Efficient Floating-point Division Quantum Circuit using Newton-Raphson Division

Gayathri S S, R. Kumar[‡], Samiappan Dhanalakshmi

Department of Electronics and Communication Engineering, College of Engineering and Technology, Faculty of Engineering and Technology, SRM Institute of Science and Technology, SRM Nagar, Kattankulathur 603203

E-mail: gayathrs@srmist.edu.in, kumarr@srmist.edu.in[‡], dhanalas@srmist.edu.in

Abstract. The development of quantum algorithms is facilitated by quantum circuit designs. A floating-point number can represent a wide range of values and is extremely useful in digital signal processing. A quantum circuit model to implement the floating-point division problem using the Newton-Raphson division algorithm is proposed in this paper. The proposed division circuit offers a significant savings in T-gates and qubits used in the circuit design when correlated with the state of art works proposed on fast division algorithms. The qubits savings are estimated around 17% and 20%, T-count savings are around 59.03% and 20.31%. Similarly, T-depth savings is estimated around 77.45% and 24.33% over the existing works.

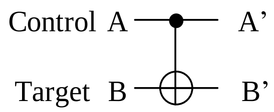
Keywords: Quantum Computing, Clifford+T gates, Floating-point division, T-count, T-depth

1. Introduction

The ability of quantum computing to solve many problems that traditional computing devices cannot handle is demonstrated by Shor's algorithm [1]. Quantum circuits have numerous applications in quantum signal processing [2, 3, 4] quantum optics [5, 6, 7], and quantum thermodynamics [8, 9]. Quantum circuits are reversible by virtue of their equal number of inputs and outputs. Ancillary inputs are said to be excess inputs and excess outputs are said to be garbage outputs [10, 11]. As qubits are highly fragile and unstable, they are prone to noise errors and their states might collapse due to the effect of the noise information [12,13,14]. Researchers prefer to use fault-tolerant quantum circuits and error correction codes to eliminate the effect of error on quantum circuits. Clifford+T quantum gates are a class of fault-tolerant gates used to build quantum circuits that have shown significant resistance to noise sources [15,16]. Table 1 contains a list of Clifford+T gates along with their corresponding symbols. Clifford+T gates do not include the Toffoli (CCNOT) gate, which is essential in the realization of arithmetic circuits. As a result, researchers have concentrated their efforts on realizing the critical CCNOT gate realization using Clifford+T gates [16-19]. T-gate implementation is overpriced when compared to other quantum gates, and it is estimated to be hundreds of times more expensive than the Hadamard gate, which produces superposition on qubits [19].

Table 1. Fault tolerant Quantum Clifford+T gates

S.no	Quantum Gate	Symbol
1	Pauli-X	$\oplus$ (Or) X
2	Hadamard	H
3	T gate	T

4	Conjugate transpose of T or $T^\dagger$	$T^\dagger$
5	Phase or S gate	S
6	Conjugate transpose of S or $S^\dagger$	$S^\dagger$
7	Controlled Not (CNOT) gate	

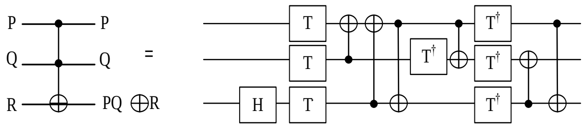


Figure 1. Resource optimized implementation of CCNOT(Toffoli) gate using T gates

Figure 1 shows the T-count optimized realization of Toffoli gate with a T-gate count of 7 and the number of T-gate layers as 3. This decomposition is considered to be an optimized Toffoli implementation for realizing logical operations [20].

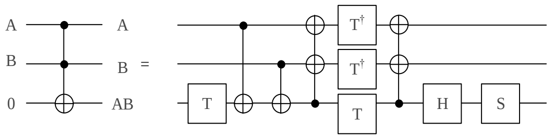


Figure 2. Resource optimized decomposition of CCNOT(Toffoli) gate to implement logical AND operation

Figure 2 depicts Gidney's proposal for a T-count optimized realization of the Toffoli gate with a T-gate count of 4 and a total of 2 T-gate layers [21]. It is well understood that quantum circuits do not produce any garbage outputs. If the circuit is built, it must run backwards in the Uncomputation section after the required outputs have been copied [14,15]. The T-count of the entire quantum circuit is calculated by adding the total number of T-gates used in the computing and uncomputing sections of the quantum circuit. Because three T-gates are eliminated when pairing, the T-count is found to be 8 whenever a CCNOT gate is paired as a combination in the computing and uncomputing section [20,21]. The benefit of Gidney's adder is that the Uncomputation section of the Toffoli operation does not use any T-gate components, so the T-gate count of Gidney's structure does not increase when the garbage outputs.

2. Related works

Quantum circuits for solving integer problems received major interest in the literature [13,17-19,22] whereas, problems that employ floating-point numbers has received less attention in literature [23-25]. Many scientific, information and communication technology, fields rely on floating-point division. In this paper, Newton-Raphson algorithm is employed to construct a division circuit to handle floating-point inputs. The algorithmic procedure to find the quotient of floating-point dividend and divisor is presented in Algorithm 1. The top-level overview of the quantum circuit model built to execute division operations on floating-point inputs is shown in Figure 1. The exponent difference between the dividend and divisor exponents is found using the quantum subtractor block. The division operation is done on the fractional part of the inputs. Finally, a normalization unit is used to normalize the result.

Algorithm 1 Algorithm to perform division on floating-point numbers

Dividend $A_SA_EA_M$ and Divisor $B_SB_EB_M$
Result $R_SR_ER_M$
$R_{S=}A_s \oplus B_s$
$I_E = E_x - E_y$
$I_M = M_x/M_y$
$R_ER_M = Normalize\ (I_E, I_M)$

3. Proposed Quantum Circuit for Division using Newton-Raphson division

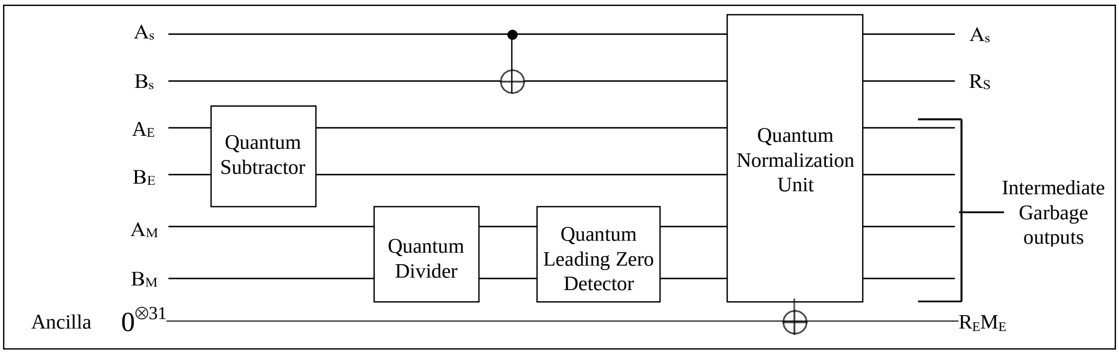


Figure 3. Quantum Circuit to Perform Floating-point division

There are two types of binary division algorithms: slow and fast division methods. SRT division, Restoringand Non-restoring division algorithms are slow division algorithms as they are digit by digit recurrence algorithms. The fast division algorithms are Goldschmidt division and Newton- Raphson division, as their number of iterations is O (log n) [25]. For computing one iterative process, the Newton-Raphson convergence time is much faster. Newton-Raphson division algorithm first finds the reciprocal of the divisor using an initial proximity. The accurate reciprocal of the divisor is calculated by an iterative formula for log n times. Then the reciprocal is multipliedwith the dividend to find the quotient. The algorithm to find the perform division on two integers is shown in Algorithm 2.

Algorithm 2 Newton-Raphson algorithm to compute division

Dividend N_x and Divisor D_x
Quotient Q_r
Assign N =length of input qubits
Assign $X_1 = 1/D_x$
for i = 1 to log N
$X_{i+1} = X_i(2 - D * X_i)$
end
$Q_r = N_x * X_{logN}$

The high-level overview of the quantum division circuit on IEEE-754 standard input is shown in Figure 3. Each iteration of a Newton-Raphson division circuit employs two quantum multipliers and one quantum subtractor unit. The reciprocal approximation is observed withthe help of a lookup table that computes the initial guess, which serves as the seed for the calculation. The iteration begins with a quantum multiplier that calculates $D_X * X_i$ after the

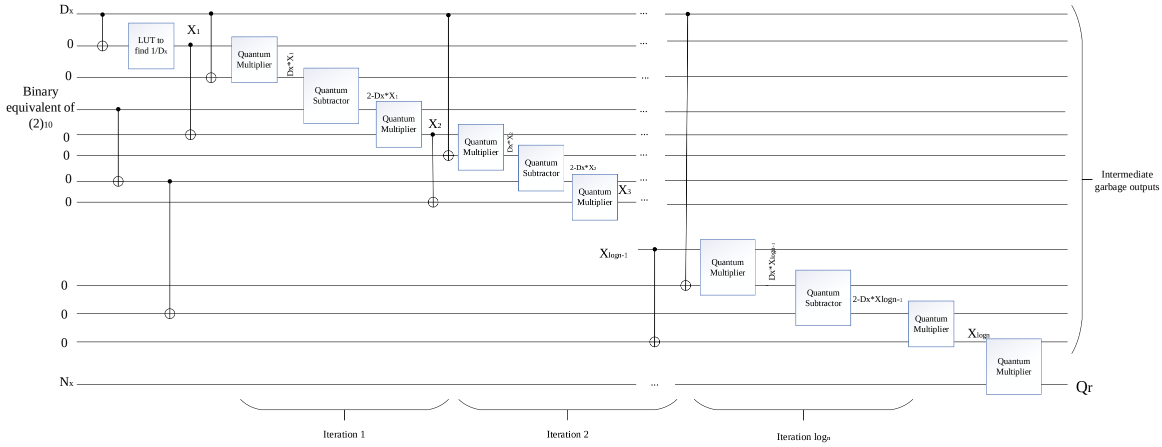


Figure 4. High level overview of integer division quantum circuit using Newton-Raphson algorithm

initial guess is made. In the algorithm, the quantum subtractor unit performs the subtraction process and generates the factor $2\text{-}D_X*X_i$. By locating the next factor X_{i+1} the second quantum multiplier aids in iteration execution. Finally, the quotient is calculated from the result of a multiplier unit wherein, the multiplier calculates the product of X_{logn} with the numerator N_x of the division circuit. The proposed division circuit is constructed using Clifford+T gates applicable for quantum systems.

4. Resource Estimation of Proposed Newton-Raphson Division Circuit

The Newton-Raphson division algorithm is applied on the given floating-point dividend and divisor and the resource utilized for the proposed design is shown in Equation 1. The subtractor and multiplier circuits are crucial components of Newton-Raphson division. The proposed circuitmodel makes use of the multiplier and subtractor designs found in [23]. The authors of the aforementioned work proposed quantum circuit designs for floating-point division circuits using three different approaches. The three approaches include two classes of numerical repeat (i.e. slow) division algorithms and one class of less iteration involved fast solving algorithm, namely the Goldschmidt (GSCH) algorithm. However, the authors did not address the quantum circuit model for the other category of fast division algorithm, Newton-Raphson division algorithm.

$$Resource - count = Resource - count_{11*multiplier+5*Subtractor} \quad (1)$$

Figure 4 shows the Top-level overview of quantum circuit model utilizing Newton-Raphson division algorithm for an input length of 32 qubits floating-point number. From the quantum circuit model, it is clear that after the initial guess is made each iterative process employs a quantum floating-point subtractor and floating-point multiplication circuit. After the final iteration one more multiplicative block is employed to calculate the quotient of the proposed division circuit model. The proposed design utilizes Craig Gidney’s T-count efficient adder [21,25] and the resource estimation is made as shown in Equation 1. The proposed division circuit model is compared with other existing quantum circuit model, which used Goldschmidt division circuits, a type of fast division algorithm [25], and the resource used is shown in Table 2. Table 3 shows that the proposed division circuit uses fewer resources and saves significantly on the qubits, T-depth, and T-count. The qubits savings are estimated around 17%, 20%, T-count savings are around 59.03% and 20.31% , Similarly, T-depth savings is estimated around 77.45% and 24.33% over the existing works on GSCH division algorithm in [25] and [27] respectively.

Table 2. Resource assessment of the proposed Newton-Raphson division circuit

Module	T-depth	Ancilla	T-count
Quantum Vedic multiplier [25]	13032	17856	71424
Subtractor [25]	410	5170	19825
Quantum LZD [25]	46	178	408
Shift circuit [26]	2	620	1687
Exponent adjustment	16	8	32
Copy circuit	Nil	128	Nil
Restoration circuit	Nil	32	Nil
Total	13506	23992	93376

Table 3. Comparison of proposed Newton-Raphson division quantum circuits with the existent designs

Designs	Qubits	T-Count	T-depth
GSCH divider 1 [25]	29074	2,27,920	59,916
GSCH divider 2 [27]	30,008	1,17,187	17850
Proposed Newton-Raphson divider	23996	93376	13506

5. Conclusion

A new design for quantum floating-point division is proposed in this paper using the Newton-Raphson algorithm. The proposed quantum circuit saves more T-counts than the existing work. The proposed design may be better suited for designing any quantum algorithm that employs floating-point division, where the main solicitude is expected to be lower T-cost.

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