Screening the Cosmological Constant in non-local gravity

Ying-li Zhang[109](a) and Misao Sasaki[110](a)

(a) *Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*

Abstract

In this paper, we consider a class of model of non-local gravity with a large bare cosmological constant, Λ, and study its cosmological solutions. In the absence of matter, we find a power-law expanding universe solution $a \propto t^n$ with $n < 1$, that is, a universe with decelarated expansion without any fine-tuning of the parameter. So the effect of the cosmological constant is effectively screened in this solution. Also it is found that the solution is ghost-free for a very wide range of parameters. Thus our solution opens up new possibilities for solution to the cosmological constant problem.

1 Introduction

Since the discovery of the accelerated expansion of the current universe, there arise extensive discussions on the mechanism to explain the observation within or beyond the Einstein gravity. One of the simplest model is to modify the right-hand side of the Einstein gravity where a cosmological constant term Λ and other unknown matter terms are added. Another popular approach involves the modification of gravity, so that the late-time accelerated expansion can be obtained 'naturally'. Among them, one of the most extensively studied of such modifications is $F(R)$ theory, where F is an arbitrary function of Ricci scalar R. Recently, inspired by the study of string/M theory or conventional quantum gravity, another type of the modification of gravity, called non-local gravity, attracts more and more attentions. In this theory, a non-local term $f(\Box^{-1}R)$ is added into the action, where f is an arbitrary function and $\Box^{-1}$ is the inverse of d'Alembertian operator. Its cosmological effect and other various aspects has been studied in [1].

Here we concern on another aspect of non-local gravity. It is proposed in [2] that there is some hope to solve the cosmological constant problem in this framework. Inspired by their claim, a detailed mechanism to screen the cosmological constant in the non-local gravity is presented in [3]. However, like the situation of any other theories with higher-derivatives, here the non-local operator $\Box^{-1}$ often involves a wrong sign in the kinetic term, that is a ghost which will lead to the instability of the system. Some attempts have been made to avoided this problem around the flat or deSitter spacetime, but now successful up to now.

In this paper, we consider a simple one-parameter family of non-local gravity models with bare cosmological constant Λ and study their cosmological solutions. The models are characterized by a function $f(\psi) = f_0 e^{\alpha\psi}$ where $\psi \equiv \Box^{-1}R$, with α being the real, dimensionless parameter. We assume the universe is spatially flat, and consider the case where there is no matter contribution. Interestingly, we find a power-law solution $a \propto t^n$, where a is the cosmic scale factor, with $n < 1$. This implies that the effect of the cosmological constant is completely shielded to render the expansion of the universe decelerated. To be specific, as α varies from $-\infty$ to $+\infty$, n increases monotonically from 0 to 1/2. Thus the universe behaves like a radiation-dominated universe for $\alpha \gg 1$. Then we examine if the solution is free from a ghost. To our happy surprise, it is found that for a very wide range of the parameter α, namely for $\alpha > \alpha_{cr} \approx 0.17$, the solution is found to be ghost-free. For this range of α, we find $n > n_{cr} \approx 0.35$.

2 Non-local gravity and ghost-free condition

We consider a class of non-local gravity whose action is given by

$$S = \int d^4x\sqrt{-g}\left\{\frac{1}{2\kappa^2}\left[R\left(1+f(\Box^{-1}R)\right)-2\Lambda\right]+\mathcal{L}_{\rm matter}\left(Q;g\right)\right\}, \tag{2.1}$$

where $\kappa^2 = 8\pi G$, f is a function that characterizes the nature of non-locality with $\Box^{-1}$ being the inverse of the d'Alembertian operator, Λ is a (bare) cosmological constant and Q stands for matter fields. For definiteness, we

[109] Email address: yingli@yukawa.kyoto-u.ac.jp
[110] Email address: misao@yukawa.kyoto-u.ac.jp

assume matter is coupled minimally to gravity. Therefore, the above may be regarded as an action in the Jordan frame.

In this simple class of non-local gravity, we rewrite the action into a local form by introducing two scalar fields ψ and ξ as

$$\begin{aligned} S &= \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left\{R\left(1+f(\psi)\right)-\xi\left(\Box\psi-R\right)-2\Lambda\right\}+\mathcal{L}_{\rm matter}\right] \\ &= \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left\{R\left(1+f(\psi)+\xi\right)+g^{\mu\nu}\partial_\mu\xi\partial_\nu\psi-2\Lambda\right\}+\mathcal{L}_{\rm matter}\right]. \end{aligned} \tag{2.2}$$

Assuming a spatially flat FLRW universe with the metric,

$$ds^2=-dt^2+a^2(t)\delta_{ij}dx^idx^j\,, \tag{2.3}$$

by varying the action with respect to $g_{\mu\nu}$, ξ and ψ, respectively, one obtains the field equations as

$$0 = -3H^2\left(1+f(\psi)+\xi\right)-\frac{1}{2}\dot\xi\dot\psi-3H\left(f'(\psi)\dot\psi+\dot\xi\right)+\Lambda+\kappa^2\rho\,, \tag{2.4}$$

$$0 = \left(2\dot H+3H^2\right)\left(1+f(\psi)+\xi\right)-\frac{1}{2}\dot\xi\dot\psi+\left(\frac{d^2}{dt^2}+2H\frac{d}{dt}\right)\left(f(\psi)+\xi\right)-\Lambda+\kappa^2P\,, \tag{2.5}$$

$$0 = \ddot\psi+3H\dot\psi+6\dot H+12H^2\,, \tag{2.6}$$

$$0 = \ddot\xi+3H\dot\xi+\left(6\dot H+12H^2\right)f'(\psi)\,, \tag{2.7}$$

where a dot denotes the time derivative $\dot{} = d/dt$, $H=\dot a/a$, and $\rho=-T^0_0$ and $P=T^i_i/3$ are the energy density and pressure of the matter fields, respectively.

To examine whether the theory contains a wrong sign in the kinetic term (ghost) or not, in the following we take an unperturbed approach, that is to make a conformal transformation of the metric to bring the action into the one in the Einstein frame, namely the conformal frame in which the gravitational part of the action (2.2) becomes purely Einstein. For this purpose, from now on, we denote the metric in the Jordan frame by $g_{\mu\nu}$ while the one in the Einstein frame is labeled by the index (E) as $g^{(E)}_{\mu\nu}$ and consider a conformal transformation,

$$g_{\mu\nu}=\Omega^2g^{(E)}_{\mu\nu}\,, \tag{2.8}$$

and identify the conformal factor as

$$\Omega^{-2}=1+f(\psi)+\xi\,, \tag{2.9}$$

then the gravitational part of the action becomes Einsteinian as

$$\begin{aligned} S = \int d^4x\sqrt{-g^{(E)}}\Big\{&\frac{1}{2\kappa^2}\Big(R^{(E)}-6\nabla^{\mu(E)}\phi\nabla^{(E)}_\mu\phi-2\nabla^{\mu(E)}\phi\nabla^{(E)}_\mu\psi-e^{2\phi}f'(\psi)\nabla^{\mu(E)}\psi\nabla^{(E)}_\mu\psi-2e^{4\phi}\Lambda\Big) \\ &+e^{4\phi}\mathcal{L}_{\rm matter}(Q;e^{2\phi}g^{(E)})\Big\}\,. \end{aligned} \tag{2.10}$$

It is now easy to derive the ghost-free condition. Since there are only two scalar fields, the condition for the absence of a ghost is that the trace and the determinant of the kinetic term matrix are both positive. In the present case, it is readily seen that only the positivity of the determinant is sufficient, that is,

$$\det\begin{vmatrix} 6 & 1 \\ 1 & e^{2\phi}f'(\psi)\end{vmatrix}>0\,. \tag{2.11}$$

In terms of the original fields, this condition is expressed as

$$f'(\psi)>\frac{1}{6}\left(1+f(\psi)+\xi\right)>0\,, \tag{2.12}$$

where $1+f(\psi)+\xi>0$ is a necessary condition from Eq. (2.9). Later the above condition is used to examine if our cosmological solutions are free from ghosts or not.

3 Cosmological solutions

As a class of simple non-local gravity models, we consider the case when $f(\psi)$ is given by an exponential function,

$$f(\psi) = f_0 e^{\alpha\psi} . \tag{3.1}$$

we look for cosmological solutions with the assumption that the scale factor is a power-law function of time t, $a \propto t^n$. We also assume the absence of matter fields, $\rho = P = 0$. By solving the system of equations (2.4) - (2.7), one obtains the following solutions for scalar fields $\psi(t)$ and $\xi(t)$:

$$\psi(t) = \frac{1}{\alpha} \ln \left[\frac{\Lambda t^2}{6 f_0 n(n+1)} \right] , \quad \xi(t) = -1 - \frac{(2n-1)\alpha\Lambda t^2}{2(3n+1)(n+1)} , \tag{3.2}$$

where the index n is determined by the second equation in terms of parameter α as

$$6\alpha n^2 + 3(1-\alpha)n - 1 = 0 . \tag{3.3}$$

so in principle there exist two solutions for n as

$$n_1 = \frac{-3 + 3\alpha + \sqrt{3(3\alpha^2 + 2\alpha + 3)}}{12\alpha} , \quad n_2 = \frac{-3 + 3\alpha - \sqrt{3(3\alpha^2 + 2\alpha + 3)}}{12\alpha} . \tag{3.4}$$

which are plotted in fig.1 in terms of real parameter α. From the figure, we notice that an expanding solution is given by $n = n_1$ for any α and by $n = n_2$ for $\alpha < 0$. In both cases, n approaches $1/2$ for $|\alpha| \gg 1$, that is, the evolution of the universe looks like a radiation-dominated one.

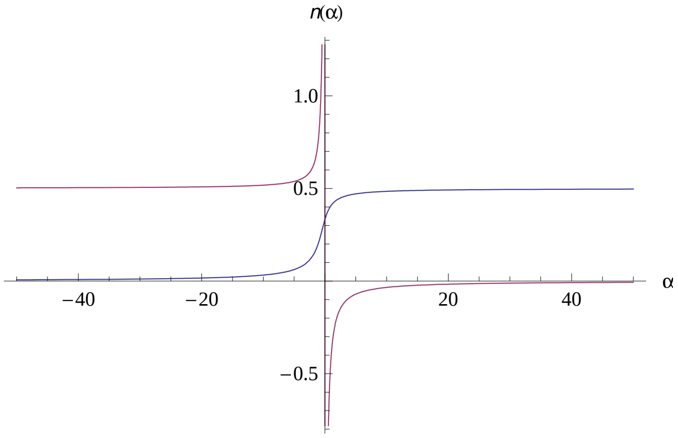

Figure 1: Index n as a function of parameter α. Here the blue line (the one in the middle, which is regular at $\alpha = 0$) denotes $n = n_1$ and the red line $n = n_2$.

Inserting solutions eq.(3.2) and (3.4) into the ghost-free condition eq.(2.12), one finds that n_2 is excluded, while for n_1, the parameter α is constrained by

$$\alpha > \alpha_{cr} = \frac{1}{15}(7 - 2^{\frac{1}{3}} - 2^{\frac{5}{3}}) \approx 0.17 \Longrightarrow 1/2 > n(= n_1) > n_{cr} \approx 0.35 , \tag{3.5}$$

which implies a decelerated expanding universe in the absence of matter. Thus without any fine-tuning, the solution successfully screens the effect of the cosmological constant that would have led to accelerated expansion. It should be noted that Eq. (2.12) implies $f'(\psi) > 0$, which reduces to the constraint on the parameter $f_0 > 0$ for a positive α. Thus the solution we have obtained in the above is meaningful only for a positive cosmological constant Λ.

To explore the behavior of corresponding solutions in the Einstein frame, under the conformal transformation eq.(2.8), the cosmic proper time t_E in the Einstein frame is related to t by $dt = \Omega dt_E$, so that

$$t_E = \int_0^t \frac{dt}{\Omega} = \frac{\sqrt{\Lambda}\, t^2}{2\sqrt{(n+1)(3n+1)}} , \tag{3.6}$$

$$H_E = \frac{1}{a_E}\frac{da_E}{dt_E} = \frac{\Omega}{a}\frac{d}{dt_E}\left(\frac{a}{\Omega}\right) = \frac{\Omega^2}{a}\frac{d}{dt}\left(\frac{a}{\Omega}\right) = \frac{n_E}{t_E} , \tag{3.7}$$

where $n_E = (n+1)/2$. Thus, inserting these relationship into solutions in the Jordan frame, eq.(3.2) and (3.5), one soon obtains the corresponding solutions in the Einstein frame as

$$\psi(t) = \frac{1}{\alpha}\ln\left[\frac{1}{3f_0(2n_E-1)}\sqrt{\frac{\Lambda(3n_E-1)}{n_E}}\,t_E\right], \tag{3.8}$$

$$\xi(t) = -1-\frac{\alpha(4n_E-3)}{2}\sqrt{\frac{\Lambda}{n_E(3n_E-1)}}\,t_E\,, \tag{3.9}$$

where $0.75 > n_E > n_{E,cr} \approx 0.675$. Thus we conclude that the screening mechanism is also valid in the Einstein frame.

Before closing this section, an important comment is in order. As we demonstrated, the theory can be recasted in the form of a scalar-tensor theory or in the form of Einstein gravity plus two scalar fields. Therefore one might consider our theory to be simply a scalar-tensor theory that gives rise to a decaying cosmological constant. Technically this is true. But it should be emphasized that the form of the action for these scalar fields in either the original frame or the Einstein frame is completely fixed by the original form of non-local gravity given by eq.(2.1): There is no freedom in maneuvering the form of the Lagrangian for these scalar fields. In other words, even if one were to regard this theory as a scalar-tenor theory with two non-minimally coupled scalar fields (in the original Jordan frame), it would be highly non-trivial to find a theory that would lead to a solution with a decaying cosmological constant which decays sufficiently fast in the original frame as well as in the Einstein frame.

4 Conclusion

We have studied cosmological solutions in a simple class of non-local gravity with cosmological constant. The model is characterized by a function $f(\psi) = f_0 e^{\alpha\psi}$ where $f_0 > 0$ and α is a real parameter, and ψ is the inverse of the d'Alembertian acting on the scalar curvature, $\psi = \Box^{-1}R$. In the absence of matter fields, we have found power-law solutions $a \propto t^n$ with $n < 1$, that is, with decelerated expansion. We have found that for $\alpha > \alpha_{cr} \approx 0.17$, the solution is ghost-free. Thus without any fine-tuning, the solution successfully screens the effect of the cosmological constant that would have led to accelerated expansion.

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