

Ground state properties of Zinc isotopes investigated following the Skyrme Hartree-Fock-Bogoliubov approach

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Introduction

The structure of nucleus is still an area of interest due to its mysterious nature. The richness in its theoretical and experimental channels adds to its beauty too. The explanation of the nuclear structure, based on the nucleons and their interactions is indeed a great challenge. Development of a unified theoretical framework that could explain the properties of nuclei and make successful predictions, especially related to exotic nuclei which could not be tackled experimentally, is the usual practice in nuclear physics. The true nature of nuclear force and the nucleon-nucleon interaction is still not known clearly. Though, Bohr, Mottelson and Pines [1] proposed a theory, considering the pairing interactions, in analogy with the BCS theory, it failed in the regions far from the stability line, since it was inadequate in incorporating the influence of pairing upon the unstable states.

Hartree-Fock-Bogoliubov (HFB) theory [2] is a successful theory, which unifies the self-consistent mean-field approach in the Hartree-Fock theory, in which the properties of nuclei are explained by treating nucleons as independent particles moving in an average potential and the pairing correlations due to the introduction of quasi-particles in a pairing field as in the BCS model. The general product wave functions are Slater determinants, which are determined by considering all the correlations in the single-particle approach by making use of the variational principle.

In this work, we investigated some of the ground-state properties of the isotopes of Zinc (Zn) ($Z=30$), with mass number ranging from 54 to 81. For this, we performed the Skyrme-Hartree-Fock-Bogoliubov calculations [3] for all the isotopes, using SLY4 as the Skyrme functional. We then compared our results with the available experimental data [4].

Formalism

The shell model suggests that each nucleon in a nucleus moves independently in an average potential generated by all the other nucleons. A single-particle potential can be derived from the two-body interactions, following the application of the variational minimization on the Hartree-Fock energy by treating the Slater determinants as trial wave functions. The resulting Hartree-Fock equation contains a density dependent self-consistent field in addition to the kinetic energy term and can be solved iteratively.

The HFB theory treats correlations in a single-particle approach by the introduction of quasi-particles in a pairing potential. The ground-state of the nucleus is considered as a vacuum of quasi-particles, which are either particles or holes in the limit of pairing correlations. To derive the required HFB equation, we start with the many-body Hamiltonian and the particle number symmetry is conserved by the introduction of Lagrange multipliers through the Lipkin-Nogami method. The HFB equation [2] is given by

$$\begin{pmatrix} h & \Delta \\ -\Delta^* & -h^* \end{pmatrix} \begin{pmatrix} U_k \\ V_k \end{pmatrix} = \begin{pmatrix} U_k \\ V_k \end{pmatrix} \cdot E_k \quad (1)$$

The study is carried out using the code HFBTHO (v2.00d) [3], which iteratively diagonalizes the Hamiltonian based on the Skyrme functional SLY4, until a self-consistent solution is reached. The pairing interaction used in the HFB approach is of the form,

$$V_{pairing}^{(n,p)}(r) = V_0^{(n,p)} \left(1 - \sigma \frac{\rho(r)}{\rho_0} \right) \delta(r - r') \quad (2)$$

where, $V_0^{(n,p)}$ is the pairing strength, $\rho(r)$ is the isoscalar nucleonic density and $\rho_0 = 0.16 \text{ fm}^{-3}$ [3]. Here, a mixed surface-volume pairing force ($\sigma = 0.5$) has been used with a quasi-particle cut-off at 60 MeV. The harmonic oscillator basis was

characterized by setting the length of the oscillator as 2.2 fm. The principal number of oscillator shells was taken as 16. The same value of pairing strength was chosen for both protons and neutrons, which is 300 MeV. The calculations for odd isotopes were carried out by blocking the quasi-particle states.

Results and Discussion

The binding energy per nucleon (BE/A) of the isotopes of Zinc having mass numbers 54 to 86 were calculated and then compared with the available experimental data and the results are plotted in Fig.1. It is evident from the plot that the results of the HFB calculations follow the same trend as that of the experimental values. Odd-even staggering for both even-even and even-odd isotopes can be clearly noticed. The maximum in the parabola corresponding to A=68, is due to the shell closure of the neutron

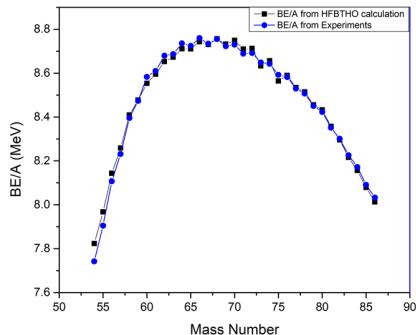


Fig.1 Plot of BE/A against mass number of Zn isotopes subshell $1f^{5/2}$.

The single (S_n) and two-neutron (S_{2n}) separation energies [5] were calculated and then compared with the experimental values. The one-neutron separation energy is defined as,

$$S_n = BE(Z, N) - B(Z, N - 1) \quad (3)$$

and the two-neutron separation energy is given by,

$$S_{2n} = BE(Z, N) - B(Z, N - 2) \quad (4)$$

It can be seen that the single-neutron separation energy of odd isotopes is less than that of their even neighbors and thus odd-even effects are reflected well in Fig.2. SLY4 functional proves

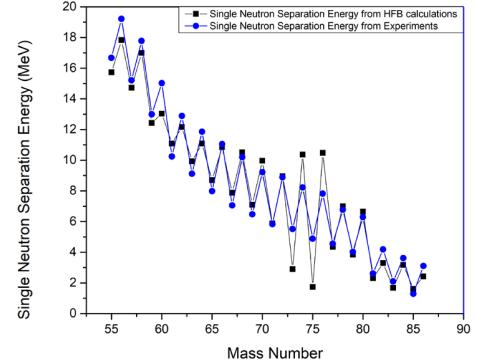


Fig. 2 Plot of single-neutron separation energy against mass number of Zn isotopes to be consistent with an excellent agreement with the experimental values.

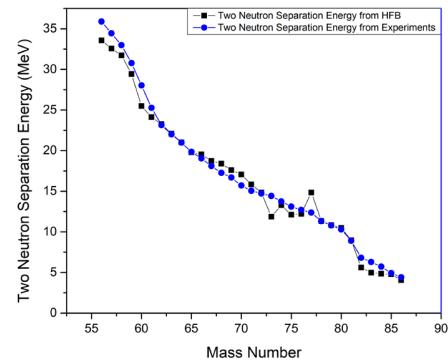


Fig. 3 Plot of two-neutron separation energy against mass number of Zn isotopes

The plot of two-neutron separation energy also shows an admirable agreement with the experimental data, as shown in Fig.3.

References

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