Brane world scenario in the presence of a non-minimally coupled bulk scalar field

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Abstract. We present our recent work on brane world models with a non-minimally coupled scalar field. In [9] we examined the stability of these models against scalar field perturbations and we discussed possible physical implications, while in [10] we developed a numerical approach for the solution of the Einstein equations with the non-minimally coupled scalar field.

1. Introduction

A possible way to extend four-dimensional physics is to consider extra dimensions. However, our world appears to be four dimensional, thus extra dimensions should be hidden in low energies. Two mechanisms for this are available: a) The Kaluza-Klein Scenario (small compact extra dimensions with compactification scale $M > 1TeV$) and b) The brane world scenario.

According to the brane world scenario, ordinary matter is assumed to be trapped in a 3D submanifold (brane world) that is embedded in a multi-dimensional manifold (bulk). Contrary to ordinary matter, gravitons are allowed to propagate in the bulk. The brane world scenario has the advantage that it predicts new phenomenology even at the TeV scale. Furthermore, it puts on a new basis fundamental problems such as the hierarchy and the cosmological constant problem.

For the localization of ordinary matter on the brane we can mention two mechanisms. According to the first one, fermions are trapped on the brane due to the existence of a topological defect toward the extra dimensions [1] . However, there is a more powerful mechanism [2], for the localization of extended structures of particles which includes: Gauge fields, fermions and bosons with Gauge charge. This mechanism is based on a specific phase structure which is known as the layer phase (Higgs phase along the brane combined with a confinement phase along the extra dimensions), see details in Refs. [3, 4].

The standard brane world scenario with gravity and flat compact extra dimensions is the ADD model [5] (Arkani-Hammed, Dimopoulos and Dvali). In the case of warped extra dimensions we have the first and second Randall-Sundrum models [6].

According to the second Randall-Sundrum model (RS2-model), we have a single brane with a positive energy density (the tension σ), whereas the bulk has a negative five-dimensional cosmological constant Λ. The corresponding Einstein equations have a solution only if a fine-tuning condition is satisfied ($\Lambda = -\frac{\sigma^2}{6}$), in units where $8\pi G_5 = 1$. An extension of the RS2-model with a second negative tension $-\sigma$ brane, is the RS1-model [6]. In this case we have an orbifolded extra dimension of radius r_c. The two branes are fitted to the fixed points of the orbifold, $z = 0$ and $z_c = \pi r_c$ with tensions σ and $-\sigma$ correspondingly. The particles of the standard model are assumed to be trapped on the negative tension brane, which is called visible, while the positive tension brane is called hidden.

There are numerous generalizations of the standard Randall-Sundrum scenario, like models with more than five dimensions, models with topological defects toward the extra dimensions, multibrane models and models with higher order curvature corrections (i.e. Gauss-Bonnet gravity). See for example [1, 7, 8] and references therein.

In our recent papers [9, 10] we have studied a generalization of the RS2-model with a nonminimally coupled bulk scalar field, via an interaction term of the form $-\frac{1}{2}\xi R\phi^2$, where ξ is a dimensionless coupling [1]. In particular, the stability properties of the model are investigated in Ref. [9], and a possible implication of the model to the layer phase mechanism for the localization of ordinary matter on the brane is discussed. Furthermore, in Ref. [10] we solve numerically the Einstein equations with the non-minimally coupled scalar field and the appropriate boundary conditions on the brane. Although our numerical approach is suitable for an arbitrary form of the potential, we have examined the simplest case of a potential $V(\phi) = \lambda\phi^4$ for the scalar field. We showed that, according to the value of the nonminimal coupling ξ, our model possesses three classes of new static solutions (for details see section 2.5).

It is worth to mention, that in the case of brane models with a non-minimally coupled scalar field, K. Tamvakis and collaborators have found analytical solutions by choosing appropriately the potential for the scalar field [11].

Furthermore, implications of the above mentioned model for the stabilization of the extra dimension are presented in [12], while for phenomenological implications the reader may consult [13, 14].

5D models in the framework of Brans-Dicke theory have been also studied. Particularly, in Ref. [15], static analytical solutions are constructed for a special class of potentials, and the stabilization of the extra dimension is discussed.

2. Brane world models with a nonminimally coupled scalar field
2.1. The action of the model
In [9, 10] we have studied brane world models with a nonminimally coupled bulk scalar field. The action of these models is:

$$S = \int d^5x \left(\mathcal{L}_{RSF} + \mathcal{L}_\phi\right) \tag{1}$$

where $d^5x = d^4x\,dz$, and z parameterizes the extra dimension.

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[1] The coupling ξ possesses two characteristic values: a) the minimal coupling for $\xi = 0$ and b) the conformal coupling for $\xi_c = 3/16$

The gravity part of the lagrangian, if we set $8\pi G_5 = 1$ (G_5 is the five-dimensional Newton constant), is given by the equation

$$\mathcal{L}_{RSF} = \sqrt{|g|}\left(F(\phi)R - \Lambda\right) - \sigma\delta(z)\sqrt{|g^{(brane)}|} \tag{2}$$

where R is the five-dimensional Ricci scalar, g is the determinant of the five-dimensional metric tensor g_{MN} ($M, N = 0, 1, ..., 4$), and g^{brane} is the determinant of the induced metric on the brane. We adopt the mostly plus sign convention for the metric [16]. Furthermore, Λ is a negative five dimensional cosmological constant, and σ is the brane tension.

The factor

$$F(\phi) = \frac{1}{2}(1 - \xi\phi^2) \tag{3}$$

corresponds to a nonminimally coupled scalar field with an interaction term of the form $\mathcal{L}_{int} = -\frac{1}{2}\xi R \ \phi^2$, and ξ is a dimensionless coupling. Note that if we set $\xi = 0$ the model is reduced to the RS2-model with a minimally coupled scalar field.

The scalar field part of the lagrangian is

$$\mathcal{L}_\phi = \sqrt{|g|}\left(-\frac{1}{2}g^{MN}\nabla_M\phi\nabla_N\phi - V(\phi)\right) \tag{4}$$

where the potential is assumed to be of the standard form $V(\phi) = \lambda\phi^4$.

2.2. An obvious solution of the model in the case of the fine tuning $\Lambda = \frac{-\sigma^2}{6}$

If the fine tuning $\Lambda = \frac{-\sigma^2}{6}$ is satisfied, the Einstein equations have a *static solution* of the form

$$ds^2 = a(z)^2(-dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2) + dz^2 \tag{5}$$

where $a(z) = e^{-k|z|}$ is the warp factor and $k = \sqrt{\frac{-\Lambda}{6}}$.

We recognize the well known solution of the RS2-model. However, in the case we examine there is a difference, as for appropriate values of the dimensionless coupling ξ this solution is unstable against perturbations of the scalar field [9], see for details the next subsection.

2.3. Spectrum of the scalar field in the presence of the RS2-metric

In Refs. [9, 10] we studied the spectrum of scalar field in the background of the RS2-vacuum. In this section we give the results of our study.

If we set

$$w = sgn(z)\frac{(e^{k|z|} - 1)}{k} \tag{6}$$

the RS2-metric of Eq. (5) can be put into the manifestly conformal to the five-dimensional Minkowski space form

$$ds^2 = \alpha(w)^2(-dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 + dw^2) \tag{7}$$

where

$$\alpha(w) = \frac{1}{k|w| + 1} \tag{8}$$

If we consider a perturbation $\hat{\phi}$ around the scalar field vacuum ($\phi = 0$) we obtained the corresponding linearized equation:

$$\frac{1}{\sqrt{|g|}} \, \partial_M \left[\sqrt{|g|} g^{MN} \partial_N \hat{\phi}(x, w) \right] + \xi R(w) \, \hat{\phi}(x, w) = 0 \tag{9}$$

We can set

$$\hat{\phi}(x, w) = e^{ipx} \frac{\psi(w)}{\alpha^{3/2}(w)} \tag{10}$$

where $\alpha(w) = 1/(k|w| + 1)$, and $m^2 = p_\mu p^\mu$ is the effective four dimensional mass.

The function $\psi(w)$ satisfies the Schrondiger-like equation

$$-\psi''(w) + \left[V(w) - m^2 \right] \psi(w) = 0 \tag{11}$$

where the potential $V(w)$ is equal to

$$V(w) = \frac{(\alpha^{3/2}(w))''}{\alpha^{3/2}(w)} + \xi \alpha^2(w) R(w) \tag{12}$$

From Eqs. (8) and (12) we get

$$V(w) = -16k(\xi - \xi_c) \left(-\delta(w) + \frac{5k}{4(k|w| + 1)^2} \right) \tag{13}$$

where $\xi_c = 3/16$ is the five dimensional conformal coupling.

Note that the coefficient in front of the potential change sign when ξ crosses the five dimensional conformal coupling. This result implies that the potential has two characteristic forms, as we see in the left-hand panel ($\xi < \xi_c$) and the right-hand panel ($\xi > \xi_c$) of Fig. 10 in [10].

If we use the above results, Eqs. (11) and (13), we can show (for details see Refs. [9, 10]) that:

(i) For $\xi < 0$ there is a unique tachyon mode, localized on the brane, plus a continuous spectrum of positive energy states.

(ii) For $\xi > \xi_c$ there is at least one tachyon mode or more than one, depending on the value of ξ, plus a continuous spectrum of positive energy states.

(iii) For $0 < \xi \leq \xi_c$ there are no tachyon modes. There is only a continuous spectrum of positive energy states.

(iv) For $\xi = 0$ there is a zero mode, plus a continuous spectrum of positive energy states.

The above results implies an instability of the RS2-metric for $\xi > \xi_c$ or $\xi < 0$.

2.4. Einstein equations with the non-minimally coupled scalar field

The Einstein equations, which correspond to the action of Eq. (1) are

$$G_{MN} + \Lambda \, g_{MN} + \sigma \delta(z) \frac{\sqrt{|g^{(brane)}|}}{\sqrt{|g|}} g_{\mu\nu} \delta_M^\mu \delta_N^\nu = T_{MN}^{(\phi)} \tag{14}$$

where the energy momentum tensor for the scalar field is

$$T_{MN}^{(\phi)} = \nabla_M\phi\nabla_N\phi - g_{MN}[\frac{1}{2}g^{P\Sigma}\nabla_P\phi\nabla_\Sigma\phi + V(\phi)] + 2\nabla_M\nabla_N F(\phi) - 2g_{MN}\Box F(\phi) + (1-2F(\phi))G_{MN}$$
(15)

The equation of motion for the scalar field is

$$\Box\phi + \frac{\partial F(\phi)}{\partial\phi}R - \frac{\partial V(\phi)}{\partial\phi} = 0$$
(16)

The above equation is not independent of the Einstein equations (7), as it is equivalent to the conservation equation $\nabla^M T_{MN}^{(\phi)} = 0$, where $T_{MN}^{(\phi)}$ is given by Eq. (15).

We are looking for static solutions of the form

$$ds^2 = a^2(z)(-dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2) + dz^2, \quad \phi = \phi(z)$$
(17)

We obtain (for detail see Ref. [10]) the following second order differential equations:

$$3(1 - \xi\phi^2(z))A''(z) + (1-2\xi)\phi'(z)^2 - 2\xi\phi(z)\phi''(z) + 2\xi A'(z)\phi(z)\phi'(z) + \sigma\delta(z) = 0$$
(18)

$$-\phi''(z) - 4A'(z)\phi'(z) - \xi\left(8A''(z) + 20A'(z)^2\right)\phi(z) + V'(\Phi) = 0$$
(19)

and the constraint equation

$$6(1 - \xi\phi^2(z))A'(z)^2 + \Lambda - \frac{1}{2}\phi'(z)^2 + V(\phi(z)) - 8\xi A'(z)\phi(z)\phi'(z) = 0$$
(20)

Note that only two of the above three equations are independent.

As this system is complicated we will not look for analytical solutions, but we will try to solve it numerically.

For the numerical integration of these equations it is necessary to know the values of $A(0)$, $\phi(0)$, $A'(0)$ and $\phi'(0)$. These values are determined by the junction conditions (see Eqs. (18), (19) and the constraint equation.

According to the junction conditions

$$A'(0) = \frac{-\sigma}{(6 - 6\xi\phi(0)^2 + 32\xi^2\phi(0)^2)}$$
(21)

$$\phi'(0) = \frac{8\xi\sigma\phi(0)}{(6 - 6\xi\phi(0)^2 + 32\xi^2\phi(0)^2)}$$
(22)

while $\phi(0)$ can be obtain by the following sixth order algebraic equation:

$$\frac{\sigma^2}{(6 - 6\xi\phi(0)^2 + 32\xi^2\phi(0)^2)} + \Lambda + V(\phi(0)) = 0$$
(23)

2.5. Numerical solutions

For the determination of the two unknown functions $A(z)$ and $\phi(z)$ we can solve the system of second order differential equations (18) and (19) for $z \geq 0$ numerically. In order to integrate it is necessary to know the values of $A(0)$, $\phi(0)$, $A'(0)$ and $\phi'(0)$. If we assume that the warp factor is normalized to unity on the brane (or $a(0) = 1$) we find that $A(0) = 0$ (note that $a(z) = e^{A(z)}$). The value of $\phi(0)$ is obtained by solving Eq. (22), and the values of $A'(0^+)$ and $\phi'(0^+)$ can be found from Eqs. (20) and (21). Then it is an easy task to use a routine of Fortran or Mathematica to extract the numerical solutions.

The model we examine has four independent parameters $\xi, \lambda, \Lambda, \sigma$. We kept fixed the parameters λ, Λ, σ, assuming that the fine tuning $\Lambda = -\frac{\sigma^2}{6}$ is satisfied, and we varied the parameter ξ. Depending on the value of ξ we obtained three classes of numerical solutions with different characteristics:

(i) For $\xi < 0$ the warp factor exhibits a naked singularity at finite proper distance z_s in the bulk, while the scalar field $\phi(z)$ is almost constant near the brane, and tends to infinity as z tends to the singularity point in the bulk. See Figs. 1,2,3 in [10].

(ii) For $\xi > \xi_c$ the warp factor $a(z)$ is of the order of unity in a small region near the brane and increases exponentially ($a(z) \to e^{kz}$ with $k = \sqrt{\frac{-\Lambda}{6}}$), as $z \to +\infty$, while the scalar field $\phi(z)$ is nonzero on the brane and tends rapidly to zero in the bulk. See Figs. 4,5,6 in [10].

(iii) For $0 < \xi < \xi_c$ the warp factor $a(z)$ of this solution tends rapidly to infinity (faster than case (2)), also the scalar field $\phi(z)$ is nonzero on the brane and tends rapidly to infinity in the bulk. Contrary to case (2), where the space-time is asymptotically AdS_5, in this case the scalar curvature tends to infinity. See Fig. 7 in [10].

Also we investigated what happens when the fine tuning is violated, and we found that in appropriate regions of the parameter space, the three classes of solutions we described above are preserved. However, there are regions of the parameters where there are no static solutions (or Eq. (22) has no real solutions). A thorough investigation of these regions is very extended and it is beyond the scope of our work.

Furthermore, in [10] we have examined the stability of the above mentioned solutions and we obtained that the first and second classes are unstable against scalar field perturbations, while the third class is stable.

Separately, we have examined the case of conformal coupling ξ_c, and we saw that the corresponding solutions are the form of the second class (see Fig. 1 for a tensionless brane). However, in this case the solutions are found to be stable. Possible physical implications are discussed in [10].

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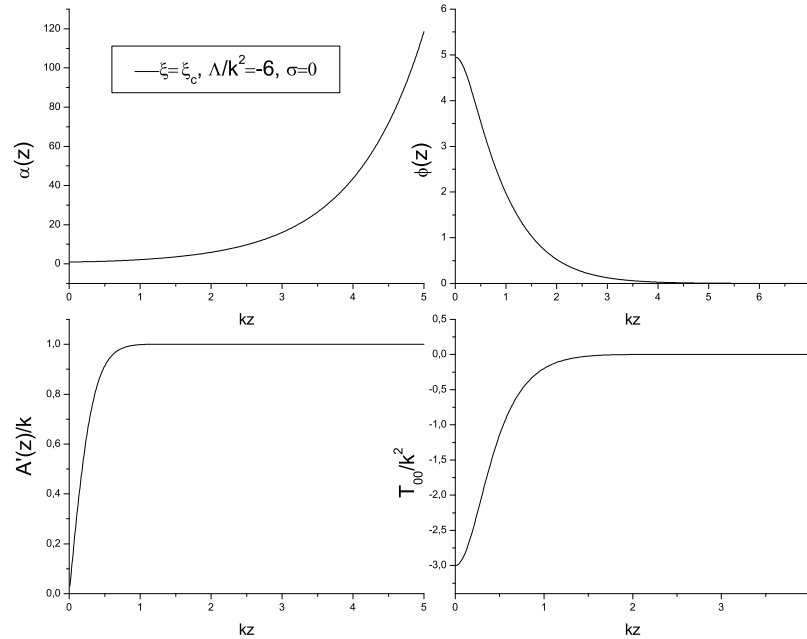
Figure 1. $a(z)$, $\phi(z)$, $A'(z)/k$ and $T_{00}(z)/k^2$, for $\xi = \xi_c$, as a function of kz for $\sigma = 0$ and $\lambda/k^2 = 0.01$, where $k = \sqrt{\frac{-\Lambda}{6}}$.

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