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Topological gravity and Chern–Simons forms in $d = 4$

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ABSTRACT

Following the construction introduced by Antoniadis and Savvidy in Refs. [1–3], we study metric-independent topological invariants on a $(2n+1)$-dimensional space–time. These invariants allow us to show that Chamseddine's even-dimensional topological gravity corresponds to a Chern–Simons–Antoniadis–Savvidy form. Starting from this result, more general four-dimensional topological gravity actions are explicitly constructed.

1. Introduction

In Refs. [4–6] A.H. Chamseddine constructed topological actions for gravity in all dimensions. He found in the odd-dimensional case the action given by $S^{(2n+1)} = k\int_{M_{2n+1}} \mathcal{L}^{(2n+1)}_{\text{ChS}}(A)$ where $\mathcal{L}^{(2n+1)}_{\text{ChS}}(A)$ corresponds to a $(2n+1)$-Chern–Simons form. All the dynamical fields are components of an algebra-valued, one-form gauge connection $A = A^A{}_\mu T_A \otimes \mathrm{d}x^\mu$, and the $(2n+1)$-Lagrangian form is given by [7]

$$\mathcal{L}^{(2n+1)}_{\text{ChS}}(A) = (n+1)\int_0^1 \mathrm{d}t \left\langle A\left(t\mathrm{d}A + t^2A^2\right)^n\right\rangle. \tag{1}$$

Under off-shell gauge transformations, $\mathcal{L}^{(2n+1)}_{\text{ChS}}(A)$ only changes by a closed form, and therefore the theory is described as 'quasi-invariant' in the literature. Perhaps the best-known example of this kind of theories is three-dimensional gravity,

$$\mathcal{L}^{(3)}_{\text{G}}(A) = \frac{1}{2}\epsilon_{abc}\left(R^{ab} + \frac{2}{3\ell^2}e^a e^b\right)e^c, \tag{2}$$

because of the famous quantization of the system due to Witten [8,9]. However, the construction can be performed in every odd dimension, and to be extended to the case of superalgebras. In higher odd dimensions, the theory has a very complex dynamics, with propagating degrees of freedom (although the proof of renormalizability hasn't been extended to this case). In the last decades, these kinds of systems have been thoroughly studied; see for instance Refs. [10–15] and a comprehensive review in Ref. [16].

In the even-dimensional case, a similar construction using as only field a 1-form gauge connection is not possible. As a matter of fact it is necessary to use in addition to the gauge field A, at least a 0-form multiplet ϕ in the fundamental representation of the gauge group. It is because the n-product of the field strength $\langle F^n\rangle$ is a group invariant $2n$-form, but also it is a topological invariant density which doesn't provide equations of motion. But when the scalar field ϕ^a in the fundamental representation is included, it is possible to construct a $2n$-dimensional action as

$$S^{(2n)}[A,\phi] = k\int_{M_{2n}} \epsilon_{a_1\ldots a_{2n+1}}\phi^{a_1}F^{a_2a_3}\cdots F^{a_{2n}a_{2n+1}}, \tag{3}$$

where $F = \mathrm{d}A + AA$. This action (even-dimensional topological gravity) was obtained by Chamseddine in [4] from an odd-dimensional Chern–Simons Lagrangian using a dimensional reduction method. This kind of action principles have attracted some attention recently. They can provide interesting cosmological dynamics, with non-vanishing torsion (see Ref. [17]).

Besides Chamseddine's dimensional reduction, topological gravity has other deep links with Chern–Simons forms. For instance, in Ref. [18] it was found that even-dimensional topological gravity action arises from odd-dimensional Chern–Simons gravity using non-linear realizations of the Poincaré group $ISO(d-1,1)$. The field ϕ^a was identified with the coset field associated with the non-linear realizations of the group.

Further explorations were developed in Ref. [19]. There it was shown that even-dimensional topological gravity actions, invariant under the Poincaré group, correspond (up to a multiplicative constant) to a gauged Wess–Zumino–Witten term (see also Ref. [20]).

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In the current article, we will analyze the relationship between Chamseddine's even-dimensional topological gravity and a particular case of Chern–Simons structures developed by Antoniadis and Savvidy in Refs. [1,2].

The plan of the article is as follows. In Section 2, a short review on Chern–Simons–Antoniadis–Savvidy forms is presented. In Section 3 the relationship between this construction and Chamseddine's even-dimensional topological gravity is presented. In Section 4 some examples of the construction are studied for the case of *AdS*, conformal and $\mathfrak{so}(D-1,1)\oplus\mathfrak{so}(D-1,2)$ algebras [21,22].

2. The Chern–Simons–Antoniadis–Savvidy forms

In this section, we briefly review some examples of the Antoniadis–Savvidy generalization of Chern–Simons polynomials including algebra-valued, higher-order gauge fields.

In order to clarify ideas, let us consider the gauge invariant $(2n+3)$-form given by

$$\Gamma_{2n+3} = \langle F^n H\rangle, \tag{4}$$

where $H = \mathrm{d}B + [A, B]$ is the 3-form 'field-strength' tensor for the generalized gauge field 2-form B. By direct computation of the derivative we can see that Γ_{2n+3} is a closed form: $\mathrm{d}\Gamma_{2n+3} = 0$. According to the Poincaré lemma, this implies that Γ_{2n+3} can be locally written as an exterior differential of a certain $(2n+2)$-form. This potential $(2n+2)$-form is given by [2], (see also [23])

$$\Gamma_{2n+3} = \langle F^n H\rangle = \mathrm{d}\mathfrak{C}^{(2n+2)}_{\text{ChSAS}}, \tag{5}$$

where the $(2n+2)$-form $\mathfrak{C}^{(2n+2)}_{\text{ChSAS}}$ is given explicitly by

$$\mathfrak{C}^{(2n+2)}_{\text{ChSAS}}(A,B) = \int_0^1 \mathrm{d}t\langle AF_t^{n-1}H_t + \cdots + F_t^{n-1}AH_t + F_t^n B\rangle. \tag{6}$$

From eq. (6), we have for the case $n=1$

$$\mathfrak{C}^{(4)}_{\text{ChSAS}} = \int_0^1 \mathrm{d}t\langle AF_t + F_t B\rangle = \langle FB\rangle. \tag{7}$$

It seems remarkably similar to the original expression for Chamseddine topological gravity, although in the eq. (7) case B is a 2-form. This is not an accident; in Section 3 it will be shown that topological gravity [4] does correspond to a $2n$-dimensional Chern–Simons–Antoniadis–Savvidy (ChSAS) form.

Depending on the dimensionality, it is possible to construct other invariants, with a richer structure. For instance, in Refs. [3] and [24] the invariants Γ_{2n+6} and Γ_{2n+8} are constructed as

$$\Gamma_{2n+6} = \langle F^n H_6\rangle + n\langle F^{n-1}H_4^2\rangle = \mathrm{d}\mathfrak{C}^{2n+5}_{\text{ChSAS}},$$

and

$$\Gamma_{2n+8} = \langle F^n H_8\rangle + 3n\langle F^{n-1}H_4H_6\rangle + n(n-1)\langle F^{n-2}H_4^3\rangle = \mathrm{d}\mathfrak{C}^{2n+7}_{\text{ChSAS}},$$

respectively.

3. Topological gravity and *ChSAS* invariants

In this section we analyze the case of 0-forms in the context of the Antoniadis–Savvidy construction and its relationship with Chamseddine's even-dimensional topological gravity.

Following the same procedure from Refs. [2,3,24], it is possible to construct the invariant $(2n+1)$-form density

$$\Gamma_{2n+1} = \langle F^n H_1\rangle, \tag{8}$$

where $H_1 = \mathrm{D}\phi = \mathrm{d}\phi + [A,\phi]$ is a 1-form 'field-strength' tensor for the 0-form 'generalized gauge field' ϕ. Since Γ_{2n+1} is a closed form, $\mathrm{d}\Gamma_{2n+1} = 0$, from the Poincaré lemma we know that locally must exist a potential $2n$-form

$$\Gamma_{2n+1} = \langle F^n H_1\rangle = \mathrm{d}\mathfrak{C}^{(2n)}_{\text{ChSAS}}. \tag{9}$$

The 2-form and 1-form 'curvatures' satisfy the Bianchi identities,

$$\mathrm{D}F = 0, \quad \mathrm{D}H + [\phi, F] = 0, \tag{10}$$

and under gauge transformations behave as

$$\delta F = \mathrm{D}(\delta A), \quad \delta H_1 = \mathrm{D}(\delta\phi) + [\delta A, \phi]. \tag{11}$$

Using (11) we find

$$\delta\Gamma_{2n+1} = \mathrm{d}\langle\delta AF^{n-1}H_1 + \cdots + F^{n-1}\delta AH_1 + F^n\delta\phi\rangle. \tag{12}$$

Following Ref. [25], we introduce a one-parameter family of potentials and strengths through the parameter t, $0\le t\le 1$, $A_t = tA$, $F_t = tF + (t^2-t)A^2$, $\phi_t = t\phi$, $H_{1t} = tH_1 + (t^2-t)[A,\phi]$, and from eq. (12), we find

$$\delta\langle F_t^n H_{1t}\rangle = \mathrm{d}\langle\delta A_t F_t^{n-1}H_{1t} + \cdots + F_t^{n-1}\delta A_t H_{1t} + F_t^n\delta\phi_t\rangle. \tag{13}$$

Since $\delta = (\partial/\partial t)\delta t$ and $\delta A_t = \delta t A$, $\delta\phi_t = \delta t\phi$ we get

$$\Gamma_{2n+1} = \langle F^n H_1\rangle = \mathrm{d}\mathfrak{C}^{2n}_{\text{ChSAS}},$$

where the $2n$-form $\mathfrak{C}^{2n}_{\text{ChSAS}}$, is a Chern–Simons–Antoniadis–Savvidy form given by

$$\begin{aligned}\mathfrak{C}^{(2n)}_{\text{ChSAS}}(A,\phi) &= \int_0^1 \mathrm{d}t\langle AF_t^{n-1}H_{1t} + \cdots + F_t^{n-1}AH_{1t} + F_t^n\phi\rangle \\ &= \langle F^n\phi\rangle.\end{aligned} \tag{14}$$

In Refs. [4–6] A.H. Chamseddine found that to construct even-dimensional topological actions for gravity it is necessary to use, in addition to the gauge fields, a scalar multiplet ϕ^a in the fundamental representation of the gauge group and that for even-dimensional spaces there is no natural geometric candidate such as the Chern–Simons form. The gauge group is $ISO(2n-1,1)$ or $SO(2n,1)$ or $SO(2n-1,2)$. To form a group invariant $2n$-form, the n-product of the field strength is not enough, but will require in addition a scalar field ϕ^a in the fundamental representation.

In Ref. [18] it was shown that the action (3) can be obtained from the $(2n+1)$-dimensional Chern–Simons gravity genuinely invariant under the Poincaré group with suitable boundary conditions, and in Ref. [19] was found that it correspond (up to a multiplicative constant) to a gauged Wess–Zumino–Witten term for the Poincaré group case.

From Eq. (14) we can see that the Chern–Simons–Antoniadis–Savvidy form $\mathfrak{C}^{(2n)}_{\text{ChSAS}}$ permit to construct even-dimensional topological actions for gravity for the $ISO(2n-1,1)$ or $SO(2n,1)$ or $SO(2n-1,2)$ gauge groups as well as another gauge groups such as the $SO(D-1,1)\oplus SO(D-1,2)$ group from Ref. [21]. For example, when the gauge group is the AdS group, the form $\mathfrak{C}^{(2n)}_{\text{ChSAS}}$ does corresponds to

$$\langle F^n\phi\rangle = \varepsilon_{a_1a_2\cdots a_{2n+1}}F^{a_1a_2}\cdots F^{a_{2n-1}a_{2n}}\phi^{a_{2n+1}}, \tag{15}$$

which coincides with Chamseddine's Lagrangian for even-dimensional topological gravity from Ref. [4].

4. Topological gravity as four-dimensional Chern–Simons–Antoniadis–Savvidy form

In this Section we construct explicitly the four-dimensional actions for topological gravity for the *AdS* group, conformal group and $SO(D-1,1)\oplus SO(D-1,2)$ group.

4.1. Topological gravity for AdS algebra

The conmutation relations of the five-dimensional *AdS* algebra

$$[J_{ab}, J_{cd}] = \eta_{bc}J_{ad} + \eta_{ad}J_{bc} - \eta_{ac}J_{bd} - \eta_{bd}J_{ac},$$
$$[J_{ab}, P_c] = \eta_{bc}P_a - \eta_{ac}P_b,$$
$$[P_a, P_b] = J_{ab},$$

where

$$\eta_{ab} = \mathrm{diag}\,(-,+,+,+,+)\,.$$

The connection A and the scalar field ϕ valued in the $SO(4,2)$ algebra are given by

$$A = A^A_\mu T_A dx^\mu = e + \omega = \frac{1}{\ell}e^a P_a + \frac{1}{2}\omega^{ab}J_{ab},$$
$$\phi = \phi^A T_A = \varphi + \beta = \frac{1}{\ell}\varphi^a P_a + \frac{1}{2}\beta^{ab}J_{ab}.$$

From eq. (14), we have that $\mathfrak{C}^{(4)}_{\mathrm{ChSAS}}$ for the *AdS* algebra is given by

$$S^{(4)}_{\mathrm{ChSAS}} = \int_{\mathcal{M}^4} \langle F^2\phi\rangle = \int_{\mathcal{M}^4} \varepsilon_{abcde}F^{ab}F^{cd}\phi^e, \tag{16}$$

which coincides with the four-dimensional action for the four-dimensional topological gravity found by Chamseddine in Ref. [4].

Since in this case the curvature can be write as

$$F = dA + A^2 = T + \mathcal{R} = \frac{1}{\ell}T^a P_a + \frac{1}{2}\left(R^{ab} + \frac{1}{\ell^2}e^a e^b\right)J_{ab},$$

we have that the Chern–Simons–Antoniadis–Savvidy form is given by $\mathfrak{C}^4_{\mathrm{ChSAS}} = \langle (T+\mathcal{R})(T+\mathcal{R})(\varphi+\beta)\rangle$, so that the corresponding Chern–Simons–Antoniadis–Savvidy action can be written explicitly as

$$S^{(4)}_{\mathrm{ChSAS}} = \frac{\kappa}{4\ell}\int \varepsilon_{abcde}\left(R^{ab}R^{cd} + \frac{2}{\ell^2}R^{ab}e^c e^d + \frac{1}{\ell^4}e^a e^b e^c e^d\right)\varphi^e, \tag{17}$$

where, for simplicity we have considered the case $T^a = 0$. This result is valid for a four-dimensional manifold which correspond to the boundary of the five-dimensional manifold where the invariant Γ_5 is defined.

The fields e^a and ω^{ab}, with $a, b = 0, 1, 2, 3, 4$, in (17) are one-forms pulled baked in 4 dimensions. Using the decomposition $e^a = (e^i, e^4)$, $\omega^{ab} = (\omega^{ij}, \omega^{i4} = \lambda e^i)$ with $i = 0, 1, 2, 3$, the curvatures R^{ab} and T^a can be written as $R^{ij} = \hat{R}^{ij} - \lambda^2 e^i e^j$, $T^i = \hat{T}^i + \lambda e^i e^4$, $R^{i4} = \lambda\left(de^i + \omega^i{}_j e^j\right) = \lambda\hat{T}^i$, $T^4 = de^4 + \lambda e_i e^i$, with $\hat{R}^{ij} = d\omega^{ij} + \omega^i{}_k\omega^{kj}$, $\hat{T}^i = de^i + \omega^i{}_j e^j$ and we find

$$S^{(4)}_{\mathrm{ChSAS}} = \int\left(\mathcal{L}_{\varphi^4} + \mathcal{L}_{\varphi^i}\right),$$

where

$$\mathcal{L}_{\varphi^4} = \frac{\kappa}{4\ell}\varepsilon_{ijkm}\left(\hat{R}^{ij}\hat{R}^{km} + \alpha\hat{R}^{ij}e^k e^m + \beta e^i e^j e^k e^m\right)\varphi,$$
$$\mathcal{L}_{\varphi^i} = \frac{\kappa\left(\lambda^2 - 1\right)}{\lambda\ell}\varepsilon_{ijkm}\left(\hat{R}^{ij} + \gamma e^i e^j\right)\hat{T}^k\varphi^m, \tag{18}$$

with $\alpha = \frac{2}{l^2} - 2\lambda^2$, $\beta = \frac{1}{l^4} - \frac{2\lambda^2}{l^2}$, $\varphi \equiv \varphi^4$, $\gamma = \frac{1}{l^2} - \lambda^2$ and where we have used the condition $T^a = 0$. From this Lagrangian it is apparent that neither the $l \longrightarrow \infty$ nor the $l \longrightarrow 0$ limit yields the Brans–Dicke term alone [26,27]. Rescaling κ properly, those limits will lead either to Gauss–Bonnet like term or to the cosmological like constant term by itself respectively (with $T^i = 0$).

4.2. Topological gravity for conformal algebra

Let us consider the algebra $\mathrm{Conf}_4 = \mathfrak{so}(4,2)$ expressed in terms of the canonical base

$$[M_{ab}, M_{cd}] = \eta_{bc}M_{ad} - \eta_{bd}M_{ac} - \eta_{ac}M_{bd} + \eta_{ad}M_{bc},$$

with the range of indexes $a, b, c, d, \ldots = 0, 1, 2, 3, 4, 5$ and a Minkowskian metric left invariant for the group

$$\eta_{ab} = \mathrm{diag}\,(-1,+1,+1,+1,+1,-1)\,.$$

One of the symmetric invariant tensors of rank three of this algebra is provided by

$$\langle M_{ab}M_{cd}M_{ef}\rangle = \varepsilon_{abcdef}\,.$$

In order to separate pieces with different physical meaning from a four-dimensional point of view, let us perform the standard change of basis defined trough

$$J_{ij} = M_{ij},$$
$$K_i = M_{i4} + M_{i5},$$
$$P_i = M_{i4} - M_{i5},$$
$$D = M_{45},$$

with the range of indexes $i, j, k \ldots = 0, 1, 2, 3$

$$A = aD + \frac{1}{\ell}h^i K_i + \frac{1}{\ell}e^i P_i + \frac{1}{2}\omega^{ij}h_{ij},$$
$$\phi = \frac{1}{\ell}\varphi D + v^i K_i + u^i P_i + \frac{1}{2}\beta^{ij}h_{ij}.$$

So that when the gauge group is the conformal group, the Chern–Simons–Antoniadis–Savvidy action is given by

$$S^{(4)}_{\mathrm{ChSAS}} = \int\left(\mathcal{L}_\varphi + \mathcal{L}_v + \mathcal{L}_\beta + \mathcal{L}_u\right),$$

with

$$\mathcal{L}_\varphi = \varepsilon_{ijkm}\left(\frac{1}{4\ell}\hat{R}^{ij}\hat{R}^{km} + \frac{2}{\ell^3}\hat{R}^{ij}h^k e^m + \frac{4}{\ell^5}h^i h^j e^k e^m\right)\varphi,$$
$$\mathcal{L}_v = \frac{2}{\ell}\varepsilon_{ijkm}\left[\hat{R}^{ij} + \frac{4}{\ell^2}h^i e^j\right]\left[\hat{T}^k - e^k a\right]v^m,$$
$$\mathcal{L}_\beta = \frac{1}{\ell}\varepsilon_{ijkm}\left\{\frac{1}{2}\left[\hat{R}^{ij} + \frac{4}{\ell^2}h^i e^j\right]\left[\mathrm{d}a - \frac{2}{\ell^2}h^f e_f\right] - \frac{1}{\ell}\left[\mathrm{D}h^i\mathrm{D}h^j - \hat{T}^i\hat{T}^j + 2\mathrm{D}h^i\hat{T}^j + 2e^i a\mathrm{D}h^j + 2h^i a\hat{T}^j\right]\right\}\beta^{km}, \tag{19}$$
$$\mathcal{L}_u = -\frac{2}{l}\varepsilon_{ijkm}\left[\hat{R}^{ij} + \frac{4}{\ell^2}h^i e^j\right]\left[\mathrm{D}h^k + h^k a\right]u^m,$$

being $\hat{R}^{ij}$ the Lorentz curvature, $\hat{T}^i$ the torsion and D the standard Lorentz covariant derivative in the spin connection.

5. Topological gravity for $\mathfrak{so}(D-1,1) \oplus \mathfrak{so}(D-1,2)$ algebra

The $\mathfrak{so}(D-1,1) \oplus \mathfrak{so}(D-1,2)$ algebra was proposed in Ref. [21]. This algebra was re-obtained in Ref. [22] from *AdS* algebra using the so-called expansion procedure and in Ref. [28] from Maxwell algebra through a procedure known as deformation of algebras. The corresponding generators satisfy the following commutation relations,

$$[J_{ab}, J_{cd}] = \eta_{bc} J_{ad} + \eta_{ad} J_{bc} - \eta_{ac} J_{bd} - \eta_{bd} J_{ac},$$
$$[J_{ab}, P_c] = \eta_{bc} P_a - \eta_{ac} P_b, \ [P_a, P_b] = Z_{ab},$$
$$[J_{ab}, Z_{cd}] = \eta_{bc} Z_{ad} + \eta_{ad} Z_{bc} - \eta_{ac} Z_{bd} - \eta_{bd} Z_{ac},$$
$$[Z_{ab}, P_c] = \eta_{bc} P_a - \eta_{ac} P_b,$$
$$[Z_{ab}, Z_{cd}] = \eta_{bc} Z_{ad} + \eta_{ad} Z_{bc} - \eta_{ac} Z_{bd} - \eta_{bd} Z_{ac}. \quad (20)$$

In order to write down the Chern–Simons–Antoniadis–Savvidy Lagrangian $\mathcal{L}^{(4)}_{\text{ChSAS}} \equiv \mathfrak{C}^{(4)}_{\text{ChSAS}}(A, \phi) = \langle F^2 \phi \rangle$ for the $\mathfrak{so}(D-1,1) \oplus \mathfrak{so}(D-1,2)$ algebra, we start from the corresponding-valued two-form curvature

$$F = \frac{1}{2} R^{ab} J_{ab} + \frac{1}{l}\left(T^a + k^a{}_b e^b\right) P_a + \frac{1}{2}\left(D_\omega k^{ab} + k^a{}_c k^{cb} + \frac{1}{l^2} e^a e^b\right) Z_{ab}, \quad (21)$$

and the 0-form

$$\phi = \frac{1}{2}\phi^{ef} J_{ef} + \frac{1}{l}\phi^e P_e + \frac{1}{2} h^{ef} Z_{ef}. \quad (22)$$

Using the invariant tensor for the $\mathfrak{so}(D-1,1) \oplus \mathfrak{so}(D-1,2)$ algebra [29]

$$\langle J_{ab} J_{cd} P_e \rangle = \frac{4}{3}\alpha_1 l^3 \varepsilon_{abcde},$$
$$\langle Z_{ab} Z_{cd} P_e \rangle = \frac{4}{3}\alpha_1 l^3 \varepsilon_{abcde}, \quad (23)$$
$$\langle J_{ab} Z_{cd} P_e \rangle = \frac{4}{3}\alpha_1 l^3 \varepsilon_{abcde},$$

where α_1 is an arbitrary constant of dimensions $[\text{length}]^{-3}$, we find

$$\mathcal{L}^{(4)}_{\text{ChSAS}}(A, \phi) = \frac{\alpha_1 l^2}{3} \varepsilon_{abcde} R^{ab} R^{cd} \phi^e + \frac{2\alpha_1}{3} \varepsilon_{abcde} R^{ab} e^c e^d \phi^e + \frac{\alpha_1}{3l^2} \varepsilon_{abcde} e^a e^b e^c e^d \phi^e,$$

where (i) $R^{ab} = d\omega^{ab} + \omega^a{}_c \omega^{cd}$, with $a, b, c, d = 0, 1, 2, 3, 4$, (ii) e^a and ω^{ab} are one-forms pulled baked in 4 dimensions, (iii) we have considered, for simplicity, $k^{ab} = 0$ and $T^a = 0$.

Using the same decomposition from Section 4, $e^a = \left(e^i, e^4\right)$, $\omega^{ab} = \left(\omega^{ij}, \omega^{i4} = \lambda e^i\right)$, $\phi^a = \left(\phi^i, \phi^4\right)$ with $i = 0, 1, 2, 3$ and rotating the basis in such way that in each point of space the field ϕ^a has components $\phi^4 = \phi$ and $\phi^i = 0$, we find

$$\mathcal{L}^{(4)}_{\text{ChSAS}}(A, \phi) = \frac{\alpha_1}{3}\phi \varepsilon_{ijkm}\left\{l^2 \hat{R}^{ij} \hat{R}^{km} + \tilde{\alpha} \hat{R}^{ij} e^k e^m + \tilde{\beta} e^i e^j e^k e^m\right\}, \quad (24)$$

with $\tilde{\alpha} = 2\left(2 - \lambda^2 l^2\right)$, $\tilde{\beta} = \frac{4}{l^2} - 4\lambda^2 + \lambda^4 l^2$. This Lagrangian correspond to the Brans–Dicke Lagrangian with cosmological term for small value of l^2.

6. Concluding remarks

In Refs. [1–3] the idea of using forms of higher degree as non-abelian gauge fields was used to construct gauge invariant Lagrangian forms which are independent of the metric. In the present work we use the formalism introduced by Antoniadis and Savvidy to construct metric-independent topological invariant on $(2n+1)$-dimensional space–time. This construction allow us to find a 'Chern–Simons–Antoniadis–Savvidy' form in $2n$ dimensions given by Eq. (14). This equation permits reobtain the even-dimensional topological actions for gravity from Refs. [4–6] and to construct topological gravities for another gauge groups such us $SO(D-1,1) \oplus SO(D-1,2)$ group from Ref. [21]. These results together with those of Ref. [19] allow us to conjecture that, at least for the case where the gauge group is the Poincaré group, the gauged Wess–Zumino–Witten term corresponds to a Chern–Simons–Antoniadis–Savvidy form.

The actions (18), (19), and (24) can be understood as a generalization of the action (6) from Ref. [17]. In this context the torsion free condition on the 4-dimensional torsion is not valid. This fact has the consequence that the torsion becomes a new source for the metric curvature, which can give rise to very interesting cosmological solutions different from the known four-dimensional Friedmann–Robertson–Walker solutions. A detailed study, for a particular case of the Lagrangian (18) (case $\varphi^n = 0$) can be found in Refs. [17] and [30].

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