35 Spontaneous leptonic CP violation and θ_{13}

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Abstract In this work one reviews a simple scenario [1] where the above three aspects (leptogenesis, leptonic mixing and spontaneous CP violation) are related. To this aim, we shall add to the Standard Model (SM) a minimal particle content: two Higgs triplets $\Delta_a\,(a = 1, 2)$ with unit hypercharge and a complex scalar singlet S with zero hypercharge.

35.1 The model

The SM is extended with two Higgs triplets $\Delta_a\,(a = 1, 2)$ of unit hypercharge and a complex scalar singlet S with zero hypercharge. In the $SU(2)$ representation:

$$\Delta_a = \begin{pmatrix} \Delta_a^0 & -\Delta_a^+/\sqrt{2} \\ -\Delta_a^+/\sqrt{2} & \Delta_a^{++} \end{pmatrix}. \tag{35.1}$$

CP invariance is imposed at the Lagrangian level and a Z_4 symmetry is introduced under which the scalar and lepton fields transform as indicated in Table. 35.1

The most general scalar potential invariant under the above symmetries can be written as

$$V^{CP\times Z_4} = V_S + V_\phi + V_\Delta + V_{S\phi} + V_{S\Delta} + V_{\phi\Delta} + V_{S\phi\Delta}, \tag{35.2}$$

where each terms are presented in [1]. Since CP invariance has been imposed at the Lagrangian level, all the parameters are assumed to be real. This symmetry can be spontaneously broken by the complex VEV of the scalar singlet S. To show that this is indeed the case, let us analyze the scalar potential for S. The tree-level potential then reads

$$V_0 = m_S^2 v_S^2 + \lambda_S v_S^4 + 2\left(\mu_S^2 + \lambda_S'' v_S^2\right) v_S^2 \cos(2\alpha) + 2\lambda_S' v_S^4 \cos(4\alpha), \tag{35.3}$$

with $\langle S\rangle = v_S e^{i\alpha}$. Besides the trivial solution $v_S = 0$, which leads to $V_0 = 0$, there are other three possible solutions to the minimization problem with $v_S \neq 0$: (i) $\alpha = 0, \pm\pi$, (ii) $\alpha \pm \frac{\pi}{2}$ and (iii) $\cos(2\alpha) = -\dfrac{\mu_S^2 + \lambda_S'' v_S^2}{4\lambda_S' v_S^2}$.

Only the last solution is of interest to us since it leads not only to the spontaneous breaking of the CP symmetry but also to a non-trivial CP-violating phase in the one-loop diagrams relevant for leptogenesis, for a review on this subject see [2].

Table 35.1: Representations of the fields under the $A_4 \times Z_4$ and $SM = SU(2)_L \times U(1)_Y$ symmetries.

Field	L	e_R, μ_R, τ_R	Δ_1	Δ_2	ϕ	S	Φ	Ψ
A_4	$\mathbf{3}$	$\mathbf{1}, \mathbf{1}', \mathbf{1}''$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$
Z_4	i	$-i$	1	-1	i	-1	i	1
SM	$(2,-1/2)$	$(1,-1)$	$(3,1)$	$(3,1)$	$(2,1/2)$	$(1,0)$	$(1,0)$	$(1,0)$

In order to generate a realistic lepton mixing pattern we shall also impose an A_4 discrete symmetry at high energies. We recall that, in a particular basis, the Clebsch-Gordan decompositions of the A_4 group are can be made with real coefficients. The spontaneous breaking of the A_4 symmetry is then guaranteed by adding to the theory two extra heavy scalar fields, Φ and Ψ, with a suitable VEV alignment. The complete symmetry assignments of the fields under $A_4 \times Z_4$ and $SU(2)_L \times U(1)_Y$ are given in Table 35.1.

Below the cut-off scale Λ, the flavour dynamics is encoded in the relevant effective Yukawa Lagrangian $\mathcal{L}$, which contains the lowest-order terms[1] in an expansion in powers of $1/\Lambda$,

$$\begin{aligned}\mathcal{L} =& \frac{y_e^\ell}{\Lambda}\left(\overline{L}\Phi\right)_{\mathbf{1}}\phi e_R + \frac{y_\mu^\ell}{\Lambda}\left(\overline{L}\Phi\right)_{\mathbf{1}''}\phi\mu_R + \frac{y_\tau^\ell}{\Lambda}\left(\overline{L}\Phi\right)_{\mathbf{1}'}\phi\tau_R \\ &+ \frac{y_2}{\Lambda}\Delta_2\left(L^T L\Psi\right)_{\mathbf{1}} + \frac{1}{\Lambda}\Delta_1\left(L^T L\right)_{\mathbf{1}}\left(y_1 S + y_1' S^*\right) + \text{H.c.}\,.\end{aligned} \tag{35.4}$$

As soon as the heavy scalar fields develop VEVs along the required directions, namely,

$$\langle\Phi\rangle = (r,0,0)\,,\ \langle\Psi\rangle = (s,s,s)\,, \tag{35.5}$$

and the scalar singlet S acquires a complex VEV, $\langle S\rangle = v_S\, e^{i\alpha}$, the Yukawa matrices become

$$\mathbf{Y}^e = \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}, \quad \mathbf{Y}^{\Delta_1} = y_{\Delta_1}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \mathbf{Y}^{\Delta_2} = \frac{y_{\Delta_2}}{3}\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}, \tag{35.6}$$

and

$$y_{e,\mu,\tau} = \frac{r}{\Lambda} y^\ell_{e,\mu,\tau}\,, \quad y_{\Delta_1} = \frac{v_S}{\Lambda'}\left(y_1 e^{i\alpha} + y_1' e^{-i\alpha}\right), \quad y_{\Delta_2} = \frac{y_2}{\Lambda} s. \tag{35.7}$$

Notice that the Yukawa matrices $\mathbf{Y}^{\Delta_1}$ and $\mathbf{Y}^{\Delta_2}$ exhibit the so-called $\mu-\tau$ and magic symmetries, respectively.

[1] In principle, one could also include the renormalizable 4-dimension term $\Delta_2 L^T L$. This term is however easily removed by imposing an additional shaping Z_4 symmetry.

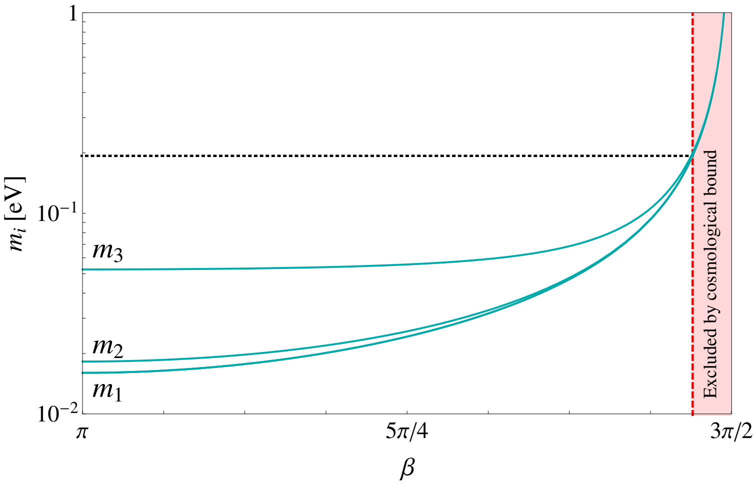

Figure 35.1: Neutrino masses m_i as a function of the high-energy phase β in the exact TBM case.

35.2 Low-energy phenomenology

In the present framework, neutrinos acquire masses through the well-known type II seesaw mechanism due to the tree-level exchange of the heavy scalar triplets Δ_a. The unitary mixing matrix $\mathbf{U}$ is given by

$$\mathbf{U} = e^{-i\sigma_1/2} \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \mathrm{diag}\left(1, e^{i\gamma_1}, e^{i\gamma_2}\right), \tag{35.8}$$

where

$$\gamma_1 = (\sigma_1 - \beta)/2, \quad \gamma_2 = (\sigma_1 - \sigma_2)/2, \quad \sigma_{1,2} = \arg\left(z_2 \pm z_1 e^{i\beta}\right). \tag{35.9}$$

Hereafter we consider the relevant CP-violating phase as being β. Since at this point there is no Dirac-type CP violation ($\mathbf{U}_{13} = 0$), the Majorana phases $\gamma_{1,2}$ are the only source of CP violation in the lepton sector.

At 1σ confidence level, the neutrino mass squared differences are [3]

$$\Delta m^2_{21} = \left(7.59^{+0.20}_{-0.18}\right) \times 10^{-5}\,\mathrm{eV}^2, \quad \Delta m^2_{31} = \left(2.50^{+0.09}_{-0.16}\right) \left[-2.40^{+0.09}_{-0.08}\right] \times 10^{-3}\,\mathrm{eV}^2, \tag{35.10}$$

for the normal [inverted] neutrino mass hierarchy. The present model cannot accommodate an inverted hierarchy for the neutrino mass spectrum. The dependence of neutrino masses on the high-energy phase β is presented in Fig. 35.1 for the exact TBM case. The light red shaded area is currently disfavoured by the recent WMAP seven-year cosmological observational data [4].

The T2K [5] and MINOS [6] neutrino oscillation data imply for the θ_{13} mixing angle

$$\sin^2\theta_{13} = 0.013^{+0.007}_{-0.005}\begin{pmatrix}+0.015\\-0.009\end{pmatrix}\begin{bmatrix}+0.022\\-0.012\end{bmatrix}, \tag{35.11}$$

at $1\sigma(2\sigma)[3\sigma]$. Recently, through the observation of electron-antineutrino disappearance, the Daya Bay Reactor Neutrino Experiment has also measured the non-zero value [7]:

$$\sin^2(2\theta_{13}) = 0.092 \pm 0.016(\text{stat}) \pm 0.005(\text{syst}), \tag{35.12}$$

with a significance of 5.2σ. In the light of these results, models that lead to tribimaximal mixing appear to be disfavored. Here we shall consider small perturbations around the TBM vacuum-alignment conditions (35.5). We consider two distinct cases (with $|\varepsilon_{1,2}| \ll 1.$):

CASE A - Small perturbations around the flavon VEV $\langle\Phi\rangle = (r, 0, 0)$ of the form $\langle\Phi\rangle = r(1, \varepsilon_1, \varepsilon_2)$;

Due to the new form of $\langle\Phi\rangle$, the charged lepton Yukawa matrix is

$$\mathbf{Y}^\ell = \begin{pmatrix} y_e & y_\tau\varepsilon_1 & y_\mu\varepsilon_2 \\ y_\tau\varepsilon_2 & y_\mu & y_e\varepsilon_1 \\ y_\mu\varepsilon_1 & y_e\varepsilon_2 & y_\tau \end{pmatrix}, \tag{35.13}$$

which implies $\mathbf{U}_\ell \neq \mathbb{1}$, where $\mathbf{U}_\ell$ is the unitary matrix which rotates the left-handed charged-lepton fields to the their physical basis. The new lepton mixing matrix $\mathbf{U} = \mathbf{U}_\ell^\dagger \mathbf{U}_{\text{TBM}}$ yields the perturbed mixing angles

$$\sin^2\theta_{12} \simeq \frac{1}{3}\left[1 - 2(\varepsilon_1 + \varepsilon_2)\right], \quad \sin^2\theta_{23} \simeq \frac{1}{2}(1 + 2\varepsilon_1), \quad \sin^2\theta_{13} \simeq \frac{(\varepsilon_1 - \varepsilon_2)^2}{2}, \tag{35.14}$$

at lowest order in $\varepsilon_{1,2}$. Obviously, the rotation of the charged lepton fields does not affect the neutrino spectrum nor generate a Dirac-type CP-violating phase. Since the flavon fields are real, the Majorana phases $\gamma_{1,2}$ also remain unaltered.

CASE B - Small perturbations around the flavon VEV $\langle\Psi\rangle = s(1, 1, 1)$ of the form $\langle\Psi\rangle = s(1, 1 + \varepsilon_1, 1 + \varepsilon_2)$;

The Yukawa couplings $\mathbf{Y}^{\Delta_2}$ contributing to the neutrino mass matrix are now given by

$$\mathbf{Y}^{\Delta_2} = \frac{y_{\Delta_2}}{3}\begin{pmatrix} 2 & -1-\varepsilon_2 & -1-\varepsilon_1 \\ -1-\varepsilon_2 & 2+2\varepsilon_1 & -1 \\ -1-\varepsilon_1 & -1 & 2+2\varepsilon_2 \end{pmatrix}. \tag{35.15}$$

Consequently, at first order in $\varepsilon_{1,2}$, the neutrino mass spectrum get small corrections. Still, as in the unperturbed case, it can be shown that an inverted neutrino hierarchy is not allowed. In the present case, the approximate analytic expressions for the mixing angles are

$$\sin^2\theta_{12} \simeq \frac{1}{3} + \frac{2}{9}(\varepsilon_1 + \varepsilon_2), \quad \sin^2\theta_{13} \simeq \frac{(\varepsilon_1 - \varepsilon_2)^2}{72\cos^2\beta}, \quad \sin^2\theta_{23} \simeq \frac{1}{2} + \frac{1}{6}(\varepsilon_1 - \varepsilon_2), \tag{35.16}$$

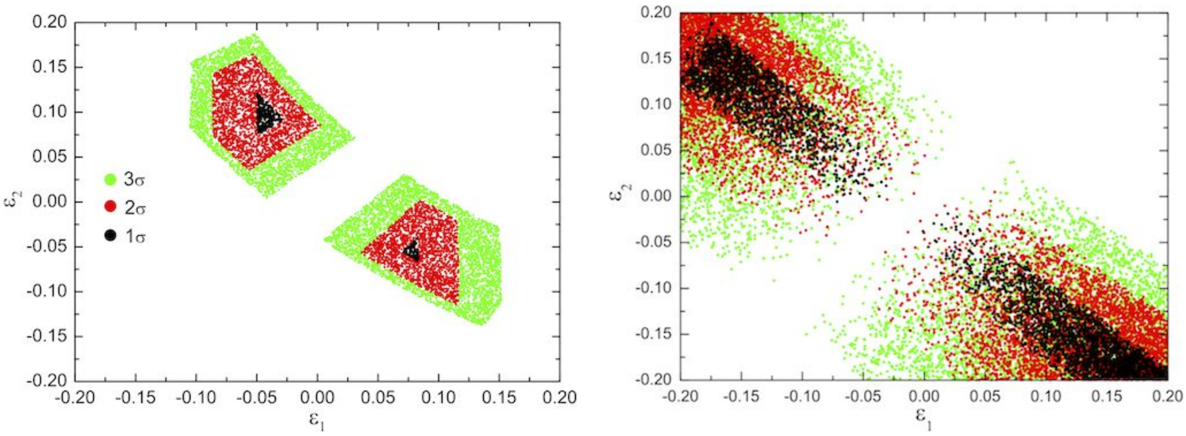

Figure 35.2: Allowed regions in the $(\varepsilon_1, \varepsilon_2)$ plane corresponding to the VEV perturbations of the flavon field $\langle\Phi\rangle = r(1, \varepsilon_1, \varepsilon_2)$ in case A (left panel) and $\langle\Psi\rangle = s(1, 1+\varepsilon_1, 1+\varepsilon_2)$ in case B (right panel). The scatter points were obtained considering the 1σ (black), 2σ (red) and 3σ (green) neutrino oscillation data.

while for the Dirac-type CP-violating invariant $J_{\rm CP}$ we have

$$J_{\rm CP} = \mathrm{Im}\left[\,\mathbf{U}_{11}\mathbf{U}_{22}\mathbf{U}^*_{12}\mathbf{U}^*_{21}\,\right] \simeq \frac{\varepsilon_2 - \varepsilon_1}{36}\tan\beta\,.$$

We now comment on the possibility of reproducing the recent Daya Bay θ_{13} value (35.12) in our framework. In the absence of a 3-neutrino global analysis of the oscillation data including the Daya Bay results, we take the 1σ values for θ_{12}, θ_{23} and $\Delta m^2_{21,31}$ obtained in [3]. One can see that the new Daya Bay value for θ_{13} is not compatible with the remaining mixing angles for case A. Instead, for case B we get a perfect agreement with all data [1].

35.3 Higgs triplet decays and leptogenesis

The mechanism of leptogenesis can be naturally realized in the present model due to the presence of the scalar triplets Δ_1 and Δ_2. In the presence of CP-violating interactions, the decay of Δ_a into two leptons generates a nonvanishing leptonic asymmetry for each triplet component $(\Delta^0_a, \Delta^+_a, \Delta^{++}_a)$. Assuming $M_a \ll M_b$, the flavoured CP asymmetry given for each triplet component can be rewritten as

$$\epsilon^{\alpha\beta}_a = c_{\alpha\beta}\,\mathbf{P}^a_{\alpha\beta}\,\epsilon^0_a\,, \quad \epsilon^0_a = \frac{1}{3\pi}\frac{z_a\, z_b\, |u_a|^2\, M^2_a\, \sin\beta}{z^2_a\, t_a v^4 + 4\,|u_a|^4 M^2_a}\,, \tag{35.17}$$

where $c_{\alpha\beta}$ is $2-\delta_{\alpha\beta}$ for Δ_a^0, Δ_a^{++} and 1 for Δ_a^+, and with $t_1 = 3$ and $t_2 = 2$. The matrix $\mathbf{P}^a$ is given by

$$\mathbf{P}^a = \frac{(-1)^a}{2}\begin{pmatrix} -2(1+\varepsilon_1+\varepsilon_2) & \varepsilon_1-\varepsilon_2 & \varepsilon_2-\varepsilon_1 \\ \varepsilon_1-\varepsilon_2 & 4\left(\varepsilon_1+\varepsilon_2\, y_\mu^2/y_\tau^2\right) & 1+\varepsilon_1+\varepsilon_2 \\ \varepsilon_2-\varepsilon_1 & 1+\varepsilon_1+\varepsilon_2 & -4\left(\varepsilon_1+\varepsilon_2\, y_\mu^2/y_\tau^2\right) \end{pmatrix}, \tag{35.18}$$

for case A, while

$$\mathbf{P}^a = (-1)^a\left[\frac{1}{2}+\delta_{a2}\frac{v^4 z_2^2(\varepsilon_1+\varepsilon_2)}{18M_2^2u_2^4+9v^4z_2^2}\right]\begin{pmatrix} -2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \tag{35.19}$$

in case B. Obviously, in the TBM limit ($\varepsilon_{1,2}=0$), there is a unique matrix $\mathbf{P}^a$. In this case, the flavour structure of $\mathbf{P}^a$ dictates that the only allowed decay channels of Δ_a are into the ee and $\mu\tau$ flavours. Once the VEV perturbations are introduced, new decay channels are opened in case A with the corresponding CP asymmetries suppressed by $\mathcal{O}(\varepsilon)$ factors.

Maximizing ϵ_a^0 with respect to the VEV of the decaying scalar triplet u_a, one obtains

$$\epsilon^0_{1,\max} \simeq \frac{M_1\sqrt{\Delta m_{31}^2}}{12\sqrt{6}\pi v^2}\sin\beta, \quad \epsilon^0_{2,\max} \simeq \frac{M_2\sqrt{\Delta m_{31}^2}}{48\pi v^2}\tan\beta\,. \tag{35.20}$$

One can see from the above equations that sufficiently large values of the CP asymmetries can be obtained in the flavoured regime, i.e. $M_a < 10^{12}$. Therefore, unlike the type-I seesaw framework [8–11], imposing to the Lagrangian a discrete symmetry do not necessarily leads to a vanishing leptonic CP asymmetry in the type II seesaw case [12].

35.4 Conclusion

A simple scenario where spontaneous CP violation, leptonic mixing and thermal leptogenesis are related was presented. We added a minimal particle content to the SM, namely, two Higgs triplets $\Delta_{1,2}$ and a complex scalar singlet S. In this framework, a single phase connects low- and high-energy CP violation.

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