STATUS OF BFKL POMERON [a]

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I summarize recent results on the small-x behaviour of QCD structure functions, which improve the formulation of the next-to-leading BFKL equation by incorporating renormalization group constraints in the collinear limits.

I have been asked to report on the status of the hard pomeron [1,2,3]. So, I should immediately say that it is recovering from a serious illness, thanks to massive injections of Renormalization Group (R.G.) [4,5].

Since last year, a somewhat paradoxical situation has arisen. On one hand, the Q^2-dependence of the small-x rise of the structure functions at HERA is perfectly explained by two-loop QCD evolution, except perhaps for a residual slope $\frac{d\log F_2}{d\log 1/x} \simeq .2$ at $Q_0^2 = 4$ (GeV)^2.

On the other hand, the original prediction $F_2(x) \sim x^{-\omega_P(Q^2)}$ of the BFKL equation, yielding too large a slope at leading level ($\omega_P = .55$ for $\alpha_s = .2$) has been pushed to NL level after many years of theoretical effort. The result is [2,3]

$$\omega_P(Q^2) = 4\log 2\bar{\alpha}_s(Q^2)\ \ (1 - 6.47\bar{\alpha}_s(Q^2) +\ \ldots), \bar{\alpha}_s = \frac{N_c\alpha_s}{\pi}\ , \qquad (1)$$

showing a huge NL coefficient, so that the small-x expansion appears essentially unstable [6].

Nevertheless, such seemingly contradictory outputs describe two different regimes of the same theory (QCD), so that a calculational method interpolating both must be found, eventually. The obvious hint is to make the small-x expansion consistent with the QCD collinear behaviour; how, is what I am going to describe.

Note first that the BFKL approach is normally set up for hard processes with two scales, that we call k (upper) and k_0 (lower) scale. For instance, we may set $k = Q$ (the photon virtuality)

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and $k_0 = p_\perp$ (the pion transverse momentum) if we look at DIS scattering with production of a forward hard pion. The total DIS cross section case obtains for k_0 = hadronic mass, so that $k \gg k_0$ (single scale limit). Roughly, one has the k-factorization formula: [7]

$$\sigma(s, \mathbf{k}, \mathbf{k}_0) = \int \frac{d\omega}{2\pi i} \left(\frac{S}{kk_0}\right)^{\omega} G_\omega(\mathbf{k}, \mathbf{k}_0) \;\; (\text{Impact Factors}), \tag{2}$$

where we have taken the symmetrical scaling variable kk_0/s and G_ω is the BFKL gluon Green's function.

The collinear properties of (1) come out in the DGLAP limit $k \gg k_0$, where RG factorization is argued to hold ($t = \log \frac{k^2}{\lambda^2}$) [8]

$$G_\omega(\mathbf{k}, \mathbf{k}_0) = \mathcal{F}_\omega(t)\tilde{\mathcal{F}}_\omega(t_0) + \text{higher twists} \,, \;\; (t - t_0 \gg 1) \tag{3}$$

Here $\mathcal{F}_\omega(t)$ is the solution of the homogeneous BFKL equation

$$K_\omega \mathcal{F}_\omega = \omega \mathcal{F}_\omega \tag{4}$$

and carries the t-dependence of the gluon density, while $\tilde{\mathcal{F}}_\omega(t_0)$ carries the Pomeron singularity ω_P and contains a nonperturbative part, depending on the details of $\alpha_s(t)$ in the strong coupling region $k^2 \simeq \lambda^2$ (where the perturbative Landau pole is).

The first improvement of Eq(4) that we propose [4] is to construct a RG invariant kernel K_ω with correct collinear behaviour to all orders. Its form is

$$K_\omega(\mathbf{k}, \mathbf{k}') = \sum_{n=0}^{\infty} (\bar{\alpha}_s(t))^{n+1} \quad K_n^\omega(\mathbf{k}, \mathbf{k}'), \tag{5}$$

where the K_n^ω's are scale-invariant kernels, with the eigenvalue functions $\chi_n^\omega(\gamma)$, to be constructed with the requirements of (i) matching the exact expressions $\chi_0(\gamma)$ and $\chi_1(\gamma)$ of the leading and NL kernels [2,3] and (ii) having the correct collinear singularities to all orders, of the form [4]

$$\begin{aligned} \chi_n^\omega(\gamma) &\simeq \frac{1 \cdot A_1 \ldots (A_1 + (n-1)b)}{(\gamma + \frac{1}{2}\omega)^{-n-1}}, \qquad \gamma \ll 1, \\ &\simeq \frac{1 \cdot (A_1 - b) \ldots (A_1 - nb)}{(1 - \gamma + \frac{1}{2}\omega)^{-n-1}}, \qquad 1 - \gamma \ll 1, \end{aligned} \tag{6}$$

where $A_1(\omega)$ is the ω-moment of the nonsingular part of the DGLAP splitting function $P_{gg} - 2N_c/z$, and b is the beta function coefficient.

The peculiar ω-dependence of the γ-singularities in Eq.(6) is due to the choice of kk_0/s as scaling variable in Eq.(2). The structure (6) insures ω-independent poles at $\gamma = 0$ for scaling variable k^2/s and $k \gg k_0$, and at $\gamma = 1$ in the opposite limit, as required by single-logarithmic scaling violations in the two cases. The ω-shift is thus a method to resum double-logs [4] in the scale-dependent contributions to the kernel [3]. The precise form of χ_0^ω and χ_1^ω can then be derived on this basis.

The second important improvement is to provide a solution for Eq.(4) with the kernel (5) by the so-called ω- expansion method [5]. That is

$$\mathbf{k}^2 \mathcal{F}_\omega(t) = \int \frac{d\gamma}{2\pi} e^{\gamma t - \frac{1}{b\omega} X(\gamma, \omega)} = \dot{g}_\omega(t), \tag{7}$$

where

$$\chi(\gamma, \omega) = \frac{\partial}{\partial \gamma} X(\gamma, \omega) = \chi_0^\omega(\gamma) + \omega \, \frac{\chi_1^\omega(\gamma)}{\chi_0^\omega(\gamma)} + O(\omega^2), \tag{8}$$

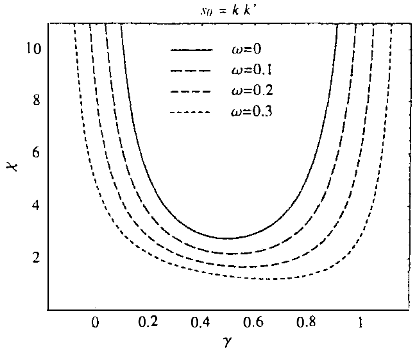

Figure 1: The effective eigenvalue function $\chi(\gamma,\omega)$ for different values of ω

and the terms of NNL order or higher have explicit form in terms of the χ_n^ω's, with $n \geq 2$.

Superficially, Eq.(8) amounts to the replacement $\bar{\alpha}_s(t) \to \omega/\chi_0^\omega$ in the NL expansion, but it is actually a general method for solving Eq(4), which is able to resum the collinear behaviour. A striking feature of (8) is that it has simple γ-poles only, and that the leading twist ones cancel from NNL level on.

The above improvements stabilize the estimate of both anomalous dimension and hard pomeron, in a nearly scheme-independent way [5]. Fig.(1) shows $\chi(\gamma,\omega)$ in Eq.(8) for various values of ω. It decreases smoothly with ω by retaining its shape, it has shifted γ-poles and a stable minimum up to sizeable ω-values.

The resummed L,NL,... anomalous dimensions are related to a saddle point value $\bar{\gamma}_\omega(t)$ of Eq.(7), given by $b\omega t = \chi(\bar{\gamma},\omega)$. The latter fails at some ω value $\omega_s = \chi_{\rm min}(t)/bt$, which signals the breakdown of the small-x expansion and is plotted in Fig.(2). Compared to its leading and NL approximations, the resummed estimate works up to low t-values, and is $\omega_s = .27 \div .32$ for $\bar{\alpha}_s = .2 \div .3$.

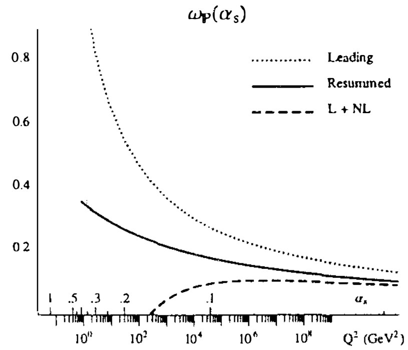

Figure 2: The leading, pure next-to-leading and resummed hard pomeron.

Furthermore, the anomalous dimension itself can be continued past the saddle point failure [5,9] by direct evaluation of Eq.(7) and of $\gamma_{\rm eff}(\gamma,t) = \dot{g}_\omega(t)/g_\omega(t)$. The result is shown in Fig. (3), and compared to L and NL approximations. The resummed expression joins smoothly to the fixed order perturbative one until very close to the singularity point, which is somewhat lower than the ω_s value (see[5] for the details).

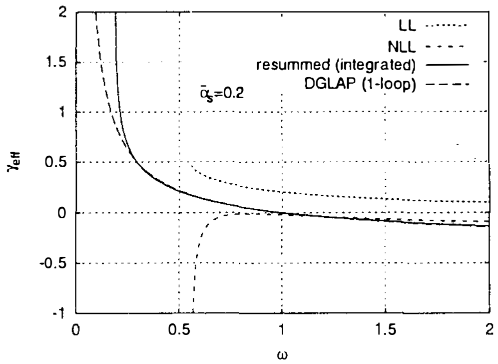

Figure 3: The anomalous dimension in various approximations.

To sum up, the good news come from an improved small-x equation, which incorporates the RG constraints, is able to predict resummed anomalous dimensions which interpolate smoothly between fixed order and hard pomeron regimes, and in able to provide a stable estimate of the latter.

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References

1. E.A. Kuraev, L.N. Lipatov and V.S. Fadin, JEPT **45** (1978) 199; Y.Y. Balitsky and L.N. Lipatov, Sov.J.Nucl.Phys. **28**(1978) 22.
2. V.S. Fadin and L.N. Lipatov, Phys.Lett.**429**B (1998) 127, and references therein.
3. G. Camici and M. Ciafaloni, Phys.Lett.**412**B (1997) 396 and **430**B (1998) 349, and references therein.
4. G. Salam, JHEB **9807** (1998) 19; M. Ciafaloni and D. Colferai, hep-ph/9812366.
5. M. Ciafaloni, D. Colferai and G. Salam, to appear.
6. J. Blümlein and A. Vogt, Phys. Rev. **D58** (1998) 014020; D.A. Ross., Phys. Lett. **431B** (1998) 161; E.M.Levin, hep-ph/9806228.
7. M. Ciafaloni, Phys.Lett.**429**B (1998) 363.
8. J.C. Collins and J. Kwiecinski, Nucl. Phys. **B 316** (1989) 307; G. Camici and M. Ciafaloni Phys.Lett.**395**B (1997) 118.
9. R.S. Thorne, hep-ph/9901331.