while such a restriction is in effect is flagged to indicate the special condition.

7. The range of magnets from which candidates will be selected by the selection algorithms can be restricted to a portion of the ring. Output of least squares options constrained in this way is also flagged.
8. The fast or the slow selection algorithm may be employed.
9. The closed orbit can be Fourier analyzed with $0 \leq \phi = \int \frac{ds}{\nu\beta} \leq 2\pi$ as an independent variable. A least squares fitting approach is used so that any number of inoperative sensors are allowable, but the validity of higher harmonics depends strongly on adequate sensor data. Any number of harmonics can be removed from the closed-orbit data.
10. A similar Fourier analysis of quadrupole positions may be performed.

The program has been checked by comparison of predicted closed orbits with those calculated by SYNCH[6] and by verification that orbits produced by known quadrupole displacements give those displacements as a solution. The comparison with SYNCH becomes practically exact as the displacement in SYNCH is taken over just a piece of a quadrupole to represent a δ-function field error. Even with the whole quad displaced the discrepancy is ~1%. When the test case consists of many displaced quads the iteration converges but at rates depending on the particular error pattern and the selection algorithm. An example taken from pre-alignment survey[7] of placement errors in the main ring is given in the table. In this case the fast technique has been quite successful.

Although the program is designed to work with beam-sensor data it may also be used in interpreting survey data. Using the measured errors a closed orbit can be calculated. This procedure automatically filters out the low harmonics of the error by the response curve of the accelerator[8] and therefore requires no arbitrary subtraction procedure on the data except for correction of the closing error. Taking the next step of seeking to improve the machine by correcting the calculated closed orbit by moving quadrupoles without recourse to beam-position data is essentially an act of faith. Because one cannot assume that the magnetic centers of the quads are known better than ±.25 mm in relation to the measured pin positions one can certainly make only larger corrections to quadrupole positions. Therefore, even without survey errors a residual closed-orbit distortion of ~±1 inch must be expected.

The disclaimer above to the contrary notwithstanding, main-ring survey data show position errors so large that the movements of $\lesssim$20 quads by as much as a centimeter seem to be indicated. The safest approach to correcting such a situation would be to do a good positioning according to the surveyed errors and then correct iteratively by beam-sensor readings. If operational considerations prohibit such a systematic approach one can probably make approximate corrections by selecting quads to correct surveyed errors. Whether the resultant correction will be adequate to hold the beam in the chamber at high momentum is questionable.

REFERENCES

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4. Two-hour cpu time on a PDP-10 (KA10).
5. L.C. Teng, private communication.
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7. E. Bleser and J. Schivell, private communication, 11-13-71.
8. See, for example, G. Bingham, TM-200 (1970) pp. 59-60.

TABLE: CORRECTION OF MAIN-RING CLOSED-ORBIT ERROR

Horizontal	Rms(cm)	Maximum(cm)
Before correction	1.40	4.25
After 10 moves (slow selection)	.39	1.57
After 10 moves (fast selection)	.40	1.58
After 32 moves (fast selection)	.21	.70

Vertical		
Before correction	.52	1.64
After 8 moves (slow selection)	.19	.83
After 8 moves (fast selection)	.19	.86
After 32 moves (fast selection)	.09	.36