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Propagation of Electromagnetic Wave in de Sitter Cosmology

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Abstract. A propagation of the electromagnetic wave in the background of de-Sitter space has been investigated. An exact analytical solutions of Maxwell vacuum equations for the electromagnetic fields and vector potential in de-Sitter spacetime has been presented. It is explicitly shown that the six components of the electromagnetic fields and/or the four components of the vector potential can be expanded in terms of the spherical harmonics with only two radial functions, respectively. It is also shown that the two radial profile functions are satisfied the Schrödinger-like wave equation with Eckart-like potential and/or Legendre equation. The radial dependence of the radial wave function and propagation of the wave has be produced as in 2D and 3D graphs.

Key words: de Sitter spacetime, Maxwell equations, Electromagnetic wave

I. INTRODUCTION

According to the principal of the fundamental of physics, there are four fundamental interactions in nature, namely, electromagnetic, gravitational, weak, and strong interactions. Among them, theory of the electromagnetic interaction is well described and a well-formulated theory. A great deal of attention has been paid to the study of the electromagnetic field theory from both classical and quantum points of view, in the last few centuries. One has to emphasize that the theory of the electromagnetic fields is described by the Maxwell field equations which are the second-order partial differential equations (PDE)s. In most realistic situations, it is sufficiently complicated to obtain an exact analytical solution to Maxwell's equations. Basic idea about the theory of the electromagnetic field can be found in the textbook in electrodynamics and electrostatics [1–13]. The study of electromagnetic fields has occupied an outstanding position in the physical literature because its solution has been of ultimate importance for the development of our understanding of nature. On the other hand, the application of the electromagnetic field is a very wide range. One of the branches of this theory is the study of electromagnetic wave which we are interested in this paper. The frequency range of the electromagnetic waves start from radio wave to gamma ray depending on their energies. Particularly, the light-ray (photon) is also electromagnetic wave in visible band. On the other hand the electric field of the axially-symmetric uniformly charged ring is quite common problem in classical electrostatics.

In general, the electromagnetic waves propagate in four-dimensional space in which spatial coordinates (x, y, z) including time, t and it is governed by Maxwell vacuum equations which are PDE for the six components of the electromagnetic fields $\mathbf{E} = (E_x, E_y, E_z)$ and $\mathbf{B} = (B_x, B_y, B_z)$ or the four components of the vector potential $A_\alpha = (A_t, A_x, A_y, A_z)$. The relations between the electromagnetic fields and vector potential read, $\mathbf{E} = -\nabla A_t + \partial \mathbf{A}/\partial t$ and $\mathbf{B} = \nabla \cdot \mathbf{A}$, where ∇ is the nabla operator. Finally, electromagnetic wave equation can be expressed as [11, 12]

$$\nabla^2 \mathbf{E} - \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 , \qquad \nabla^2 \mathbf{B} - \frac{\partial^2 \mathbf{B}}{\partial t^2} = 0 , \tag{1}$$

or

$$\nabla^2 \mathbf{A} - \frac{\partial^2 \mathbf{A}}{\partial t^2} = 0 , \qquad \nabla^2 A_t - \frac{\partial^2 A_t}{\partial t^2} = 0 , \tag{2}$$

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where nabla square in Cartesian coordinates is defined as

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \, . \tag{3}$$

The curved spacetime due to the presence of source of strong gravity strongly affects the structure and properties of the electromagnetic field. Consequently, the electromagnetic wave propagation becomes sensitive to the background gravity and spacetime curvature. The general Relativity proposed by Albert Einstein in 1916 proposed that the spacetime curvature strongly depends on the gravity. The Maxwell equations, initially prpsed in flat spacetime, can be now modified to the case of curved spacetime background using the definition of covariant derivations. Among the big number of the solutions of the Einstein field equations for the curved spacetime the de-Sitter spacetime attracts special interest of the modern astrophysicists. De-Sitter spacetime is the special cosmological solution of the Einstein equations which contains the non-zero cosmological constant. The current cosmological and astrophysical observations confirm the existence of the dark energy and dark matter in the Universe. Particularly, dark energy is responsible for the repulsive gravitational force dominated in the Universe in cosmological scale. The cosmological constant appearing in the solution of de-Sitter spacetime corresponds to dark energy and investigation of the properties of this solution is very important task of the modern cosmology and astrophysics. In this paper we are interested to consider the background de-Sitter spacetime in order to study the electromagnetic field equation.

In the spherical coordinates $x^\alpha = (t, r, \theta, \phi)$, de-Sitter spacetime can be described as

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta = -f dt^2 + \frac{1}{f} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right) \, , \tag{4}$$

where $f = 1 - r^2/R^2$, R is the cosmological horizon of the spacetime related to the Hubble constant as $R = 1/H$ with $H \simeq 67.8 \mathrm{km/s/Mpc}$.

This research paper is organized as follows. In Sect. II, we study Maxwell field equations in vacuum in the background of de-Sitter space. In Sect. III, we present the general solution of the Maxwell equation for the electromagnetic wave. In Sect. IV, we present the exact analytical solution for radial wave equation. Finally, in Sect. V, we summarize our results and give a future outlook related to the present work.

Throughout the paper we use a space-like signature $(-,+,+,+)$, a system of units in which $G = c = \hbar = 1$ and, we restore these constants when obtaining results are needed to compare with observational data. Greek indices run from 0 to 3 and Latin indices from 1 to 3.

II. MAXWELL VACUUM EQUATIONS

It is well known that when one studies fundamental problems or equations in curved spacetime, it is necessary to include the general relativistic effect and/or the effects arises due to gravitational field. The pairs of Maxwell vacuum equations for the electromagnetic fields $(B^{\hat{r}}, B^{\hat{\theta}}, B^{\hat{\phi}})$ and $(E^{\hat{r}}, E^{\hat{\theta}}, E^{\hat{\phi}})$ in curved space are given as [14]

$$\frac{\partial}{\partial r}\left(\frac{B^{\hat{r}}}{\sqrt{g^{\theta\theta}g^{\phi\phi}}}\right) + \frac{\partial}{\partial \theta}\left(\frac{B^{\hat{\theta}}}{\sqrt{g^{rr}g^{\phi\phi}}}\right) + \frac{\partial}{\partial \phi}\left(\frac{B^{\hat{\phi}}}{\sqrt{g^{rr}g^{\theta\theta}}}\right) = 0 \, , \tag{5}$$

$$\frac{\partial}{\partial t}\left(\frac{B^{\hat{r}}}{\sqrt{g^{\theta\theta}g^{\phi\phi}}}\right) = \frac{\partial}{\partial \phi}\left(\frac{E^{\hat{\theta}}}{\sqrt{-g^{tt}g^{\theta\theta}}}\right) - \frac{\partial}{\partial \theta}\left(\frac{E^{\hat{\phi}}}{\sqrt{-g^{tt}g^{\phi\phi}}}\right) \, , \tag{6}$$

$$\frac{\partial}{\partial t}\left(\frac{B^{\hat{\theta}}}{\sqrt{g^{rr}g^{\phi\phi}}}\right) = \frac{\partial}{\partial r}\left(\frac{E^{\hat{\phi}}}{\sqrt{-g^{tt}g^{\phi\phi}}}\right) - \frac{\partial}{\partial \phi}\left(\frac{E^{\hat{r}}}{\sqrt{-g^{tt}g^{rr}}}\right) \, , \tag{7}$$

$$\frac{\partial}{\partial t}\left(\frac{B^{\hat{\phi}}}{\sqrt{g^{rr}g^{\theta\theta}}}\right) = \frac{\partial}{\partial \theta}\left(\frac{E^{\hat{r}}}{\sqrt{-g^{tt}g^{rr}}}\right) - \frac{\partial}{\partial r}\left(\frac{E^{\hat{\theta}}}{\sqrt{-g^{tt}g^{\theta\theta}}}\right) \, . \tag{8}$$

and

$$\frac{\partial}{\partial r}\left(\frac{E^{\hat{r}}}{\sqrt{g^{\theta\theta}g^{\phi\phi}}}\right)+\frac{\partial}{\partial \theta}\left(\frac{E^{\hat{\theta}}}{\sqrt{g^{rr}g^{\phi\phi}}}\right)+\frac{\partial}{\partial \phi}\left(\frac{E^{\hat{\phi}}}{\sqrt{g^{rr}g^{\theta\theta}}}\right)=0\,, \tag{9}$$

$$\frac{\partial}{\partial t}\left(\frac{E^{\hat{r}}}{\sqrt{g^{\theta\theta}g^{\phi\phi}}}\right)=\frac{\partial}{\partial \theta}\left(\frac{B^{\hat{\phi}}}{\sqrt{-g^{tt}g^{\phi\phi}}}\right)-\frac{\partial}{\partial \phi}\left(\frac{B^{\hat{\theta}}}{\sqrt{-g^{tt}g^{\theta\theta}}}\right)\,, \tag{10}$$

$$\frac{\partial}{\partial t}\left(\frac{E^{\hat{\theta}}}{\sqrt{g^{rr}g^{\phi\phi}}}\right)=\frac{\partial}{\partial \phi}\left(\frac{B^{\hat{r}}}{\sqrt{-g^{tt}g^{rr}}}\right)-\frac{\partial}{\partial r}\left(\frac{B^{\hat{\phi}}}{\sqrt{-g^{tt}g^{\phi\phi}}}\right)\,, \tag{11}$$

$$\frac{\partial}{\partial t}\left(\frac{E^{\hat{\phi}}}{\sqrt{g^{rr}g^{\theta\theta}}}\right)=\frac{\partial}{\partial r}\left(\frac{B^{\hat{\theta}}}{\sqrt{-g^{tt}g^{\theta\theta}}}\right)-\frac{\partial}{\partial \theta}\left(\frac{B^{\hat{r}}}{\sqrt{-g^{tt}g^{rr}}}\right)\,, \tag{12}$$

while in de Sitter spacetime metric (4) the pair of Maxwell equations can rewritten as [14]

$$\frac{\sqrt{f}}{r}\frac{\partial}{\partial r}\left(r^2B^{\hat{r}}\right)+\frac{1}{\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta B^{\hat{\theta}}\right)+\frac{\partial}{\partial\phi}B^{\hat{\phi}}\right]=0\,, \tag{13}$$

$$\frac{\partial}{\partial t}B^{\hat{r}}=\frac{\sqrt{f}}{r\sin\theta}\left[\frac{\partial}{\partial\phi}E^{\hat{\theta}}-\frac{\partial}{\partial\theta}\left(\sin\theta E^{\hat{\phi}}\right)\right]\,, \tag{14}$$

$$\frac{\partial}{\partial t}B^{\hat{\theta}}=\frac{\sqrt{f}}{r\sin\theta}\left[\sin\theta\frac{\partial}{\partial r}\left(r\sqrt{f}E^{\hat{\phi}}\right)-\frac{\partial}{\partial\phi}E^{\hat{r}}\right]\,, \tag{15}$$

$$\frac{\partial}{\partial t}B^{\hat{\phi}}=\frac{\sqrt{f}}{r}\left[\frac{\partial}{\partial\theta}E^{\hat{r}}-\frac{\partial}{\partial r}\left(r\sqrt{f}E^{\hat{\theta}}\right)\right]\,, \tag{16}$$

and

$$\frac{\sqrt{f}}{r}\frac{\partial}{\partial r}\left(r^2E^{\hat{r}}\right)+\frac{1}{\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta E^{\hat{\theta}}\right)+\frac{\partial}{\partial\phi}E^{\hat{\phi}}\right]=0\,, \tag{17}$$

$$\frac{\partial}{\partial t}E^{\hat{r}}=-\frac{\sqrt{f}}{r\sin\theta}\left[\frac{\partial}{\partial\phi}B^{\hat{\theta}}-\frac{\partial}{\partial\theta}\left(\sin\theta B^{\hat{\phi}}\right)\right]\,, \tag{18}$$

$$\frac{\partial}{\partial t}E^{\hat{\theta}}=-\frac{\sqrt{f}}{r\sin\theta}\left[\sin\theta\frac{\partial}{\partial r}\left(r\sqrt{f}B^{\hat{\phi}}\right)-\frac{\partial}{\partial\phi}B^{\hat{r}}\right]\,, \tag{19}$$

$$\frac{\partial}{\partial t}E^{\hat{\phi}}=-\frac{\sqrt{f}}{r}\left[\frac{\partial}{\partial\theta}B^{\hat{r}}-\frac{\partial}{\partial r}\left(r\sqrt{f}B^{\hat{\theta}}\right)\right]\,. \tag{20}$$

It must be pointed out that equations (13)-(20) are the eight coupled partial differential equations (PDEs) for the six variables $E^{\hat{i}}$ and $B^{\hat{i}}$, $(i = r, \theta, \phi)$. It seems to be difficult to get an analytical solutions of these equations for all components of the electromagnetic fields, in particular, when the metric function, f, exists as shown in equations. However, there are several techniques to reduce these equations in the form of usual wave equations. Our analyses show that one can obtain wave equation for, at least, for radial component(s) of the electromagnetic fields.

Indeed, hereafter acting the time derivative operator $\partial/\partial t$ onto both side of equation (14) and using the equations (19), (20), and (13), one get the following second order partial differential equation for the radial magnetic field:

$$\frac{\partial^2}{\partial t^2}B^{\hat{r}}=\frac{f}{r^2}\frac{\partial}{\partial r}\left[f\frac{\partial}{\partial r}\left(r^2B^{\hat{r}}\right)\right]+\frac{f}{r^2}\Delta_\Omega B^{\hat{r}}\,. \tag{21}$$

Similarly, acting again the time derivative operator $\partial/\partial t$ onto both side of the equation (18) and taking into account equations (15), (16), and (17), one can get the second order partial differential equation for radial electric field

$$\frac{\partial^2}{\partial t^2}E^{\hat{r}}=\frac{f}{r^2}\frac{\partial}{\partial r}\left[f\frac{\partial}{\partial r}\left(r^2E^{\hat{r}}\right)\right]+\frac{f}{r^2}\Delta_\Omega E^{\hat{r}}\,, \tag{22}$$

where Δ_Ω is the angular part of Laplacian operator satisfied the following equation

$$\Delta_\Omega Y_{\ell m}\equiv\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]Y_{\ell m}=-\ell(\ell+1)Y_{\ell m}\,, \tag{23}$$

here $Y_{\ell m}(\theta,\phi)$ is the spherical harmonics with given by the following multipole and azimuthal numbers, $\ell = 0,1,2,...$ and $|m| \leq \ell$.

As we can see that equations (21) and (22) are the same differential equations but for the different quantities which means that the solutions of these equations should be very similar. We can immediately guess that the solutions of these equations must be expressed in terms of the spherical harmonics and the general solutions can be expanded in the form of Fourier transform as

$$\{E^{\hat{r}}, B^{\hat{r}}\} = \frac{1}{2\pi}\sum_{\ell m}\int d\omega R_\ell(r,\omega)Y_{\ell m}(\theta,\phi)e^{-i\omega t}\ , \tag{24}$$

where ω is the frequency of the electromagnetic wave and $R_\ell(r,\omega)$ is the radial part of the wave function.

At this point, at least, we have an idea about solutions of the radial equations of the electromagnetic wave. The next task is to find the general solutions of the angular parts of electromagnetic fields. The details of the solutions of the electromagnetic wave equations will be discussed in the next section III.

III. GENERAL SOLUTIONS OF WAVE EQUATIONS

Now, we focus on the solutions of the radial and angular components of the electromagnetic wave. The general solutions of the wave equations for each components of the electromagnetic fields can be given by [14]:

$$B^{\hat{r}}(t,\mathbf{r}) = \frac{1}{2\pi}\sum_{\ell,m}\int\frac{d\omega}{r^2}U_{\ell m}Y_{\ell m}e^{-i\omega t}\ , \tag{25}$$

$$B^{\hat{\theta}}(t,\mathbf{r}) = \frac{1}{2\pi}\sum_{\ell,m}\int\frac{\sqrt{f}d\omega}{r\ell(\ell+1)}\left[\frac{\partial U_{\ell m}}{\partial r}\frac{\partial Y_{\ell m}}{\partial\theta} - \frac{i\omega}{f\sin\theta}V_{\ell m}\frac{\partial Y_{\ell m}}{\partial\phi}\right]e^{-i\omega t}\ , \tag{26}$$

$$B^{\hat{\phi}}(t,\mathbf{r}) = \frac{1}{2\pi}\sum_{\ell,m}\int\frac{\sqrt{f}d\omega}{r\ell(\ell+1)}\left[\frac{\partial U_{\ell m}}{\partial r}\frac{1}{\sin\theta}\frac{\partial Y_{\ell m}}{\partial\phi} + \frac{i\omega}{f}V_{\ell m}\frac{\partial Y_{\ell m}}{\partial\theta}\right]e^{-i\omega t}\ , \tag{27}$$

and

$$E^{\hat{r}}(t,\mathbf{r}) = \frac{1}{2\pi}\sum_{\ell,m}\int\frac{d\omega}{r^2}V_{\ell m}Y_{\ell m}e^{-i\omega t}\ , \tag{28}$$

$$E^{\hat{\theta}}(t,\mathbf{r}) = \frac{1}{2\pi}\sum_{\ell,m}\int\frac{\sqrt{f}d\omega}{r\ell(\ell+1)}\left[\frac{\partial V_{\ell m}}{\partial r}\frac{\partial Y_{\ell m}}{\partial\theta} + \frac{i\omega}{f\sin\theta}U_{\ell m}\frac{\partial Y_{\ell m}}{\partial\phi}\right]e^{-i\omega t}\ , \tag{29}$$

$$E^{\hat{\phi}}(t,\mathbf{r}) = \frac{1}{2\pi}\sum_{\ell,m}\int\frac{\sqrt{f}d\omega}{r\ell(\ell+1)}\left[\frac{\partial V_{\ell m}}{\partial r}\frac{1}{\sin\theta}\frac{\partial Y_{\ell m}}{\partial\phi} - \frac{i\omega}{f}U_{\ell m}\frac{\partial Y_{\ell m}}{\partial\theta}\right]e^{-i\omega t}\ , \tag{30}$$

where $\mathbf{r} \equiv (r,\theta,\phi)$ is the radius vector, $U_{\ell m} = U_{\ell m}(r,\omega)$ and $V_{\ell m} = V_{\ell m}(r,\omega)$ are the radial wave functions, respectively. Notice that the solutions (25)-(30) are fully satisfied Maxwell vacuum equations (13)-(20), and it is enough to find the radial profile functions $U_{\ell m}(r)$ and $V_{\ell m}(r)$ by solving the radial equations.

A. The four-vector potential

Before move on further, it is also interesting to find the expressions for the four-vector potential of the electromagnetic fields, i.e. $A_\alpha = (A_t, A_r, A_\theta, A_\phi)$, it is better to introduce the electromagnetic field tensor, $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$, and on the other hand, the components of the tensor can be expressed as

$$F_{\alpha\beta} = \begin{pmatrix} 0 & -E^{\hat{r}} & -\sqrt{f}rE^{\hat{\theta}} & -\sqrt{f}r\sin\theta E^{\hat{\phi}} \\ E^{\hat{r}} & 0 & \frac{r}{\sqrt{f}}B^{\hat{\phi}} & -\frac{r\sin\theta}{\sqrt{f}}B^{\hat{\theta}} \\ \sqrt{f}rE^{\hat{\theta}} & -\frac{r}{\sqrt{f}}B^{\hat{\phi}} & 0 & r^2\sin\theta B^{\hat{r}} \\ \sqrt{f}r\sin\theta E^{\hat{\phi}} & \frac{r\sin\theta}{\sqrt{f}}B^{\hat{\theta}} & -r^2\sin\theta B^{\hat{r}} & 0 \end{pmatrix}\ , \tag{31}$$

and

$$F_{r\theta} = \frac{\partial}{\partial r}A_\theta - \frac{\partial}{\partial \theta}A_r = \frac{r}{\sqrt{f}}B^{\hat{\phi}} , \tag{32}$$

$$F_{r\phi} = \frac{\partial}{\partial r}A_\phi - \frac{\partial}{\partial \phi}A_r = -\frac{r\sin\theta}{\sqrt{f}}B^{\hat{\theta}} , \tag{33}$$

$$F_{\theta\phi} = \frac{\partial}{\partial \theta}A_\phi - \frac{\partial}{\partial \phi}A_\theta = r^2\sin\theta B^{\hat{r}} , \tag{34}$$

$$F_{rt} = \frac{\partial}{\partial t}A_r - \frac{\partial}{\partial r}A_t = E^{\hat{r}} , \tag{35}$$

$$F_{\theta t} = \frac{\partial}{\partial \theta}A_t - \frac{\partial}{\partial t}A_\theta = \sqrt{f}rE^{\hat{\theta}} , \tag{36}$$

$$F_{\phi t} = \frac{\partial}{\partial \phi}A_t - \frac{\partial}{\partial t}A_\phi = \sqrt{f}r\sin\theta E^{\hat{\phi}} , \tag{37}$$

Using the general solutions (25)-(30) and after performing simple algebraic manipulations, the solution of the vector potential are

$$A_t(t,\mathbf{r}) = -\frac{1}{2\pi}\sum_{\ell,m}\int\frac{d\omega}{\ell(\ell+1)}f\frac{\partial}{\partial r}V_{\ell m}(r)Y_{\ell m}(\theta,\phi)e^{-i\omega t} , \tag{38}$$

$$A_r(t,\mathbf{r}) = -\frac{1}{2\pi}\sum_{\ell,m}\int\frac{d\omega}{\ell(\ell+1)}\frac{i\omega}{f}V_{\ell m}(r)Y_{\ell m}(\theta,\phi)e^{-i\omega t} , \tag{39}$$

$$A_\theta(t,\mathbf{r}) = \frac{1}{2\pi}\sum_{\ell,m}\int\frac{d\omega}{\ell(\ell+1)}U_{\ell m}(r)\frac{1}{\sin\theta}\frac{\partial}{\partial\phi}Y_{\ell m}(\theta,\phi)e^{-i\omega t} , \tag{40}$$

$$A_\phi(t,\mathbf{r}) = -\frac{1}{2\pi}\sum_{\ell,m}\int\frac{d\omega}{\ell(\ell+1)}U_{\ell m}(r)\sin\theta\frac{\partial}{\partial\theta}Y_{\ell m}(\theta,\phi)e^{-i\omega t} . \tag{41}$$

IV. EXACT SOLUTION FOR RADIAL FUNCTION

Here we discuss the exact analytical solution for the radial function. In the previous section, we showed that the general solution of Maxwell equations for the electromagnetic fields can be expressed in term of two radial functions $U_{\ell m}(r)$ and $V_{\ell m}(r)$. Substituting the the general solutions (25)-(30) into wave equations (21) and (22), one can obtain the following the second order ordinary differential equations

$$\left[f\frac{\partial}{\partial r}\left(f\frac{\partial}{\partial r}\right) + \omega^2 - f\frac{\ell(\ell+1)}{r^2}\right]\{U_{\ell m}, V_{\ell m}\} = 0 , \tag{42}$$

Since the equation is the second order differential equation, one can get two solutions with two constants of the integration. As one can see that the equations for functions $U_{\ell m}$ and $V_{\ell m}$ are the same, that is why the solutions will be equivalent to each other and for simplicity we consider the solution for the function $U_{\ell m}$. Before move on further, let us consider wave motion in a flat space. It is easy to check that the solution of the equation (42), in a flat space i.e. $f = 1$ can be expressed in terms of the Bessel functions:

$$U_{\ell m}(r,\omega) = \sqrt{r}\left[c_1 J_{l+\frac{1}{2}}(\omega r) + c_2 Y_{l+\frac{1}{2}}(\omega r)\right] , \tag{43}$$

where c_1, c_2 are the constants of the integration, respectively.

A. Hyperbolic function as solution of wave equation

Hereafter introducing new variable $x = r/R$ and performing simple algebraic manipulations wave equation (42) reduces to

$$(1-x^2)\frac{\partial}{\partial x}\left((1-x^2)\frac{\partial U_{\ell m}}{\partial x}\right) + \omega^2R^2 - (1-x^2)\frac{\ell(\ell+1)}{x^2}U_{\ell m} = 0 , \tag{44}$$

B. Jacobi function as solution of wave equation

Hereafter introducing new variable x,

$$x = \int \frac{d(r/R)}{f} = \int \frac{d(r/R)}{1 - r^2/R^2} = \frac{1}{2} \ln \frac{1 + r/R}{1 - r/R} , \quad \rightarrow \quad r = R \tanh 2x , \tag{45}$$

We obtain the following Schrodinger-like equation

$$\left(\partial_x^2 + \omega^2 - \frac{\ell(\ell+1)}{\sinh^2 2x} \right) \{U_{\ell m}, V_{\ell m}\} = 0 , \tag{46}$$

C. Legendre function as solution of wave equation

Hereafter introducing new variable $y = R/r$, we obtain the associated Legendre equation as

$$\partial_y \left[(1 - y^2) \partial_y U(y) \right] + \ell(\ell+1) U(y) - \frac{\omega^2 R^2}{1 - y^2} U(y) = 0 , \tag{47}$$

The solution is given by $U_{\ell m}(x) = C_{\ell m} Q_\ell^{\omega R}(x)$.

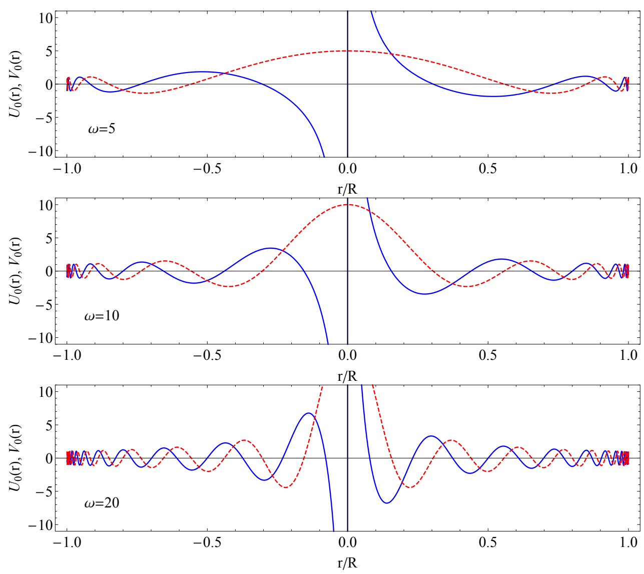

FIG. 1. Radial dependence of the profile functions $U_{\ell m}(r)$ and $V_{\ell m}(r)$.

V. CONCLUSIONS

In the present research paper, we have studied the propagation of the electromagnetic wave on a cosmological scale. We derived the Maxwell vacuum equations for the electromagnetic fields in the background de Sitter spacetime. For simplicity, we investigated the Maxwell vacuum equations for the six components of the electromagnetic field in de Sitter spacetime. Our results can be summarized as follows: The explicit form of the general solution for the electromagnetic wave has been obtained in the de Sitter spacetime. Three different types of solutions for the radial wave equations have been derived. Additionally, the luminosity of the electromagnetic wave has been determined.

These solutions provide important insights into the behavior of electromagnetic waves in an expanding universe. In particular, the varying nature of the electromagnetic field components and their dependence on the cosmological parameters offer potential applications in cosmological observations, especially in understanding the propagation of light from distant sources across an expanding space. Moreover, the different types of solutions obtained may shed light on the complex interactions between electromagnetic radiation and the curved spacetime, with implications for high-energy astrophysics and cosmology.

The study of the luminosity of the electromagnetic wave in the de Sitter background also opens up avenues for exploring the redshift effects and the impact of cosmic expansion on the observed intensity of radiation. Future work could extend these findings by considering more complex spacetime geometries, such as those with matter content, or by applying these results to specific astrophysical scenarios, such as the propagation of radiation from early universe sources like quasars or gamma-ray bursts.

Ultimately, this research contributes to the broader understanding of wave propagation in curved spacetime, providing theoretical groundwork that may support observational cosmology and help in the development of more accurate models of light propagation across cosmological distances.

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