Effect of QCD Phase Transition on Axion Properties for Two Flavors of NJL Model

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Abstract. The effect of chiral phase transition on the properties of the axion in the Nambu-Jona-Lasinio two-flavor model was studied at finite temperature. We tend to study and analyze the changes of thermodynamic potential, and note that the changes of temperature, coupling parameters and quark chemical potential have great influence on thermodynamic potential. Then the variations of axion mass, self-coupling and quark condensation are given by numerical calculation.

1. Introduction

The strong CP (charge conjugation and parity) problem was initially addressed by the axion in a kinetic manner [1-4]. Axions can thermalize and form Bose-Einstein condensates, as shown by recent research [5-10], which proves the accuracy of the axion's limiting temperature expansion. One of the physically strongest excited particles outside of the Standard Model is the QCD axion (SM). The absence of CP breaking in hadron interactions [11], in particular the electric dipole moment of neutrons [12-15], is explained by axion, which renders the effective QCD theta angle nonphysical and enables QCD dynamics to solve its strong CP problem. Because the axions are bosons with very weak interactions and very light masses, they are considered to be one of the leading candidates for cold dark matter [3][16-18]. In the reference, the impact of axions on star evolution has also been discussed [6-10][19-20]. Due to the weak interaction of non-leptons with changes in singularity during the evolution of neutron stars into singularities, a considerable amount of axion production is possible. Axions may significantly alter the star's energy budget as well [21]. Axion may be produced in hot, high-density astrophysical plasma or transmit energy from stars [22-25]. Therefore, the estimation of axion masses is important to study the effect of axions on stellar cooling.

NJL model is usually used to study the CP violation effect and the influence of vacuum environment on QCD phase transition. Axion masses and self-coupling at finite temperatures in the absence and presence of magnetic fields were investigated in the literature [26]. The calculations show that the axion mass and self-coupling corrections are quite close to the chiral transition temperature. The NJL model has been previously accustomed to study the CP breaking effect and the θ effect of vacaum on QCD [11]. So the NJL model is used to study the axions in this paper.

2. Axion in NJL Model

We will show the combined effect of quark chemical potential and interaction constant on the properties of the axions in a nonlocal two-flavor NJL model with enhanced' t hooft interaction [29-30].

Describing strong interactions, with $U(1)$ anomalies and spontaneous breaking of chiral symmetry, the quantum chromodynamics Lagrangian contains a θ term due to transient effects that lead to CP breaking [25].

$$L_\theta = \frac{g^2\theta}{32\pi^2} F\overline{F} \tag{1}$$

Where g is a coupling parameter of strong interaction. F and $\overline{F}$ are the gluon field strength and its dual[26]. The Lagrangian expression of the axion quark interaction can be obtained by adding interaction terms.

$$L_a = \theta F\overline{F} + \frac{a}{f_a} F\overline{F} \tag{2}$$

Equation (2) can be effectively expressed as the interaction between QCD axion field a and quark.

$$\mathcal{L}_a = 8G_2\left[e^{i\frac{a}{f_a}} \det\left(\psi_R \psi_L\right) + e^{-i\frac{a}{f_a}} \det\left(\psi_L \psi_R\right) \right] \tag{3}$$

The effective Lagrangian density of the isospin symmetric two-flavor NJL model with CP-breaking term is given by the following equation [29].

$$\mathcal{L} = \overline{\psi}\left(i\gamma^\mu \partial_\mu - m_0\right)\psi + \mathcal{L}_q + \mathcal{L}_a \tag{4}$$

Where Ψ denotes the fermion field and m_0 denotes the current quark mass. The fermionic interaction part of the Lagrangian density is given by the following equation [30].

$$\mathcal{L}_q = G_1\left[(\overline{\psi}\tau_0\psi)^2 + \left(\overline{\psi}\tau_i i\gamma_5\psi\right)^2 \right] \tag{5}$$

where τ_0 is the unit matrix, τ_i is the bubble matrix of $(i = 1,2,3)$.

2.1. Thermodynamic Potential for Isospin Symmetry

In NJL model with quark chemical potential and different interaction constants at finite temperature, the thermodynamic potential of QCD axions can be expressed as

$$\Omega(\sigma,\eta,\mu,T,a) = -G_2\left(\eta^2 - \sigma^2\right)\cos\frac{a}{f_a} + G_1\left(\eta^2 + \sigma^2\right) - 2G_2\sigma\eta\sin\frac{a}{f_a} + \Omega_q \tag{6}$$

Where $\sigma = \langle \overline{q}q \rangle$ and $\eta = \langle \overline{q}i\gamma_5 q \rangle$ are the chiral and $SU(2)$ isospin single heavy state pseudoscalar condensates, respectively. Where Ω_q has the following expression [25].

$$\Omega_q = -N_C \sum_{i=1}^{4} \int \frac{d^3p}{(2\pi)^3} \left\{ E_p + 2T\ln\left(1 + e^{-E_i/T}\right) \right\} \tag{7}$$

$$E_p = \sqrt{p^2 + M^2}, \quad M = \sqrt{\left(m + \alpha_0\right)^2 + \beta_0^2} \tag{8}$$

Where $N_c = 3$ denotes the quark color number, $G_1 = (1-c)G_0$ and $G_2 = cG_0$, $E_i = E_p \pm \mu$ is the excitation energy o-f quark and antiquark, α_0 and β_0 in the effective mass M [27].

$$\alpha_0 = -2\left(G_1 + G_2 \cos\frac{a}{f_a}\right)\sigma + 2G_2\eta \sin\frac{a}{f_a}$$
$$\beta_0 = -2\left(G_1 - G_2 \cos\frac{a}{f_a}\right)\eta + 2G_2\sigma \sin\frac{a}{f_a} \tag{9}$$

The integral in equation (7) can be divided into two parts: the zero temperature part and the finite temperature part [25]. We truncate the integral for the zero temperature part and relax the integral for the finite temperature part to infinity [26].

For a given value of $a = 0$, the thermodynamic potential is a function of the condensate σ and η, and the physical values of the condensate at different temperatures correspond to the solution of the gap equation[27], i.e.

$$\left.\frac{\partial\Omega}{\partial\sigma}\right|_{\sigma=\bar{\sigma}} = 0 \quad ; \quad \left.\frac{\partial\Omega}{\partial\eta}\right|_{\eta=\bar{\eta}} = 0 \tag{10}$$

In the thermal medium and NJL model, the effective thermodynamic potential of the QCD axon is given by the following equation.

$$\mathcal{V} = \Omega\left[\bar{\sigma}(a,T,\mu),\bar{\eta}(a,T,\mu),\overline{\mu_e}(a,T,\mu),a,T\right] \tag{11}$$

The axion mass and self-coupling are defined as the second-order derivative of the effective potential at $a = 0$, the fourth-order derivative, respectively [31].

$$m_a^2 = \left.\frac{d^2\mathcal{V}}{da^2}\right|_{a=0} = \frac{\chi_{\text{top}}}{f_a^2} \quad ; \quad \lambda_a = \left.\frac{\mathrm{d}^4\mathcal{V}}{\mathrm{d}a^4}\right|_{a=0} \tag{12}$$

3. Results and discussion

In this part, we show results for different quantities related to QCD axion thermodynamics in the NJL model under isospin symmetry. The parameters of NJL model are $\Lambda = 590MeV$, $c = 0.8$, $G_0 = 2.435/\Lambda^2$ and $m_u = m_d = 6MeV$. In the present work, our choice of $c = 0.8$ gives the results obtained, while comparing them with those obtained for $c = 0.2$ in Ref [25].

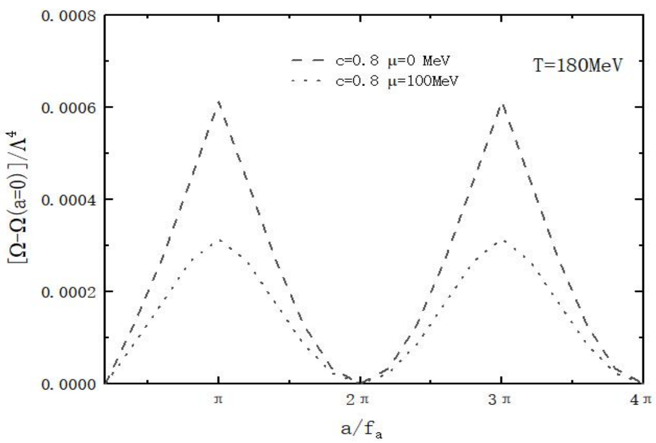

Figure 1. The change of thermodynamic potential with angle under different parameters and chemical potential.

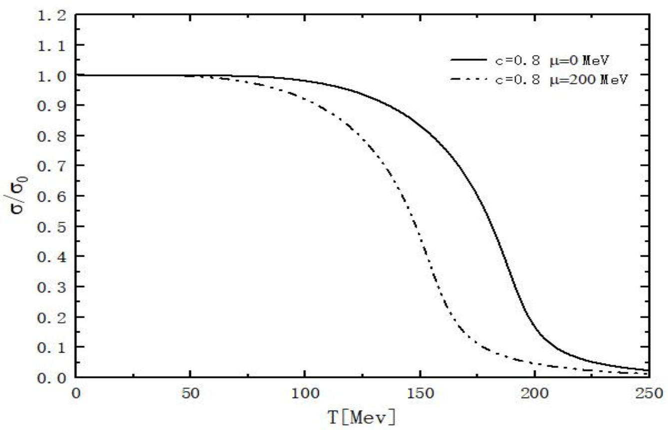

Figure 2. Change of the normalized scalar coalescence σ / σ_0 with temperature for a = 0.

The difference of thermodynamic potential between $0\,MeV$ and $100\,MeV$ at fixed temperature is shown in Figure.1. Since the form in M gives the results a certain sinusoidality, the effective potential takes the shape of a valley and a channel. We find that at a fixed temperature and angle the value of the thermodynamic potential becomes smaller due to the addition of the chemical potential, the image becomes flatter as the temperature becomes larger.

Figure. 2 gives the temperature dependence of quark condensation for different chemical potential environments at a fixed $a = 0$. We show that quark condensation decreases with increasing temperature in the temperature range considered and can see that near the critical temperature $T = 140MeV$, where the chemical potential is equal to zero, the reduction of quark condensation becomes significantly faster, reflecting the effective recovery of chiral symmetry. At the same time, we increase the chemical potential and find that the addition of the chemical potential accelerates the recovery of chiral symmetry.

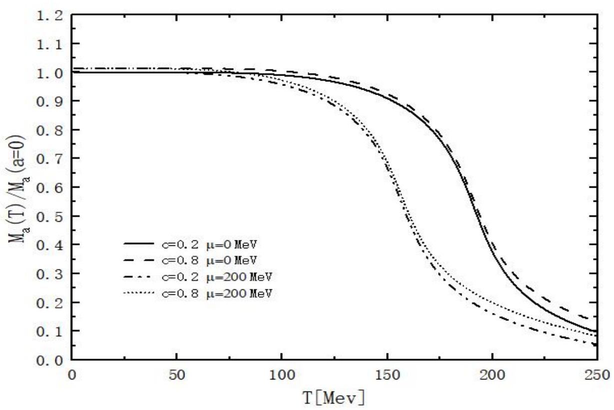

Figure 3. Temperature dependence of standard axion mass and quark chemical potential at finite temperature.

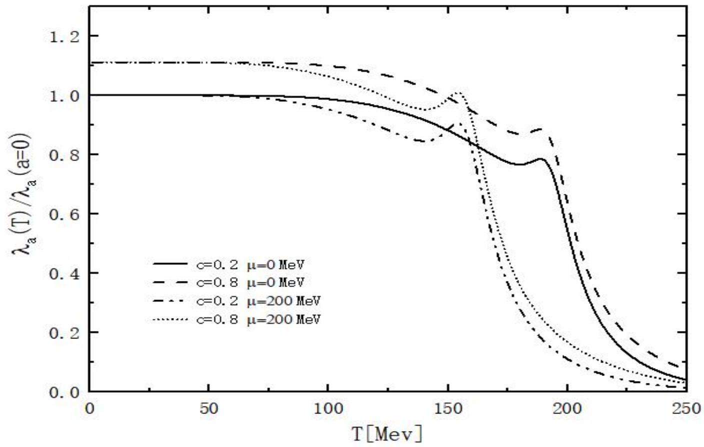

Figure 4. The standardized axion self-coupling and quark chemical potential likely change with temperature at finite temperature.

Figure 3 shows the axion mass begins to decrease significantly at the temperature node and moves towards lower temperatures as the chemical potential increases, and the temperature at which the axion mass decreases significantly moves to low temperature and the axion mass increases significantly due to the increase in the control interaction strength parameter c. As the chemical potential increases, the critical temperature of the chiral phase transition decreases, thus affecting the axion mass. The relationship between the topological magnetisation rate and axion mass given in equation (12) suggests that this is just a different representation of the rapid decrease in topological magnetisation rate in the NJL model over this temperature interval.

Figure 4 axion self-coupling variation curve. For the normalised self-coupling, as the chemical potential increases, corresponding to a leftward shift in temperature, the self-coupling begins to decrease. As the interaction strength parameter c increases, the self-coupling of the axion also increases significantly. This behaviour of axion self-coupling is consistent with the fact that the chiral phase change temperature decreases with increasing chemical potential. It is worth noting that there is a small peak near the phase change temperature, which may be a limitation of the mean field approximation in the evaluation of the thermodynamic potential [27].

4. Conclusion

In the present work, we show the changes of QCD axion in the NJL model of two flavors at finite temperature under different parameters. First, we study and analyse the difference of the thermodynamic potential with temperature and find that the variation of temperature and interaction parameters and chemical potential has a strong influence on the thermodynamic potential. Subsequently, we give the variation of quark condensation, axion mass and self-coupling with the interaction parameter c and the fixed chemical potential, respectively, and the results show that the differentiation of quark condensation has an important influence on the axion mass and self-coupling parameters.

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