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Gravitomagnetic Effects in the Kerr-Newman Spacetime

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Abstract

In this work we consider gravitomagnetic effects in the context of the Kerr-Newman solution of the General Relativity theory. Firstly, the gravitoelectric and gravitomagnetic fields are defined with the aid of the expression of the gravitational force, which is a Lorentz-type force. Then, as an application, we study the frame dragging effect, the light deflection and the gravitomagnetic time delay, exhibiting the electric charge contribution in each case and comparing the results obtained with those predicted in the Kerr spacetime.

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1 Introduction

In the context of the General Relativity theory, using the weak field approximation and considering a material source with low rotating motion, we can work in the gravitoelectromagnetic framework, a formal analogy with the electrodynamics. In this analogy, the rotation of a mass creates the gravitomagnetic field, while the rest mass only generates the gravitoelectric field. Consequently, one defines the gravitomagnetic field in the setting of the Kerr metric, which describes the curved spacetime geometry around a rotating mass [12]. Indeed, the rotation of the Earth produces a gravitomagnetic field that causes a precession in gyroscopes orbiting around the planet. This effect, called the Lense-Thirring effect, was verified by the GP-B experiment with an accuracy of 19% [6]. In recent years, several aspects of gravitomagnetism have been studied, taking into account effects produced by the gravitational field of rotating astronomical sources [13, 2, 10].

In this paper, we consider the spacetime of a rotating, electrically charged body, which can represent a Kerr-Newman black hole [1, 11]. Initially, we define the gravitoelectric and gravitomagnetic fields in the Kerr-Newman spacetime, exhibiting their dependence in relation to the electric charge of the source. To this end, we calculate the Lorentz-type gravitational force produced by the central body. Thereafter, we investigate gravitomagnetic effects as frame dragging and gravitomagnetic time delay; also, we examine if the gravitomagnetic field contributes to the light deflection. The results obtained are compared with those predicted in the Kerr solution context.

The gravitoelectric and gravitomagnetic fields are defined in the next section and the expressions are utilized in Section 3 to investigate the frame dragging and light deflection. Then, in Section 4, the study of gravitomagnetic time delay is developed. Finally, Section 5 is devoted to our conclusions.

2 Gravitoelectric and gravitomagnetic fields

Let us consider the Kerr-Newman metric, that describes the gravitational field of a central mass M rotating with angular momentum $\overrightarrow{j}$ and electric charge Q. The line element of Kerr-Newman, taking into account: (i) the weak field approximation conditions

$$\frac{GM}{c^2r'} \ll 1 \quad \text{and} \quad \frac{GQ^2}{c^4r'^2} \ll 1, \tag{1}$$

where G is the gravitational constant and c is the speed of light in free space, and (ii) a localized and slowly rotating source that satisfies the relation

$$\frac{j}{cMr'} \ll 1, \tag{2}$$

is given by [4]

$$\begin{aligned} ds^2 \;&=\; -c^2\left(1-\frac{2GM}{c^2r'}+\frac{GQ^2}{c^4r'^2}\right)dt^2+\left(1+\frac{2GM}{c^2r'}-\frac{GQ^2}{c^4r'^2}\right)dr'^2 \\ &\quad +r'^2(d\theta^2+sen^2\theta d\varphi^2)-\frac{4Gj}{c^3r'^2}\left(1-\frac{Q^2}{2c^2Mr'}\right)sen\theta \\ &\quad \times\left(r'sen\theta d\varphi\right)\left(cdt\right). \end{aligned} \tag{3}$$

It is useful to introduce the transformation $r' = r\left(1 + \frac{GM}{c^2 r} - \frac{GQ^2}{4c^4 r^2}\right)$. Thus, we obtain the expression of equation (3) in isotropic coordinates:

$$\begin{aligned} ds^2 &= -c^2\left(1 - \frac{2GM}{c^2 r} + \frac{GQ^2}{c^4 r^2}\right)dt^2 + \left(1 + \frac{2GM}{c^2 r} - \frac{GQ^2}{2c^4 r^2}\right) \\ &\quad \times[dr^2 + r^2(d\theta^2 + sen^2\theta d\varphi^2)] \; - \frac{4Gj}{c^3 r^2}\left(1 - \frac{Q^2}{2c^2 Mr}\right) sen\theta \\ &\quad \times (rsen\theta d\varphi)(cdt)\,. \end{aligned} \tag{4}$$

In this way, for a Cartesian-like coordinate system $x^\mu = (ct, \overrightarrow{r})$ with $\overrightarrow{r} = (x, y, z)$ and $\mu = 0, 1, 2, 3$, we have the line element

$$\begin{aligned} ds^2 &= -c^2\left(1 - \frac{2GM}{c^2 r} + \frac{GQ^2}{c^4 r^2}\right)dt^2 + \left(1 + \frac{2GM}{c^2 r} - \frac{GQ^2}{2c^4 r^2}\right)\delta_{ij}dx^i dx^j \\ &\quad - \frac{4}{c^2}\left(1 - \frac{Q^2}{2c^2 Mr}\right)\left(\overrightarrow{A}_0 \cdot d\overrightarrow{x}\right)(cdt)\,, \end{aligned} \tag{5}$$

being $\overrightarrow{A}_0 = \frac{G}{cr^3}(\overrightarrow{j} \times \overrightarrow{r})$ and $\overrightarrow{j} = j\widehat{z}$.

The equation (5) can be written as

$$ds^2 = -c^2\left(1 - \frac{2\Phi_1}{c^2}\right)dt^2 + \left(1 + \frac{2\Phi_2}{c^2}\right)\delta_{ij}dx^i dx^j - \frac{4}{c}\left(\overrightarrow{A} \cdot d\overrightarrow{x}\right)dt, \tag{6}$$

where we define

$$\Phi_1 = \frac{GM}{r} - \frac{GQ^2}{2c^2 r^2}, \tag{7}$$

$$\Phi_2 = \frac{GM}{r} - \frac{GQ^2}{4c^2 r^2}, \tag{8}$$

$$\overrightarrow{A} = \left(1 - \frac{Q^2}{2c^2 Mr}\right)\overrightarrow{A}_0. \tag{9}$$

To first order in Φ_1, Φ_2 and $\overrightarrow{A}$, the Lagrangian for the motion of a test particle of mass m is

$$L = -mcds/dt = -mc^2\left(1 - \frac{v^2}{c^2}\right)^{1/2} + m\gamma\Phi_1 + m\gamma\frac{v^2}{c^2}\Phi_2 - 2m\frac{\gamma}{c}\overrightarrow{v} \cdot \overrightarrow{A}, \tag{10}$$

where $\gamma = 1/\sqrt{1 - v^2/c^2}$. Considering that in the presence of a weak gravitational field the particle has a small velocity ($v^2/c^2 \ll 1$) [9], one finds

$$L = -mc^2\left(1 - \frac{v^2}{c^2}\right)^{1/2} + m\Phi_1 - \frac{2m}{c}\overrightarrow{v} \cdot \overrightarrow{A}. \tag{11}$$

This equation is analogous to the electromagnetic case [7], so that the gravitational force $\overrightarrow{F} = d\overrightarrow{p}/dt$, with $\overrightarrow{p} = \gamma m \overrightarrow{v}$, takes the form

$$\overrightarrow{F} = -m\overrightarrow{E} - 2m\frac{\overrightarrow{v}}{c} \times \overrightarrow{B}, \tag{12}$$

where the gravitoelectric field $\overrightarrow{E}$ and the gravitomagnetic field $\overrightarrow{B}$ are expressed by

$$\overrightarrow{E} = -\nabla\Phi_1 = \left(\frac{GM}{r^2} - \frac{GQ^2}{c^2r^3}\right)\widehat{r}, \tag{13}$$

$$\overrightarrow{B} = \nabla \times \overrightarrow{A} = \overrightarrow{B}_0 - \frac{Q^2}{2c^2Mr}\left[\frac{G}{c}\left(\frac{4\widehat{r}(\widehat{r}\cdot\overrightarrow{j}) - 2\overrightarrow{j}}{r^3}\right)\right], \tag{14}$$

being

$$\overrightarrow{B}_0 = \nabla \times \overrightarrow{A}_0 = \frac{G}{cr^3}\left[3\widehat{r}(\widehat{r}\cdot\overrightarrow{j}) - \overrightarrow{j}\right] \tag{15}$$

the usual gravitomagnetic field of a rotating mass [4]. If $\overrightarrow{j} = 0$, we have that the contribution for gravitational effects is only due to the gravitoelectric field, given by (13), corresponding to the Reissner-Nordström spacetime [3]. On the other hand, equation (14) explicitly shows the dependence of the gravitomagnetic field with the electrical charge of the rotating mass.

3 Frame dragging and light deflection

Now, considering (14), we can obtain the following expression to the angular velocity of precession of gyroscopes relative to distant stars [4]

$$\overrightarrow{\Omega} = \frac{\overrightarrow{B}}{c} = \overrightarrow{\Omega}_0 - \frac{Q^2}{2c^2Mr}\left[\frac{G}{c^2}\left(\frac{4\widehat{r}(\widehat{r}\cdot\overrightarrow{j}) - 2\overrightarrow{j}}{r^3}\right)\right], \tag{16}$$

with $\overrightarrow{\Omega}_0 = \dfrac{\overrightarrow{B}_0}{c}$. This precession is equivalent to a dragging of inertial frames caused by the gravitomagnetic field. The result obtained shows the electric charge contribution to the gravitational effect of frame dragging [14].

An expression for the light deflection angle by a Kerr-Newman mass in the equatorial plane, which considers higher order terms of M, $\overrightarrow{j}$ and Q^2, was recently calculated by Chakraborty and Sen [3, equation (34)]. From their formula, we can apply the approximations of weak field and slow rotation of the source. In this case, the deflection angle stays

$$\alpha = \frac{1}{c^2}\left[(3\pi - 4)\,\Phi_1(R) - (3\pi - 8)\Phi_2(R)\right], \tag{17}$$

where $\Phi_1(R)$ and $\Phi_2(R)$ are the gravitoelectric potentials defined by (7) and (8), being R the distance of closest approach. Therefore, taking into account the approximations adopted, the gravitomagnetic field does not influence the light deflection phenomenon.

4 Gravitomagnetic time delay

Let us consider a ray of electromagnetic radiation that propagates from a point $P_1 : (ct_1, \vec{r}_1)$ to a point $P_2 : (ct_2, \vec{r}_2)$ in the spacetime given by (6), where $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and $\eta_{\mu\nu} = diag(-1, 1, 1, 1)$ is the Minkowski metric. The total time spent by the ray in the path is [5]

$$\int_{t_1}^{t_2} dt = \frac{1}{c}\int_{P_1}^{P_2} \mid d\vec{r} \mid + \frac{1}{2c}\int_{P_1}^{P_2} h_{\mu\nu}k^{\mu}k^{\nu}dl, \tag{18}$$

where $k^{\mu} = (1, \hat{k})$, $\hat{k}$ is the constant unit propagation vector of the signal and $dl =\mid d\vec{r} \mid= (\delta_{ij}dx^i dx^j)^{1/2}$ denotes the Euclidean length element along the straight line that joins P_1 to P_2. In turn, the gravitational time delay is defined as

$$\Delta = \frac{1}{2c}\int_{P_1}^{P_2} h_{\mu\nu}k^{\mu}k^{\nu}dl \tag{19}$$

and the gravitomagnetic time delay is expressed by

$$\Delta_B = \frac{1}{c}\int_{P_1}^{P_2} h_{0i}k^0 k^i dl. \tag{20}$$

Using (6), we have

$$\Delta_B = \frac{1}{c}\int_{P_1}^{P_2} \left[-\frac{2}{c^2}\left(1 - \frac{Q^2}{2c^2 Mr}\right)\overrightarrow{A}_0\right]\cdot d\overrightarrow{r}, \tag{21}$$

with $\widehat{k}dl = d\overrightarrow{r}$. One can now write that

$$\Delta_B = -\frac{2}{c^3}\int_{P_1}^{P_2} \overrightarrow{A}_0 \cdot d\overrightarrow{r} + \frac{Q^2}{c^5 M}\int_{P_1}^{P_2} \frac{\overrightarrow{A}_0}{r}\cdot d\overrightarrow{r}. \tag{22}$$

The above equation exhibits explicitly the dependence of the gravitomagnetic time delay with the electric charge. On the other hand, if $Q = 0$ we recover the known expression in the Kerr spacetime [5]. It is interesting to note that the gravitomagnetic time delay could have a noticeable participation in the gravitational lensing delay of extragalactic sources, so that should be considered in the analysis of observational data [8].

5 Conclusion

We study the gravitoelectromagnetic formalism in the context of the Kerr-Newman spacetime. In this sense, the expression of the gravitational force was obtained and, then, we define the gravitomagnetic field, which includes the electric charge contribution for gravitational effects. In sequence, we examine some gravitomagnetic effects, such as frame dragging and gravitomagnetic time delay, exhibiting the electric charge terms in each case. In the particular case of light deflection, we verify that only gravitoelectric potentials affect the deflection angle.

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