Renyi entropy analysis of compound-hadrons in hadron-nucleus interaction

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Entropy is important characteristic of systems having several degrees of freedom. It seems quite natural to use it in description of high energy multiparticle production processes. In particular, entropy of multiplicity distribution is an effective variable characterizing inelastic collisions with many particles produced.

A convenient means to study the energy dependence of multiplicity in an integrated form is provided by the so-called information-entropy. Information entropy of charged particle multiplicity distribution P_n is defined as [1, 2]

$$S = -\sum_n P_n \ln P_n$$

Here, P_n is the probability of having "*n*" produced particles in the final state such that, $\sum_n P_n$ = 1. Information entropy is a measure of the uncertainty associated with a multiplicity distribution. A wide distribution gives more uncertainty and a larger value of S than a sharply peaked one.

Apart from information entropy there is also Renyi entropy [3]. It can play an important role in investigating fractal properties of multiparticle production process. This method can be applied to events having higher as well as lower multiplicity.

It is generally believed that medium energy knocked out protons, which manifest themselves as grey particles in nuclear emulsion, are supposed to carry relevant information about the hadronization mechanism, since the time scale of emission of these particles is of the same order as that of the produced shower particles (pions). One may consider pions and medium energy protons together in equal footings without making any distinction between them and termed them as "compound hadrons". Therefore if one combines the number of grey (n_g) and shower particles (n_s) per event in a collision as a new parameter, named "compound multiplicity" ($n_c = n_g + n_s$), it could also play an important role in understanding the reaction dynamics in high energy nuclear interactions.

Here the fractal properties have been studied using the concept of Renyi entropy for the produced compound hadrons and pions separately for $\pi^- - AgBr$ interactions at 350 GeV in $(\cos\theta)$ phase space.

In terms of the probability of multiplicity distribution P_n the q th order Renyi entropy can be defined as

$$H_q = \frac{1}{(q-1)} \ln\left[\sum_n (P_n)^q\right]$$

Generalised fractal dimension D_q can be obtained from the relation

$$D_q = \frac{H_q}{Y_m}$$

where Y_m is the central rapidity value in the centre of mass frame and is given by

$Y_m = \ln(\sqrt{s}$ - 2 m_n <n_p> / m_π) . Here m_π is the pion rest mass, and $\sqrt{s}$ is the centre of mass energy of the collision process, <n_p> denotes the average number of participating nucleons.

D_q is related to anomalous fractal dimension d_q by the relation $d_q = 1 - D_q$

Renyi entropy and its dependence on multiplicity and incident energy have been studied by very few groups to understand the mechanism of particle production.

Here we have calculated the Renyi entropy values of order q=2 to 5 for the present interaction. The calculated values are tabulated in the Table 1. It is seen from the table that Renyi entropy values are found to decrease with order number q. Then we have calculated the values of generalized fractal dimension D_q and listed in Table 1. Table 1 shows that D_q decreases with increasing q. It exhibits multifractality for production of compound hadrons.

Using the values of D_q anomalous fractal dimension d_q and hence the ratio d_q / d_2 has been calculated and shown in Table 1.

The spiky structure of density distribution of produced particles can also be investigated with the help of a set of branching parameters. The higher order branching parameters can be expressed in terms of lower order parameters resulting in a linear expression for d_q.

$$\frac{d_q}{d_2} = (1 - r) + \frac{qr}{2}$$

A non-zero value of the r implies the multifractal behavior. The variation of d_q / d_2 with q has been done and from the slope parameter calculated value of r has been listed in Table 1. The value of r signifies the degree of multifractality. For our interaction the value of r is greater than zero. This reveals the multifractal nature of multiparticle production mechanism. The whole analysis is repeated for pions also and the corresponding results are tabulated in Table 1 for the sake of comparison.

Degree of multifractality is found to be higher for compound hadrons than pions revealing that compound hadrons play an important role in multiparticle production mechanism. The r value characterizing the degree of multifractality is also greater for compound hadrons than pions.

Table 1: Values of different parameters

Particle	Order	Hq	Dq	dq / d2	r
Compound hadrons	2	2.544 ± 0.044	0.654 ±0.087	1.00 ±0.04	0.128 ±0.004
	3	2.452 ±0.042	0.631 ±0.086	1.067 ±0.04	
	4	2.391 ±0.038	0.614 ±0.079	1.116 ±0.03	
	5	2.282 ±0.041	0.586 ±0.085	1.197 ±0.04	
Pions	2	2.132 ±0.051	0.548 ±0.095	1.00 ±0.05	0.104 ±0.002
	3	2.057 ±0.044	0.529 ±0.086	1.042 ±0.04	
	4	1.946 ±0.041	0.501 ±0.084	1.104 ±0.04	
	5	1.876 ±0.041	0.482 ±0.084	1.146 ±0.04	

References

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