Collapse of differentially rotating relativistic stars: Post black hole formation stage

Motoyuki Saijo[1] and Ian Hawke[2]

School of Mathematics, University of Southampton, Southampton SO17 1BJ, United Kingdom

Abstract

We investigate the collapse of differentially rotating supermassive stars (SMSs) by means of 3+1 hydrodynamic simulations in general relativity. We particularly focus on the onset of collapse to understand the final outcome of collapsing SMSs. We find that the estimated ratio between the black hole (BH) and the surrounding disk from the equilibrium star is roughly the same as the results from numerical simulation. This statement suggests that the picture of axisymmetric collapse is adequate in the absence of nonaxisymmetric instabilities for illustrating the final state of the collapse. We also find that when the newly formed BH is almost an extreme Kerr, a corotation resonance can be triggered by the oscillation of a BH. In this case, nonaxisymmetric instabilities are triggered by corotation resonance and make a significant difference in the gravitational waveforms. This alternative scenario for the collapse of differentially rotating SMSs might be observable by LISA.

There exists plenty of evidence that supermassive black holes (SMBHs) exists in the centre of galaxies, but their actual formation process has been a mystery for many decades [1]. Several different scenarios have been proposed, some based on stellar dynamics, others on gas hydrodynamics, and still others that combine the processes. Here we consider a possibility of forming an SMBH from the collapse of a supermassive star (SMS).

There are two categories of collapsing rotating SMSs based on their angular momentum distribution. One is the collapse of a uniformly rotating SMS. This happens when momentum transport is large, either through viscous turbulence or magnetic process, which drives the star to rotate uniformly. The other is the collapse of differentially rotating SMSs. This happens when the viscous and the magnetic effects are small, which allows the star to rotate differentially. One of the representative scenarios for forming a differentially rotating star is as follows. First, a gas cloud gathers in an almost spherical configuration with some amount of angular momentum in the system. Next the almost spherical star contracts, conserving the specific angular momentum due to the lack of viscosity, to form a differentially rotating star, and possibly a disk at the end of the contraction.

During the contraction of the differentially rotating SMS, prior to forming a supermassive disk, two possible instabilities may arise that terminate the contraction. One is the post-Newtonian gravitational instability, which leads the star to collapse dynamically. The other is the dynamical bar mode instability, which changes the angular momentum distribution of the star to form a bar, and possibly leads to the central core of the star collapsing to a black hole (BH) due to the angular momentum loss.

Here we focus on the post-Newtonian gravitational instability in differentially rotating SMSs. We particularly focus on the case where the final estimated BH is very close to the extreme Kerr BH, which potentially leads to rotational instabilities if they occur. In particular, we plan to answer the following questions. Does the BH form coherently? What are the features of the dynamics? Does the newly formed disk lead to contain various instabilities? Can this system act as an efficient source of gravitational waves (GWs)? In order to answer these questions, three dimensional general relativistic hydrodynamics are desirable. A more detailed discussion will be presented in the forthcoming paper [2]. Throughout this paper, we use the geometrized units with $G = c = 1$ and adopt Cartesian coordinates (x, y, z) with the coordinate time t.

[1]Email address: ms1@maths.soton.ac.uk
[2]Email address: I.Hawke@soton.ac.uk

Table 1: Four different rotating equilibrium SMSs for evolution.

Model	$\rho_0^{\max(a)}$	$M^{(b)}$	$J/M^{2(c)}$	$M/R_{\rm e}{}^{(d)}$	$m_{\rm disk}{}^{(e)}$	$(a/M)^{({\rm BH})(f)}$
I	1.56×10^{-5}	4.88	0.99	2.56×10^{-2}	0.044	0.98
II	1.56×10^{-5}	5.07	1.03	2.63×10^{-2}	—	$\gtrsim 1$
III	1.56×10^{-5}	5.31	1.07	2.78×10^{-2}	—	$\gtrsim 1$
IV	1.56×10^{-5}	5.75	1.10	3.47×10^{-2}	—	$\gtrsim 1$

(a): Maximum rest mass density

(b): Gravitational mass

(c): J: Total angular momentum

(d): R_e: Equatorial proper radius

(e): Ratio of the estimated rest mass of the disk from the equilibrium star to the rest mass of the equilibrium star

(f): Estimated Kerr parameter of the final hole from the equilibrium star

We perform 3+1 hydrodynamic simulations in general relativity using CACTUS[3] (gravitational physics), CARPET[4] (mesh refinement of space and time), WHISKY[5] (general relativistic hydrodynamics). Spacetime is evolved using the BSSN formulation with generalised hyperbolic K-driver for the lapse and generalised hyperbolic $\tilde{\Gamma}$-driver for the shift (e.g. [2]). We set the outermost boundary of the computational grid for all direction as $x_{\rm max} = 111 - 131M$, imposing plane symmetry across the $z = 0$ plane, and use $4 - 10$ refinement levels.

We first investigate the onset of collapse by evolving four differentially rotating equilibrium stars. We use the perfect fluid approximation with a Γ-law equation of state, choosing $\Gamma = 4/3$ to represent a SMS (the pressure is dominated by radiation pressure). We also impose a high degree of differential rotation, $\Omega_{\rm c}/\Omega_{\rm e} \approx 10$, to construct the equilibrium star, where Ω_c and Ω_e represents the angular velocity at the center and the equatorial surface, respectively. We choose the z-axis as the rotational one of the equilibrium star. The character of the equilibrium stars is summarized in Table 1. Since we use the polytropic equation of state $P = \kappa \rho_0^{\Gamma}$ (P: pressure, κ: constant, ρ_0: rest mass density, Γ: adiabatic exponent) when constructing initial data sets, all physical quantities are rescalable in terms of κ. Therefore, we represent all physical quantities in a nondimensional one in this paper, which is equivalent to setting $\kappa = 1$. To trigger collapse we deplete pressure by 1%. Checking the maximum rest mass density of the rotating stars throughout the evolution, we conclude that models I and II are radially unstable, while models III and IV are stable [2].

Next we trace the mass and angular momentum of the newly formed BH throughout the evolution using the technique of dynamical horizon. A dynamical horizon is defined as the spacelike marginally trapped tube which is composed of future-marginally trapped surface, i.e. apparent horizon. In order to compute the gravitational mass and the total angular momentum of the BH locally, we need to construct the timelike and the rotational Killing vectors intrinsic to the horizon, should they exist on the horizon numerically (e.g. [3]). Using these Killing vectors, we monitor the gravitational mass, total angular momentum and the Kerr parameter of the newly formed BH throughout the evolution (Fig. 1). The BH mass, the spin and the Kerr parameter increase monotonically after the BH has formed, by swallowing much of the surrounding material. This stage lasts roughly until the matter is swallowed, located inside the radius of the innermost stable circular orbit of the final BH.

We have also confirmed that the estimated mass and spin of the BH from the equilibrium configuration of the collapsing SMS are in good agreement with the results from the dynamics. For instance, the estimated Kerr parameter from the equilibrium star of model I is 0.98 (Table 1), while the result of numerical simulation is ≈ 0.97 (Fig. 1). Also the ratio between the estimated rest mass of the disk and the rest mass of the equilibrium star of model II is 0.044 (Table 1), while the result of numerical simulation is ≈ 0.05 (Fig. 2).

[3]http://www.cactuscode.org

[4]http://www.carpetcode.org

[5]http://www.whiskycode.org

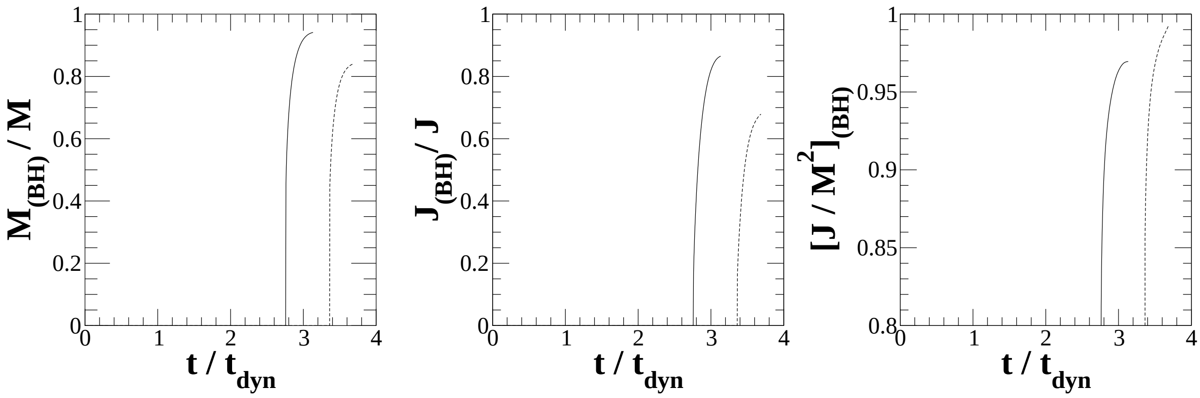

Figure 1: Gravitational mass ($M_{(\rm BH)}$), total angular momentum ($J_{(\rm BH)}$) and Kerr parameter ($(J/M^2)_{(\rm BH)}$) of a newly formed BH as a function of time. Solid and dashed line represent models I and II, respectively. Hereafter $t_{\rm dyn}$ represents the dynamical time defined as $t_{\rm dyn} = \sqrt{R_e^3/M}$.

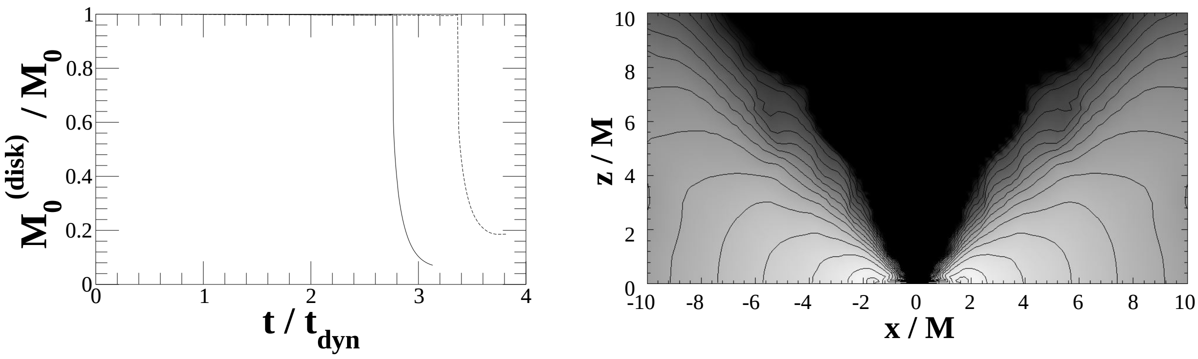

Figure 2: (Left panel) Disk mass as a function of time. We defined the disk mass as the rest mass outside the apparent horizon of the newly formed BH. Solid and dashed line represent models I and II, respectively.
(Right panel) Snapshot of the rest mass density in the meridional plane for model III at $t = 3.82 t_{\rm dyn}$. The contour lines denote $\rho_0/\rho_0^{\rm max} = 10^{-0.4(20-i)}$ ($i = 1, \cdots, 20$), where $\rho_0^{\rm max} = 8.56 \times 10^{-5}$. Note that the radius of the apparent horizon in the equatorial plane is $r \approx 0.16M$ in coordinate units.

We furthermore study the formation of a massive disk from the collapse of differentially rotating SMSs. We trace the rest mass of the disk by defining the rest mass outside the apparent horizon of the newly formed BH for models I and II (Fig. 2). The rest mass of the disk monotonically decreases once the BH has formed, since the newly formed BH grows monotonically by swallowing the surrounding materials. One noticeable feature in Fig. 2 is that there is a plateau at the final stage of model II. This indicates that the self gravity, the centrifugal force, and the pressure gradient are roughly balanced so that the disk can maintain Keplarian orbital motion around the BH. We also illustrate the snapshot of the rest mass density in the meridional plane of model II (Fig. 2). The maximum of the rest mass density is located around $r \approx 2M$ in coordinate units.

Finally we investigate the gravitational waveform from the collapsing object. We introduce the Weyl scalar Ψ_4 to study the outgoing gravitational waves. If we put the observer sufficiently far from the source, the Weyl scalar Ψ_4 roughly represents the outgoing gravitational waves, ignoring the radiation back scattered by the curvature. We observe the waveform (the real component of Ψ_4) along the x-axis in the equatorial plane at coordinate location $r \approx 60M$ for models I and II. Note that the equatorial radius of the equilibrium star is $r \approx 38.0M - 39.1M$ for models I and II. We find that the waveform contains three different stages. The first stage is the burst. This happens around horizon formation of

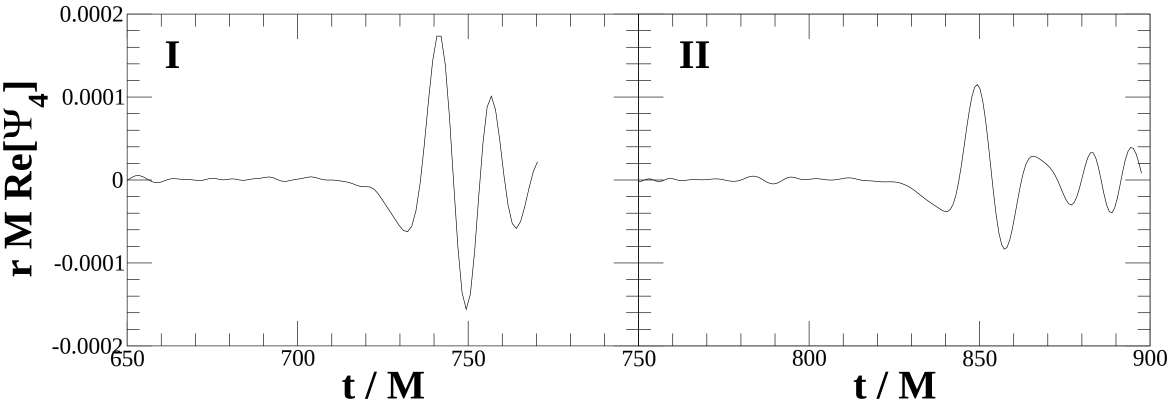

Figure 3: Gravitational waveform measured with the Weyl scalar Ψ_4 observed along the x-axis in the equatorial plane at $r = 61.43M$ for model I and $r = 59.20M$ for model II. Note that the time at which the apparent horizon is first detected is $t = 676M$ for model I and $t = 788M$ for model II, respectively. Taking the wave propagation time from the source to the observer into account, the apparent horizon formation in the waveform is roughly just before the peak due to the burst.

collapsing SMS. The dominant contribution of the burst comes from the axisymmetric mode due to the radial instability. The second stage is the quasinormal ringing of a newly formed BH. The dominant contribution is again the axisymmetric mode. The final stage might be related to an instability in the disk, and only appears in the model II. In fact when the spin of the newly formed BH is very close to the extreme Kerr, the amplitude of the gravitational wave signal gradually grows after the quasinormal ringing. We also check the azimuthal m modes of the rest mass density traced at the certain radius in the equatorial plane, and found that the $m = 2$ mode starts growing exponentially after the ringdown. One possible explanation for the exponential growth of the $m = 2$ mode at late times is the existence of corotation resonance of the newly formed disk triggered by the vibration of the hole. The dominant quasinormal mode of the BH has the frequency is $M\omega_{\rm qnm} = 0.43$ for $a/M = 0.9998$ [4], where $\omega_{\rm qnm}$ is a frequency of the quasinormal mode and a is the Kerr parameter. If the corotation resonance is triggered by quasinormal ringing, the necessary condition for triggering a corotation resonance is $\omega_{\rm qnm} = m\Omega$ (e.g. [5]) at a certain radius of the star, where Ω is the angular velocity. Since the inner part of the disk has $M\Omega \approx 0.38$ and $r \approx 0.79M$ in coordinate unit (Fig. 2), there exists a radius inside the disk which satisfies the above condition. In order to confirm the growth in the amplitude and the possible interpretation as corotation resonance, further time integration from our termination time of the three dimensional hydrodynamics in general relativity is necessary. Since we terminate the time integration by hand, there is no obstacle of continuing our simulation except for the computational time.

We investigate the collapse of differentially rotating SMSs, especially focusing on the post BH formation stage, by means of three dimensional hydrodynamic simulations in general relativity. We particularly focus on the onset of collapse to form a rapidly rotating hole as the final outcome.

We have found that the evolutional results about the feature of the final hole and the disk behaves quite similar to the estimation from the equilibrium configuration when the estimated, final BH has $J_{\rm (BH)}/M_{\rm (BH)}^2 < 1$. This result suggests that in the absence of a nonaxisymmetric instability, the estimation of the BH mass and the disk mass agree with a simple axisymmetric picture that the specific angular momentum is conserved throughout the evolution and the newly formed BH swallows the matter up to the radius of the innermost stable circular orbit.

We have also found that when the newly formed BH is "very" close to the extreme Kerr with sufficient matter around it, a corotational resonance of the matter may be triggered by the quasi-normal mode of the BH. As the Kerr parameter goes to 1, the radius of the innermost stable circular orbit coincides with that of the event horizon. Hence the necessary condition for the corotation resonance triggered by the vibration of the BH is satisfied. Therefore the system potentially emits gravitational waves effectively due to the resonance. However, further time integration of the post BH formation stage is necessary to confirm the above statement.

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