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Decoupling Limits in Effective Field Theories via Higher Dimensional Operators

Andrea Quadri

Istituto Nazionale di Fisica Nucleare, Sez. di Milano, Via Celoria 16, I-20133 Milan, Italy; andrea.quadri@mi.infn.it

Abstract: The non-decoupling effects of heavy scalars and vector fields play an important role in the indirect search for Beyond the Standard Model (BSM) physics at the LHC. By exploiting some new differential equations for the 1-PI amplitudes, we show that such non-decoupling effects are absent for quite a general class of effective field theories involving dimension six two-derivative and dimension eight four-derivative operators, once the resummation in certain BSM couplings is taken into account and some particular regimes of the relevant couplings are considered.

Keywords: renormalization; models beyond the standard model; extensions of electroweak Higgs sector

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1. Introduction

In the absence of any direct signal of new physics at the LHC, the search for Beyond the Standard Model (BSM) effects can be addressed within an Effective Field Theory (EFT) approach [1–11], either in terms of Standard Model Effective Theories (SMEFT) or Higgs Effective Field Theory (HEFT) (see e.g., the reviews in references [9,12]).

The high-energy dynamics can, in fact, induce measurable effects in low-energy observables, despite the fact that new particles are too heavy to be directly detected in experiments.

This is due to the so called non-decoupling effects (see e.g., [13–18]), induced by loop corrections, that survive in the large mass limit of the BSM particles living in the theory assumed to be valid at high energies.

There is no fundamental reason why the latter theory should be power-counting renormalizable. Therefore, it is interesting to investigate whether some higher-dimensional operators in that theory might affect the non-decouping contributions to the low-energy physical observables one can directly measure at colliders.

In the usual EFT treatment, computations are mostly limited to the first few terms in the small coupling expansion. In the present study, we will, on the contrary, consider a particular set of dimension six two-derivative and dimension eight four-derivative operators, for which a full resummation is possible.

The impact of these operators is quite dramatic, since, in some particular regimes, a complete decoupling of high-energy dynamics from the low-energy observables occurs.

This result holds true for the fully resummed amplitudes that exhibit a qualitatively different behavior from their small coupling expansion.

The technical tool that allows one to study such a regime is the use of dynamical (i.e., propagating inside loops) gauge-invariant variables.

The construction of the gauge-invariant dynamical counter-part of a scalar particle has been studied in [19–22].

In the present study, we extend the analysis to the case of vector fields. For the sake of simplicity, we will only work in the Landau gauge. The formalism in a generic R_ξ-gauge is technically more involved but does not change the physical content of the analysis. This will be presented elsewhere.

The treatment of fermions will also be presented in a separate publication, the reason being that, for fermions, it is not always possible to carry out a field transformation implementing a change in variables to their gauge-invariant description. This can be achieved only if the fermionic fields have some specific charge under the relevant gauge group, allowing for the building of a gauge-invariant combination out of the fermionic multiplet and the scalar containing the Higgs mode.

This is at variance with scalars and gauge fields, for which the construction is always possible, provided that spontaneous symmetry breaking happens in the model at hand.

We will work within the Algebraic Renormalization approach [23–25].

The key remark is that some operators, that in the ordinary formalism modify both the quadratic and the interaction terms in the Lagrangian, are represented by purely quadratic contributions if gauge-invariant variables are used.

Therefore, they only affect propagators and, consequently, in some particular cases, one can write down a differential equation controlling the dependence of the one-particle irreducible (1-PI) amplitudes on the BSM couplings [20].

These differential equations, in turn, can be exactly solved and lead to homogeneous Euler functions in the relevant couplings. This result holds true for all orders in perturbation theory and provides useful information on the structure of the fully resummed amplitudes.

In particular, one can easily identify some regimes in which complete decoupling happens. In those regimes, the small coupling expansion does not make any sense.

From a physical point of view, these results cast a shadow on the feasibility of extracting physical information from non-decoupling effects, at least within perturbation theory. It might, in fact, be that the physically relevant high-energy dynamics are affected by the presence of such higher-dimensional operators.

In that case, perturbative computations, that are limited to the first few terms in the small coupling expansion, are quite misleading, since they point to low-energy effects that are, indeed, not present in the full theory.

This study is organized as follows. In Section 2, we set our notations and consider, for illustration purposes, a simple Higgs portal model, connecting a SU(2) spontaneously broken gauge theory at high energy to the SM. In Section 3, the gauge-invariant variables for the scalar and the vector fields are constructed. In Section 4, the differential equations for the 1-PI amplitudes are derived and the non-decoupling limit is analyzed. Finally, conclusions are presented in Section 5. Appendix A collects the functional identities of the theory, while Appendix B.2 contains the derivation of the propagators in the Landau gauge.

2. A Simple Higgs Portal Model

For the sake of definiteness, we will discuss a simple Higgs portal model with Lagrangian

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{ext} \,, \tag{1}$$

where $\mathcal{L}_{SM}$ is the SM Lagrangian and $\mathcal{L}_{ext}$ is given by

$$\begin{aligned}\mathcal{L}_{ext} = & -\frac{1}{4}G^2_{a\mu\nu} + \frac{1}{4}\mathrm{Tr}(D^\mu\phi)^\dagger D_\mu\phi - \frac{\lambda}{2}\left[\frac{1}{2}\mathrm{Tr}(\phi^\dagger\phi) - v^2\right]^2 \\ & + g_1 \mathrm{Tr}\left[(\phi^\dagger\phi) - v^2\right]\Phi^\dagger\Phi \,. \end{aligned}\tag{2}$$

Φ denotes the SM Higgs doublet. The model describes a SU(2) spontaneously broken theory of massive gauge fields and one physical scalar at high energy. The high-energy scale Λ is set by the v.e.v. of the field ϕ, denoted by v. The extra gauge fields and the scalar are singlets under the electroweak $SU(2)_L \times U(1)_Y$ group. The coupling to the low-energy SM dynamics happens via the interaction in the second line of Equation (2).

λ is the quartic potential of the high-energy scalar field, while g_1 is the portal coupling describing the simplest interaction between the high-energy scalar and the Higgs field.

The singlet gauge fields are $A_\mu = A_{a\mu}\frac{\tau_a}{2}$, τ_a being the Pauli matrices, while ϕ is represented in matrix form by $\phi = \phi_0 + i\tau_a\phi_a$, $\phi_0 = v + \sigma$. v is the vacuum expectation value of ϕ_0. ϕ_a are the high-energy pseudo-Goldstone fields.

Under an infinitesimal gauge transformation of parameters α_a, the fields transform as follows (g is the coupling constant of the extra SU(2) group):

$$\delta A_{a\mu} = \partial_\mu \alpha_a + g\epsilon_{abc}A_{b\mu}\alpha_c\,, \quad \delta\phi_a = \frac{g}{2}\phi_0\alpha_a + \frac{g}{2}\epsilon_{abc}\phi_b\alpha_c\,, \quad \delta\phi_0 = -\frac{g}{2}\alpha_a\phi_a\,. \tag{3}$$

We also define the field strength in the usual way:

$$G_{a\mu\nu} = \partial_\mu A_{a\nu} - \partial_\nu A_{a\mu} + g\epsilon_{abc}A_{b\mu}A_{c\nu}\,. \tag{4}$$

The covariant derivative $D_\mu\phi$ is defined by

$$D_\mu\phi = \partial_\mu\phi - igA_{a\mu}\frac{\tau_a}{2}\phi\,. \tag{5}$$

Of course, in a BSM approach, other operators can be considered, yet the essence of our analysis is unaffected by the particular choice of such operators, so we will limit ourselves to the simplest case of Equation (2).

This theory is an example of the so-called Higgs-portal models, see, e.g., refs. [10,26–35]. The extended sector affects the low-energy physics via loop effects, so one might hope to extract some signals of BSM physics via non-decoupling effects (i.e., contributions that survive in the large mass limit of the extra BSM particles).

If only power-counting renormalizable interactions are allowed, a detailed analysis of these models can be consistently studied [16,17,35].

Yet there is no fundamental reason why only dimension four operators should enter into BSM physics.

In the present study, we will show that, if some suitable dimension six and eight operators are introduced, non-decoupling effects vanish once the resummed amplitudes are considered.

3. Gauge-Invariant Variables

In a recent series of papers [19–22,36,37], the construction of a dynamical gauge-invariant field for the scalar mode was presented. For the sake of completeness, we report here a detailed discussion of the main results.

The key idea is to introduce quantum fields that are gauge-invariant and are in one-to-one correspondence with the original fields of the theory.

Let us consider the field ϕ. Its gauge-invariant counter-part is the composite operator

$$\frac{1}{4v}\mathrm{Tr}(\phi^\dagger\phi) - \frac{v}{2} = \sigma + \ldots \tag{6}$$

where dots stand for higher dimensional terms in the fields. The combination in the l.h.s. of the above equation is gauge-invariant and the normalization is chosen in such a way that, in the linearized approximation, it reduces to the real scalar σ.

One could try to study a model where the path-integral is carried out over the original fields σ, ϕ_a, while the composite operator in Equation (6) is defined by coupling it to an external source $\beta_{\phi^\dagger\phi}$. This approach has been widely studied in refs. [38–40].

On the other hand, one could try to construct a model where the gauge-invariant field becomes propagating, i.e., the path-integral is carried out over those fields. For that purpose, one makes use of the Lagrange multiplier technique as presented in Refs. [19–22,36,37].

Therefore, we introduce the gauge-invariant field h together with the Lagrange multiplier field X. By being on-shellwith X, h must reduce to the gauge-invariant combination in the l.h.s. of Equation (6), i.e.,

$$h \sim \frac{1}{4v}\mathrm{Tr}(\phi^\dagger\phi) - \frac{v}{2} = \sigma + \ldots \tag{7}$$

where the dots stand for higher dimensional terms in the fields and $\sim$ denotes on-shell equivalence.

In the relevant scalar sector, the path integral is originally over σ, ϕ_a; then, it must be carried out over σ, ϕ_a and additionally h, X.

This procedure is best implemented in a standard way by the Lagrange multiplier technique in the BRST formalism. For that purpose, we introduce the set $\bar{c}, c$ of antighost and ghost fields [20] associated with the above field redefinition. The constraint BRST differential $\tilde{s}$ reads

$$\tilde{s}\bar{c} = h - \frac{1}{4v}\mathrm{Tr}(\phi^\dagger\phi) + \frac{v}{2}\,, \quad \tilde{s}X = c\,, \quad \tilde{s}c = 0\,, \quad \tilde{s}h = 0\,. \tag{8}$$

Notice that $\tilde{s}$ is nilpotent due to the gauge invariance of the right hand side (r.h.s.) of Equation (7). It anticommutes with the ordinary BRST differential s associated with the gauge group. Under s, all fields $X, h, \bar{c}, c$ are invariant, since they are singlet under both the electroweak $SU(2)_L \times U(1)_Y$ group and the high-energy BSM $SU(2)$ group. We denote by $s' = s + \tilde{s}$ the full BRST differential of the model.

Then, we add to the action of the model in the conventional formalism the following BRST-exact term ($m^2 = 4\lambda v^2$ is the mass of the σ-field in Equation (2)):

$$\begin{aligned} S_{\text{aux,scalar}} &= \tilde{s}\int d^4x\,\Big[\bar{c}(\Box + m^2)X\Big] = s'\int d^4x\,\Big[\bar{c}(\Box + m^2)X\Big] \\ &= \int d^4x\,\Big\{X(\Box + m^2)\Big[h - \frac{1}{4v}\mathrm{Tr}(\phi^\dagger\phi) + \frac{v}{2}\Big] - \bar{c}(\Box + m^2)c\Big\}\,. \end{aligned} \tag{9}$$

The (local) physical observables of the theory are identified by the cohomology of the full BRST differential s' in the sector with zero ghost number, i.e., two operators $\mathcal{O}, \mathcal{O}'$ are physically equivalent if they differ by a BRST-exact term $s'\mathcal{R}$, $\mathcal{O}' = \mathcal{O} + s'\mathcal{R}$ [41].

The physical content of the theory is therefore not affected by the introduction of $S_{\text{aux,vect}}$ in Equation (20), since the latter is a BRST-exact term with ghost number zero.

Integrating out the extra degrees of freedom $h, \bar{c}, c$ in the path-integral provides a useful insight into the mechanism at work. One finds

$$\begin{aligned} \int \mathcal{D}\bar{c}\mathcal{D}c\mathcal{D}X \exp\Big(iS_{\text{aux,scalar}}\Big) &= \int \mathcal{D}\bar{c}\mathcal{D}c \exp\Big(-i\int d^4x\,\bar{c}(\Box + m^2)c\Big) \\ &\quad \times \int \mathcal{D}X \exp\Big(i\int d^4x\; X(\Box + m^2)\Big[h - \frac{1}{4v}\mathrm{Tr}(\phi^\dagger\phi) + \frac{v}{2}\Big]\Big) \\ &\sim \int \mathcal{D}\bar{c}\mathcal{D}c \exp\Big(-i\int d^4\,\bar{c}(\Box + m^2)c\Big) \\ &\qquad \times \delta\Big[(\Box + m^2)\Big[h - \frac{1}{4v}\mathrm{Tr}(\phi^\dagger\phi) + \frac{v}{2}\Big] \\ &\sim \det(\Box + m^2))\frac{1}{|\det(\Box + m^2)|}\delta\Big[h - \frac{1}{4v}\mathrm{Tr}(\phi^\dagger\phi) + \frac{v}{2}\Big] \\ &\sim \delta\Big[h - \frac{1}{4v}\mathrm{Tr}(\phi^\dagger\phi) + \frac{v}{2}\Big]\,, \end{aligned} \tag{10}$$

i.e., the determinant arising from the δ-function is exactly compensated (modulo inessential multiplicative factors) by the integration over the Grassmann variables $\bar{c}, c$.

Another way to approach the problem is to go on shell with X in Equation (9). One obtains a Klein–Gordon equation:

$$(\Box + m^2)\Big[h - \frac{1}{4v}\mathrm{Tr}(\phi^\dagger\phi) + \frac{v}{2}\Big] \Rightarrow h = \frac{1}{4v}\mathrm{Tr}(\phi^\dagger\phi) - \frac{v}{2} + \eta\,, \tag{11}$$

with η being a free scalar field with squared mass m^2 whose correlators can be proven to vanish in perturbation theory [37]. Formally, this follows from the cancellation of the determinants in Equation (10), giving back the constraint in the last of Equations (10). A detailed rigorous analysis of the on-shell reduction is presented in ref. [37]. For this reason, the η mode can be safely set to zero in the following discussion.

So far, the introduction of the extra fields $h, X, \bar{c}, c$ has not changed the physical content of the model. One can prove that the physical observables are exactly the same and the S-matrix elements are those of the original theory, as follows from the previously reported general cohomological argument that the classical action has been modified by a BRST-exact term. Diagrammatically, this can be understood as follows. One can diagonalize the quadratic part in the σ, X-sector by the linear field redefinition

$$\sigma = \sigma' + X_1 + h\,. \tag{12}$$

The propagators of σ' and X_1 have an overall opposite sign (see Equation (A12)). Diagrams involving σ' and X_1 cancel out against each other. The remaining diagrams give rise to different off-shell amplitudes than in the conventional formalism, yet once one changes toon-shell, one recovers the same physical observables as the original theory, as guaranteed by the BRST invariance of the theory and by the fact that $S_{aux,scalar}$ is BRST-exact.

Now, the biggest advantage of the extra field formalism becomes apparent.

We remark that quadratic terms in the gauge-invariant field h will only affect the propagator of h (remember that h is a propagating field, over which the path-integral is carried out) and thus one can derive a scaling equation for the one-particle (1-PI) irreducible amplitudes by introducing a suitable differential operator whose eigenvector is the h-propagator itself. The differential operator characterizes the h-propagator as a homogeneous Euler function of weight -1, while 1-PI amplitudes with l_h h-propagators will be Euler functions of weight $-l_h$.

Adding such quadratic terms amounts, of course, to changing the physical content of the theory. The corresponding operators in the conventional formalism are obtained by being on-shell with the extra fields and these give rise to complicated dimension six operators that affect both the quadratic terms and the interaction vertices. For those operators, resummation in their BSM couplings becomes a hard (if not impossible) task, while, if one uses gauge-invariant fields, resummation in the BSM couplings under discussion is a simple consequence of the scaling differential equations, as will be shown in Section 4.

To be more specific, one can add the quadratic mass and kinetic terms to the classical action

$$\int d^4x \Big(-\frac{M^2 - m^2}{2} h^2 - \frac{z}{2} h \Box h \Big)\,. \tag{13}$$

These are physical gauge-invariant operators and they modify the physical content of the theory.

By substituting back the solution for h in Equation (11) at $\eta = 0$ into the classical vertex functional Equation (A2), we obtain their counter-parts in the conventional formalism:

$$\int d^4x \Big[-\frac{M^2}{32v^2} \Big(\mathrm{Tr}(\phi^\dagger \phi) - 2v \Big)^2 - \frac{z}{32v^2} \mathrm{Tr}(\phi^\dagger \phi) \Box \mathrm{Tr}(\phi^\dagger \phi) \Big]\,. \tag{14}$$

We remark that the m^2-dependent term cancels out against the corresponding contribution in the classical action in Equation (A2), i.e., the only physical parameters are M and z. The cancellations involving m^2 have been discussed in ref. [37].

Notice that a dimension six operator has appeared via the kinetic term in h. This will play a crucial role in the construction of the decoupling limit.

As anticipated, the ordinary formalism operators in Equations (14) yield a complicated set of interactions that is hard to treat beyond the small coupling regime. On the other hand, the scaling differential equation for the operators in Equation (14) will give us, for free, the resummation in z (and important phenomenological consequences for the SM effective field theory program, as we will explain in the next sections).

Gauge Field

We now move to the construction of a dynamical gauge-invariant variable for the massive gauge field $A_{a\mu}$. For the sake of simplicity, we will consider the Landau gauge. The complete analysis in an arbitrary R_ξ-gauge will be presented elsewhere.

In order to set the stage, we first need to fix the gauge *à la* BRST, so we add to the Lagrangian in Equation (2) the following gauge-fixing term:

$$S_{\text{g.f. + ghost}} = \int d^4x \Big[-b_a \partial A_a + \bar{c}_a \partial^\mu D_\mu c_a \Big] . \tag{15}$$

The covariant derivative acts on the ghost fields c_a as

$$D_\mu c_a = \partial_\mu c_a + g\epsilon_{abc} A_{b\mu} c_c . \tag{16}$$

The relevant gauge-invariant counter-part of the gauge field A_μ is

$$\begin{aligned} a_\mu &\sim \frac{i}{gv^2}\Big[2\phi^\dagger D_\mu \phi - \partial_\mu(\phi^\dagger\phi)\Big] \\ &= \Big(A_{a\mu} - \frac{2}{gv}\partial_\mu \phi_a\Big)\tau_a + \ldots \end{aligned} \tag{17}$$

where the dots stand for terms of higher dimension in the fields.

The procedure to enforce the on-shell constraint in Equation (17) follows the same lines as in the scalar case. The additional anti-ghost $\bar{c}_\mu$ is now a vector field, $\bar{c}_\mu = \bar{c}_{a\mu}\frac{\tau_a}{2}$, transforming under the constraint BRST differential $\tilde{s}$ as

$$\tilde{s}\bar{c}_\mu = a_\mu - \frac{i}{gv^2}\Big[2\phi^\dagger D_\mu \phi - \partial_\mu(\phi^\dagger\phi)\Big] . \tag{18}$$

The constraint ghost and Lagrange multiplier are, respectively, $c_\mu = c_{a\mu}\frac{\tau_a}{2}$ and $X_\mu = X_{a\mu}\frac{\tau_a}{2}$. They form a BRST doublet [41–43] under $\tilde{s}$:

$$\tilde{s}X_\mu = c_\mu , \qquad \tilde{s}c_\mu = 0 . \tag{19}$$

The nilpotency of $\tilde{s}$ again follows from the gauge invariance of the r.h.s. of Equation (17).

The additional terms to be added to the action are

$$\begin{aligned} S_{\text{aux,vect}} &= \int d^4x\, \tilde{s}\mathrm{Tr}\Big(\bar{c}_\mu \Sigma^{\mu\nu} X_\nu\Big) \\ &= \int d^4x\, \mathrm{Tr}\Big\{ -\bar{c}_\mu \Sigma^{\mu\nu} c_\nu + X_\mu \Sigma^{\mu\nu}\Big[a_\nu - \frac{i}{gv^2}\Big(2\phi^\dagger D_\nu\phi - \partial_\nu(\phi^\dagger\phi)\Big)\Big]\Big\} , \end{aligned} \tag{20}$$

where the symmetric tensor $\Sigma^{\mu\nu}$ denotes the two-point 1-PI amplitude of the gauge field A_μ in the Landau gauge and is given by

$$\Sigma^{\mu\nu} = (\Box g^{\mu\nu} - \partial^\mu\partial^\nu) + M_A^2 g^{\mu\nu} , \tag{21}$$

with $M_A = gv/2$ being the mass of the vector field.

In the Landau gauge, the gauge field propagator is transverse and the pseudo-Goldstone field is massless. The physical unitarity in this gauge has been studied in detail in ref. [44].

The quadratic part in the relevant sector reads

$$\int d^4x \Big[\frac{1}{2}A_{a\mu}(\Box g^{\mu\nu} - \partial^\mu\partial^\nu)A_{a\nu} + \frac{M_A^2}{2}\Big(A_{a\mu} - \frac{1}{M_A}\partial_\mu\phi_a\Big)^2 - b_a\partial A_a\Big]. \tag{22}$$

The propagators can be obtained by diagonalizing the two-point 1-PI amplitudes in the sector spanned by $A_{a\mu}, a_{a\mu}, X_{a\mu}, b_a, \phi_a$. The derivation is presented in Appendix B.2.

We notice that the mass eigenstate $a'_{a\mu}$ in Equation (A18) is also BRST-invariant, since according to Equation (A14), it is given by

$$a'_{a\mu} = a_{a\mu} - \frac{1}{M_A^2}\partial_\mu b_a\,, \tag{23}$$

i.e., a linear combination of gauge-invariant variables. Hence, one can freely add an independent mass term

$$\int d^4x \frac{M'^2 - M_A^2}{2} {a'_{a\mu}}^2 \tag{24}$$

as well as a transverse combination

$$\int d^4x \frac{z'}{2} a'_{a\mu}(\Box g^{\mu\nu} - \partial^\mu\partial^\nu)a'_{a\nu} \tag{25}$$

while preserving gauge-invariance. M' and z' are additional BSM couplings, as well as M and z in Equation (13).

Other choices involving the longitudinal parts are also possible (e.g., $(\partial a'_a)^2$), yet when these operators are switched on and one moves to on-shell, quadratic higher derivative terms in the pseudo-Goldstone fields arise and, consequently, negative norm states are theoreticallyintroduced [45,46]. For this reason, we limit to the contributions in Equations (24) and (25).

The a'_μ-propagator is correspondingly modified as

$$\Delta_{a'_{a\mu}a'_{b\nu}} = \frac{i\delta_{ab}}{-(1+z')p^2 + M'^2}T_{\mu\nu} + \frac{i\delta_{ab}}{M'^2}L_{\mu\nu}\,. \tag{26}$$

while all other propagators are unaffected.

Even at $z' = 0$, the shift in the mass term induces a violation of power-counting renormalizability, since now the A_μ-propagator develops a constant longitudinal part

$$\Delta_{A_{a\mu}A_{b\nu}} = \frac{i\delta_{ab}}{-p^2 + M'^2}T_{\mu\nu} + i\delta_{ab}\frac{M'^2 - M_A^2}{M_A^2 M'^2}L_{\mu\nu} \tag{27}$$

unless $M' = M_A$.

The violation of power-counting renormalizability by the a'_μ-mass term can be understood by noticing that there are two contributions to the mass term

$$\begin{aligned}\int d^4x \frac{M'^2 - M_A^2}{2}{a'_{a\mu}}^2 &= \int d^4x \frac{M'^2 - M_A^2}{2}\Big(a_{a\mu} - \frac{1}{M_A^2}\partial_\mu b_a\Big)^2 \\ &= \int d^4x \Big(\frac{M'^2 - M_A^2}{2}a_{a\mu}^2 - \frac{M'^2 - M_A^2}{M_A^2}a_{a\mu}\partial^\mu b_a + \frac{M'^2 - M_A^2}{2M_A^4}\partial^\mu b_a\partial_\mu b_a\Big)\,. \end{aligned} \tag{28}$$

As a consequence of the gauge invariance of a_μ, the last two terms in the above equation can be removed by adding the BRST-exact term

$$\frac{M'^2 - M_A^2}{M_A^2}\int d^4x \tilde{s}\Big[\partial_\mu \bar{c}_a\Big(a_{a\mu} - \frac{1}{2M_A^2}\partial_\mu b_a\Big)\Big]\,. \tag{29}$$

and they are thus unphysical. The first term is the on-shell equivalent of the dim.6 operator

$$\int d^4x \, \frac{M'^2 - M_A^2}{2} {a_{a\mu}}^2 = \int d^4x (M'^2 - M_A^2) \, \mathrm{Tr} \, a_\mu^2 \sim \\ \frac{M'^2 - M_A^2}{4v^2 M_A^2} \int d^4x \, \mathrm{Tr} \left\{ \phi^\dagger \phi \left[4D^\mu \phi \, (D_\mu \phi)^\dagger + 2\partial^\mu (\phi^\dagger D_\mu \phi + (D_\mu \phi)^\dagger \phi) - \Box \phi^\dagger \phi \right] \right\} . \tag{30}$$

The classical action is thus modified by a non-renormalizable interaction. The relevant term giving a mass contribution to the gauge field is the first one in the r.h.s. of Equation (30), belonging to the family of operators

$$C_n \equiv \int d^4x \mathrm{Tr} \, [(\phi^\dagger \phi)^n (D^\mu \phi)^\dagger D_\mu \phi] \, . \tag{31}$$

All of them contribute to the gauge field mass term. As is very well known, only C_0 leads to a power-counting renormalizable theory.

By the same argument, the additional kinetic term corresponds to a dimension eight operator with four derivatives

$$\int d^4x \, \frac{z'}{2} a'_{a\mu} (\Box g^{\mu\nu} - \partial^\mu \partial^\nu) a'_{a\nu} \sim -\frac{4z'}{g^2 v^4} \int d^4x \mathrm{Tr} \left[\phi^\dagger D_\mu \phi (\Box g^{\mu\nu} - \partial^\mu \partial^\nu)(\phi^\dagger D_\nu \phi) \right] . \tag{32}$$

It contributes both to the quadratic part and to the interaction terms, as also happens for the terms in Equation (30).

In the standard formalism, it is difficult to compute the radiative corrections induced by those operators beyond the small coupling expansion and it is very hard to guess the form of the resummation.

On the other hand, by using the dynamical gauge-invariant fields, the additional operators are rewritten in a form that only contributes to the quadratic part.

This paves the way for the derivation of some novel differential equations, allowing for the determination of the functional dependence of the amplitudes on the new parameters in an exact way. This will be discussed in the next section.

4. The Decoupling Limit

The parameters z, M^2 and z', M'^2 only enter in the propagators Δ_{hh} and $\Delta_{a'_{a\mu} a'_{b\mu}}$, respectively, and never in the interaction vertices. Moreover, the propagator Δ_{hh} is an eigenvector of eigenvalue -1 of the differential operator

$$\mathcal{D}_z^{M^2} \equiv (1+z) \frac{\partial}{\partial z} + M^2 \frac{\partial}{\partial M^2} \, , \tag{33}$$

while the propagator $\Delta_{a'_{a\mu} a'_{b\mu}}$ is an eigenvector of eigenvalue -1 of the differential operator

$$\mathcal{D}_{z'}^{M'^2} \equiv (1+z') \frac{\partial}{\partial z'} + M'^2 \frac{\partial}{\partial M'^2} \, , \tag{34}$$

i.e.,

$$\mathcal{D}_z^{M^2} \Delta_{hh} = -\Delta_{hh} \, , \qquad \mathcal{D}_{z'}^{M'^2} \Delta_{a'_{a\mu} a'_{b\mu}} = -\Delta_{a'_{a\mu} a'_{b\mu}} \, . \tag{35}$$

Let us now consider an n-th loop 1-Pi amplitude $\Gamma^{(n)}_{\varphi_1 \dots \varphi_r}$ with r φ_i external legs, $\varphi_i = \varphi(p_i)$ denoting a generic field or external source of the theory with incoming momentum p_i.

$\Gamma^{(n)}_{\varphi_1 \dots \varphi_r}$ can be decomposed as the sum of all diagrams with (amputated) external legs $\varphi_1 \dots \varphi_r$ with zero, one, two, … l_h internal h-propagators and zero, one, two, … l_a internal a'-propagators:

$$\Gamma^{(n)}_{\varphi_1\dots\varphi_r} = \sum_{l_h,l_a\geq 0} \Gamma^{(n;l_h,l_a)}_{\varphi_1\dots\varphi_r} \tag{36}$$

Then, each $\Gamma^{(n;l_h,l_a)}_{\varphi_1\dots\varphi_r}$ is clearly an eigenvector of $\mathcal{D}^{M^2}_z$ of eigenvalue $-l_h$ and of $\mathcal{D}^{M'^2}_{z'}$ of eigenvalue $-l_a$, namely

$$\mathcal{D}^{M^2}_z \Gamma^{(n;l_h,l_a)}_{\varphi_1\dots\varphi_r} = -l_h \Gamma^{(n;l_h,l_a)}_{\varphi_1\dots\varphi_r}\,, \qquad \mathcal{D}^{M'^2}_{z'} \Gamma^{(n;l_h,l_a)}_{\varphi_1\dots\varphi_r} = -l_a \Gamma^{(n;l_h,l_a)}_{\varphi_1\dots\varphi_r}\,. \tag{37}$$

According to Euler's theorem, the most general solution to the above differential equations is a homogeneous function in the variables $M^2/(1+z)$ and $M'^2/(1+z')$, i.e., a function of the form

$$\Gamma^{(n;l_h,l_a)}_{\varphi_1\dots\varphi_r}(z, M^2, z', M'^2) = \frac{1}{(1+z)^{l_h}} \frac{1}{(1+z')^{l_a}} \Gamma^{(n;l_h,l_a)}_{\varphi_1\dots\varphi_r}\left(0, \frac{M^2}{1+z}, 0, \frac{M'^2}{1+z'}\right). \tag{38}$$

Notice that this result holds true for all orders in the loop expansion.

This is preserved by renormalization, provided that the finite normalization conditions are chosen in such a way to fulfil Equation (38) [20].

Equation (38) predicts the structure of the fully resummed amplitudes, i.e., it contains the exact dependence on the parameters z, z', M^2, M'^2.

In particular, one can consider trajectories in the couplings space where the ratios $M^2/(1+z)$ and $M'^2/(1+z')$ are kept fixed while letting z, z' tend towards infinity.

According to Equation (38), one sees that, in such a limit, only the contribution $\Gamma^{(n;0,0)}$ will survive. This is equivalent to saying that all contributions generated by diagrams, where at least one internal line is a Δ_{hh} or a$\Delta_{a'a'}$-propagator, vanish, i.e., all non-decoupling effects are washed out.

This is quite a surprising result. In fact, it implies that the high-energy dynamics are totally decoupled from the infrared regime. In some sense, physical particles of the high-energy theory act as classical background sources, influencing the low-energy physics only by tree-level contributions.

It is worth commenting on the potential phenomenological impact of such decoupling limits on BSM fits.

The EFT description of the low-energy effects, arising from integrating out the gauge and scalar fields of the extended Lagrangian (2), can be arranged as a set of operators of increasing dimension in the high-energy scale Λ.

In the present case, the typical energy scale Λ is set by v, i.e., the scale controlling the spontaneous symmetry breaking in the high-energy sector.

The EFT description will give rise to an effective low-energy Lagrangian of the form

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \frac{1}{\Lambda}\mathcal{L}_5 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \frac{1}{\Lambda^3}\mathcal{L}_7 + \frac{1}{\Lambda^4}\mathcal{L}_8 + \dots, \qquad \mathcal{L}_d = \sum_i c_i^{(d)} \mathcal{O}_i^{(d)}\,. \tag{39}$$

$\mathcal{O}_i^{(d)}$ are local $SU(3)_c \times SU(2)_L \times U(1)_Y$-invariant operators of canonical dimension d, involving only the light SM fields.

$\mathcal{L}_{SM}$ is the SM Lagrangian, which is known to be a very good description of electroweak physics, with the exception of the small neutrino masses that are taken into account by $\mathcal{L}_5$.

The higher order terms $\mathcal{L}_{d\geq 5}$ yield contributions to physical processes that are suppressed by factors $(E/\Lambda)^{d-4}$, where E is the relevant energy scale of the process under investigation. The virtual effects due to the propagation of the heavy degrees of freedom inside loops are captured by the Wilson coefficients $c_i^{(d)}$. The latter can be computed by matching with the UV complete high-energy theory.

The effects described by $\mathcal{L}_{d\geq 5}$ must be small, due to the success of $\mathcal{L}_{SM}$ in describing experimental data.

This hints at the fact that either the scale of new physics Λ is very large or the structure of $\mathcal{L}_{d\geq 5}$ is particularly elaborated, or perhaps a combination of both [47].

In the standard treatment of SM Effective Field Theories, one takes the assumption that Λ is large and carries out a perturbative treatment of amplitudes in the small coupling regimes with respect to the BSM parameters.

The mechanism described in the present study offers a precise new way to ensure that the effects of $\mathcal{L}_{d\geq 5}$ are small based on resummation; Equation (38) implies that the full amplitudes (beyond the small coupling approximation in z, M^2, z', M'^2) are suppressed in the *strong* coupling regime for z, z'.

This is a non-trivial result that points towards the necessity of going beyond the small coupling expansion in the SM effective field theory program.

In view of the fact that HL-LHC will allow for studying anomalous couplings with a precision of a few percent, and that future colliders can significantly improve that precision [47,48], a comparison with the experimental data of the SM effective field theory fits must take into account the resummation effects.

5. Conclusions

In the present study, we have shown how to construct dynamical gauge-invariant variables for the gauge fields, by extending the procedure already obtained for scalars. The method works whenever spontaneous symmetry breaking occurs.

Dynamical gauge-invariant variables are quantum fields over which the path-integral is carried out, at variance with the approach based on composite gauge-invariant operators [38–40].

One of the main advantages of the gauge-invariant dynamical fields is that they allow for representing certain operators, involving complicated interactions in the standard formalism, by purely quadratic contributions to the classical action.

This, in turn, allows one to derive powerful differential equations controlling the dependence of the 1-PI amplitudes on their coefficients.

For the special choice of dimension six and dimension eight two derivative operators for the scalar and the vector fields, given in Equations (13), (24) and (25), (respectively, and Equations (14), (30) and (32) in the ordinary formalism), this implies that fully resummed amplitudes have a fairly simple dependence on such BSM couplings (see Equation (38)).

Then, it is easy to identify trajectories in the coupling space where non-decoupling effects from the propagation of high-energy particles are washed out, while keeping the (tree-level) pole masses of such particles at fixed values (constant ratios $M^2/(1+z)$ and $M'^2/(1+z'))$.

The argument is very general and does not depend on the particular interactions in the high-energy theory. It applies whenever the spectrum only contains ordinary particles (no higher derivatives terms in the quadratic part) and when operators exist in the high-energy theory, such as those given in Equations (14), (30) and (32) in the standard formalism.

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Abbreviations

The following abbreviations are used in this manuscript:

BRST	Becchi–Rouet–Stora–Tyutin
l.h.s., r.h.s	left hand side, right hand side

Appendix A. The Model and Its Symmetries

The complete classical vertex functional reads

$$\begin{aligned}\Gamma^{(0)} = \Gamma^{(0)}_{\rm SM} + \int d^4x \Big\{ & -\frac{1}{4}G^2_{a\mu\nu} + \frac{1}{4}{\rm Tr}(D^\mu\phi)^\dagger D_\mu\phi - \frac{\lambda}{2}\Big[\frac{1}{2}{\rm Tr}(\phi^\dagger\phi) - v^2\Big]^2 \\ & + g_1 {\rm Tr}\Big[(\phi^\dagger\phi) - v^2\Big]^2 \Phi^\dagger\Phi \\ & - \frac{M^2 - m^2}{2}h^2 - \frac{z}{2}h\Box h + \frac{M_a^2 - M_A^2}{2}{a'_\mu}^2 + \frac{z_a}{2}a'_\mu(\Box g^{\mu\nu} - \partial^\mu\partial^\nu)a'_\nu \\ & - b_a\partial A_a + \bar{c}_a\partial^\mu D_\mu c_a \\ & + X(\Box + m^2)\Big[h - \frac{1}{4v}{\rm Tr}(\phi^\dagger\phi) + \frac{v}{2}\Big] - \bar{c}(\Box + m^2)c \\ & + {\rm Tr}\Big\{X_\mu\Sigma^{\mu\nu}\Big[a_\nu - \frac{i}{gv^2}\Big(2\phi^\dagger D_\nu\phi - \partial_\nu(\phi^\dagger\phi)\Big)\Big] - \bar{c}_\mu\Sigma^{\mu\nu}c_\nu\Big\} \\ & + \bar{c}^*\Big(h - \frac{1}{4v}{\rm Tr}(\phi^\dagger\phi) + \frac{v}{2}\Big) + {\rm Tr}\Big\{\bar{c}^{*\mu}\Big[a_\mu - \frac{i}{gv^2}\Big(2\phi^\dagger D_\mu\phi - \partial_\mu(\phi^\dagger\phi)\Big)\Big] \\ & + A_a^{*\mu}D_\mu c_a - \frac{g}{2}\phi_0^* c_a\phi_a + \phi_a^*\Big(\frac{g}{2}\phi_0 c_a + \frac{g}{2}\epsilon_{abc}\phi_b\omega_c\Big)\Big\}\,. \end{aligned} \tag{A1}$$

In the above equation, $\Gamma^{(0)}_{\rm SM}$ denotes the classical vertex functional of the SM, including the SM classical action, the gauge-fixing and ghost terms, as well as the external sources (the so-called antifields [42]) required to define, at the quantum level, the BRST transformations of the fields generated by the electroweak gauge group.

The additional higher dimensional operators written in terms of gauge-invariant dynamical fields are reported in the third line of Equation (A2).

a'_μ is defined by

$$a'_{a\mu} = a_{a\mu} - \frac{1}{M_A^2}\partial_\mu b_a\,.$$

The fourth and fifth lines contain the Lagrange multipliers together with the constraint ghost and antighost fields (that remain free). The differential operator $\Sigma^{\mu\nu}$ is

$$\Sigma^{\mu\nu} = (\Box g^{\mu\nu} - \partial^\mu\partial^\nu) + M_A^2 g^{\mu\nu}\,. \tag{A2}$$

Finally, in the last two lines, the antifields for the constraint BRST transformations $\tilde{s}\bar{c}, \tilde{s}\bar{c}_\mu$, as well as for the SU(2) gauge group BSM extension, are introduced with the convention

$$\bar{c}^{*\mu} = \bar{c}_a^{*\mu}\frac{\tau_a}{2}\,.$$

The classical action in Equation (A2) obeys several functional identities, in addition to the usual ones (Slavnov–Taylor identities, b-equation, ghost equation) valid for the SM part $\Gamma^{(0)}_{\rm SM}$:

- The X- and X_a^μ-equations:

$$\begin{aligned}\frac{\delta\Gamma^{(0)}}{\delta X} &= (\Box + m^2)\frac{\delta\Gamma^{(0)}}{\delta\bar{c}^*}\,, \\ \frac{\delta\Gamma^{(0)}}{\delta X_a^\mu} &= \frac{1}{2}\Sigma^{\mu\nu}\frac{\delta\Gamma^{(0)}}{\delta\bar{c}_a^{*\nu}}\,;\end{aligned} \tag{A3}$$

- The b-equation and ghost equation for the high-energy SU(2) gauge group:

$$\frac{\delta\Gamma^{(0)}}{\delta b_a} = \partial A_a\,, \qquad \frac{\delta\Gamma^{(0)}}{\delta\bar{c}_a} = \partial^\mu \frac{\delta\Gamma^{(0)}}{\delta A_a^{*\mu}}\,; \tag{A4}$$

- The h and a_μ-equations:

$$\begin{aligned}\frac{\delta\Gamma^{(0)}}{\delta h} &= -(M^2 - m^2)h - z\Box h + (\Box + m^2)X + \bar{c}^*\,,\\ \frac{\delta\Gamma^{(0)}}{\delta a_{a\mu}} &= (M_a^2 - M_A^2)a'_{a\mu} + z_a(\Box g^{\mu\nu} - \partial^\mu\partial^\nu)a'_{a\nu} + \frac{1}{2}\Sigma^{\mu\nu}X_{\nu a} + \bar{c}^*_{a\mu}\,.\end{aligned} \tag{A5}$$

  Notice that the r.h.s. is linear in the quantum fields and, therefore, no further external source is required to renormalize these identities.
- The high-energy SU(2) Slavnov–Taylor identity:

$$\mathcal{S}(\Gamma^{(0)}) = \int d^4x \Big[\frac{\delta\Gamma^{(0)}}{\delta A^*_{a\mu}}\frac{\delta\Gamma^{(0)}}{\delta A_{a\mu}} + \frac{\delta\Gamma^{(0)}}{\delta\sigma^*}\frac{\delta\Gamma^{(0)}}{\delta\sigma} + \frac{\delta\Gamma^{(0)}}{\delta\phi_a^*}\frac{\delta\Gamma^{(0)}}{\delta\phi_a} + b_a\frac{\delta\Gamma^((0)}{\delta\bar{c}_a}\Big] = 0\,; \tag{A6}$$

- The b-equation

$$\frac{\delta\Gamma^{(0)}}{\delta b_a} = -\partial A_a; \tag{A7}$$

- The ghost equation

$$\frac{\delta\Gamma^{(0)}}{\delta\bar{c}_a} = \partial^\mu\frac{\delta\Gamma^{(0)}}{\delta A^*_{a\mu}}; \tag{A8}$$

- The constraint Slavnov–Taylor identities:

$$\begin{aligned}\mathcal{S}_{\tilde{s},\text{scal}}(\Gamma^{(0)}) &= \int d^4x\Big[c\frac{\delta\Gamma^{(0)}}{\delta X} + \frac{\delta\Gamma^{(0)}}{\delta\bar{c}^*}\frac{\delta\Gamma^{(0)}}{\delta\bar{c}}\Big] = 0\,,\\ \mathcal{S}_{\tilde{s},\text{vect}}(\Gamma^{(0)}) &= \int d^4x\Big[c_{a\mu}\frac{\delta\Gamma^{(0)}}{\delta X_{a\mu}} + \frac{\delta\Gamma^{(0)}}{\delta\bar{c}^*_{a\mu}}\frac{\delta\Gamma^{(0)}}{\delta\bar{c}_{a\mu}}\Big] = 0\,;\end{aligned} \tag{A9}$$

- The ghost equations for the constraint ghosts:

$$\frac{\delta\Gamma^{(0)}}{\delta\bar{c}} = -(\Box + m^2)c\,, \qquad \frac{\delta\Gamma^{(0)}}{\delta\bar{c}_{a\mu}} = -\frac{1}{2}\Sigma^{\mu\nu}c_a\,. \tag{A10}$$

  By using Equation (A10) in Equation (A9), one obtains the X-equations (A3).

Appendix B. Propagators

Appendix B.1. Scalar Fields

The diagonalization of the quadratic part in the scalar sector spanned by σ, X, h is achieved by setting [20]

$$X = X_1 + h\,, \qquad \sigma = \sigma' + X_1 + h\,. \tag{A11}$$

The propagators in the mass eigenstate (diagonal) basis are

$$\Delta_{\sigma'\sigma'} = -\Delta_{X_1X_1} = \frac{i}{p^2 - m^2}\,, \qquad \Delta_{hh} = \frac{i}{(1+z)p^2 - M^2}\,. \tag{A12}$$

Appendix B.2. Landau Gauge

One must diagonalize the quadratic part given by

$$\int d^4x \Big\{ \frac{1}{2} A_{a\mu}(\Box g^{\mu\nu} - \partial^\mu \partial^\nu) A_{a\nu} + \frac{M_A^2}{2} \Big(A_{a\mu} - \frac{1}{M_A} \partial_\mu \phi_a \Big)^2 - b_a \partial A_a + X_{a\mu} \Big[(\Box g^{\mu\nu} - \partial^\mu \partial^\nu) + M_A^2 g^{\mu\nu} \Big] \Big(a_{a\nu} - A_{a\nu} + \frac{1}{M_A} \partial_\nu \phi_a \Big) \Big\} . \tag{A13}$$

One first removes the $\phi - A_\mu$-mixing via the redefinition

$$b_a = b'_a + M_A \phi_a \,, \tag{A14}$$

followed by the cancellation of the $b' - A_\mu$-mixing by the replacement

$$A_{a\mu} = A'_{a\mu} - \frac{1}{M_A^2} \partial_\mu b'_a \,. \tag{A15}$$

A further set of field redefinitions

$$A'_{a\mu} = A''_{a\mu} + X_{a\mu} \,, \qquad X_{a\mu} = X'_{a\mu} + a_{a\mu} \tag{A16}$$

take care of the $X_\mu - A_\nu$ and $X_\mu - a_\nu$ mixing. One is eventually left with

$$\int d^4x \Big\{ \frac{1}{2} A''_{a\mu} [(\Box + M_A^2) g^{\mu\nu} - \partial^\mu \partial^\nu] A''_{a\nu} - \frac{1}{2} X'_{a\mu} [(\Box + M_A^2) g^{\mu\nu} - \partial^\mu \partial^\nu] X'_{a\nu} + \frac{1}{2} a_{a\mu} [(\Box + M_A^2) g^{\mu\nu} - \partial^\mu \partial^\nu] a_{a\nu} - \frac{1}{2M_A^2} \partial^\mu b'_a \partial_\mu b'_a + \frac{1}{2} \partial^\mu \phi_a \partial_\mu \phi_a + (X'_{a\mu} + a_{a\mu}) \partial^\mu (b'_a + M_A \phi_a) \Big\} . \tag{A17}$$

The mixing terms in the last line of the above equation can be removed by the local field redefinition

$$X'_{a\mu} = X''_{a\mu} + \frac{1}{M_A^2} \partial_\mu b'_a + \frac{1}{M_A} \partial_\mu \phi_a \,, \qquad a_{a\mu} = a'_{a\mu} - \frac{1}{M_A^2} \partial_\mu b'_a - \frac{1}{M_A} \partial_\mu \phi_a \,. \tag{A18}$$

No new $b' - \phi$-mixing is generated. The diagonal propagators in momentum space (mass eigenstates) are finally given by

$$\Delta_{A''_{a\mu} A''_{b\nu}} = \Delta_{a'_{a\mu} a'_{b\nu}} = -\Delta_{X''_{a\mu} X''_{b\nu}} = \frac{i\delta_{ab}}{-p^2 + M_A^2} T_{\mu\nu} + \frac{i\delta_{ab}}{M_A^2} L_{\mu\nu} \qquad \Delta_{b'_a b'_b} = -\frac{i\delta_{ab} M_A^2}{p^2} \,, \;\; \Delta_{\phi_a \phi_b} = \frac{i\delta_{ab}}{p^2} \,. \tag{A19}$$

In the symmetric basis $(A_\mu, \phi, b, X_\mu, a_\mu)$

$$b_a = b'_a + M_A \phi_a \,, \qquad A_{a\mu} = A''_{a\mu} + X''_{a\mu} + a'_{a\mu} - \frac{1}{M_A} \partial_\mu b'_a \,, \qquad X_{a\mu} = X''_{a\mu} + a'_{a\mu} \,, \qquad a_{a\mu} = a'_{a\mu} - \frac{1}{M_A^2} \partial_\mu b'_a - \frac{1}{M_A} \partial_\mu \phi_a \tag{A20}$$

the non-vanishing propagators are given by

$$\Delta_{A_{a\mu}A_{b\nu}} = \frac{i\delta_{ab}}{-p^2+M_A^2}T_{\mu\nu}\,, \qquad \Delta_{A_{a\mu}b_b} = -\frac{\delta_{ab}p_\mu}{p^2}\,, \qquad \Delta_{A_{a\mu}a_{b\nu}} = \frac{i\delta_{ab}}{-p^2+M_A^2}T_{\mu\nu}\,,$$
$$\Delta_{X_{a\mu}a_{b\nu}} = \frac{i\delta_{ab}}{-p^2+M_A^2}T_{\mu\nu} + \frac{i\delta_{ab}}{M_A^2}L_{\mu\nu}\,, \qquad \Delta_{a_{a\mu}a_{b\nu}} = \frac{i\delta_{ab}}{-p^2+M_A^2}T_{\mu\nu} + \frac{i\delta_{ab}}{M_A^2}L_{\mu\nu}\,,$$
$$\Delta_{a_{a\mu}\phi_b} = \frac{\delta_{ab}}{M_A}\frac{p_\mu}{p^2}\,, \qquad \Delta_{b_a\phi_b} = \frac{i\delta_{ab}M_A}{p^2}\,, \qquad \Delta_{\phi_a\phi_b} = \frac{i\delta_{ab}}{p^2}\,. \tag{A21}$$

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