



# On the cancellation of radiative corrections to the cross section of electron-proton scattering <sup>☆</sup>



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## ABSTRACT

The largest radiative corrections to the cross section of electron-proton scattering at high energies are associated with emission of photons, real and virtual, by electron. They contain large logarithms coming from soft and collinear photons. Cancellation of the contributions of the soft photons to virtual and real corrections is well known. Less known is the fact that the contributions of the photons collinear to scattered electrons are also canceled for the most of experiments. On the contrary, contributions of the photons collinear to initial electrons as a rule are not canceled. It is shown, however, that these contributions are canceled for the experimental set up suggested by A.A. Vorobev for measurement of proton radius.

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## 1. Introduction

The striking difference in the proton radius values extracted at the Paul Scherrer Institute from the  $2S - 2P$  transition in muonic hydrogen [1,2] and obtained from electron-proton scattering and hydrogen spectroscopy [3] (for a review, see Ref. [4]) caused a surge of interest of theorists and experimentalists to the problem and got the name “proton radius puzzle” [5,6]. Latest electron scattering experiments at JLab [7] and MAMI [8] and hydrogen spectroscopy experiments [9,10] not only did not resolve the puzzle, but made it even more confusing.

Currently new scattering experiments are being prepared. A distinctive feature of one of them [11], which was suggested by A.A. Vorobev and has to be performed with a low-intensity electron beam at MAMI, is that instead of detecting a scattered electron, as in previous experiments, it is supposed to detect with a high precision a recoil proton in the region of low momentum transfer squared  $Q^2$  from 0.001 to 0.04 GeV<sup>2</sup>. The aim is to extract the proton radius with 0.6 percent precision, which could be decisive in solving the proton radius puzzle. To this end, it is planned

to achieve 0.2 percent accuracy of the cross section  $d\sigma/(dQ^2)$  measurement.

Such accuracy requires precise account of radiative corrections. Although calculation of the radiative corrections to the electron-proton scattering cross section has a long history (see, for example, Refs. [12–15] and recent reviews [16–18]),<sup>1</sup> the results obtained before cannot be completely applied to the experiment discussed above. The reason is that they were obtained for experiments in which scattered electrons were detected (honestly speaking, there was the experiment [20] where the recoil proton was detected; but calculation of the radiative corrections to this experiment was not explained). Since the radiative corrections include contributions of inelastic processes with photon emission, they depend strongly on experimental conditions, so that the corrections calculated for experiments with detection of scattered electrons are not suitable for experiments with detection of recoil proton. It occurs that the radiative corrections for experiments with detection of recoil proton have a new and pleasant property – cancellation of the most important pieces. This paper is dedicated to explanation and derivation of this property.<sup>2</sup>

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<sup>1</sup> Higher order corrections to the lepton line were considered for the standard experimental set-up with scattered electron measurement in [19].

<sup>2</sup> Cancellation of leptonic radiative corrections to deep inelastic scattering was discussed in [21] and [22].

## 2. One-loop corrections

We will denote four-momenta of initial and final electron (proton) as  $l$  ( $p$ ) and  $l'$  ( $p'$ );  $l^2 = l'^2 = m^2$ ,  $p^2 = p'^2 = M^2$ , and will use the designations  $Q^2 = -q^2$ ,  $q = p - p'$  both for elastic and inelastic processes. In the following we assume electrons to be ultrarelativistic in both the initial and final states and also neglect  $m^2$  compared with  $M^2$  and  $Q^2$ .

In the Born approximation the electron-proton cross section is known since 1950 [23]. It can be written as

$$\frac{d\sigma_B}{dQ^2} = \frac{2\pi\alpha^2}{Q^4} \frac{2M(E_l^2 + E_l'^2 + M(E_l - E_l'))}{E_l^2(2M + E_l - E_l')} \times (\tau G_M^2(Q^2) + \epsilon G_E^2(Q^2)), \quad (1)$$

where  $E_l$  and  $E_l'$  are initial and final electron energies in the rest frame of the initial proton,

$$\epsilon = \frac{2E_l E_l' - M(E_l - E_l')}{E_l^2 + E_l'^2 + M(E_l - E_l')}, \quad (2)$$

$\tau = Q^2/(4M^2)$ ,  $G_E(Q^2)$  and  $G_M(Q^2)$  are electric and magnetic form factors, which are expressed through the Dirac and Pauli form factors  $f_1(Q^2)$  and  $f_2(Q^2)$ , defined by the matrix element of the electromagnetic current  $J^\mu$  between initial and final proton states

$$\langle P' | J^\mu(0) | P \rangle = \bar{u}(p') \left( f_1(Q^2) \gamma^\mu + f_2(Q^2) \frac{[\gamma^\mu, \gamma^\nu] q_\nu}{4M} \right) u(p), \quad (3)$$

as

$$G_M(Q^2) = f_1(Q^2) + f_2(Q^2), \quad (4)$$

$$G_E(Q^2) = f_1(Q^2) - \frac{Q^2}{4M^2} f_2(Q^2).$$

With elastic scattering  $E_l'$  and  $Q^2$  are not independent variables and are expressed through the initial electron energy  $E_l$  and the scattering angle  $\theta$  in the proton rest frame:

$$E_l' = \frac{E_l}{1 + (2E_l/M) \sin^2(\theta/2)}, \quad Q^2 = 4E_l E_l' \sin^2(\theta/2). \quad (5)$$

The energies of the scattered electron  $E_l'$  and the recoil proton  $E_p'$  can be expressed also through  $E_l$  and  $Q^2$ :

$$E_p' = \frac{Q^2}{2M} + M, \quad E_l' = E_l - \frac{Q^2}{2M}. \quad (6)$$

Experimentally observed cross section  $d\sigma_{exp}$  usually is presented as

$$d\sigma_{exp} = d\sigma_B(1 + \delta), \quad (7)$$

where the radiative correction  $\delta$  is given by the sum of the correction  $\delta_{virt}$  accounting contributions of higher orders in the electromagnetic coupling  $\alpha$  to the elastic scattering cross section and the correction  $\delta_{real}$  due to inelastic processes admitted by experimental conditions:  $\delta = \delta_{virt} + \delta_{real}$ . In the one-loop approximation the virtual corrections  $\delta_{virt}$  is determined by the interference of the Born amplitude with the one-loop amplitudes, and the real correction  $\delta_{real}$  is related to the one-photon emission.

Taking separately, the virtual and real contributions has no sense, neither from theoretical, nor from experimental point of view; in the first case – because of infrared divergencies, in the

second – because only processes with emission of any number of sufficiently soft photons are experimentally observed.

The most important virtual correction is so called vertex correction. In the one-loop approximation it is well known (see, for example, [24])

$$\delta_{vertex}^e = \frac{\alpha}{\pi} \left[ - \left( \ln \left( \frac{Q^2}{m^2} \right) - 1 \right) \ln \left( \frac{m^2}{\lambda^2} \right) - \frac{1}{2} \ln^2 \left( \frac{Q^2}{m^2} \right) + \frac{3}{2} \ln \left( \frac{Q^2}{m^2} \right) + \frac{\pi^2}{6} - 2 \right]. \quad (8)$$

Here  $\lambda$  is the “photon mass” introduced for regularization of the infrared divergencies.

Photons emitted in the scattering process by an electron can be conveniently divided, as is usually done, into soft, non-affecting the elastic kinematics of the process, and hard. The soft photons can be defined as those which have energy less than  $\omega_0$  (with a sufficiently small  $\omega_0$ ) in some reference frame. For them, the factorization theorem [25] can be used and their contribution  $\delta_{soft}^e$  to  $\delta_{real}$  can be found in any frame using the integrals calculated in [26]. If the limitation on the photon energy  $\omega < \omega_0$  is imposed in the rest proton system, then (see, for example, [27])

$$\delta_{soft}^e = \frac{\alpha}{\pi} \left[ \left( \ln \left( \frac{Q^2}{m^2} \right) - 1 \right) \ln \left( \frac{4\omega_0^2}{\lambda^2} \right) - \frac{1}{4} \ln^2 \left( \frac{4E_l^2}{m^2} \right) - \frac{1}{4} \ln^2 \left( \frac{4E_l'^2}{m^2} \right) + \frac{1}{2} \ln^2 \left( \frac{4E_l E_l'}{Q^2} \right) + \ln \left( \frac{4E_l E_l'}{m^2} \right) + \text{Li}_2 \left( 1 - \frac{Q^2}{4E_l E_l'} \right) - \frac{\pi^2}{3} \right]. \quad (9)$$

The sum of the vertex and the soft photon corrections is free from infrared singularities

$$\delta_{vertex+soft}^e = \frac{\alpha}{\pi} \left[ - \left( \ln \left( \frac{Q^2}{m^2} \right) - 1 \right) \ln \left( \frac{E_l E_l'}{\omega_0^2} \right) + \frac{3}{2} \ln \left( \frac{Q^2}{m^2} \right) - 2 - \frac{1}{2} \ln^2 \left( \frac{E_l}{E_l'} \right) + \text{Li}_2 \left( 1 - \frac{Q^2}{4E_l E_l'} \right) - \frac{\pi^2}{6} \right], \quad (10)$$

so that the hard photon contribution to  $\delta_{real}$  can be calculated with zero photon mass. The cancellation of the infrared singularities in (10) is well known. It is a consequence of the general statement [25] about the cancellation of infrared singularities in the sum of virtual corrections and corrections due to the emission of soft photons. However, the sum (10) contains  $\ln \left( \frac{Q^2}{m^2} \right)$ , i.e. so called collinear singularities. In general, the terms with  $\ln \left( \frac{Q^2}{m^2} \right)$  remain also after account of contributions of hard photons, because the theorem on the cancellation of collinear singularities [28,29] evidently can not be applied to radiative corrections to electron-proton scattering. However, in the experimental conditions of Ref. [11] these terms disappear. It is amusing enough that their cancellation may be related to the Kinoshita–Lee–Nauenberg theorem [28,29]. It will be explained in the next section. And here we will demonstrate it using less advanced arguments.

We have to add to (10) the correction from the hard photon emission. With logarithmic accuracy, this correction has a clear physical interpretation. It consists of two parts, corresponding to photon emission by initial and final electrons. Both of them can be

calculated using the quasi-real electron method [30]. For the final electron emission it gives

$$\frac{\omega d\sigma^{f.e.e.}}{d^3k} = \frac{\alpha}{4\pi^2} \left( \frac{E_l'^2 + (E_l' - \omega)^2}{\omega E_l'(kl')} - \frac{m^2}{(kl')^2} \frac{(E_l' - \omega)}{E_l'} \right) d\sigma_B, \quad (11)$$

where  $k$  and  $\omega$  are the photon 4-momentum and energy,  $l'$  and  $E_l'$  are the 4-momentum and energy of the final electron in the elastic process, and  $d\sigma_B$  is the cross section of the elastic process. With the logarithmic accuracy, the upper limit of the integration over photon emission angle should be taken equal to the scattering angle in the elastic process, so that we obtain with this accuracy

$$\frac{x d\sigma^{f.e.e.}}{dx} = \frac{\alpha}{2\pi} \ln \frac{Q^2}{m^2} (1 + (1-x)^2) d\sigma_B, \quad (12)$$

where  $x = \omega/E_l'$ . Integration over  $x$  can be performed from  $\omega_0/E_l'$  to 1, that gives

$$d\sigma^{f.e.e.} = \frac{\alpha}{\pi} \ln \frac{Q^2}{m^2} \left( \ln \frac{E_l'}{\omega_0} - \frac{3}{4} \right) d\sigma_B. \quad (13)$$

Comparing it with (10) one sees that the contribution (13) cancels in (10) the collinear singular terms with the coefficient  $\ln \frac{E_l'}{\omega_0}$  and a half of the terms with the energy independent coefficient.

For the correction due to radiation of the initial electron we have, with the same accuracy

$$\frac{\omega d\sigma^{i.e.e.}}{d^3k} = \frac{\alpha}{4\pi^2} \left( \frac{E_l^2 + (E_l - \omega)^2}{\omega E_l(kl)} - \frac{m^2}{(kl)^2} \frac{(E_l - \omega)}{E_l} \right) d\sigma_B \Big|_{\vec{l} \rightarrow \vec{l} - \vec{k}}, \quad (14)$$

and after integration with the logarithmic accuracy over emission angles

$$d\sigma^{i.e.e.} = \frac{\alpha}{2\pi} \ln \frac{Q^2}{m^2} \int_{\omega_0/E_l}^{x_m} \frac{dx}{x} (1 + (1-x)^2) d\sigma_B \Big|_{\vec{l} \rightarrow \vec{l}(1-x)}, \quad (15)$$

where  $x = \omega/E_l$  and  $x_m$  is its upper kinematic bound,  $x_m = 1 - (\sqrt{Q^2(Q^2 + 4M^2)} + Q^2)/(4E_l M)$ .

Emission of a photon by an initial electron changes its energy, and thus cross sections of processes initiated by the electron after radiation (quasi-real electron). This, along with the difference  $x_m$  from 1, is the reason why virtual corrections are not canceled by the real ones in the most experiments. But in the proposed experiment [11], where the differential in the momentum transferred to the final proton,  $d\sigma/(dQ^2)$ , is supposed to be measured in the region

$$Q^2 \ll E_l^2, \quad Q^2 \ll E_l M, \quad (16)$$

one has  $x_m \simeq 1$ ,  $E_l' \simeq E_l$  and

$$\frac{d\sigma_B}{dQ^2} \simeq \frac{4\pi\alpha^2}{Q^4} f_1^2(Q^2), \quad (17)$$

i.e.  $d\sigma_B/(dQ^2)$  does not depend on  $E_l$  at fixed  $Q^2$ , so that  $d\sigma^{i.e.e.}$  equal to  $d\sigma^{f.e.e.}$  and in the sum they cancel the terms with  $\ln \frac{Q^2}{m^2}$  in (10).

In fact, the cancellation of the virtual and real corrections turns out to be even stronger. It occurs that in the one-loop approximation the cancellation takes place not only with logarithmic

accuracy, and the terms not having the collinear singularities (constant terms) are canceled as well. Of course, such cancellation can not be proved by some approximate method and requires more strict approach. It can be seen using exact results of Ref. [31] for a photon emission in collisions of muons with electrons. Making the necessary replacements and substitutions one can obtain for the cross section of photon emission by electron in the experimental conditions discussed above (detailed output will be given elsewhere [32])

$$d\sigma^{e.e.} = \frac{4\alpha^3}{Q^4} f_1^2(Q^2) \int_{\omega_0/E_l}^1 \frac{dx}{x} \left[ (1 + (1-x)^2) \ln \frac{Q^2}{m^2} - 2(1-x) \right] \\ = \frac{d\sigma_B}{dQ^2} \frac{\alpha}{\pi} \left[ 2 \left( \ln \frac{Q^2}{m^2} - 1 \right) \ln \frac{E_l}{\omega_0} - \frac{3}{2} \ln \frac{Q^2}{m^2} + 2 \right]. \quad (18)$$

Comparing it with (10) at  $E_l' \simeq E_l$ ,  $Q^2 \ll E_l^2$ , one sees that the contribution (18) cancels it completely.

### 3. Higher order corrections

The cancellation demonstrated above is not restricted by the one-loop approximation, and takes place also in higher orders, at least with logarithmic accuracy. It can be shown using the parton picture, developed for the theoretical description of the deep inelastic electron-proton scattering [33–37], but applying it to “deep inelastic proton-electron scattering”.

The cross section of electron-proton scattering with radiative corrections due only to electron interaction can be considered as inclusive proton-electron scattering cross section. At fixed  $Q^2$  it can be written with logarithmic accuracy in terms of the “parton distribution functions”  $f_e^e(x, Q^2)$  and  $f_e^e(x, Q^2)$ . More precisely, the cross section of the deep inelastic proton-electron scattering can be written as

$$(2\pi)^3 2E_p' \frac{d^3\sigma}{d^3p'} = \frac{\pi e^4}{Q^4} \frac{1}{\sqrt{(pl)^2 - m^2 M^2}} J^{\mu\nu}(p, p') W_{\mu\nu}(l, q), \quad (19)$$

where  $J^{\mu\nu}(p, p')$  is the proton current tensor

$$J^{\mu\nu}(p, p') = \overline{\sum}_{pol} J^\mu J^{*\nu}, \quad (20)$$

$\overline{\sum}_{pol}$  means summation over final polarizations and averaging over initial ones,

$$J^\mu = u(p') \left( f_1(Q^2) \gamma^\mu + f_2(Q^2) \frac{[\gamma^\mu, \gamma^\nu] q_\nu}{4M} \right) u(p), \quad (21)$$

and  $W_{\mu\nu}(l, q)$  is the deep inelastic scattering tensor,

$$W_{\mu\nu}(l, q) = \frac{1}{4\pi} \overline{\sum}_X \langle l | j_\nu^{(e)}(0) | X \rangle \langle X | j_\mu^{(e)}(0) | l \rangle \\ \times (2\pi)^4 \delta(q + l - p_X). \quad (22)$$

Here  $|l\rangle$  is the initial electron state,  $X$  is any state which can be produced in photon-electron collisions,  $\overline{\sum}_X$  means averaging over initial electron polarizations and summation over discrete and integration over continuous variables of  $X$ ,  $j_\mu^{(e)}(x)$  is the electron electromagnetic current operator. Taking into account the conservation of the current, one can represent  $W_{\mu\nu}$  in the form

$$W^{\mu\nu}(l, q) = - \left( g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) F_1(x, Q^2) + \frac{1}{(lq)} \left( l^\mu - \frac{(lq)}{q^2} q^\mu \right) \left( l^\nu - \frac{(lq)}{q^2} q^\nu \right) F_2(x, Q^2), \quad (23)$$

where  $Q^2 = -q^2$ ,  $x = Q^2/(2(lq))$ . Calculating the tensor  $J^{\mu\nu}$

$$J^{\mu\nu} = G_M^2(Q^2) \left( g_{\mu\nu} q^2 - q_\mu q_\nu \right) + \frac{Q^2 G_M^2(Q^2) + 4M^2 G_E^2(Q^2)}{4M^2 + Q^2} p^\mu p^\nu, \quad (24)$$

where  $P = p + p'$ ,  $G_E(Q^2)$  and  $G_M(Q^2)$  are proton electric and magnetic form factors, performing tensor convolution and using

$$\frac{d^3 p'}{2E'_p} = \frac{\pi}{4} \frac{Q^2 dx}{x^2 \sqrt{(pl)^2 - m^2 M^2}}, \quad (25)$$

we obtain

$$\frac{d\sigma}{dQ^2 dx} = \frac{\pi \alpha^2}{2x^2 Q^2 ((pl)^2 - m^2 M^2)} \left[ (2Q^2 G_M^2 - 4M^2 G_E^2) F_1 + \left( -G_M^2 (m^2 Q^2 + (lq)^2) + \frac{Q^2 G_M^2 + 4M^2 G_E^2}{4M^2 + Q^2} (pl)^2 \right) \frac{F_2}{(lq)} \right]. \quad (26)$$

where

$$(lq) = \frac{Q^2}{2x}, \quad (pl) = 2ME_l - \frac{Q^2}{2x}. \quad (27)$$

The region of variation of  $x$  at fixed  $Q^2$  is determined by conditions  $M_X^2 \geq m^2$  and  $(lq) \leq E_l q_0 + \sqrt{E_l^2 - m^2} \sqrt{q_0^2 + Q^2}$  with  $q_0 = M - E'_p = -Q^2/(2M)$ , i.e.

$$1 \geq x \geq \frac{MQ^2}{\sqrt{E_l^2 - m^2} \sqrt{Q^2(4M^2 + Q^2)} - E_l Q^2}. \quad (28)$$

Till now we did not make any approximation. Using the conditions (16) of the proposed experiment and the Callan-Gross relation [38]  $F_2 = 2xF_1$ , we obtain

$$\frac{d\sigma}{dQ^2} = \frac{4\pi \alpha^2}{Q^4} f_1^2(Q^2) \int_{x_0}^1 \frac{dx}{x} F_2(x, Q^2), \quad (29)$$

where

$$x_0 = \frac{Q}{2E_l} \ll 1. \quad (30)$$

In the parton picture [33–37], which can be used with the logarithmic accuracy, the structure functions are expressed through the parton distributions in the initial electron. The charged partons in this case are electrons and positrons, so that

$$F_2 = x(f_e^e + f_e^{\bar{e}}), \quad (31)$$

where  $f_e^e$  and  $f_e^{\bar{e}}$  are the electron and positron distributions in the initial electron.

The positron distribution  $f_e^{\bar{e}}$  appears in the two-loop approximation, so that in the one-loop approximation only  $f_e^e$  does contribute. Moreover, in this approximation  $f_e^e$  is equal to the valence electron distribution,  $f_e^v$ , which is not singular at small  $x$ , so that

the lower limit  $x_0$  of the integration in (29) can be taken equal to zero. Therefore the cancellation of the logarithmic radiative corrections discussed in the previous section has a simple explanation in the language of parton distributions: it takes place because

$$\int_0^1 dx f_e^v(x, Q^2) = 1 \quad (32)$$

regardless of the value of  $Q^2$ . Remind that (32) is the consequence of the charge conservation.

Surprisingly enough, this cancellation can be explained with the help of the Kinoshita–Lee–Nauenberg theorem. Of course, this theorem is not applicable to any processes with initial electron, contrary to a fairly common delusion that the theorem can be applied to inclusive cross sections (see, for example, [22]), i.e. for the cancellation of collinear singularities summation over final states is sufficient. As explicitly shown in [29], this is wrong, and summation over initial degenerate states is also necessary. But the theorem can be applied to completely inclusive production of electrons, positrons and photons by the photon with the virtuality  $Q^2$ . Using the famous “reciprocal relation” [33–37,39,40], which means in our case equality of the parton distribution function  $f_e^e(x, Q^2)$  and the “parton fragmentation function”  $\bar{f}_e^e(x, Q^2)$  for the inclusive cross section of electron production by the photon with the virtuality  $Q^2$ , one can connect two processes and to prove the cancellation (for details see [32]).

Starting from the two-loop approximation the situation becomes not so simple. The radiative corrections depend strongly on what is really measured, and if the fully inclusive cross section (29) is measured, they become large. The reason is that in this approximation the target proton can interact not only with the scattering electron, but also with one of components of the electron-positron pair produced by this electron. In this case the total cross section for interaction of the virtual photon emitted by the target proton with incoming electron does not decrease with energy, in contrast with the one-loop approximation, and the large contribution come from the region of small  $(lq)$ , or small  $x$  in (29). On the language of the parton distributions it means that both  $f_e^e(x, Q^2)$  and  $f_e^{\bar{e}}(x, Q^2)$  becomes singular at  $x = 0$  and the lower limit  $x_0$  of the integration in (29) can not be taken equal to 0. Evidently, such experimental conditions are not the best one. It seems that more preferable are the conditions at which production of electron-positron pairs is forbidden. In this case in Eq. (31) only  $f_e^v$  does contribute, and due to its property (32) the major pieces of the radiative corrections cancel also in higher orders of perturbation theory (for details see [32]).

#### 4. Conclusion

It turns out that the setting of the experiment suggested by A.A. Vorobev [11] has an interesting feature – cancellation of main radiative corrections. We showed it in this paper by several methods and with various accuracy. The most simple and physically transparent is the quasi-real electron method [30] which can be used in the one-loop approximation and has logarithmic accuracy. It turns out, however, that in the one-loop approximation the cancellation of the virtual and real radiative corrections is not restricted by the logarithmic accuracy, but takes place for the terms not having the collinear divergence (constant terms) as well. It was shown with the help of the exact results of Ref. [31] for a photon emission in collisions of muons with electrons. Moreover, it turns out, that for the experimental conditions at which production of electron-positron pairs is forbidden, the cancellation is not restricted by the one-loop approximation and with the logarithmic

accuracy it takes place in the higher orders of perturbation theory also. It was demonstrated using the parton distribution method [33–37] developed for the theoretical description of the deep inelastic electron-proton scattering. It is interesting that the famous reciprocal relation [33–37,39,40] permits to prove the cancellation of the terms with  $\ln \frac{Q^2}{m^2}$  in the one-loop radiative corrections with the help of the Kinoshita–Lee–Nauenberg theorem [28,29].

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