AN EFFECTIVE STRING THEORY DESCRIBING THE VORTICES OF THE ABELIAN HIGGS MODEL

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Abstract

Superconducting vortices with quarks on their ends provide a concrete picture for confinement in QCD. Explicit calculations with models of this type have been compared both with experimental data and with lattice simulations of QCD. However, these calculations have all been carried out in the approximation where the vortex line at the center of the QCD flux tube is a straight line connecting the quarks. In this talk, we show how consideration of fluctuations in the shape of the vortex leads to an anomaly free effective string theory describing these vortices, and we discuss its applications to hadron physics.

Introduction

ı this talk we will first review the dual superconductor mechanism of confinement. [1] We will then ıow how the theory of superconducting vortices can be expressed as an effective string theory. Finally, e show that this theory possesses features of old fashioned string theory, such as Regge trajectories, nd indicate the corrections due to the finite thickness of the vortices.

Dual Superconductivity

ı the dual superconductor model of confinement proposed by Nambu, Mandelstam, and 't Hooft, [1] ›ng distance QCD is described by an effective dual theory, governed by a Lagrangian $\mathcal{L}_{\text{eff}}$, which is a ınction of dual potentials C_μ and Higgs fields ϕ carrying monopole charge. The fields ϕ and C_μ couple ›cally with a coupling constant $g = \frac{2\pi}{e}$, where e is the Yang–Mills coupling constant ($\frac{e^2}{4\pi} = \alpha_s$). The ıonopole field ϕ develops a nonvanishing vacuum expectation value ϕ_0, which gives the dual gluon eld C_μ a mass $M_C = g\phi_0$. Dual potentials couple to electric color charge like ordinary potentials ›uple to monopoles. The potentials C_μ thus couple to a $q\bar{q}$ pair via a Dirac string tensor $G^S_{\mu\nu}$, which nonvanishing along a line L connecting the $q\bar{q}$ pair. The Higgs field ϕ vanishes on L and approaches s vacuum value ϕ_0 at large transverse distances. A dual Meissner effect confines the electric color ux ($\mathbf{Z_3}$ flux) to a narrow tube (Abrikosov–Nielsen–Olesen vortex) [2] surrounding L. As a result, ıe energy of the $q\bar{q}$ pair increases linearly with their separation R, and the quarks are confined in adrons. For small R, the color field generated by the quarks expels the monopole condensate from ıe bulk of the region between them, and a Coulomb potential develops.

Calculations with explicit models[3] of this type have been compared both with experimental data nd with Monte Carlo simulations of QCD.[4] To a very good approximation, the Abelian Higgs model vith a suitable color factor) can be used to describe the flux tube along the z axis connecting the $q\bar{q}$ air. The Euclidean Lagrangian $\mathcal{L}_{\text{eff}}$ has the form

$$\mathcal{L}_{\text{eff}} = \frac{4}{3}\left\{\frac{1}{4}\left(\partial_\mu C_\nu - \partial_\nu C_\mu + G^S_{\mu\nu}\right)^2 + \frac{1}{2}|(\partial_\mu - igC_\mu)\phi|^2 + \frac{\lambda}{4}(|\phi|^2 - \phi_0^2)^2\right\}. \tag{1}$$

'he Dirac string tensor $G^S_{\mu\nu}$ coupling the quarks to the dual potentials in (1) is given by

$$G^S_{\mu\nu}[\tilde{x}^\mu] = -e\epsilon_{\mu\nu\alpha\beta}\int d^2\sigma \frac{1}{2}\epsilon^{ab}\frac{\partial\tilde{x}^\alpha}{\partial\sigma^a}\frac{\partial\tilde{x}^\beta}{\partial\sigma^b}\delta^{(4)}\left(x^\mu - \tilde{x}^\mu(\sigma)\right). \tag{2}$$

he integration in (2) is over the surface $\tilde{x}^\mu(\sigma)$ swept out by the line L attached to the $q\bar{q}$ pair moving ›ong world lines Γ_1 and Γ_2. This surface is bounded by the loop Γ. (See Fig. 1)

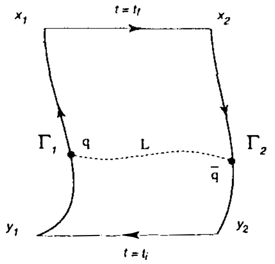

Fig. 1: The Loop Γ

'he monopole field ϕ vanishes on the surface $\tilde{x}^\mu(\sigma)$, parameterized by σ^a, $a = 1, 2$,

$$\phi(\tilde{x}^\mu(\sigma)) = 0. \tag{3}$$

'he surface $x^\mu = \tilde{x}^\mu(\sigma)$ is the location of the Abrikosov–Nielsen–Olesen vortex. The Landau-Ginzburg arameter λ/g^2 of the dual theory [5] is approximately equal to 1/2. This corresponds to the border between a type I and type II superconductor. The monopole mass $M_\phi = \sqrt{2\lambda}\phi_0$ is the same as the dual gluon mass $M_C \equiv M$, the string tension is $\mu = \frac{4}{3}\pi\phi_0^2$, and the flux tube radius is $a = \frac{\sqrt{2}}{M}$.

Finally, we note that recent studies[6] of lattice QCD indicate that, at long distances, QCD is in a sense equivalent to a dual Abelian Higgs model with λ/g^2 close to $1/2$.

3 Contributions of Fluctuations of the Flux Tube to the Energy of a $q\bar{q}$ Pair

All previous calculations[4] with the dual theory have been carried out in the classical approximation corresponding to a flat vortex sheet $\tilde{x}^\mu(\sigma)$. Fluctuations in the shape of the flux tube at distances greater than its radius a also contribute to the $q\bar{q}$ interaction. (see Fig. 1) To account for these contributions, we must express the Wilson loop of the dual theory $W_{\text{eff}}(\Gamma)$ as an integral over vortex sheets.[7] The action $S[C_\mu, \phi, G^S_{\mu\nu}]$ of the dual theory is

$$S[C_\mu, \phi, G^S_{\mu\nu}] = \int d^4x \left[\mathcal{L}_{\text{eff}} + \mathcal{L}_{GF}\right] , \tag{4}$$

and the Wilson loop $W_{\text{eff}}(\Gamma)$ is

$$W_{\text{eff}}(\Gamma) = \frac{\int \mathcal{D}C_\mu \mathcal{D}\phi \mathcal{D}\phi^* e^{-S[C_\mu,\phi,G^S_{\mu\nu}]}}{\int \mathcal{D}C_\mu \mathcal{D}\phi \mathcal{D}\phi^* e^{-S[C_\mu,\phi,G^S_{\mu\nu}=0]}} . \tag{5}$$

The action (4) includes a gauge fixing term $\mathcal{L}_{GF}$, and $S[C_\mu, \phi, G^S_{\mu\nu} = 0]$ is the action in the absence of a quark-antiquark pair. For brevity, we suppress the denominator of (5) in the rest of the paper.

The integration in the numerator of (5) goes over all field configurations which include a vortex sheet $\tilde{x}^\mu(\sigma)$ bounded by the loop Γ. We will carry out the integrations over C_μ and ϕ in the following way:
(1) We will first fix the location of a vortex sheet $\tilde{x}^\mu(\sigma)$, and integrate only over field configurations for which $\phi(\tilde{x}^\mu(\sigma)) = 0$.
(2) We will then integrate over all possible vortex sheets $\tilde{x}^\mu(\sigma)$, so that W_{eff} takes the form

$$W_{\text{eff}}(\Gamma) = \int \mathcal{D}\tilde{x}^\mu e^{-S_{\text{eff}}[\tilde{x}^\mu(\sigma)]} . \tag{6}$$

In the rest of this talk we will show how to obtain the transformation from the field representation (5) to the string representation (6), and will give the form of the action S_{eff} and the meaning of the integration over all surfaces in (6). Akhmedov, Chernodub, Polikarpov, and Zubkov[7] have obtained the string representation of the Abelian Higgs model in the London limit $\lambda \to \infty$, where $|\phi|$ is fixed. The work described here can be regarded as an extension of the results of this paper to the full Abelian Higgs model for all values of λ.

4 Effective String Theory of Vortices

To obtain to the string representation (6) from the field representation (5), we introduce into Eq. (5) the factor one written in the form,

$$1 = J[\phi] \int \mathcal{D}\tilde{x}^\mu \delta\left[\text{Re}\phi(\tilde{x}^\mu(\sigma))\right] \delta\left[\text{Im}\phi(\tilde{x}^\mu(\sigma))\right] . \tag{7}$$

Eq. (7) defines the Jacobian $J[\phi]$. Given ϕ, the integral (7) selects the surface $\tilde{x}^\mu(\sigma)$ on which ϕ vanishes. Inserting (7) into (5) yields

$$W_{\text{eff}}(\Gamma) = \int \mathcal{D}C_\mu \mathcal{D}\phi \mathcal{D}\phi^* e^{-S} J[\phi] \int \mathcal{D}\tilde{x}^\mu \delta\left[\text{Re}\phi(\tilde{x}^\mu(\sigma))\right] \delta\left[\text{Im}\phi(\tilde{x}^\mu(\sigma))\right] . \tag{8}$$

'he field integration in (8) is over all field configurations $\phi(x^\mu)$ which contain a vortex, while the ıtegral over all surfaces forces $\tilde{x}^\mu$ to lie on the surface $\phi(x^\mu) = 0$. We now reverse the order of the eld integrals and the string integral in (8). This gives

$$W_{\text{eff}} = \int \mathcal{D}\tilde{x}^\mu \int \mathcal{D}C_\mu \mathcal{D}\phi\mathcal{D}\phi^* J[\phi]\delta\left[\text{Re}\phi(\tilde{x}^\mu(\sigma))\right]\delta\left[\text{Im}\phi(\tilde{x}^\mu(\sigma))\right]e^{-S}. \tag{9}$$

'he string integral in (9) is over all surfaces $\tilde{x}^\mu(\sigma)$, while the field integral is over only those field onfigurations $\phi(x^\mu)$ for which $\phi(\tilde{x}^\mu(\sigma)) = 0$.

Eq. (9) has the form (6), with S_{eff} given by

$$e^{-S_{\text{eff}}[\tilde{x}^\mu]} = \int \mathcal{D}C_\mu \mathcal{D}\phi\mathcal{D}\phi^* J[\phi]\delta\left[\text{Re}\phi(\tilde{x}^\mu(\sigma))\right]\delta\left[\text{Im}\phi(\tilde{x}^\mu(\sigma))\right]e^{-S}. \tag{10}$$

'he δ functions in (10) force ϕ to vanish on the surface $\tilde{x}^\mu$. Hence the field integrations are over only ıose configurations which have a vortex at $\tilde{x}^\mu$. This differs from the original field representation (5) f $W_{\text{eff}}(\Gamma)$, where the integrals are over all field configurations which contain a vortex on any sheet.

To calculate W_{eff} we must then evaluate:

l) $J[\phi]$, Eq. (7).

2) The field integration in (10) determining S_{eff}.

3) The integration over all surfaces (9) determining W_{eff}.

Evaluating the Jacobian $J[\phi]$

'he Jacobian $J[\phi]$ in (10) is evaluated for field configurations which vanish on a specific surface $\tilde{x}^\mu(\sigma)$. o distinguish this surface $\tilde{x}^\mu(\sigma)$ from the integration variable in the integral (7) defining $J[\phi]$, we ewrite (7) as

$$J^{-1}[\phi] = \int \mathcal{D}\tilde{y}^\mu \delta\left[\text{Re}\phi(\tilde{y}^\mu(\tau))\right]\delta\left[\text{Im}\phi(\tilde{y}^\mu(\tau))\right], \tag{11}$$

here $\phi(\tilde{x}^\mu(\sigma)) = 0$. The integration over $\tilde{x}^\mu$ defining the action (9) and the integrations defining the acobian in (11) must be carried out in the same way, since they are both determined by the definition f the integral over all surfaces specified in (7). From (11) we see that the Jacobian $J[\phi]$ is itself the ıverse of a "string theory," defined by the integration over all surfaces $\tilde{y}^\mu(\tau)$. The representation)) of the field theory Wilson loop is therefore a ratio of two string theories. String theories contain nomalies,[8] which must not be present in field theories,[7,9]. Hence, in (9), the anomalies of the two ring theories must cancel.

The δ functions in (11) will select those surfaces $\tilde{y}^\mu(\tau)$ which lie in the neighborhood of $\tilde{x}^\mu(\sigma)$. o evaluate (11), we separate $\tilde{y}^\mu(\tau)$ into components lying on the surface $\tilde{x}^\mu(\sigma)$ and components (of ıagnitude ρ) lying along the normal to the surface. The integral over the normal components is etermined by the normal derivatives $\left.\frac{\partial|\phi(\sigma,\rho)|}{\partial\rho}\right|_{\rho=0}$ of the magnitude of the Higgs field evaluated at the ırface $\tilde{x}^\mu$. The integral over the components of $\tilde{y}^\mu$ lying on the surface corresponds to an integration ver coordinate reparameterizations $\sigma \to \sigma(\tau)$ of the surface $\tilde{x}^\mu(\sigma)$. The resulting integral for $J^{-1}[\phi]$ an be written in the factorized form

$$J^{-1}[\phi] = J_\perp^{-1}[\phi]J_{||}^{-1}[\tilde{x}^\mu], \tag{12}$$

here

$$J_\perp^{-1} = \text{Det}_\sigma^{-1}\left[\left(\frac{\partial|\phi(\sigma,\rho)|}{\partial\rho}\right)^2\bigg|_{\rho=0}\right] \equiv \prod_\sigma \left(\frac{\partial|\phi(\sigma,\rho)|}{\partial\rho}\right)^{-2}\bigg|_{\rho=0}. \tag{13}$$

'he product is over all points σ on the worldsheet $\tilde{x}^\mu(\sigma)$. The quantity $J_{||}^{-1}$ in (12) is the integral ver the coordinate parameterizations $\sigma(\tau)$, given by

$$J_{||}^{-1}[\tilde{x}^\mu] = \int \mathcal{D}\sigma \text{Det}_\tau\left[\sqrt{g(\sigma(\tau))}\right], \tag{14}$$

where $\sqrt{g}$ is the square root of the determinant of the induced metric $g_{ab} = \partial_a \bar{x}^\mu \partial_b \bar{x}^\mu$ evaluated on the worldsheet ($\partial_a \equiv \frac{\partial}{\partial \sigma^a}$).

Up to now, we have not specified how either the integral over $\sigma(\tau)$ in (14) or the integral over the parameterizations of the surface $\bar{x}^\mu(\sigma)$ in (9) is to be carried out. The only important thing is that they be done in a consistent way. We have carried out these integrations using the techniques of Polyakov.[8] The result for the integral (14), which has the form of a two dimensional string theory, is:

$$J_{\|}^{-1}[\bar{x}^\mu] = \mathrm{Det}_\sigma^{-1}[-\nabla_\sigma^2]\Delta_{FP} \,. \tag{15}$$

The quantity $-\nabla_\sigma^2$ is the two dimensional Laplacian on the surface $\bar{x}^\mu(\sigma)$,

$$-\nabla_\sigma^2 = -\frac{1}{\sqrt{g}}\partial_a g^{ab}\sqrt{g}\partial_b \,, \tag{16}$$

and

$$\Delta_{FP} \equiv \exp\left\{-\frac{26}{48\pi}\int d^2\sigma \frac{1}{2}(\partial_a \ln\sqrt{g})^2 - \mu\int d^2\sigma\sqrt{g}\right\} \,, \tag{17}$$

is a Faddeev-Popov determinant arising from fixing the nonphysical parameterization degrees of freedom in (14). We have used the notation of Akhmedov et. al.[7] Eqs. (12), (13), and (14) then give $J[\phi]$. All the dependence of $J[\phi]$ on the field ϕ is contained in $J_\perp[\phi]$.

6 Calculating the Field Integrals

The dual theory is an effective theory describing the $q\bar{q}$ interaction at distances greater than the flux tube radius a. The only fluctuations at such distances are string fluctuations described by the integral (9) over all surfaces $\bar{x}^\mu(\sigma)$. The field integrations in (10) determining the effective string interaction must then be evaluated in the steepest descent approximation around the classical solution C_μ^{class}, ϕ^{class}. The boundary condition on this solution is $\phi^{\mathrm{class}}(\bar{x}^\mu(\sigma)) = 0$. The action S evaluated at the classical solution is

$$S^{\mathrm{class}}[\bar{x}^\mu] = S[\bar{x}^\mu, \phi^{\mathrm{class}}, C_\mu^{\mathrm{class}}] \,. \tag{18}$$

The fields ϕ^{class}, C_μ^{class} minimize the action for a fixed location of the vortex sheet $\bar{x}^\mu$.

The result of the steepest descent calculation of $e^{-S_{\mathrm{eff}}[\bar{x}^\mu]}$ around the solution ϕ^{class}, C_μ^{class} is the following:

$$e^{-S_{\mathrm{eff}}[\bar{x}^\mu]} \equiv J_{\|}[\bar{x}^\mu]\int \mathcal{D}C_\mu \mathcal{D}\phi^* \mathcal{D}\phi e^{-S} J_\perp[\phi]\delta\left[\mathrm{Re}\phi(\bar{x}^\mu(\sigma))\right]\delta\left[\mathrm{Im}\phi(\bar{x}^\mu(\sigma))\right] = e^{-S^{\mathrm{class}}}\mathrm{Det}^{-1/2}[G^{-1}]J_{\|}[\bar{x}^\mu] \,, \tag{19}$$

where G^{-1} is the inverse Green's function determined by the quadratic terms in the expansion of the action about the classical solution. The δ functions in (19), which specify the location of the vortex, cause the field integration to produce a Jacobian which cancels $J_\perp$, so that only $J_{\|}$ appears on the right hand side of Eq. (19). In obtaining (19), we have also divided by the integral over vacuum configurations appearing in the denominator of (5).

The effect of the determinant of G^{-1} is to renormalize the parameters in S^{class}. Short distance renormalization effects are cut off in the dual theory at the scale of the string radius a. These renormalizations are unimportant, as all the modes in G^{-1} have masses larger than a^{-1}.

7 Parameterizing the Integral Over All Surfaces

In order to carry out the integration $\mathcal{D}\bar{x}^\mu$ of $e^{-S_{\mathrm{eff}}}$ over all surfaces, it is convenient to choose particular coordinates. We select some fixed sheet $\bar{x}^\mu$, and define vectors $\bar{n}_{\mu A}$, $A = 3, 4$, normal to the sheet, which satisfy the equations

$$\bar{n}_{\mu A}(\sigma)\partial_a \bar{x}^\mu(\sigma) = 0 \,, \quad a = 1, 2 \,, \quad A = 3, 4 \,. \tag{20}$$

For points x^μ close to the sheet $\bar{x}^\mu$ we can write,

$$x^\mu = \bar{x}^\mu(\sigma) + \bar{n}^\mu_A(\sigma)x^A_\perp . \tag{21}$$

Eq. (21) defines the coordinate transformation $x^\mu \to \sigma, x^A_\perp$.

We now use these coordinates to parameterize the surface $\tilde{x}^\mu(\sigma)$. Doing this will allow us to break up the integral (9) over $\tilde{x}^\mu$ into an integral over distinct surfaces and an integral over parameterizations of the surface $\tilde{x}^\mu$. For a given parameterization $\tilde{x}^\mu(\sigma)$, we choose a reparameterization $f(\sigma)$ defined so that

$$\tilde{x}^\mu(f(\sigma)) = \bar{x}^\mu(\sigma) + n^\mu_A(\sigma)\tilde{x}^A_\perp(\sigma) . \tag{22}$$

Eq. (22) requires that the point $\tilde{x}^\mu(f(\sigma))$ lie on the line normal to the surface $\bar{x}^\mu$ at the point $\bar{x}^\mu(\sigma)$. The term $n_{\mu A}(\sigma)\tilde{x}^A_\perp(\sigma)$ then represents the displacement of the surface $\tilde{x}^\mu$ from the surface $\bar{x}^\mu$. We can then write $\tilde{x}^\mu(\sigma)$ as

$$\tilde{x}^\mu(\sigma) = \bar{x}^\mu(\tilde{\sigma}(\sigma)) + n^\mu_A(\tilde{\sigma}(\sigma))\tilde{x}^A_\perp(\tilde{\sigma}(\sigma)) , \tag{23}$$

where $\tilde{\sigma}(\sigma) \equiv f^{-1}(\sigma)$. This allows us to write the integration (9) over $\tilde{x}^\mu(\sigma)$ as an integration over distinct surfaces (labeled by $\tilde{x}^A_\perp$) and an integration over parameterizations $\tilde{\sigma}(\sigma)$. The integral over $\tilde{\sigma}(\sigma)$ produces a factor $\Delta_{FP}\mathrm{Det}^{-1}_\sigma[-\nabla^2_\sigma] = J^{-1}_{||}$, which cancels the factor $J_{||}$ in $e^{-S_{\mathrm{eff}}}$, and we obtain,

$$W_{\mathrm{eff}} = \int \mathcal{D}\tilde{x}^A_\perp e^{-S^{\mathrm{class}}[\tilde{x}^\mu]}\mathrm{Det}^{-1/2}[G^{-1}] . \tag{24}$$

Eq. (24) gives the string representation of the Abelian Higgs model.

The action $S^{\mathrm{class}}[\tilde{x}^\mu]$ does not depend on the parameterization $\tilde{\sigma}(\sigma)$ and hence is expressed in terms only of $\bar{x}^\mu$ and $\tilde{x}^A_\perp$ via (23). This result (24) could also have been obtained by introducing a fixed surface $\bar{x}^\mu$ at an earlier stage and replacing the right hand side of Eq. (7) by the product of $J_\perp[\phi]$ and an integral over $\mathcal{D}\tilde{x}^A_\perp$. We have chosen a more general approach to obtain (24), because we can also derive, from this approach, a string representation which does not contain a fixed surface $\bar{x}^\mu$.

8 The Curvature Expansion

To describe long distance fluctuations, we need S^{class} only for vortex sheets which have a radius of curvature R_F greater than the flux tube radius a. The expansion of S^{class} in the parameter a/R_F is carried out by expanding S^{class} in powers of the extrinsic curvature:

$$\mathcal{K}^A_{ab}(\sigma) = -(\partial_a n_{\mu A}(\sigma))(\partial_b \tilde{x}^\mu(\sigma)) . \tag{25}$$

The $n_{\mu A}$ are normal vectors to the sheet $\tilde{x}^\mu$, satisfying $n^A_\mu(\sigma)\partial_a\tilde{x}^\mu(\sigma) = 0$.

The lowest order term S_0 in this expansion of (18) is the value of the action, evaluated at the solution $\phi^{(0)}$,$C^{(0)}_\mu$ of the approximate classical equations obtained by neglecting terms containing the curvature:

$$S_0 \equiv S[\tilde{x}^\mu, \phi^{(0)}, C^{(0)}_\mu] = \mu \int d^2\sigma \sqrt{g} , \tag{26}$$

where $\mu = \frac{4}{3}\pi\phi_0^2$ is the string tension calculated for a flat vortex. Eq. (26) is the Nambu–Goto action. As is well known, it gives rise to linear Regge trajectories with slope $\alpha' = 1/2\pi\mu$. Using the value $\alpha' \approx .9(\mathrm{GeV})^{-2}$ for the slope of the ρ trajectory gives $\phi_0 \approx 210\mathrm{MeV}$.

The difference,

$$\delta S = S^{\mathrm{class}} - S_0 , \tag{27}$$

evaluated to second order in the extrinsic curvature has the form,

$$\delta S = \beta \int d^2\sigma \sqrt{g}\mathcal{K}^A_{ab}\mathcal{K}^{Aab} , \tag{28}$$

where β is called the rigidity. [10] Eq. (27) gives an expression for β in terms of the Green's function G^{-1} evaluated at the flat solution. Since S^{class} is the value of the action at an exact solution of the equations of motion, and S_0 is its value at an approximate solution, we must have $\beta < 0$. We have evaluated (27) explicitly in the limit $g^2/\lambda \to 0$, and obtain $\beta \to -\pi/2g^2$. This result has been obtained previously in the London limit, where $|\phi|$ is fixed and a short distance cutoff is introduced. [11]

Using the classical action (18), omitting the factor $\mathrm{Det}^{-1/2}[G^{-1}]$ from (24), and choosing $\bar{x}^\mu$ to be a flat sheet, we obtain the following expression for the Wilson loop describing the string fluctuations of superconducting vortices,

$$W_{\text{eff}} = \int \mathcal{D}\bar{x}_\perp^A \exp\left\{-\int d^2\sigma\sqrt{g}\left(\mu + \beta\mathcal{K}_{ab}^A\mathcal{K}^{Aab}\right)\right\}. \tag{29}$$

The integration over $\bar{x}_\perp^A$ must be cut off at distances of the order of the string radius a. (If we refrain from expanding S^{class} in powers of the extrinsic curvature, and use the exact classical action as in (18), such a cutoff might not be necessary, since the unexpanded classical action itself could damp out the short distance $\bar{x}_\perp^A$ fluctuations.)

9 Conclusions

(1) The dual superconducting description of long distance QCD yields the effective string theory given by (24). This has an anomaly free action which, when expanded in powers of the extrinsic curvature, yields (29), the Nambu-Goto action and a rigidity term which is negative. Thus, consequences of string models used to describe Regge trajectories and spectra of hybrid mesons can also be regarded as consequences of a dual superconducting description.
(2) Eq. (29) for W_{eff} does not account for effects due to the boundaries of the string. However, as mentioned at the beginning of this talk, previous calculations [4] of the $q\bar{q}$ potential (with a straight line vortex) are applicable also at small quark–anti-quark separations, where a Coulomb potential develops. At these distances string fluctuations should not be important. We could then combine the previous calculations for a flat vortex with the long distance contributions of string fluctuations to give a more complete treatment of the dual superconductor description of long distance QCD.

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