MEASUREMENT OF Q OF SUPERCONDUCTING TE_{011} CAVITIES BY FREQUENCY SWEEP TECHNIQUE*

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ABSTRACT

A frequency sweep technique for Q measurements of superconducting cavities is described. A fast adiabatic passage through resonance shock-excites the cavity which relaxes afterwards in the form of decaying eigen oscillations. The procedure suggested permits measurements with negligible effects due to rf heating or electrical breakdown. It is suggested to use a transmission-type cavity and to measure the relaxation process of the cavity directly. The theory, which is developed in the paper, indicates that the loaded Q can be determined from the relaxation time and the coupling coefficient from the amplitude of the output signal. The frequency sweep technique has been applied to Q measurements of a superconducting TE_{011} cavity resonant at 2.868 GHz. The cavity is made of OFHC copper on which a 2.5 μm lead layer is electroplated from a lead fluoborate bath. An unloaded $Q_o = (196 \pm 15) \times 10^6$ at 4.2°K was measured. Possible systematic errors were examined and were found to be inconsequential. The temperature dependence of Q_o was also determined.

I. Introduction

Most conventional methods[1,2] for measuring the quality factor Q of a resonant cavity are no longer suitable for the extremely high Q's encountered in superconducting cavities. The decrement method, however, was used successfully by Grebenkemper and Hagen[3] and has since become a standard procedure if the Q exceeds about 10^6. In this method, the cavity is excited at its resonant frequency ω_o and then allowed to undergo eigen oscillations whose rate of decay is inversely proportional to the loaded Q. The theory underlying this method may be found, for example, in the article by Wilson[4] or by Zimmer.[5] The decrement method is applicable to transmission-type as well as reaction-type cavities. Pippard[6] discussed the relative merits of both configurations and it would seem to us that a transmission-type cavity is preferable, because it is less sensitive to small mismatches and stray reflections and because the measurements remain accurate over a wider range of coupling coefficients. In particular, it is possible to use loose input and output coupling, and errors due to inaccurate values of the coupling coefficients are reduced.

The decrement method commands a signal source with small frequency drift and extreme short-term stability. To overcome this requirement, Nguyen Tuong Viet[7] suggested using the cavity itself as the frequency determining element in a resonant loop, which consists of a transmission-type cavity, amplifier (TWT), and phase shifter for frequency adjustment. Schmitt and Zimmer[8] suggested an alternate way to avoid frequency drift and noise interference of the signal source by using a relaxation method derived from the conventional frequency sweep method.[1] A fast adiabatic passage through the resonance shock-excites the cavity which relaxes afterwards in the form of an eigen oscillation. In the experimental arrangement of Ref. 8 the relaxation signal is mixed with the drive signal and the beat phenomenon is observed.

In the present paper we describe a modified frequency sweep technique for Q measurements which eliminates effects due to rf heating or electrical breakdown. We suggest using a transmission-type cavity and measuring directly the relaxation process at the cavity output. It will be shown that the loaded Q can be determined from the relaxation time and the coupling coefficient from the amplitude of the output signal. In Section II, we present the theory of a fast adiabatic passage to the extent necessary for a quantitative interpretation of the observed signals. In Section III, the apparatus used is described in some detail, and in the last section we present experimental results obtained at 2.868 GHz with a lead-plated TE_{011} cavity.

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II. Theory

The analysis of a transmission-type resonant cavity is conveniently carried out by representing it symbolically through the equivalent circuit[9] with lumped elements of Fig. 1. The components of the equivalent circuit are normalized for simple notation; the unloaded quality factor $Q_o = (2\delta_o)^{-1}$ and the coupling coefficients $\varkappa_1$ and $\varkappa_2$ have their usual meaning. Note that the products $2\delta_o \varkappa_1$ and $2\delta_o \varkappa_2$ are geometrical constants. The basic differential equation of the problem is best written in terms of the voltage across the condenser as

$$v'' + 2\delta v' + v = u \tag{1}$$

where $\delta = \delta_o (1 + \varkappa_1 + \varkappa_2)$. The general solution of (1) with the initial conditions $v(-\infty) = v'(-\infty) = 0$ is well known.[10] The expression for the current i follows immediately as

$$i = v' = v_1' \int_{-\infty}^{\tau} \frac{u\, v_2\, d\tau}{v_2 v_1' - v_1 v_2'} - v_2' \int_{-\infty}^{\tau} \frac{u\, v_1\, d\tau}{v_2 v_1' - v_1 v_2'} \tag{2}$$

where v_1 and v_2 are two linearly independent solutions of the homogeneous differential equation. We chose $v_1 = \exp(j\Omega_1\tau)$ and $v_2 = \exp(j\Omega_2\tau)$ with $\Omega_1, \Omega_2 = j\delta \pm \sqrt{(1 - \delta^2)}$. The current then takes the form

$$i = \sum_{1}^{2} g_n\, e^{j\Omega_n\tau} \int_{-\infty}^{\tau} u\, e^{-j\Omega_n\tau}\, d\tau \tag{3}$$

with $g_1 = \Omega_1/(\Omega_1 - \Omega_2)$ and $g_2 = \Omega_2/(\Omega_2 - \Omega_1)$.

In this paper we are interested in the response of the resonant circuit to a frequency swept signal. We assume that the drive signal is given by $u = \exp j(\tau - \frac{1}{2}\dot{\Omega}\tau^2)$, which represents a quasi-sinusoidal signal ($\dot{\Omega} \ll 1$) with instantaneous frequency $\Omega = 1 - \dot{\Omega}\tau$. The unconventional minus sign assures that the first passage through resonance ($\Omega_1 \approx 1$) occurs at $\tau = 0$. The problem under consideration was attacked previously by various authors;[8,11-13] the present treatment, which was worked out by Hahn, gives explicit formulas valid in the case of a fast adiabatic passage through resonance.* The general solution of (3) may be found in terms of the function

$$F(z) = \int_0^z e^{x^2}\, dx\ ,$$

which is tabulated in the complex domain.[14] For the current follows rigorously

$$i = \sum_{1}^{2} \frac{g_n\, A_n(\tau)}{\sqrt{-j\frac{1}{2}\dot{\Omega}}}\, e^{j\frac{1}{2}(1-\Omega_n)^2/\dot{\Omega}}\, e^{j\Omega_n\tau} \tag{4}$$

with the time-dependent amplitude function

$$A_n(\tau) = F\left(\left[\tau - (1 - \Omega_n)/\dot{\Omega}\right]\sqrt{-j\tfrac{1}{2}\dot{\Omega}}\right) + j\tfrac{1}{2}\sqrt{\pi}\ .$$

*"Fast" means that the effects of cavity losses during the passage are negligible ($\dot{\Omega}^{\frac{1}{2}} \gg \delta$). "Adiabatic" means roughly that the passage through resonance lasts many periods and that the phase of the drive signal is irrelevant to the problem ($\dot{\Omega} \ll \delta$).

It may easily be verified that for a fast adiabatic passage (i.e., $\dot{\Omega} \ll \delta \ll \sqrt{\dot{\Omega}} \ll 1$) the term n = 1 mainly contributes and that the time average of the current behaves approximately as follows:

$$\tfrac{1}{2}\, i^* i \approx \begin{cases} 0\ ; & \tau\sqrt{\dot{\Omega}} < 0 \\ (\pi/4\dot{\Omega})\, e^{-2\delta\tau}\ ; & \tau\sqrt{\dot{\Omega}} > 0 \end{cases} \tag{5}$$

At microwave frequencies, the observable quantity is the output power P in the load of resistance $2\delta_o \varkappa_2$, that is, $P = \delta_o \varkappa_2\, i^* i$. The power available from the source is $P_o = (16\ \delta_o\ \varkappa_1)^{-1}$ and the time-dependent power transmission through the cavity is given, for times $t > 0$, by the unnormalized formula

$$\frac{P}{P_o} = \frac{\pi}{2\dot{\Omega} Q^2} \left\{ \frac{4\varkappa_1\varkappa_2}{(1+\varkappa_1+\varkappa_2)^2} \right\} e^{-\omega_o t/Q} \tag{6}$$

where $\dot{\Omega} = \dot{\omega}/\omega_o^2$ with $\dot{\omega} = \Delta\omega/\Delta t$ being the sweep rate in real time and ω_o is the resonant frequency of the cavity.

The interpretation of the observed response is facilitated if $\varkappa_1 = \varkappa_2 = \varkappa$, but it can be shown that the results are insensitive to a small inequality in the coupling coefficients. The loaded Q of the cavity is determined from the time constant T of the decaying output signal as

$$Q = \omega_o T \tag{7}$$

and, subsequently, the coupling coefficient is obtained from the peak output power $\hat{P} = P(t{=}0)$ according to

$$2\varkappa = \frac{1}{\rho - 1} \tag{8}$$

where

$$\rho^2 = \frac{\pi}{2\dot{\Omega} Q^2} \frac{P_o}{\hat{P}} . \tag{9}$$

The unloaded quality factor then follows directly as

$$Q_o = Q \frac{\rho}{\rho - 1} . \tag{10}$$

It was mentioned above that $2\varkappa/Q_o$ is a geometrical constant, whence ensues the important conclusion that at constant geometry $\hat{P} \propto P_o/\dot{\Omega}$ independent of the actual value of Q_o or $\varkappa$.

III. Apparatus

The frequency sweep technique suggested has been applied to Q measurements of a superconducting TE_{011} cavity. The setup of the electrical apparatus is given in Fig. 2, which is self-explanatory. The internal dimensions of the cavity (diameter = height = 13.843 cm) are selected for a resonance at about 2.856 GHz. An undercut lowers the degenerate mode TM_{111} by 8.8 MHz without affecting the desired fields. The coupling into the cavity is provided by 5 mm coaxial cables* through coupling holes with Ø = 6 mm. The center conductor of the coaxial cable is bent into a coupling loop. The loops are fully retractable, permitting an easy change of the coupling coefficient.

The mechanical construction of the cavity is shown schematically in Fig. 3, and details of the coupling mechanism are depicted in Fig. 4. The spring rings are inserted to prevent rf leakage bypassing the cavity. Cooling and mechanical alignment of the coaxial cable is provided by spring fingers. The cavity is vacuum sealed by indium gaskets and it is evacuated through the coupling holes. A vacuum in the range of 10^{-6} to 10^{-5} mm Hg is typically achieved. The cavity is inserted into a helium Dewar and, during the rf measurements, completely immersed in liquid helium; level indicators assure coverage of the cavity. A pump installation is provided to reduce the pressure above the helium bath and, thus, to lower the cavity temperature. The actual temperature is continuously monitored by means of a Germanium thermometer. The earth magnetic field is reduced by a μ-metal shield around the Dewar.

The superconductive material is 99.999% pure lead, which is deposited as a layer about 2.5 μm thick on the OFHC cavity by electroplating from a lead fluoborate bath. Great care was taken to produce a shiny, uniform, metallic Pb deposit without pinholes inclusive of the coupling holes. Details of the surface preparation are beyond the scope of this paper and will be published elsewhere.

IV. Results and Discussion

A typical detected response signal of the superconducting cavity to a frequency sweep through resonance is shown in Fig. 5. The square-law detection of the rf power is achieved by means of HP 420 crystal detectors. It was verified by a beat method that the frequency of the output signal is constant and equal to the resonant frequency of the cavity, that is, 2867.7 MHz at 4.2°K. The time constant T and the peak power $\hat{P}$ of the decaying signal is obtained from a semilogarithmic plot shown in Fig. 6.

*Identical results were obtained with a 7 mm coaxial cable. Its increased heat leak makes it less desirable, despite the smaller losses.

It should be pointed out that no deviation from the exp(-t/T) law is noticeable. The unloaded quality factor Q_o is derived according to Section II.

To test the sweep method for a possible dependence on power, loop position (x is defined in Fig. 4), or sweep rate, a series of Q measurements were performed on one cavity. A scrutiny of the results permits the following conclusions: (1) No variation with power is detectable. (2) In this run, no dependence on the loop position was observed; it could be corrected for by plotting Q_o versus $\varkappa$ and extrapolating to $\varkappa = 0$. (3) No apparent dependence on $\dot{\Omega}$ is seen, although FM noise causes a small jitter of the output signal. (4) Finally, the average of the quality factor follows to be $Q_o = (196 \pm 15) \times 10^6$ at 4.2°K.

Disturbing effects resulting from errors in $\dot{\Omega}$ may be eliminated by two methods. The first consists of reducing the coupling coefficient to a degree that $Q \approx Q_o$. Measurable output signals are still obtained by insertion of a high gain rf amplifier between cavity and diode or by use of a sensitive receiver. The second method is based on the fact that $2\varkappa/Q_o = G$ is a geometrical constant. Because the coupling mechanism is essentially a piston attenuator, the position dependence is known from theory to be $G \propto \exp(2\alpha x)$ with $\alpha \approx j_{11}'/a$, where j_{11}' is the first zero of J_1' and a is the radius of the cutoff tube. An experimental point for G was determined in a setup similar to Ref. 7, leading to the curve which is plotted in Fig. 7. G from this curve together with the measured loaded Q yield the corrected results for Q_o and $2\varkappa$ which are independent of the sweep rate, and which are computed according to $2\varkappa = GQ/(1 - GQ)$ and, of course, $Q_o = Q(1 + 2\varkappa)$. The corrected average follows as $Q_o = (192 \pm 16) \times 10^6$ in agreement with the uncorrected value.

The temperature dependence of Q_o was determined in a series of measurements which were interpreted in the described fashion, and the results are depicted in Fig. 8. The Q's obtained are at lower temperatures slightly below the theoretical[14,15] and experimental[16] values of the Stanford group, but this aspect is irrelevant to the subject of this paper.

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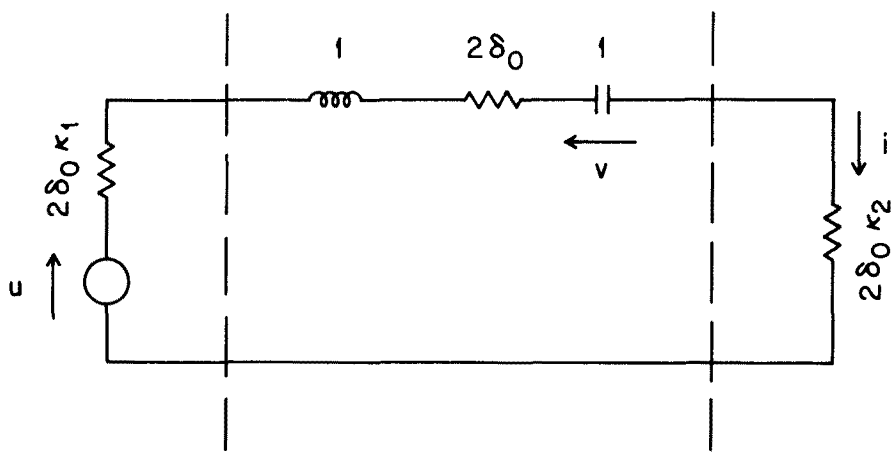

Fig. 1. Equivalent circuit of resonant cavity.

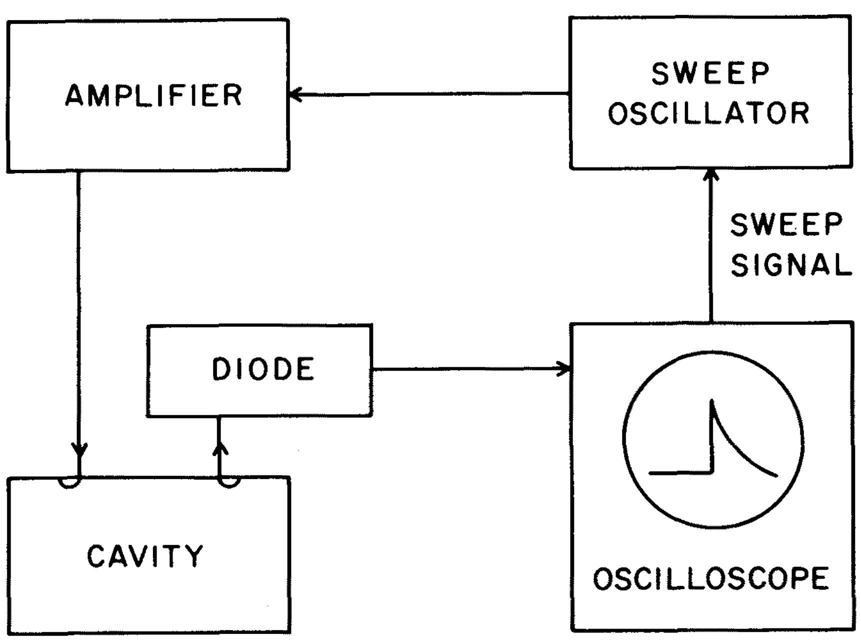

Fig. 2. Schematic diagram of test setup for Q measurements by frequency sweep technique.

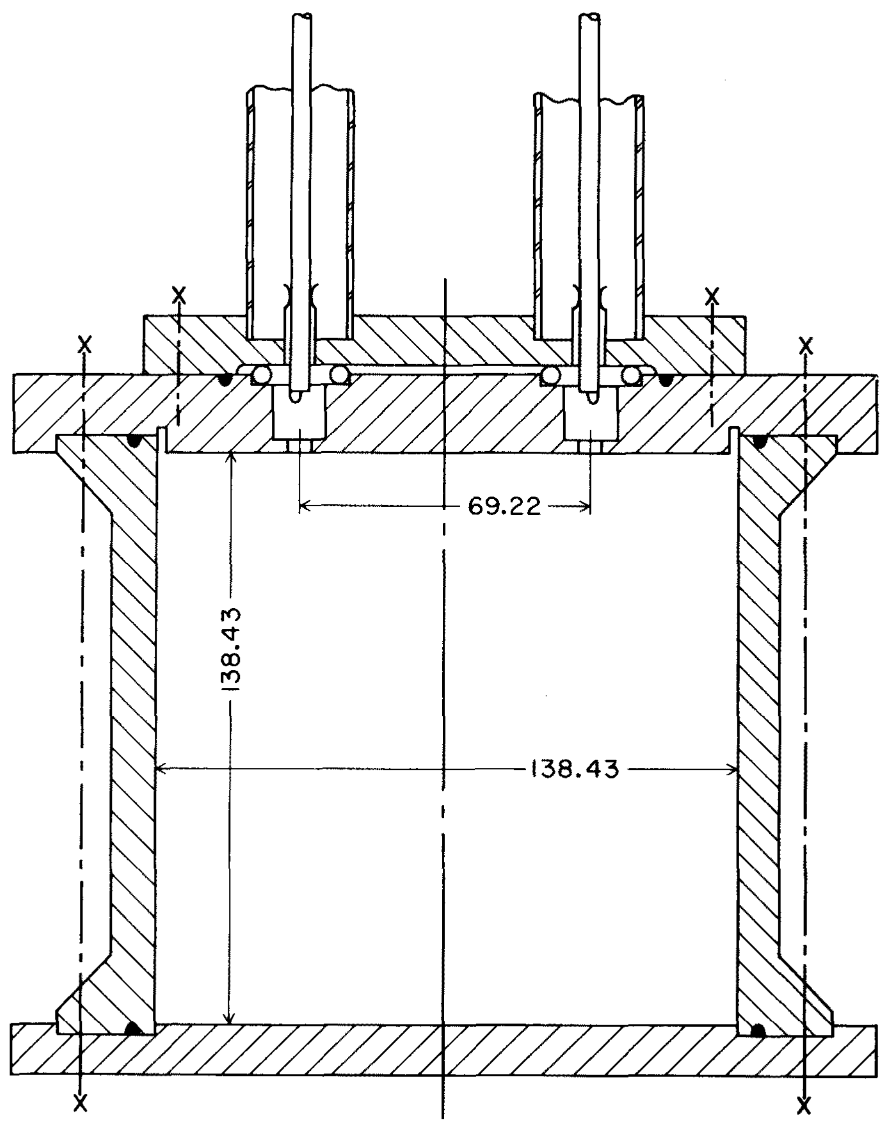

Fig. 3. Geometry of TE_{011} cavity. Dimensions are in mm.

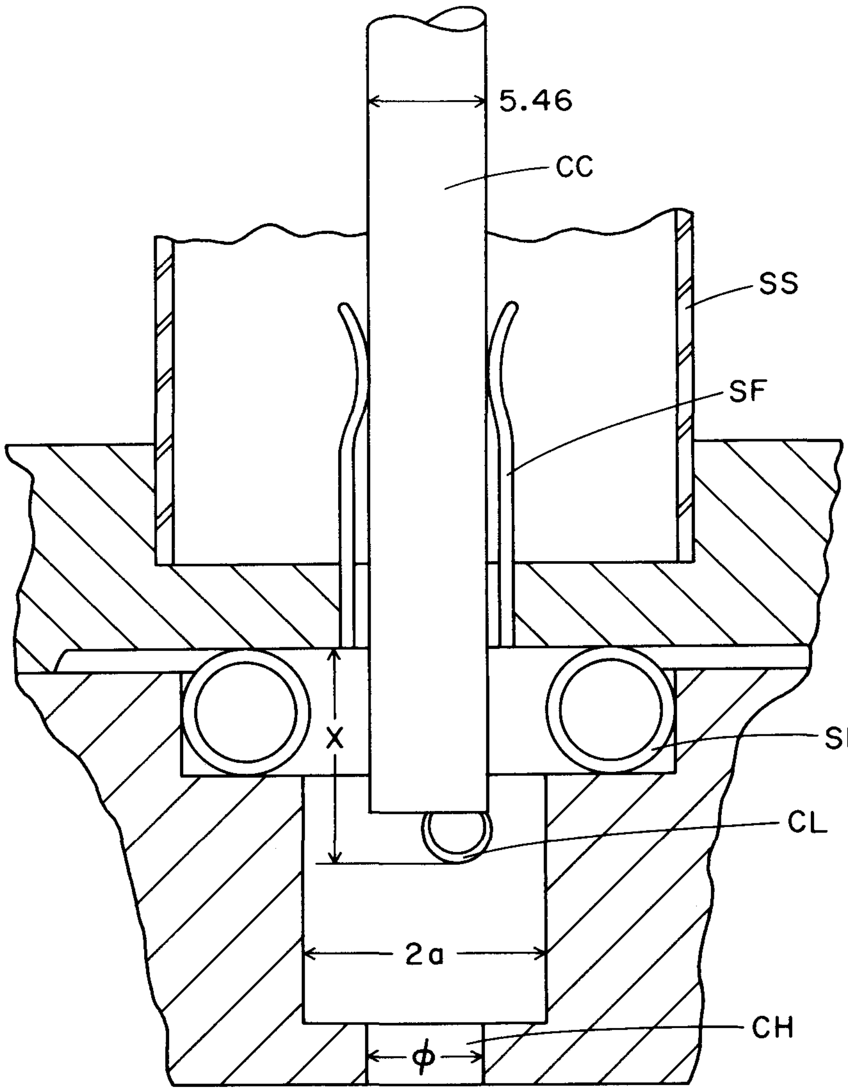

Fig. 4. Details of the coupling mechanism. Legend: CL = coupling loop, CH = coupling hole, SR = spring ring, SF = spring finger, CC = coaxial cable, SS = stainless steel vacuum pumpout, x = position of loop.

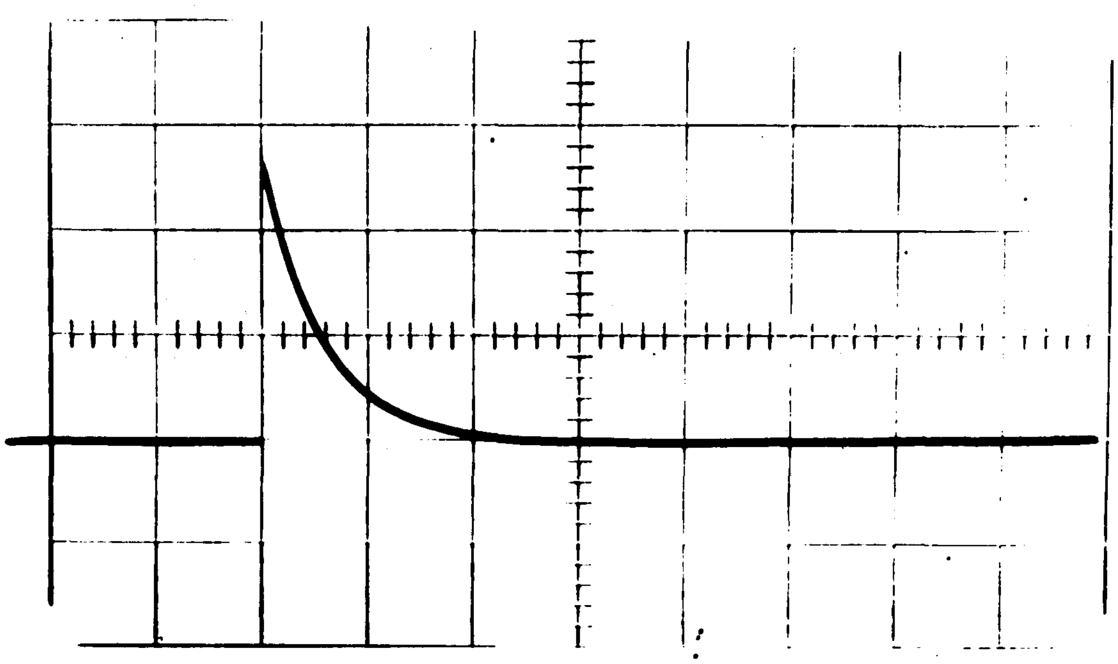

Fig. 5. Typical response of a superconducting cavity to a fast adiabatic passage through resonance. Horizontal: 10 ms/cm, vertical: 1 mV/cm.

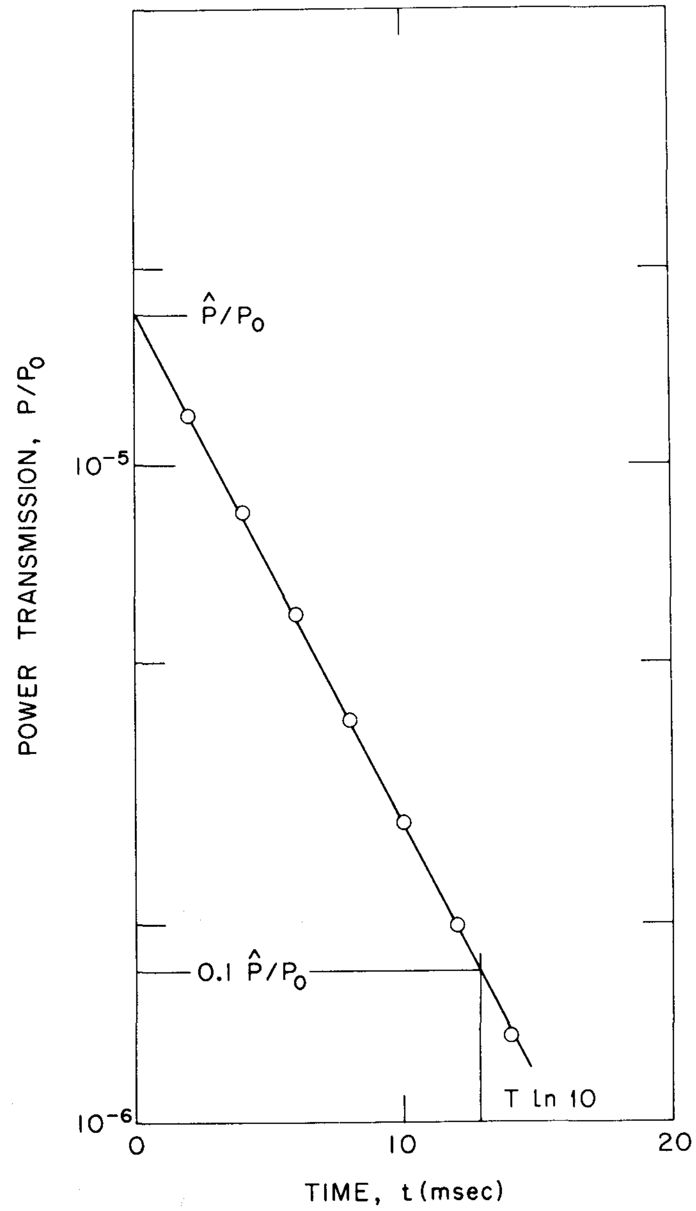

Fig. 6. Semilogarithmic plot of cavity output power versus time.

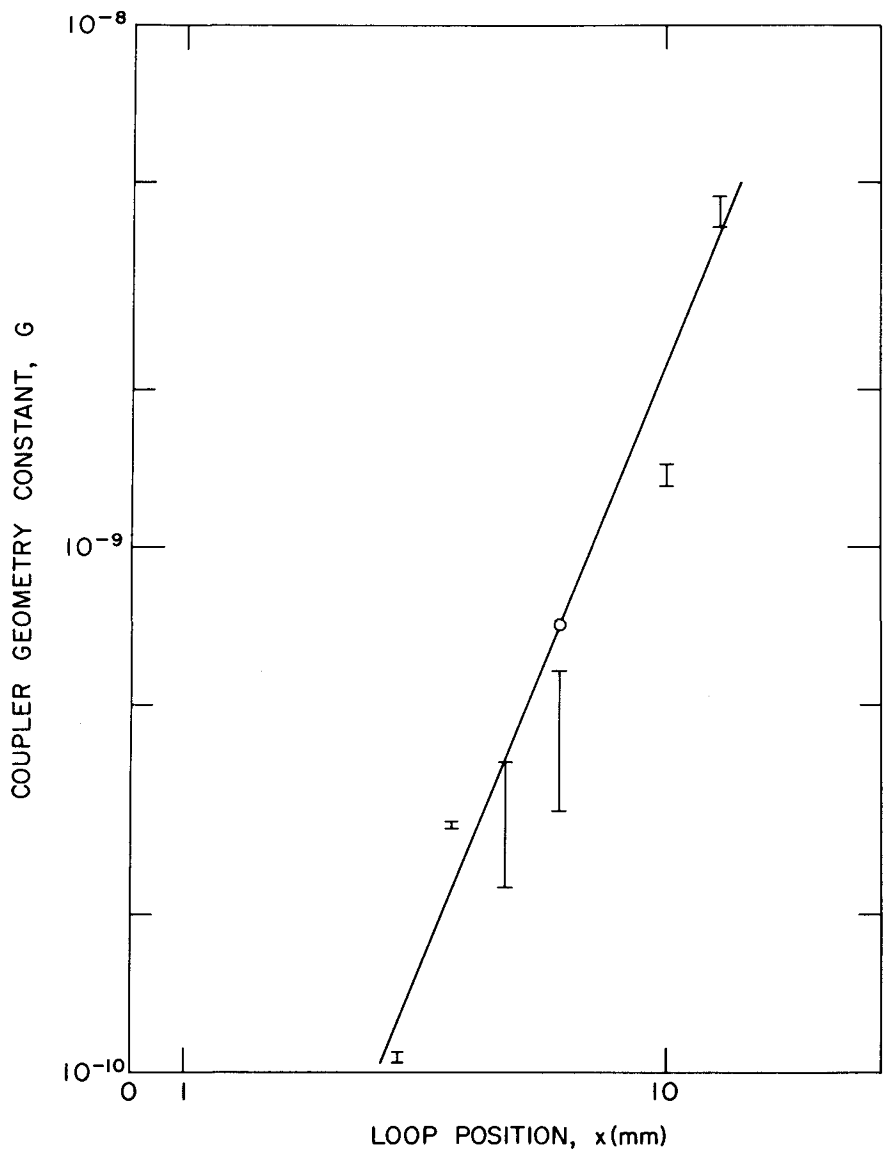

Fig. 7. Coupler geometry constant as function of loop position. Solid line is obtained by use of theoretical slope $G \propto \exp(2\alpha x)$ and one experimental point (o) from self-oscillating cavity. Experimental points from sweep method are plotted as I.

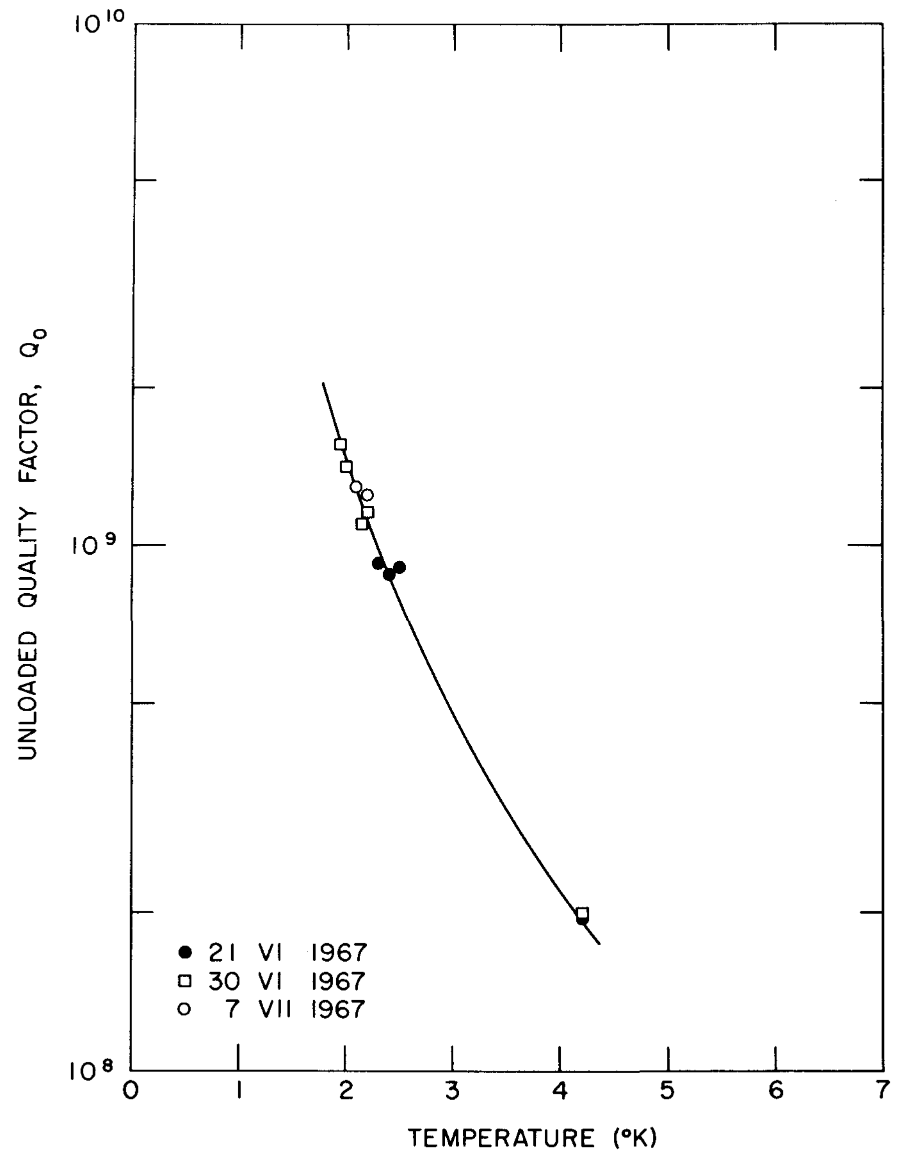

Fig. 8. Temperature dependence of unloaded quality factor. Higher values were obtained recently by reducing the coupling hole size, by electropolishing the copper surface, and by improving the magnetic shield. These results will be published soon.