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REVIEW PAPER

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Evidence for the fifth element

Astrophysical status of dark energy

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Abstract Evidence for an accelerated expansion of the universe as it has been revealed 10 years ago by the Hubble diagram of distant type Ia supernovae represents one of the major modern revolutions for fundamental physics and cosmology. It is yet unclear whether the explanation of the fact that gravity becomes repulsive on large scales should be found within general relativity or within a new theory of gravitation. However, existing evidences for this acceleration all come from astrophysical observations. Before accepting a drastic revision of fundamental physics, it is interesting to critically examine the present situation of the astrophysical observations and the possible limitation in their interpretation. In this review, the main various observational probes are presented as well as the framework to interpret them with special attention to the complex astrophysics and theoretical hypotheses that may limit actual evidences for the acceleration of the expansion. Even when scrutinized with skeptical eyes, the evidence for an accelerating universe is robust. Investigation of its very origin appears as the most fascinating challenge of modern physics.

Keywords Cosmology, Dark energy, Cosmological models

1 Introduction

Modern cosmology has achieved remarkable progresses during the last 50 years. The general picture originally designed as the “Primeval atom” by Lemaître and which has become the “Big Bang” model according to the word of one of its most famous opponent, F. Hoyle, is now recognized as the successful scientific repre-

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sentation of the world at the large scales (in space and in time) we can measure.[1] Construction of this picture has necessitated the successive abandonment of philosophical and scientific ideas, some of which are not only those of physicists but are also shared by the more general public. Maybe the first to be given up was the idea that it is hopeless to try to figure out a global picture of the universe. Although heroic pioneers can be traced back since a long time ago, undoubtedly Einstein is the first one to directly address the question of handling the universe as a single physical object with an appropriate tool in hands, general relativity (GR). The fact that one of the very first applications of his theory was Cosmology is an evidence that the cosmological question was central in his thoughts. This is reinforced by the fact that he proposed to modify the initial formulation of his theory given the problems he encountered. It is commonly said that Einstein introduced the cosmological constant to obtain a static universe because he was reluctant to the idea of an expanding picture. This formulation is inappropriate: it merely suggests that Einstein was willing to avoid an expanding universe, while he actually wanted to find at least one solution to the cosmological problem with an initial formulation which assumes the universe to be stationary, there is no indication in his seminal paper that he wished to reject a non-stationary universe: he actually started his paper by discussing the problem of having mass at large distances in the Newtonian approach, noticing that this leads to divergence of the potential. He insisted that this would lead to unacceptably large velocities for stars. He also quoted that this can be cured by assuming a correcting term to the Newtonian potential equation:

$$\nabla^2\varphi - \lambda\varphi = 4\pi G\rho \tag{1}$$

and then proposed to modify his initial theory with the addition of the cosmological term Λ. This allowed him to construct the first relativistic cosmological model, the Einstein solution, which is spatially closed (because being spherical) and static. In 1919, de Sitter discovered a new solution to Einstein equation which was written in a stationary form[2] and contains no matter (but a non-zero cosmological constant). It is only a few years after, in 1925, that Lemaître identified the de Sitter solution to an homogeneous expanding universe (Lemaître 1925). Friedmann (1922, 1924) found the general homogeneous solutions, providing the equation for the scale factor $R(t)$ and recognisee their expanding nature. It is somewhat surprising that his work has remained totally unnoticed, despite a controversy with Einstein. During this period it is clear that the nature of the redshift of what Hubble had identified as extragalactic nebulae became a question addressed by many astronomers. Slipher's discovery was probably much more intriguing now that the nature of nebulae had been identified. Eddington is often mentioned as

[1] It is fair to say that few scientists are still opposed to the "Big Bang" picture. Most of the serious opponents try to demonstrate that some observational facts, most often only one, which are coming in support of the Big Bang may be interpreted in a different way and therefore the whole construction has to be questioned. It is useful to remember that the success of a scientific model is—in some sense—measured by the number of predictions it leads to and how many are successful. Newton theory of gravity is wrong, but nevertheless it remains a high quality scientific theory because of its past (and present) successes. It is in this sense that modern cosmology should be regarded as successful, and this will remain in the future, even if it might be regarded as being "wrong"...

[2] The choice of the coordinates system lead to a form of the metric for which the coefficients are constant.

the first astronomer to have noticed a possible connection between redshift and the de Sitter solution (Lemaître 1925). Wirtz (1922) was clearly looking for a relation between redshift and distance and had in mind the possible cosmological information it might provide (Wirtz 1924; Seitter and Duerbeck 1990). Lemaître reestablished the equations that Friedmann derived, showing that expanding solutions were leading to a redshift proportional to the distance. He proposed that this effect was the origin of known redshift and provided the first estimation of the Hubble constant (Lemaître 1927). In 1929, Robertson published the now so called Robertson–Walker metric. In the same paper he mentioned that distant sources appear to have a frequency shift.

2 Basics of Friedmann–Lemaître models

The fundamental idea of the geometrical theory of gravity starts from the fact that we can assign four coordinates to any event observed in our vicinity, for instance in Cartesian coordinates: (x,y,z,t). Locally, space appears flat to be. However, this does not determine the geometry of space at larger scales: local observations put us in the same situation that led people to think the earth was flat: the fact that we can describe our vicinity by a flat map does not determine the actual geometry on larger scales. Let us take the line element of a homogeneous 3D space which can be shown[3] to be:

$$\mathrm{d}l^2 = \tilde{r}^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2) + \frac{\mathrm{d}\tilde{r}^2}{1-k\left(\frac{\tilde{r}}{R}\right)^2} \tag{2}$$

where k is $-1,0,1$ according to whether space is hyperbolic, flat, or spherical. R is a characteristic size (in the spherical case, that is the radius of the 3D-sphere embedded in a 4D space).

We then add the time as the fourth coordinate to build the equivalent of the Minkowski space–time element of special relativity and get the Robertson–Walker (RW) line element after the change of variables $\frac{\tilde{r}}{R} \to r$:

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + R(t)^2\left[r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2) + \frac{\mathrm{d}r^2}{1-kr^2}\right] \tag{3}$$

2.1 Topology

The above line element describes the local shape of space: the curvature is only a local property of space, but does not tell us about the *global* shape of space. For instance, the Euclidean plane is an infinite flat surface, while the surface of a cylinder is a 2D-space which is flat everywhere but is finite in one direction. GR will in principle allow us to derive the local geometry of space and its dynamics, but does not specify of the global topology of space. Only direct observations would

[3] It is an instructive exercise to start from an Euclidean 4D space x,y,z,u and derive the line elements $\mathrm{d}l^2$ on the 3D sphere ($x^2+y^2+z^2+u^2=R^2$) in internal spherical coordinates ($\tilde{r}=\sqrt{x^2+y^2+z^2},\theta,\phi$).

allow to test what the topology actually is. Of course this will not be possible on scales much larger than what can be observed (the horizon). We can therefore hope to prove that the universe is finite, if it is small enough, but we could not know whether we are in a finite universe of which the scale is larger than the horizon, or whether we are in an infinite universe. The interest in the topic of the cosmic topology, with possible observational signature, has been recently revived (Lachièze-Rey and Luminet 1995; Luminet et al. 2003).

2.2 Dynamics

The function $R(t)$, which appears in the RW line element, is totally independent of any further geometrical consideration. It can be specified only within a theory of gravity. The basic equation of GR relates the geometrical tensor G_{ij} to the energy-momentum tensor T_{ij}

$$G_{ij} = R_{ij} - \frac{1}{2} g_{ij} R = 8\pi G T_{ij} \tag{4}$$

where g_{ij} is the metric tensor, R_{ij} is the Ricci tensor, R the Ricci scalar. For a perfect fluid, there exists a coordinates system, called the comoving coordinates, in which the matter is at rest and the tensor T_{ij} is diagonal with $T_{00} = \rho$ and $T_{11} = T_{22} = T_{33} = p$, ρ being the density and p the pressure. A fundamental aspect of GR is that the source of gravity includes explicitly a term coming from the pressure: $\rho + 3p/c^2$. Finally, there is an analog of the Gauss theorem, i.e., the Birkhoff's theorem (1923)[4]: if the matter distribution is spherical then the evolution of the radius of a given shell of matter depends only on its content.

From the above rules, we can easily derive the equation for $R(t)$. Let us consider a spherical region of radius a in a homogeneous distribution of matter. The equivalent Newtonian acceleration is:

$$\frac{\mathrm{d}^2 a}{\mathrm{d}t^2} = g \tag{5}$$

with the acceleration being generated by the "mass" $M(a)$ of the above spherical region:

$$g = -\frac{GM(a)}{a^2} = -\frac{4}{3}\pi G(\rho + 3p/c^2)a \tag{6}$$

The density term includes the effect of kinetic energy ($E = mc^2$!). Writing total energy (E_t) conservation inside the volume of the sphere from elementary thermodynamics gives:

$$\mathrm{d}(E_t) = \mathrm{d}(\rho V c^2) = -p\mathrm{d}V \tag{7}$$

leading to:

$$\dot{\rho} = -3\left(\frac{p}{c^2} + \rho\right)\frac{\dot{a}}{a} \tag{8}$$

[4] Apparently, this theorem should be named Birkhoff–Jebsen, as it has been published 2 years earlier by an Norwegian physicist, Jebsen (Johansen and Ravndal 2006).

From these two equations, the pressure can be eliminated, and, after having multiply both terms by $\dot{a}$, the differential equation can be easily integrated. This leads to the following equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{Kc^2}{a^2(t)} \tag{9}$$

The last term corresponds to the constant of integration. Its value cannot be specified, depending on the initial conditions. The form of the above equation is independent of the radius a of the sphere and the solution $a(t)$ should be proportional to the quantity $R(t)$. $R(t)$ should also be solution of an equation of the same form, the constant K, which depends on the radius a_0, being related to the constant k which is involved in the Robertson–Walker metric element, something which can be established only within GR:

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{R^2(t)} \tag{10}$$

This relation expresses the link within the framework of GR between the geometry and the material content of the universe. In order to specify completely the function $R(t)$, one needs an equation of state for the content of the universe. The two cases often seen in cosmology are the dust case ($p = 0$) and the radiation dominated regime ($p = \frac{1}{3}\rho c^2$).

2.3 Vacuum and the cosmological constant

Vacuum is a particular medium, and one should wonder what is the equation of state of this medium. Naively, one would think that the equation will be $p_\mathrm{v} = 0$ and $\rho_\mathrm{v} = 0$. However, let us try to derive the vacuum equation of state from first principles. As in classical thermodynamics let us assume that we have a piston with vacuum in it. We also assume that simple vacuum ($p = 0, \rho = 0$) is present outside.

The energy inside the piston is $E = \rho_\mathrm{v} c^2 V$. If the volume changes by a small amount the net energy change is:

$$\mathrm{d}E = \mathrm{d}(\rho_\mathrm{v} V c^2) = \rho_\mathrm{v} c^2 \mathrm{d}V \tag{11}$$

this change is equal to the work of the pressure:

$$\mathrm{d}E = -p_\mathrm{v} \mathrm{d}V \tag{12}$$

so the equation of state is:

$$p_\mathrm{v} = -\rho_\mathrm{v} c^2 \tag{13}$$

As one can see, the conditions $p \geq 0$ and $\rho \geq 0$ ensure that the simple solution is the only one. However, there is nothing which imposes these conditions for the vacuum, and we can therefore decide to keep such a possible term. This can be directly translate in the equations governing $R(t)$ by introducing the following constant:

$$\Lambda = 8\pi G\rho_{\mathrm{v}} \tag{14}$$

Such a term is called the cosmological constant and has been historically introduced by Einstein as a modification of his original theory. It appears as an additional term in the left-hand side of Eq. 4. We have recovered the two usual Einstein–Friedmann–Lemaître (EFL) equations:

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{R^2} + \frac{\Lambda}{3} \tag{15}$$

and

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda}{3} \tag{16}$$

There are several ways to write the EFL equations. For instance, we used energy conservation, Eq. 8, which can be kept instead of one of the above equations. It can also be useful to use:

$$\dot{H} = -4\pi G\left(\rho + \frac{p}{c^2}\right) + \frac{kc^2}{R^2} \tag{17}$$

Only two independent equations were obtainable, while three unknown quantities are to be determined $(R(t), \rho(t), p(t))$; therefore, we need a further constraint which is provided by the equation of state $F(\rho, p) = 0$.

It is quite usual to write the fundamental cosmological quantities as non-dimensional quantities which depend on redshift. The following notations are very common:

$H = \frac{\dot{R}}{R}$, the Hubble parameter,
$\Omega_M = \Omega = \frac{8\pi G\rho}{3H^2}$ the density parameter,
$q = -\frac{\ddot{R}R}{\dot{R}^2}$, the deceleration parameter,
$\Omega_v = \Omega_\lambda = \lambda = \frac{\Lambda}{3H^2}$, the (reduced) cosmological constant,
$\Omega_c = -\frac{kc^2}{H^2R^2} = -\alpha$, the curvature parameter.

Quantities are labeled with a 0 when they are referred to their present value. For instance, the present day value of the density parameter Ω is Ω_0. With the above notations, the first EFL equation 15 then reads:

$$\Omega_M + \Omega_c + \Omega_\lambda = 1 \tag{18}$$

or:

$$\alpha = \Omega + \lambda - 1 \tag{19}$$

so that the "radius of the universe" can be written:

$$R = \frac{c}{H}\frac{1}{\sqrt{|\alpha|}} \tag{20}$$

while the Hubble constant evolution is given by:

$$H^2 = H_0^2(\Omega_M(1+z)^3 + \Omega_c(1+z)^2 + \Omega_\lambda) \tag{21}$$

2.4 Important quantities needed for observations

In this section, we only need to work in the framework of a geometrical theory of space–time, in which the trajectories of light rays are assumed to be the null geodesics. Let us have a comoving spherical coordinate system (r,θ,ϕ,t) the observer being at the origin of the spatial coordinates $(r=0,\theta=0,\phi=0,t_0)$, let assume that the observed source is emitting light at the coordinates $(r_S,\theta=0,\phi=0,t_S)$, and let $r(t)$ be the trajectory of the emitted photons. As this trajectory is a null geodesic, we have:

$$c^2\mathrm{d}t^2 - R^2(t)\frac{\mathrm{d}r^2}{1-kr^2} = 0 \tag{22}$$

so the variables can be separated and the integration over r is analytical:

$$\int_{t_S}^{t_0} \frac{c\mathrm{d}t}{R(t)} = \int_0^{r_S} \frac{\mathrm{d}r}{(1-kr^2)^{1/2}} = S_k^{-1}(r_S) \tag{23}$$

with:

$$S_k(r_S) = \begin{cases} \sin(r_S) & \text{if } k=+1 \\ r_S & \text{if } k=0 \\ \sinh(r_S) & \text{if } k=-1 \end{cases} \tag{24}$$

When the distance is small with respect to R_0 we just have $S_k^{-1}(r) \sim r$.

2.5 The Redshift

In order to derive the observed frequency ν_0 of the light from a source emitted at the frequency ν, we consider the trajectory of a second light ray emitted at the time $t_S + \frac{1}{\nu}$. As the source is comoving its coordinate is unchanged and we have:

$$S_k^{-1}(r_S) = \int_{t_S}^{t_0} \frac{c\mathrm{d}t}{R(t)} = \int_{t_S+1/\nu}^{t_0+1/\nu_0} \frac{c\mathrm{d}t}{R(t)} \tag{25}$$

which implies:

$$\frac{\nu_0}{\nu} = \frac{\lambda_S}{\lambda_0} = \frac{R_S}{R_0} = \frac{1}{1+z} \tag{26}$$

where z is the redshift. This is the standard formula for the cosmological shift of the frequencies. This result shows that the redshift z is a natural consequence of the expansion.

2.6 The proper distance

In GR, space changes with time, and there is no proper time, so that the "intuitive" notion of distance between two points is not a well-defined quantity. Therefore, the various methods to measure the distance between an observer and a given source give different answers. The proper distance—between the source and the observer—can be seen as a distance measured by a set of rulers at time t. The distance element is given by:

$$\mathrm{d}l^2 = \mathrm{d}s^2 = R(t)^2 \frac{\mathrm{d}r^2}{1-kr^2} \tag{27}$$

so that the proper distance is:

$$D_\mathrm{p} = R(t) S_k^{-1}(r_\mathrm{S}) \tag{28}$$

The fact that this distance changes with time is the direct consequence of the expansion of the universe. We can now examine how this length changes with time :

$$\dot{D}_\mathrm{p} = \dot{R} S_k^{-1}(r) \tag{29}$$

so that the source is *actually receding* from the observer with a speed:

$$V = \dot{D}_\mathrm{p} = \frac{\dot{R}}{R} D_\mathrm{p} = H D_\mathrm{p} \tag{30}$$

The fact that this speed could be larger than the speed of light should not be considered as a problem: this speed can be measured but cannot transport information faster than light. When the distance is small, the Doppler frequency shift is:

$$\frac{\delta \nu}{\nu} = \frac{\dot{R}}{R}\delta t = H\frac{D}{c} = \frac{V}{c} \tag{31}$$

so that the shift is the one corresponding to the Doppler shift associated with the above velocity. For large distances, the total shift results from the product of small Doppler shifts and the redshift is therefore purely kinematic. The physical nature of the expansion has been recently the subject of interesting discussions (Chodorowski 2007; Abramowicz et al. 2007; Peacock 2008; Cook and Burns 2009).

2.6.1 Comoving distances

It is sometimes useful to refer to comoving distances.[5] The comoving distance $D^\mathrm{c}(z)$ associated to the distance $D(z)$ is :

$$D^\mathrm{c}(z) = \frac{R_0}{R} D(z) = (1+z) D(z) \tag{32}$$

In the case of the proper distance, this becomes:

$$D^\mathrm{c}_\mathrm{p}(z) = R_0 S_k^{-1}(r) = \int_{t_\mathrm{S}}^{t_0} \frac{c\,\mathrm{d}t}{R(t)/R_0} = c \int_0^z \frac{\mathrm{d}z}{H(z)} \tag{33}$$

[5] This could also be confusing!

2.7 The angular distance

Let us suppose that we observe a ruler orthogonal to the line of sight. The extremities of the ruler have the coordinates $(r, 0, 0, t_S)$ and $(r, \theta, 0, t_S)$. The proper length l between the extremities is:

$$l^2 = \mathrm{d}s^2 = R(t_S)^2 r^2 \theta^2 \tag{34}$$

which provides the relation between the angle θ and the length l and thereby the angular distance defined by:

$$D_{\mathrm{ang}} = \frac{l}{\theta} = R(t_S) r \tag{35}$$

2.8 The luminosity distance

Let us assume that we observe a source with an absolute luminosity L through a telescope with a diameter d and let us choose a coordinates system which is centered on the source. Let θ be the angle between two rays reaching two points diametrically opposite on the telescope. We have $d = R(t_0) r \theta$. The energy emitted by the source that reaches the telescope is:

$$s = \frac{L}{4\pi} \times \frac{\pi \theta^2}{4} \tag{36}$$

When observed, the energy of photons has been shifted by $1/(1+z)$ but also the frequency at which they arrive is reduced by the same factor. Therefore, the flux (energy per unit time and unit surface) one gets is:

$$f = \frac{s}{\pi l^2/4} \frac{1}{(1+z)^2} = \frac{L}{4\pi R(t_0)^2 r^2 (1+z)^2} = \frac{L}{4\pi D_{\mathrm{lum}}^2} \tag{37}$$

This relation provides the luminosity distance:

$$D_{\mathrm{lum}} = R(t_0) r (1+z) = R(t_S) r (1+z)^2 = D_{\mathrm{ang}} (1+z)^2 \tag{38}$$

2.9 Distance along the line of sight

We consider here the length along the path of a photon trajectory. The length element is

$$\mathrm{d}l = c\mathrm{d}t = c\frac{\mathrm{d}R}{\dot{R}} = -\frac{c}{H(z)} \frac{\mathrm{d}z}{1+z} \tag{39}$$

This relation is useful to write the volume element.

2.10 The age of the universe

The general expression of time interval is:

$$\mathrm{d}t = \frac{\mathrm{d}R}{\dot{R}} = -\frac{1}{H(z)}\frac{\mathrm{d}z}{1+z} \tag{40}$$

3 Some solutions of the EFL equations: relativistic cosmological models

The various possible theories of gravitation provide different functions $R(t)$ and through the above tests may in principle be distinguished by observations. However, it is easy to check that the difference only occurs at high redshift. In practice, these tests may not be discriminant because the observations of distant objects are difficult and because the universe at high redshift is younger, so any object at high redshift is likely to be different from today because of evolution. In other words, general geometrical tests rely on the assumption that the typical evolution time scale of the objects under study is much *larger* than the age of the universe.

One must also underline that as already mentioned the EFL equations can be solved only once the equation of state is specified, i.e., a relation between p and ρ is adopted. It is now quite common to specify this relation by the w parameter:

$$p = w\rho c^2 \tag{41}$$

If w is constant, then Eq. 8 allows to find the evolution of the field density:

$$\rho(z) = \rho_0(1+z)^{3(1+w)} \tag{42}$$

while for a general function $w(z)$ one has:

$$\rho(z) = \rho_0 \exp\left(\int_0^z 3(1+w(z))\frac{\mathrm{d}z}{1+z}\right) \tag{43}$$

There are three particular regimes: the matter dominated one, the radiation-dominated one and the vacuum-dominated one.

In the matter-dominated case one has $p = 0$, so $w = 0$ and the mass (per comoving volume) is conserved :

$$\rho a^3 = \rho(1+z)^{-3} = \mathrm{cste} \tag{44}$$

while in the pressure-dominated case $p = \frac{1}{3}\rho c^2$, so $w = \frac{1}{3}$

$$\rho a^4 = \rho(1+z)^{-4} = \mathrm{cste} \tag{45}$$

Finally, in the vacuum-dominated case $p = -\rho c^2$, so $w = -1$ and

$$\rho = \mathrm{cste} \tag{46}$$

In some models $w < -1$, and therefore the density of the universe increases when it expands.

Fig. 1 Evolution of the scale-factor $R(t)$ with time, in FL models according to the values of the parameters Ω_M and Ω_Λ. One notes that above the critical line universes has no initial singularity; on the right side of the second branch of the critical line are the re-collapsing models

3.1 Case $(\lambda_0 = 0, p = 0)$

When the cosmological constant is zero, there are three types of solutions:

(a) when the density is above the critical density:

$$\rho > \rho_c = \frac{3H_0^2}{8\pi G} = 2.\ 10^{-29}\,\mathrm{h}^2\,\mathrm{g/cm}^3 \tag{47}$$

the spatial solution is the spherical space. The function $R(t)$ grows from zero to a maximum value then a collapse phase follows to zero.

(b) when the density is equal to the critical density, the solution is named the Einstein–de Sitter universe and $R(t)$ is simple:

$$R(t) = R_0 \left(\frac{3}{2} H_0 t\right)^{2/3} = R_0 \left(\frac{t}{t_0}\right)^{2/3} \tag{48}$$

with $t_0 = \frac{2}{3} H_0^{-1} = 1/(6\pi G \rho_c)^{1/2}$

(c) when the density is below the critical density, the function $R(t)$ grows from zero to infinity. It is easy to check from the EFL equations that the function $R(t)$ behaves like t when R is large.

The behavior of $R(t)$ can be found when $t \to 0$ independently of the model: $R(t) \propto t^{2/3}$.

Finally, the relation between the comoving coordinate r and the redshift can be expressed:

$$R_0 r = \frac{c}{H_0} \frac{2}{\Omega_0^2} \frac{\Omega_0(1+z) + 2 - 2\Omega_0 - (2-\Omega_0)\sqrt{1+\Omega_0 z}}{1+z}$$

This is known as the Mattig relation. Others useful quantities can be found in Weinberg recent text book (Weinberg 2008).

3.2 Case $(\lambda_0 > 0, p = 0)$

There are many possibilities when a cosmological constant is allowed. To specify a cosmological model, it is customary to specify two "observables": Ω_0 and Ω_λ. For instance, the cosmological view of the Friedmann–Lemaître models is summarized in Fig. 1.

The look back time, i.e., the time since the epoch corresponding to the redshift z can be obtained from (40):

$$\tau(z) = \frac{1}{H_0} \int_1^{1+z} \frac{\mathrm{d}u}{u\sqrt{\Omega_0 u^3 - \alpha_0 u^2 + \lambda_0}} \tag{49}$$

where $u = 1 + z$. When $\lambda \leq 0$ it can be shown from the expression of $\ddot{R}$ that

$$t_0 = \tau(\infty) \leq \frac{1}{H_0} \tag{50}$$

3.3 Radiation-dominated case

As we have seen the density associated with radiation evolves according to $\rho_\gamma a^4 =$ cste. It is clear that this radiation term will be dominant over the matter term for small a, i.e., at the “beginning”. In this regime, if we neglect other terms in the EFL equation, we have:

$$\dot{R} = \left(\frac{8\pi G\rho_1}{3}\right)^{1/2} \frac{R_1^2}{R} \tag{51}$$

so that $R \propto t^{1/2}$

3.4 Basics of structure formation

Structure formation in an expanding universe through gravitational instability can be traced back to the early works of Lemaître (1933), who established the spherical solution and discussed its relevance to cluster formation. In modern cosmology, linear and non-linear regime ($\delta\rho\rho \gg 1$) are used to obtain useful constraints.

3.4.1 The gravitational instability picture

The linear regime of small perturbations (both in size and in amplitude) in the matter field ($\rho(x) = \rho(1+\delta(x))$) in the absence of pressure can be derived from Newtonian equations (Peebles 1980):

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G\rho_m\delta = \frac{3}{2}\Omega_m H^2\delta \tag{52}$$

Because there is no spatial derivatives in the equation, the evolution with time is independent of the scale (this does not hold any more when the pressure is taken into account). There are generally two distinct solutions to this equation, one is a decaying mode and the second one is a growing mode which is of relevance for cosmology. The growing mode is usually written as $D(t)$ with a normalization such that the present day value is one: $D(t_0) = 1$. The evolution of a perturbation can then be written:

$$\delta(t) = D(t)\delta_0 \tag{53}$$

where δ_0 is the linear amplitude that the fluctuation would have today. The qualitative behavior of solutions of Eq. 52 is easy to understand: if $\Omega_m \ll 1$ the right-hand side is zero and the growing mode is frozen, while if $\Omega_m \sim 1$ there is a growing mode with $D(t) \propto a(t)$. The form of Eq. 52 implies that once the expansion rate history $(H(z))$ and the present day density of the universe ρ_0 are known, the solution $D(t)$ is entirely determined (the matter density parameter can be inferred from its definition $\Omega_m(z) = \frac{8\pi G\rho_m}{3H^2} = \Omega_0 \frac{(1+z)^3}{H(z)^2}\Big)$. Therefore, having a measure of the growing rate of perturbations is a direct test of the relevance of Newtonian gravity on the scale under consideration. Theories beyond GR might lead to growing rates that depend on the scale and departs from their standard values.

Non-Baryonic dark matter and cold dark matter (CDM) scenario In the early universe, the growth rate of fluctuations is more complex: pressure has to be taken into account, and there are at least two matter contents to take into account: baryons and the non-baryonic matter. These ingredients will introduce scales in the dynamics of fluctuations. Ultimately these scales will leave visible imprints in observable quantities: the fluctuations of the cosmic background radiation and the power spectrum of the matter distribution (or equivalently in the correlation function). The first scale comes from the difference in the dynamics for fluctuations larger than the horizon when compared to the ones that are smaller. Indeed, when photons (and other relativistic particles) dominated the density of the universe the right-hand side of Eq. 52 is negligible, and therefore the growing mode is frozen. However, on scales larger than the horizon the dynamics should be evaluated within a full GR treatment. Intuitively, on these large scales the radiation acts as a source of gravity which will allow the fluctuations density to grow. Therefore, a scale appears corresponding to the scale of the horizon. When the universe enters the matter-dominated area, fluctuations on all scales will grow provided that the pressure term is negligible, i.e., for the non-baryonic matter. The scale which is imprinted is therefore the size of the horizon at the time of equality between matter and radiation and is characteristic of CDM. However, the baryons are still tightly coupled to the photons and thereby their dynamics is under the control of the pressure term to be added in the right-hand side of Eq. 52. This coupling stops when the matter becomes neutral enough ($z \approx 1500$). A second scale is therefore imprinted in the baryonic component corresponding to the scale of the horizon of sound's velocity at the epoch of recombination. Although the baryonic matter is subdominant, the distribution of dark matter will nevertheless reflect this scale. This feature appears as oscillations in the power spectrum, but more distinctly as a peak in the correlation function (Matsubara 2004). In practice, the dark mater distribution is described by its linear power spectrum $P_{\rm DM}(k)$ at present epoch corresponding to the initial power spectrum $P_i(k)$ modified by the evolution. In the linear regime, the various wave numbers are independent so:

$$P_{\rm DM}(k) = P_i(k) T_{\rm DM}(k) \tag{54}$$

The nature of the initial fluctuations, the nature of dark matter (hot, warm, ...) are other ingredients that determine the shape of the transfer function. In practice, the computation of the transfer function is to be done numerically. In the CDM case, analytical functional forms have been provided (Peebles 1982; Bardeen et al. 1986; Eisenstein and Hu 1998) including the detailed baryonic features (Eisenstein and Hu 1998).

4 The quest for cosmological parameters

4.1 Hubble diagram

Looking for a possible relation between apparent velocities and distances was a scientific question that few astronomers addressed before Hubble's discovery. The determination of Hubble's constant have been the subject of considerable effort over the next 80 years following his discovery. One of the major scientific question that the Hubble Space Telescope was aimed to address was precisely the determination of the distance of close galaxies and thereby to offer an accurate measurement of the Hubble constant. The final value obtained from the HST key project to measure the Hubble constant concluded $H_0 = 72 \pm 8$ km/s/Mpc (Freedman 2001), also the debate is not entirely closed as Sandage and Tammann still concluded to a lower value: $H_0 = 62.3 \pm 4$ km/s/Mpc (Tammann et al. 2008) although these values are marginally consistent given the quoted—systematic—uncertainties.[6]

4.2 Going further and other geometrical tests

The first cosmological tests which have been proposed were based on the relation between some observable quantity to a corresponding intrinsic property of the source (like the relation between the apparent luminosity and the intrinsic luminosity). These tests are essentially geometrical in nature, they involve the coordinate r given by Eq. 25 or the relation between the coordinate r and the redshift z:

$$r = S_k\left(\frac{c}{R_0}\int_0^z \frac{\mathrm{d}z}{H(z)}\right) \tag{55}$$

the dependence on cosmological parameters coming from the difference in the expansion rate according to Eq. 15. The Hubble diagram which has been extended to high redshift thanks to type-Ia supernovae is the most popularized example of these geometrical tests. It is also rather intuitive that the Hubble diagram which expressed the speed of the expansion will provide direct information on the rate of acceleration/deceleration when extended to objects distant enough to be seen at appreciably early epoch of the history of the universe. A reliable extension of the Hubble diagram to high redshift has been made possible thanks to the use of type-Ia supernovae. SNIa could have a maximum luminosity ($M \sim -19.5$) comparable to that of an entire galaxy. Furthermore, there is a relation between the decline rate and the intrinsic luminosity making them suitable for distance measurements at cosmological scale. Because SNIa are rare, large sky area have to be surveyed on a regular basis to collect samples of SNIa. At the end of last century, two groups have independently investigated the distant SNIa Hubble diagram and concluded that supernovae at redshift $\sim$0.5 were dimmer by $\sim$0.2 mag when compared to

[6] The Hubble constant is traditionally noticed: $H_0 = h100$ km/s/Mpc.

what was expected in a unaccelerated universe. This was interpreted as an evidence for an accelerated expansion (Riess et al. 1998; Goldhaber and Perlmutter 1998; Filippenko and Riess 1998; Perlmutter et al. 1999).[7]

4.3 The age of the universe

Formally, we have no direct information on the actual age of the universe t_0. But there are several astrophysical objects for which an age can be derived. The most common one is probably the age of globular clusters. There are different limitations which make determinations of absolute ages problematic. Distance estimations is for instance one limitation. Hipparcos results have helped, but the uncertainty remains significant. Recent estimate for the age of globular clusters was given to be $12,6^{-2.2}_{+3.4}(95\%)$ Gyr (Krauss and Chaboyer 2003), an age consistent with that of the star CS 31082-001 estimated to be $14. \pm 2.4$ Gyr, based on the decay of U-238 (Hill et al. 2002). It has been noticed since the beginning of cosmology that age estimations were high given the preferred value of the Hubble constant. This has been noticed by Lemaître when debating with Einstein, Peebles (1993) and many others since, providing thereby an argument in favor of the cosmological constant which holds until the late 90s (Krauss and Turner 1995; Ellis 2002). However, this argument is relatively weak because of the difficulty to achieve reliable astrophysical estimates of H_0. With modern best values, the product $H_0 t_0$ can be evaluated numerically:

$$H_0 t_0 \approx 0.102 h t_{\mathrm{Gyr}} \tag{56}$$

with $h = 0.72$ one gets $H_0 t_0 \sim 0.93$, while only values greater than 1 would request an accelerated universe. Although, it is important to have a consistency check, improvement by one order of magnitude in precision will be necessary in order that age and Hubble constant values could be combined to provide constraints on cosmological parameters competitive with current constraints.

4.4 The cosmic microwave radiation

4.4.1 Spectrum and uniformity

The measurement of the CMB spectrum to check for the black-body shape of this spectrum was one of the most important cosmological test awaited for since the

[7] There is an intense debate on the "discovery" of acceleration from SNIa Hubble diagram in order to specify who did what and who said what. The history of the research program developed within the SCP is summarized by Goldhaber (2009). An early popular scientific report on SCP talk at AAS meeting January 1998 meeting has been published by John Glanz (Science, 279, 651), while some view from the High Z team by Kirshner (2002) is available at `http://www.cfa.harvard.edu/~rkirshner/whowhatwhen/Thoughts.htm`. It is also interesting to mention that claim for evidence of an accelerated expansion from a totally different technique has been presented at the same AAS meeting: `http://www.bk.psu.edu/faculty/daly/PU98.pdf`. These data taken at face value were pointing to a low density universe, but only the flatness of the universe as already evidenced by CMB fluctuations would have allowed to conclude on the actual acceleration of the expansion.

discovery of the background radiation. Two high-quality measurements have been published in 1990 (Gush et al. 1990; Mather et al. 1990), while the FIRAS final results were remarkably accurate (Fixsen et al. 1996): any departure from a black-body should represent only a tiny perturbation of the total energy: $\delta E/E \leq 10^{-4}$.

An interesting application of cosmic background radiation is to provide a test of the reality of the expansion: if we are able to measure the temperature of the background at higher redshift it should scale according to:

$$T(z) = T_0(1+z) \tag{57}$$

It is actually possible to measure the temperature of the background radiation through the observations of the ratio of molecular lines: the ratio of the population on two levels for which the difference in energy is only a few Kelvin is sensitive to the cosmic radiation field and therefore provides a way to actually measure the temperature of the background. Such lines can be detected in the optical domain. Actually, the first detection of the CBR was obtained by this method with CN lines (McKellar 1941; `http://www.astro.ucla.edu/~wright/CMB.html`). It has also been successfully applied to distant QSO's to measure the variation of the temperature of the background with redshift and the result are consistent with the expanding picture (Songaila et al. 1994; Srianand et al. 2008).

These measurements provide a fundamental test of the Big Bang picture but do not provide a source of constraint on the cosmological parameters.

4.4.2 Fluctuations

Since the discovery of the CMB fluctuations by COBE (Smoot et al. 1992) the idea that early universe physics has left imprints revealed by these fluctuations has gained an enormous attention. In this respect, DMR results have played a fundamental role in modern cosmology comparable to the discovery of the expansion of the universe or the discovery of the microwave background by Penzias and Wilson, and indeed this has motivated the delivering of the Nobel prize to Smoot and Mather for this discovery. One of the fundamental reasons for this is that fluctuations on scales larger than one degree in the microwave background radiation correspond to scales greater than the horizon at last scattering epoch and cannot therefore been altered by any physical process and should therefore reflect primordial fluctuations (Weinberg 1972). This also means that the very existence of these fluctuations could be explained only from yet undiscovered physics, probably relevant to the very early universe (Guth 1981), for which the expansion law has been modified when compared to the standard picture. The DMR results were providing some constraints on cosmological models (Wright et al. 1992) but it has been realized that the measure of fluctuations on smaller scales will provide much stringent information. Early detection of fluctuations on degree scales allowed to

Fig. 2 The amplitude of angular fluctuations of the CMB is expressed through their angular power spectrum. Data are WMAP, Boomerang, ACBAR Reichardt et al. (2008). A simple minimal six parameter models including a cosmological constant provides an excellent fit to the data. This is one of the most important successes of modern cosmology set interesting constraints and provided the first evidence for a flat geometry of space (Lineweaver et al. 1997; Lineweaver and Barbosa 1998). If estimations of low matter density were to be regarded as robust, this was inevitably leading to a non-zero cosmologcial constant. Even before the availability of the WMAP data, considerable progresses have been achieved on the measurement of fluctuations on all angular scales. Archeops (Benoît et al. 2003a) and Boomerang (de Bernardis et al. 2000), as well as many other small scale measurements, already provided measurements allowing tight constraints on cosmological parameters (Benoît et al. 2003b). Although the observed fluctuations were consistent with a Λ-dominated universe, a cosmological constant was not explicitly requested by the CMB data alone. Indeed, even the WMAP data were consistent with a vanishing cosmological constant, provided the Hubble constant was left as an entirely free parameter. A positive detection of a cosmological constant could be obtained only using some additional data in conjunction with CMB (Fig. 2), like the measurement of the Hubble constant. A further restriction came from the fact that the constraint on cosmological parameters were obtained within the standard CDM picture, and that many ingredients were specified without being necessarily confirmed by observations: for instance initial fluctuations are supposed to be adiabatic and to follow some power law. Therefore the “concordance” (Ostriker and Steinhardt 1995a,b) cosmology was an appropriate terminology: the model was consistent with most existing data, but the introduction of a cosmological constant was not requested by any single data, and it was far from being clear whether relaxing some of the input hypotheses would not allow for solutions without the introduction of a cosmological constant.

4.5 Clusters of galaxies

Most talks on clusters of galaxies start with stating the fact that they are the largest virialized structures in the universe. Another reason for the strong interest they represent is due to the fact that clusters are the only astronomical large objects for which we have such a wide range of information: their total mass is in principle accessible from X-ray spectroscopy, velocity dispersion, weak lensing signal, their gas content can be investigated through X-ray observations but also through their imprint on the microwave background, the thermal Sunyaev–Zel’dovich effect. Obviously, their star content can be evaluated, but also the metal content of their gas. Clusters are therefore the astrophysical objects relevant to cosmology on which the most comprehensive set of data can be obtained.

4.5.1 Baryon fraction

X-ray data allow to determine at the same time the mass of the gas content of clusters, through imagery, as well as the gas temperature, through spectroscopy. The temperature can be used to estimate their total mass. It is therefore straightforward then to infer their baryon fraction (most of the baryons being in the gas, the star even if uncertain can be taken into account without making much difference). Because, the gravitational collapse is expected to be identical for dark matter and for baryons, at least before the first shocks occur, the baryon fraction should reflect the primordial baryon fraction:

$$\frac{M_b}{M_t} = \Upsilon \frac{\Omega_b}{\Omega_M} \tag{58}$$

where Υ is expected to be close to one. Numerical simulations shows indeed that Υ should lie in the range 0.85–1. Ω_b can be known from primordial nucleosynthesis results, and therefore constraints can be obtained on Ω_M (White et al. 1993). Although this test is attractive in its principle, its application, especially in the context of precision cosmology, is somewhat problematic. There are several reasons for that: for instance, in numerical simulations, as well as in the observations, Υ seems to depend on the radius and is therefore a function $\Upsilon(r)$ on radius. The precise value of $\Upsilon(r)$ is likely to depend on the physics of the non-gravitational heating and cooling processes which are necessary to reproduce observed properties of clusters in numerical simulations. However, these possible limitations still allow to infer an upper limit on Ω_M from Eq. 58. A final limitation comes from the clumping of the gas which would lead to an underestimation of Ω_M. On the observational side, it is unclear to which accuracy cluster masses are known: agreement between lensing masses and X-ray mass is under debate. Estimations of total mass and baryon mass in the outer parts of clusters, which are expected to provide the most reliable estimation of the primordial baryon fraction, are subject to uncertainties: the X-ray emission of the gas at the virial radius is low and only for few clusters do we have a firm detection. At this outer part, the gas is eventually clumped which would bias the gas mass estimates.

4.5.2 Geometrical tests

Another strategy is to use clusters as standard candles in some sense and to constrain the cosmological parameters from the observed properties of distant clusters. For instance, it has been proposed to use cluster X-ray radius as a standard ruler (Mohr et al. 2000). Additional information may allow direct distance measurements. This is what the Sunyaev–Zel'dovich (SZ) signal provides: because the X-ray emissivity goes as the square of the gas density, while the SZ signal just goes as the density the distance can be directly obtained. Not only this method can be used to estimate the Hubble constant, but also possible to extend the Hubble diagram to distant clusters and thereby to get the same information provided by the SNIa diagram. Another variant of these tests has been proposed from the baryon fraction of clusters against redshift: because the gas fraction inferred from observations depends on the angular distance to the cluster, requesting that the gas fraction does not evolve with redshift will result on constraints on the cosmological parameters (Sasaki 1996). This test has deserved recent attention (Allen et al. 2008), but uncertainties mentioned about the baryon fraction are likely to alter this test either and it is difficult to draw robust conclusions from it (Sadat et al. 2005; Ferramacho and Blanchard 2007; Ettori et al. 2009).

4.5.3 The use of cluster abundance

The theoretical mass function A theoretical expression for the abundance of virialized structures in hierarchical picture has been first proposed by Press and

Schechter in (1974). From scaling arguments, a general expression for the mass function can be derived (Blanchard et al. 1992):

$$N(M,z) = -\frac{\overline{\rho}}{M^2\sigma_z(M)}\delta_{\rm NL}(z)\frac{{\rm d}\ln\sigma}{{\rm d}\ln M}\mathscr{F}(\nu) \tag{59}$$

$\delta_{\rm NL}(z)$ being the threshold for non-linear collapse of spherical density perturbations (in an EdS universe $\delta_{\rm NL}(z) = 1.68$), $\sigma_z(M)$ the amplitude of mass fluctuations on the mass scale M ($\sigma_z(M) = \sigma_0(M)D(z)$, $D(z)$ being the normalized growing rate of linear fluctuations) and $\nu_{\rm NL}$ the normalized threshold ($\nu_{\rm NL} = \delta_{\rm NL}/\sigma(M)$) for non-linear collapse, $\int_\nu^{+\infty}\mathscr{F}(\nu){\rm d}\nu$ is the fraction of space covered by spheres of mass $\geq M$ satisfying the non-linear criteria. Such an expression can be rigorously justified only for a given power law spectrum in an Einstein–de Sitter universe. However, the numerical investigations of the mass function have shown that it does follow the above scaling to a very high accuracy (Efstathiou et al. 1988; Sheth and Tormen 1999; Jenkins et al. 2001). Press and Schechter used for the function $\mathscr{F}$:

$$\mathscr{F}(\nu) = \sqrt{\frac{2}{\pi}}\exp\left(-\frac{\nu^2}{2}\right) \tag{60}$$

while the expression proposed by Steh and Tormen (1999; 2001), possibly extending the non-linear condition to take into account ellipsoidal collapse, was found to produce a more accurate agreement with numerical simulations:

$$\mathscr{F}(\nu) = \sqrt{\frac{2A}{\pi}}C\exp(-0.5A\nu^2)(1.+(1./(A\nu)^2)^Q) \tag{61}$$

with $A = 0.707$, $C = 0.3222$, and $Q = 0.3$. Jenkins et al. (2001) did provide another fitting formula in which the amplitude of $\delta_{\rm NL}$ is set constant (independently of the cosmology) which was found to provide an accurate fit to within 20% in relative amplitude.[8] This accuracy is sufficient for present-day available data to be compared with. More recently, larger numerical simulations were used (Warren et al. 2006; Tinker et al. 2008; Crocce et al. 2010) to investigate the mass function to a precision higher than 20%, finding some departures from strict universality and providing more accurate fitting formula, but cosmological data are not yet at a precision level for which such departures could be observed.

Local abundance The observed local abundance of clusters of a given mass M can be used to infer directly the average amplitude of matter fluctuations, $\sigma(M)$, through the expression of the mass function. Such a derived value depends weakly on cosmology (Blanchard and Bartlett 1998) as cosmology enters only through $\delta_{\rm NL}$, which as we have mentioned is almost independent on Cosmology. However, the traditional way to express the amplitude of matter fluctuations is through σ_8 and therefore the dependence of this quantity appears on cosmology and on the

[8] Another complication happens from the definition of “an object with a mass M”. Two algorithms are commonly used, the friend of friend one (FOF) and the spherical overdensity one (SO). In addition, the non-linear contrast density Δ which is used could be set to an arbitrary fixed value or to depend on Cosmology according to the virial value Δ_v evaluated within the spherical collapse model.

initial spectrum. There is a source of uncertainty due to the fact that the mass is not directly an observable quantity. Therefore, one has to use a relation between mass and some observable quantity. At the same time, the selection function of clusters within a given sample has to be known to infer the actual number per unit volume. X-ray clusters are a priori the easiest choice, as the selection function from X-ray surveys is easy to model and the X-ray temperature is a priori a reliable and physically motivated proxy of the total mass. There are various arguments from which one expects that $T_x \propto GM/r$. This leads to a scaling relation between mass and temperature:

$$T = A_{TM} M_{15}^{2/3} (\Omega_M (1+\Delta)/179)^{1/3} h^{2/3} (1+z)\,\mathrm{keV} \tag{62}$$

an expression in which the contrast density Δ is the relative density when compared to the background density of the universe. An other density contrast is also used which refers to the contrast density Δ_c relative to the critical density of the universe $\rho_c(z)$. The above scaling reads then:

$$T = A_{TM} M_{15}^{2/3} (\Omega_M (1+\Delta_c)/179)^{1/3} (E(z)h)^{2/3}\,\mathrm{keV} \tag{63}$$

where the redshift-dependent Hubble constant is written $H(z) = 100hE(z)$ km/s/Mpc. There has been much debate on which value of the normalization A_{TM} is to be used and this translates in an uncertainty in σ_8 (Pierpaoli et al. 2003). The various values published for σ_8, ranging from 0.6 to 1.05 for a ΛCDM model with $\Omega = 0.3$ (Blanchard and Douspis 2005) are consistent with each other when the various values of the normalization constant A_{TM} are taken into account. However, because of this uncertainty the actual value of σ_8 inferred from cluster abundance is still subject to some debate (Reiprich 2006; Evrard et al. 2008; Rines et al. 2007).

Abundance evolution Peebles et al. (1989) and Evrard (1989) discussed the constraints that the existence of high redshift clusters were setting on CDM models, at a time when Einstein–de Sitter cosmology was fashionable. These works followed some earlier ones on the use of cluster abundance in cosmology (Perrenod 1980). The evolution of the abundance of clusters of a given mass is a sensitive function of the growing rate of fluctuations therefore offering a powerful cosmological test (Lilje 1992; Oukbir and Blanchard 1992). Rather obviously, this test needs that local abundance to be adequately normalized. This test is primarily sensitive to the cosmological density of the universe, very weakly depending on other quantities (Blanchard and Bartlett 1998). Attempts to apply directly the test of the evolution of the temperature distribution function of clusters have been performed but still from a very limited number of clusters (typically 10 at redshift 0.35) and lead to somewhat apparently conflicting results (Henry 1997; Viana and Liddle 1999; Blanchard et al. 2000).

On the other hand, redshift number counts (without temperature information available) allow one to use samples comprising many more clusters. Indeed using simultaneously different existing surveys one can use information provided by more than 300 clusters with $z > 0.3$ (not necessarily independent). In order to model clusters number counts, for which temperatures are not known, it is necessary to have a good knowledge of the L–T relation over the redshift range which

Fig. 3 The low and high redshift temperature distributions compared to predictions of an Einstein de Sitter Universe (left side, $\sigma_8 = 0.55$, $A_{TM} = 6.35$ in Eq. 62, $\Gamma = 0.135$ is a shape parameter for the spectrum) and to predictions of the concordance model (right side, $\sigma_8 = 0.81$, $A_{TM} = 7.75$, $\Gamma = 0.168$). *Upper* (*blue*) losanges are the local data. *Lower* (*red*) losanges are the temperature distribution function ($z \sim 0.33$) derived from EMSS, (*yellow*) squares are high redshift ($z \sim 0.5$) clusters from MACS and the 400^2 deg survey

is investigated (Oukbir and Blanchard 1997). Early investigations indicated the absence of evolution of the *L–T* relation or a slight positive evolution, indicative of a high density universe (Sadat et al. 1998; Reichart et al. 1999; Borgani et al. 1999). The *L–T* relation at high redshift has been more recently determined by the XMM-Ω project (Bartlett et al. 2001; Lumb 2004) and Chandra (Vikhlinin et al. 2002). Number counts can then be computed:

$$\begin{aligned} N(> f_x, z, 2\Delta z) &= \Omega \int_{z-\Delta z}^{z+\Delta z} \frac{\partial N}{\partial z}(L_x > 4\pi D_l^2 f_x)\, \mathrm{d}z \\ &= \Omega \int_{z-\Delta z}^{z+\Delta z} N(> T(z))\, \mathrm{d}V(z) \\ &= \Omega \int_{z-\Delta z}^{z+\Delta z} \int_{M(z)}^{+\infty} N(M,z)\, \mathrm{d}M \mathrm{d}V(z) \end{aligned} \qquad (64)$$

where $T(z)$ is the temperature threshold at redshift z corresponding to the flux f_x as given by the observations, being therefore independent of the cosmological model. This approach has been used to model all available ROSAT surveys and using the latter *L–T* measurement provided by XMM (Vauclair et al. 2003). All existing X-ray cluster surveys systematically point toward high Ω_M. A combination allowed a determination of Ω_M with a 15% precision: $0.85 < \Omega_M < 1.15(1\sigma)$ (depending somewhat on the calibration issue, this is part of the systematic). During this analysis numerous possible sources of systematics were investigated with great detail and are representing roughly an additional 15% uncertainty. This means that global uncertainty is roughly 20%. This gives unambiguous evidence that the observed high redshift cluster abundance compared to local one is inconsistent with the one modeled within the concordance model. This is confirmed by the recently availability of a large sample of high redshift clusters with measurements provided by Chandra (Vikhlinin et al. 2009) and a sample of high redshift massive clusters Ebeling et al. (2007). The temperature distributions drawn from these samples is shown in Fig. 3: in agreement with previous results these temperature distributions are inconsistent with the concordance cosmology. A critical ingredient in the modeling of X-ray clusters is the mass temperature relation 62. A possible strong evolution of this relation, as proposed to reconcile redshift number counts with predictions (Vauclair et al. 2003), would allow the agreement of the concordance model with the observed temperature distributions as well.

Fig. 4 Large scale correlation function of the SDSS Luminous Red Galaxies Eisenstein et al. (2005). Three ΛCDM models ($\Omega_M h^2 = 0.12, 0.13, 0.14$) are shown with a baryonic content evaluated from the CMB ($\Omega_b h^2 = 0.024$). A model without baryon corresponds to the bottom (*magenta*) line

4.6 Large scale structure of the universe

The properties of the galaxy distribution on large scales has been a major source of constraints for cosmology in the past. The most often quoted quantity is the correlation function of galaxies. Until the 1990s the correlation function was relatively well know only on scales which are non-linear or nearly so. Therefore, the correlation function could be adjusted only by means of numerical simulations. Two major breakthroughs enabled important progresses in this area: the first one was the measurement of the correlation function of galaxy from the APM survey allowing to exclude the standard CDM picture (Maddox et al. 1990). Furthermore, the data were consistent with the predictions of a low density CDM model with $\Omega_M \sim 0.3$ and it has been argued (Efstathiou et al. 1990) that the introduction of a cosmological constant was a way to reconcile CDM with inflation predictions, in the spirit of an earlier proposition (Peebles 1984). The second one was the discovery of a simple analytical formalism allowing to construct the correlation function in the non-linear regime from the initial spectrum (Hamilton et al. 1991). More recently, the most critical advance resulted from the availability of very large galaxy surveys, the 2Df redshift survey and the SDSS survey, allowing to measure the amplitude of galaxy fluctuations on scales as large as $100h^{-1}$ Mpc (Percival et al. 2001; Tegmark et al. 2004; Eisenstein et al. 2005; Percival et al. 2007). This has provided a remarkable success to the ΛCDM picture because the amplitude of the correlation function could be predicted for models that already match the CMB fluctuations measured by WMAP: not only ΛCDM model reproduces the shape of the correlation function, but also the specific presence of a bump in the correlation function at scale of the order of $100\,\mathrm{h}^{-1}$ Mpc due to the detailed dynamics of fluctuations when the baryons are taken into account, the so called acoustic peak (Fig. 4), corresponding to the "peak" in the $\mathscr{C}_l$ of the CMB. More detailed use of power spectrum measurements might be limited by our understanding of the exact relation between galaxies distribution and the underlying dark matter distribution (Sánchez and Cole 2008).

5 Possible origin of the apparent acceleration

5.1 An Einstein-de Sitter Model

5.1.1 Supernovae Hubble diagram

The first convincing evidence for acceleration is generally considered as coming from the SNIa Hubble diagram. However, use of geometrical tests based on the assumption of no-evolution of the parent population of the test is always possibly subject to produce biased values because of un-anticipated evolution. One possible way to cure this problem is to assume some evolution and see whether the data

Fig. 5 Fitting the SNIa Hubble diagram with two free parameters, one being the cosmological constant in a flat cosmological model and the second being a parameter describing a possible time evolution of the luminosity of distant supernovae ($\Delta m(z) = K(t_0 - t(z))$) leads to the following constraints Ferramacho et al. (2009). Contours are 1, 2, and 3 sigma regions. There is a severe degeneracy between the two parameters which prevents an unambiguous determination of the cosmological constant. A vanishing cosmological constant is entirely consistent with SNIa diagram provided that a significant but not unrealistic amount of evolution is allowed for. The major issue is that it is extremely difficult from a purely observational point of view to reject such a possibility

Fig. 6 The TT spectrum of the first year WMAP data compared to three different models: one is the concordance, the two others are Einstein de Sitter models, one of which comprises neutrino contribution of $\sim 10\%$ corresponding to three degenerate families with $m_\nu \sim 0.7$eV. From Blanchard et al. (2003)

still provide evidence for the claim. For instance, an evolution term like:

$$\Delta m_e \propto z \tag{65}$$

cannot mimic the observed Hubble diagram. The limitation of this approach is due to the fact that we have no model for this evolution and therefore we are left with a purely empirical approach. Other forms of evolution may therefore lead to different conclusions. Indeed, it has been suggested that the correction term might be (Wright 2002):

$$\Delta m_e \propto \Delta t \tag{66}$$

such term leads to large degeneracy between cosmology and possible evolution (Ferramacho et al. 2009; Linden et al. 2009) (Fig. 5). Undoubtfully, despite its possible limitation, the determination of the Hubble diagram from SNIa has led to a major and rapid change of paradigm in modern cosmology. However, this change has been possible because the previous situation was problematic. Although some observational indications were favoring a low density universe, the first detections of fluctuations on degree scales were in conflict with open low density universe (Lineweaver and Barbosa 1998).

5.1.2 Fluctuations of the cosmological background radiation

The remarkable results of the WMAP experiment, with accurate measurements of the $\mathscr{C}_l$ and additional measurements on the polarization, are often quoted as providing a direct evidence for an accelerating universe. This is incorrect: cosmologi-

Fig. 7 Data from the SDSS have allowed to measure the amplitude of galaxy fluctuations on large scales. In this respect, Luminous Red Galaxies (LRG) provided measurement of the power spectrum on the largest scales. *Green crosses* correspond to Tegmark et al. (2006) and *black crosses* correspond to the latest measurements of the power spectrum of LRG from the SDSS Data Release 5 by Percival et al. (2007). The *red* continuous curve is the predicted spectrum for a typical concordance model, while the *dotted* and *dashed lines* correspond to the power spectrum for Einstein de Sitter models consistent with the WMAP fluctuation angular power spectrum $\mathscr{C}_l$ (Blanchard et al. 2003; Hunt and Sarkar 2007) cal constraints established from CMB entirely rely on the spectrum shape assumptions, which is commonly assumed to be described by a single power law. Therefore, the conclusions on the high precision obtained on cosmological parameters could be erroneous (Kinney 2001). Indeed, relaxing this hypothesis, i.e., assuming a non-power-law power spectrum, it is possible to produce $\mathscr{C}_l$ curve within an Einstein de Sitter cosmological model which provides a fit as good as the concordance model. This is illustrated in Fig. 6 on which three models are compared to the WMAP data, two being Einstein de Sitter models. Such models not only reproduce the TT (temperature–temperature) spectrum, but are also extremely close in terms of ET (polarization–temperature) and EE (polarization–polarization) spectra. Furthermore, the matter power spectra are similar on scales probed by current galaxy surveys before the availability of the SDSS LRG sample. An un-clustered component of matter like a neutrino contribution or a quintessence field with $w \sim 0$ is necessary to obtain an acceptable amplitude of matter fluctuations on clusters scales (Blanchard et al. 2003). Such models require a low Hubble constant ∼46 km/s/Mpc. Such a value might be viewed as terribly at odd with canonical HST key program value (∼72 km/s/Mpc) but is actually only $\sim 3\sigma$ away from this value, this can certainly not be considered as a fatal problem for an Einstein–de Sitter universe. The introduction of a non-power law power spectrum might appear as unnatural. However, such a feature can be produced by some models of inflation to match the $\mathscr{C}_l$ curve (Hunt and Sarkar 2007). Therefore, the amplitude and shape of the CMB fluctuations as measured by WMAP is certainly a success for the ΛCDM model but cannot be regarded as a direct indication of the presence of dark energy.

5.1.3 Large scale structure

Once an Einstein de Sitter model is built to reproduce the CMB $\mathscr{C}_l$, the amplitude of the matter fluctuations on large scales is set up and the measurement of the matter fluctuations on large scales in the present day universe is a critical way to distinguish models which are otherwise degenerated in their $\mathscr{C}_l$. The comparison of the power spectrum from the SDSS LRG with the predicted spectra for Einstein de Sitter models is clearly in favor of the concordance model (Blanchard et al. 2006), see Fig. 7. One should add some caution here: it might be possible that the biasing mechanism leads to a power spectrum at small k (large scales) which is not proportional to the actual matter power spectrum (Durrer et al. 2003), in which case the above comparison might not be a fatal failure of the Einstein de Sitter models. However, biasing mechanisms systematically lead to a correlation function on large scales which is still proportional to the matter correlation function on large scales. Comparison of the correlation function on large scales is therefore less ambiguous and its measurement should be unambiguously discriminant. Hunt and Sarkar (2007) have provided a comprehensive MCMC investigation of the Einstein de Sitter parameter space, finding models which acceptably fit the correlation function on scales below $70\,\mathrm{h}^{-1}$Mpc, but were nevertheless systematically negative on scales of the BAO peak. This is a strong evidence that there is no way in an Einstein de Sitter universe to fit simultaneously the $\mathscr{C}_l$ and the observed distribution of galaxies on large scales. This should be regarded as a remarkable success of the concordance cosmological model: although there were little doubts that this model could fit accurately most of the major existing observational facts in cosmology, the ability to produce predictions that are verified a posteriori is the signature of a satisfying scientific theory. Number counts of X-ray clusters were found to match the expectations of an EdS universe and conflict with the concordance model. However, if the standard scaling of the *M–T* relation 62 is broken by some non-gravitational processes, the number counts and temperature distribution function could be altered in a way that the concordance model can accommodate data (Vauclair et al. 2003).

5.2 Einstein cosmological constant or vacuum contribution

The most direct explanation one can provide to the cosmic acceleration is that it is due to a true cosmological constant appearing in the geometrical part of Einstein's equation, i.e., the left-hand side of Eq. 4. However, it is much more common to believe that this term arises from some contribution to the energy-momentum tensor on the right-hand side. As we have seen, from a classical point of view the vacuum might have a non-zero density and behaves identically to a cosmological constant. In addition, quantum mechanics provides an intriguing hint in this direction. The possible energy levels of an harmonic oscillator are known to be:

$$E_n = \left(n + \frac{1}{2}\right) h\nu \tag{67}$$

and so the state of lowest possible energy, the zero point, is not zero but $h\nu/2$. This non-zero value is often noticed in standard text book of quantum mechanic, but, because observable quantities correspond to transitions from one state to an other one, is not regarded as being problematic. However, as soon as gravitational interaction has to be added, one cannot avoid to take the absolute energy into account. Summing all the contributions of modes of the fluctuations of the electromagnetic field up to some wave number k_c gives a density ρ_V:

$$\rho_V = \int_0^{k_c} \frac{4\pi k^2 \mathrm{d}k}{8\pi^3} \frac{1}{2} k \sim \frac{{k_c}^4}{8\pi^2} \tag{68}$$

The total contribution therefore depends on the cutoff scale k_c. If the Planck scale is taken, this leads to a density which is something like 10^{120} too large. For all energy scales in physics does ρ_V end up with an unacceptable large value, which looks like a fundamental problem.[9] An elegant solution to this problem is obtained from supersymmetry: the contribution to vacuum from fermion is negative and therefore with an equal number of modes in fermions and bosons one gets a cancelation. However, because the supersymmetry should be broken at energy below 1 TeV or so, the problem of the vacuum density is still not solved, even if its strength has been noticeably reduced. An other possibility is that the vacuum actually behaves differently from (68) (Branchina and Zappalà 2010). Others arguments to reject this option have proposed (Perivolaropoulos 2008).

[9] It has been suggested that the zero-point fields should not be regarded as real, despite the fact that they are at the basis of the calculation of the Casimir effect (Michel 1996).

5.2.1 Detecting the cosmological constant in a laboratory experiment

The above considerations might lead to a fascinating possibility. If the observed dark energy corresponds to an actual contribution of the electromagnetic field there will be a frequency ν_c associated to the cut off in (68), and the value associated with the present value of dark energy is:

$$\nu_c \sim 1.7\ 10^{12}\,\mathrm{Hz} \tag{69}$$

Vacuum fluctuations produce noise which can be detected experimentally with Josephson junctions (Koch et al. 1980), it is therefore possible that the frequency cut-off k_c could be measured in a laboratory experiment (Beck and Mackey 2005). This point of view has been criticized by (Jetzer and Straumann 2005). A further problem arises from the fact that the existence of a cut-off like (69) could modify the classical Casimir force (Bressi et al. 2002), in a way which is excluded by existing data (Mahajan et al. 2006). However, the origin of the noise current in the Josephson junction may have nothing to do with the electromagnetic field in the device (Branchi et al. 2009).

5.3 The effect of inhomogeneities

The standard formalism of Friedmann–Lemaître models relies on the assumption that matter is uniformly distributed. Since the early work of Sachs and Wolfe (1967), perturbations are commonly written as departure $h_{\alpha\beta}$ from the Robertson–Walker metric element $g_{\alpha\beta}$:

$$\tilde{g}_{\alpha\beta} = g_{\alpha\beta} + h_{\alpha\beta}. \tag{70}$$

This is a qualitative expression, as there are many ways to write perturbations depending first on the choice of the initial form of the metric and coordinate systems. The analysis of linear perturbations ($|h_{\alpha\beta}| \ll \langle g_{\alpha\beta} \rangle$) is rich and not as trivial as one might naively think. However, the dynamics of relativistic linear perturbations to first order is well understood. Three distinct physical modes are possible: scalar perturbations which represent fluctuations of the density field ρ, vector perturbations which represent the vorticity of the velocity field, and tensor perturbations which represent gravitational waves. Only this last term is obviously not present in Newtonian dynamics. This has legitimated the use of Newtonian theory to describe evolution and average effect of perturbations on scales much smaller than the Hubble scale and for velocities much smaller than the speed of light. It is therefore common to use the Newtonian perturbed RW metric:

$$\mathrm{d}s^2 = -(1+2\psi)\mathrm{d}t^2 + a(t)^2(1-2\psi)\gamma_{ij}\,\mathrm{d}x^i\mathrm{d}x^j \tag{71}$$

ψ being the Newtonian potential. However, the fact that we live in an inhomogeneous universe has always left open the possibility that average observable quantities may be definitively different from what can be obtained in a rigorous homogeneous world, this question discussed in the context of relativistic world models, can be traced back to thirties (Eddington 1930; Tolman 1934). One early worry of this kind was about the average effect of gravitational lensing. It has been suggested that the average magnification from inhomogeneities in the universe might produce an apparent redshift–magnitude relation which behaves differently from that in a homogeneous universe and would therefore bias inferred values of the cosmological parameters (Dyer et al. 1972). Weinberg (1976) did provide a general argument to show that there is an integral constraint that guarantees the average relation in inhomogeneous worlds to be identical to the homogeneous case but that the way perturbations are handled may lead to lose this general integral constraint on average quantities even to the first order in perturbation (while it is valid at all orders). Although Weinberg's theorem is extremely clever, it relies on some hypotheses, one is that the universe still behaves as a FLRW model with its density being equal to the average density, and the subject has still been addressed in recent years (Holza and Wald 1998; Kibble and Lieu 2005; Barausse et al. 2005; Biswas and Notari 2008).

In this approach, the consequences of the presence of inhomogeneities were analyzed assuming the background evolution of the universe was still following the usual Friedmann–Lemaître equations. However, beyond the problem of properly evaluating the observable consequences of the presence of inhomogeneities (as illustrated by the calculation of the fluctuations of the CMB), one fundamental question is whether the Friedmann–Lemaître equations describing the expansion could be significantly altered by the presence of inhomogeneities, an effect named "back reaction" (Futamase 1989). At first look one would naively expect that the average effect of linear perturbations to first order is zero and that to the second order one would get something like (for the case of perturbations around an Einstein de Sitter universe):

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3}\left(1+F(h_{\alpha\beta})\right) \tag{72}$$

with :

$$F(h_{\alpha\beta}) \propto \langle h^2 \rangle \tag{73}$$

For the observable part of the universe, as we are within some perturbation with a typical size corresponding to the horizon, one expects that

$$\rho_{\rm obs} \sim \rho(1 \pm h_{\alpha\beta}) \tag{74}$$

(this is written in a very loose way that would probably horrify relativity experts, but it is done for the purpose of a qualitative illustration on how the problem is settled). Now, astrophysical data show that $\langle h \rangle$ is always of the order of 10^{-5}, or less, except in the vicinity of neutron stars and black holes, departures at this level would therefore not be very surprising. Actually, the non-linear collapse of structures like clusters or galaxies modestly enhances initial metric perturbations by a factor $\Delta^{1/3}$, but lensing measurements of galaxies and clusters directly prove that on these scales the metric perturbations remain tiny and one expects the linear approximation to be sufficient even when the structures are becoming non-linear ($\frac{\delta\rho}{\rho} \gg 1$). Of course it is clear that some observables would be distorted at a level of $h \sim 10^{-5}$. However, the possible non-negligible contribution of inhomogeneities to the global dynamics of the present universe, the so called back reaction has been proposed[10] with the idea that the actual dynamics of the universe is much more distorted by non-linear terms than naively anticipated. Ironically enough, early propositions of back reaction were suggested as a way to reduce or even cancel the presence of a large cosmological constant (Tsamis and Woodard 1993; Mukhanov et al. 1997; Brandenberger 2002), while nowadays, the important question is whether this back reaction could explain the apparent cosmological constant. This perspective has attracted a lot of attention (see Buchert 2008 for a recent review on this issue): it may even offer a possible solution to explain the acceleration of the universe, within the context of general relativity and without the introduction of any further exotic ingredient. Buchert (2000) derived the exact dynamical equations governing the average size $a_{\mathscr{D}}$ of a domain $\mathscr{D}$ in an inhomogeneous matter distribution. These equations can be written in the following form:

$$\left(\frac{\dot{a}_{\mathscr{D}}}{a_{\mathscr{D}}}\right)^2 = \frac{8\pi G\rho}{3} - \frac{\mathscr{Q}}{6} - \frac{\mathscr{R}}{6} \tag{75}$$

$$\left(\frac{\ddot{a}_{\mathscr{D}}}{a_{\mathscr{D}}}\right) = -\frac{4\pi G\rho}{3} + \frac{\mathscr{Q}}{3} \tag{76}$$

for irrotational dust, and after dropping the cosmological constant for clarity. In this expression, $\mathscr{R}$ is the average curvature over the domain and $\mathscr{Q}$ is an average quantity over the domain $\mathscr{D}$:

$$\mathscr{Q} = \frac{2}{3}\left(\langle\theta^2\rangle - \langle\theta\rangle^2 - 2\langle\sigma^2\rangle\right) \tag{77}$$

θ being $3(\dot{a}_{\mathscr{D}}/a_{\mathscr{D}})$ and σ is the rate of shear $\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma_{ij}$ (σ_{ij} being the shear tensor).

Given the analogy with the classical EFL equations, the possible consequence of back reaction has been discussed on this ground (Buchert et al. 2000; Buchert and Carfora 2003; Buchert 2008).

Following Buchert's work, several authors regard the back reaction problem as an essentially non perturbative question which cannot be solved without an appropriate treatment (Rasanen 2008; Kolb et al. 2006).

The question was handled from an other point of view by looking at second-order effect from superhorizon perturbations generated during inflation (Barausse et al. 2005; Kolb et al. 2005). The conclusion was that some acceleration could be generated in this way. However, this result has been disproved (Flanagan 2005; Geshnizjani et al. 2005; Hirata and Seljak 2005; Giovannini 2006; Räsänen 2006). The same conclusion holds for superhorizon perturbations in presence of a scalar field (Kumar and Flanagan 2008). The possibility remains open that back reaction from subhorizon scales induces a modification of the equations governing the expansion (Räsänen 2006; Kolb et al. 2006) and observational relations. However, from the astrophysical point of view what matters precisely is whether back reaction could alter significantly cosmological observations and fools the traditional

[10] The idea that the small scale evolution may have a connection with large scale dynamics in relativistic cosmology can be traced back to Eddington in (1930).

interpretation. Clearly, if the back reaction has to modify cosmological quantities by a factor $h^2 \approx 10^{-10}$ the study of back reaction would remain of pure academic interest. Actually, modifications at the level of $h \approx 10^{-5}$ are expected from linear perturbations at the scale of our horizon. Indeed, (Wetterich 2003) examined back reaction in a perturbative approach and concluded that gravitational energy of structure contributes as a back reaction term, but unless dark matter is essentially made of black holes, no appreciable change beyond the 10^{-5} level is to be anticipated. Such a term should be called a weak back reaction, because it can be anticipated from simple dimensional arguments. The important question is: are there strong back reaction terms? i.e., terms which arise from non-linear effects and which could contribute at a level much larger than the naive order of magnitude $\approx 10^{-10}$ or even be of the order of being dynamically as important as the density ρ. At this level, we are facing traditional difficulties when handling with relativistic effects. For instance, Buchert equation are written in a comoving frame in which particles are at rest. Therefore, the non-zero "mass" associated with the kinetic energy of particles should appear as a "back reaction" term in the EFL equations. Identically, the energy associated to a gravitational field has to be taken into account. These are weak back reaction terms. Using Newtonian perturbed RW metric (Gruzinov et al. 2006) showed that back reaction term can entirely disappear from the second EFL equation 16 when the density is properly evaluated, but with a different choice of coordinates, and concluded that we are very close to a "no-go" theorem: "no cosmic acceleration occurs as a result of the non-linear back reaction via averaging" (on this question a major source of complication arises from the gauge issue and different authors may find apparently discrepant results because of the use of different gauges). The present formulation of Buchert's equation is likely to depend explicitly on the choice of the coordinate system and therefore the relevance to observations is unclear. It is not even clear whether a clever gauge transformation would not allow to suppress entirely this back reaction term. Similar conclusions were reached by Kasai et al. (2006), who emphasized again that gravitational energy is a source of the gravity field which can appear as a "back reaction" term, i.e., is a weak back reaction term, and could be qualified as such, in agreement with Wetterich (2003), but which remains small in practice. In addition, they showed in some exact solutions the absence of observable "back reaction". In this respect, the Swiss cheese-model (Schücking 1954) is an interesting exact solution in which the presence of inhomogeneities (that could even be non-linear, i.e., $h \sim 1$) does not lead to any unexpected behavior, i.e., in which there is no strong back reaction effect. From a second-order perturbation analysis (Kasai et al. 2006) also argued that as long as the Newtonian conditions are satisfied:

$$l \ll \frac{c}{H_0} \quad \text{and} \quad v \ll c \tag{78}$$

"there always exist gauges where the metric differs from the RW metric only at order $\langle h \rangle$." Of course, when the size of the perturbations becomes comparable to the Hubble radius, one expects significant alteration of the standard relations (Kolb et al. 2008). A more troubling paper has been put on the ARXIV, but remains unpublished which: Nambu and Tanimoto (2005) investigated the solution of Buchert' equations in a exact spherical solution (the LTB that is described in the next section), actually a very interesting approach. They concluded that at some point “the universe starts accelerated expansion,” a similar conclusion was drawn by Chuang et al. (2008), Mansouri (2005, 2006), Moffat (2006), while it is clear that from spherical solution no physical acceleration is possible thanks to Birkhoff’s theorem and as it has been confirmed by Alnes et al. (2007), Enqvist and Mattsson (2007). This is a direct indication that acceleration could appear in some quantities but which is by no means related to the observed acceleration. The question has fallen in the arena of subtle general relativity issues, but it seems that there are serious claims that back reaction cannot produce an acceleration comparable to what is requested from the observations. There are also convincing indications that no correction to standard EFL equations is expected above the amplitude of first order perturbation 10^{-5}. There is therefore an important “cosmological no-go theorem” which remains by now only a conjecture that for the standard CDM fluctuations spectrum, there is no significant back reaction and the classical description is sufficient. It is by now the duty of advocates of real back reaction effect to demonstrate that interesting terms may exist beyond standard known linear and non-linear contributions. It remains important to examine whether weak back reaction could raise changes of the order of 10^{-3} or more, as such an effect would start to be of the order of the precision anticipated for the determination of the cosmological parameters from future experiments. The fact that Eq. 75 looks like standard one, i.e., first EFL equation, does not at all mean that terms appearing in it should be identified with similar looking standard cosmological parameters. These equations are valid for a given special choice of coordinates and it is therefore far from trivial to identify which observable consequences would follow. Even the term $\frac{\dot{a}_{\mathscr{D}}}{a_{\mathscr{D}}}$ should not a priori be identified to the standard Hubble expansion rate nor should $\ddot{a}_{\mathscr{D}}$ be used in comparison with to $-q_0$. Strictly speaking when observers determine the Hubble constant H_0 they compare the observed linear relation between luminosity distance and redshift from some galaxy sample, which are both observable quantities, and q_0 is related to the leading second-order term in z. Identically, when the cosmological density parameter (for instance) is determined by fitting the CMB $\mathscr{C}_l$, they do not proceed with the measurement of the actual density of the present day universe. A further problem is the fact that quantities entering Eqs. 75 and 76 are averaged over some volume with some characteristic scale. They are therefore expected to vary with this scale. Indeed, virulent criticisms have been raised up against the (strong) back reaction program, pointing out that the metric Eq. 70, with Friedmann–Lemaître equations have been fully successful to describe and to predict master pieces of observational cosmology; therefore, any alternative model should demonstrate its ability to reproduce these data as well with the same level of *concordance* (Ishibashi and Wald 2006).

Claims have been made recently for the detection of observable effects that could be attributed to the back reaction (Li and Schwarz 2008). Some models have been built (Mattsson 2010; Wiltshire et al. 2007) which were compared, successfully, to the main cosmological data (Leith et al. 2008; Larena et al. 2009), in agreement with the “low” Hubble constant found by Tammann et al. (2008). One can hope that this is the sign that the actual importance of back reaction will soon be clarified; although, Cosmology is a field where there is a tradition for opponents to the standard paradigm not to resign easily (Narlikar et al. 2008).

5.4 Large scale void(s)

After the discovery of an apparent acceleration from the SNIa Hubble diagram it has been emphasized that the Hubble diagram in its own was not a direct proof of an actual acceleration: even if the assumption that SNIa are standard candles is correct, an apparent acceleration could be obtained from the Hubble diagram within an inhomogeneous cosmological model (Célérier 2000) when interpreted with the presumption of an homogeneous world. The possible existence of large scale structure up to 200 Mpc suggests that this option was to be considered (Tomita 2001a). The existence of very large scale structure has been a subject of regular claims in cosmology (de Lapparent et al. 1986; Einasto et al. 1997). Such a possibility was regarded as a potential major failure of the standard view, and would even support the idea of a fractal distribution of matter on large scales (Fournier d'Albe 1907; Mandelbrot 1979; Grujic 2002). The availability of very large scale surveys like the SDSS and the 2DF has almost entirely closed this issue: although the visual impression from galaxy surveys might be that structures occasionally exist on very large scales, they are not the signature of average (r.m.s.) fluctuations on large scales greater than what could be anticipated for let say a standard ΛCDM spectrum, when measured with appropriate tools like the correlation function or the power spectrum. There is therefore no serious piece of data that suggests that the average level of fluctuations on large scales (as long as they remain smaller than the Hubble scale) is much larger than anticipated from the standard picture. The anomalous character of some rare high fluctuations is still possible (Vielva et al. 2004; Hunt and Sarkar 2010) and could for instance be due to some non-Gaussian features of the primordial fluctuations, but the tendency of the human brain (Peebles 1993) to identify insignificant patterns is a serious worry when dealing with these questions.

Given that observations tell us that the universe is isotropic around us, if some large scale inhomogeneity has to exist and be in agreement with present day observations it has to be nearly spherical and we should occupy a very specific place in this universe: for instance it may happen that we are close to the center of a nearly spherical perturbation. Such a possibility might have some theoretical motivation (Linde et al. 1995). If we actually live inside a gigantic void, close to its center (to preserve the observed isotropy of the sky), this requires the abandonment of the Einstein cosmological principle that postulates a homogeneous matter distribution on large scales, and of the Copernican principle, that we are not in a special location in the universe.

The homogeneity of the universe is directly observable in principle on scales much smaller than the Hubble radius, it is by no way obvious that it can actually be tested from observations on the largest scales (standard textbooks of relativity often mentioned that it cannot be actually tested, but this is assuming a restricted set of observable quantities). Detailed investigations of inhomogeneous models were carried out earlier, but not essentially with the purpose to provide an alternative explanation of the apparent acceleration, see for instance (Barrett and Clarkson 2000) for a description of various class of inhomogeneous cosmological solutions and references to earlier works on this subject.

The exact solution of the spherical inhomogeneity in general relativity was first published by Lemaître (1933). Tolman (1934) and Bondi (1947), aware of

Lemaître's work published studies on the same solutions. These solutions have been extensively used to examine closely whether the apparent acceleration could actually be due to such an inhomogeneous spherical solution (Tomita 2001a,b; Iguchi et al. 2002; Bolejko 2008; Chung and Romano 2006; Garfinkle 2006; Moffat 2006; Enqvist and Mattsson 2007). More complex inhomogeneous world solutions to Einstein equations have also been proposed to reproduce the observed SNIa Hubble diagram (Ishak et al. 2008). Interestingly enough, Biswas et al. (2007) found that some "onion" structure could leave no strong apparent effect. Qualitatively speaking, a void is a region with a typical size $\mathscr{L}$ where the density is lower than the average, in order for the initial singularity to happen at the same "initial time" in the past, the expansion rate in the region should be higher. This requirement is however not mandatory and it is possible to build isotropic inhomogeneous solutions with singularity starting at a different time (Célérier and Schneider 1998). Usually the "true" universe is an Einstein de Sitter $\Omega_M = 1$ and the parameters inside the void are such that it looks like a low density universe. So that inside the void:

$$\tilde{\Omega}_M \sim 0.25 \quad \text{and} \quad \tilde{H}_0 \sim 0.72 \tag{79}$$

while asymptotically (outside the void):

$$\Omega_M = 1 \quad \text{and} \quad H_0 \sim 0.50 \tag{80}$$

the value of the Hubble constant being set by the "initial time" constraint. Roughly speaking in such models the distant universe has a Hubble constant lower than the local one, which is interpreted as an acceleration. The minimal size of the void to produce an apparent acceleration from supernovae at $z \sim 0.5$ is typically of the order $300\,\mathrm{h}^{-1}$ Mpc, or slightly less, i.e., up to $z \sim 0.1$ (Alexander et al. 2007). However, this kind of consideration is not sufficient to offer a satisfying alternative: the Hubble flow is known to be quite smooth and in agreement with an uniform expansion. A significant change of the Hubble constant over a volume of size smaller than the horizon size will lead to an appreciable change of the apparent Hubble constant. Actually, such effect has been found (Jha and Riess 2007), who notice a Hubble constant 6.5% smaller on scales greater than $75\,h^{-1}$ Mpc, a small number that might be due to a problem in the calibration of close supernovae (Conley et al. 2007). No other significant departure from an uniform Hubble constant has been recently reported, actually the Hubble diagram appears to be remarkably regular from a few Mpc up to redshift where cosmological correction are needed (Madore and Freedman 1998; Kowalski et al. 2008; Tammann et al. 2008). Therefore, one can expect that voids witch size $\mathscr{L}$ is significantly smaller than the Hubble radius will not mimic adequately the present day Hubble diagram. Appreciable difference would be easy to check in the near future. However, if the voids are large, comparable to the size of the Hubble radius, it is intuitive that a good match to the supernova data could be achieved. Vanderveld et al. (2006) showed that to match closely the apparent acceleration of the Hubble diagram, a singularity at the origin should be present in the metric and other pathologies may exist. It is therefore far from being obvious that dust-filled LTB can reproduce the apparent acceleration in detail. Indeed, (Clifton et al. 2008) concluded, that if one keeps the constraint that the curvature has to remain smooth around the origin, the Hubble diagram from an LTB will be different from the one in ΛCDM and could be distinguished thereby with enough statistics at low redshit or intermediate redshift.

Convincing examples of successive models have been obtained by Garcia-Bellido and Haugbølle (2008a), who also provided a public code: `http://www.phys.au.dk/~haugboel/software.shtml`. For their best LTB model differences in the Hubble diagram between ΛCDM and LTB appear to be ¡0.05 m. Such differences are probably due to the choice of the analytical profile, and might therefore being reduced (or fitted to future data), even if this would be at the price of introducing further free parameters. The local Hubble constant is rather low ($H_0 \sim 60$ km/s/Mpc) but still acceptable. It would be interesting to examine whether this behavior is generic or specific to their model. However, the confidence in the ΛCDM relies on other pieces of evidence and the question of whether the CMB and the BAO feature can be reproduced has been also incorporated (Alnes et al. 2006; Alexander et al. 2007; Enqvist 2008; Garcia-Bellido and Haugbølle 2008a). Because the outer cosmology is an Einstein de Sitter universe with a small Hubble constant, an appropriate fit can be obtained relatively easily provided that the angular distance to the CMB is matched appropriately. These are rough evaluations, as a detailed formalism to deal with perturbations in a LTB model is lacking. However, this might soon become available (Clarkson 2007; Zibin 2008), and it is reasonable to believe that an adequate fit can be obtained. It is nevertheless fair to say that these models are not yet as impressively good to fit cosmological data as the concordance is, perhaps because of their intrinsic complexity (but on the other hand, they contain many more degrees of freedom). The situation looks like that this type of hypotheses (large void) could be adjusted to the data and could not therefore be rejected from observations if enough tuning is allowed. However, fortunately, this is not the case! Uzan (2008) have shown that in a LTB model, the time drift of the cosmological redshift can be different from what it is in homogeneous worlds, providing a possible test of the Copernican principle. The kinematic Sunyaev–Zel'dovich (kSZ) effect on distant clusters will be significantly different for distant clusters offering a different method of testing LTB (Garcia-Bellido and Haugboelle 2008b). Another impressive tool for constraining LTB models has been proposed by Caldwell and Stebbins (2008): we know that the intergalactic medium which probably contains most of the baryon is highly ionized up to redshifts ¿5; CMB photons are scattered by the electrons of this plasma, so photons we collect from the CMB are a mixture of photons which have traveled straight towards us and of photons which have been scattered at lower redshifts, if these electrons are within the void region, they are scattering a very different CMB because they are moving rapidly. In addition, even electrons which are out of the void will see a different CMB sky because the void itself and will be a further source of a distorted CMB. Therefore, contrary to the homogeneous case, an observer in the center will observe a combination of black-bodies with different temperatures, resulting in a distorted spectrum. Present day limits on possible spectral distortion of the CMB already provide interesting constraints and could become critically more stringent if limits could be improved by an order of magnitude (Caldwell and Stebbins 2008).

5.5 Quintessence

A true cosmological constant is an object that is largely unwanted from the theoretical point of view. Ratra and Peebles introduced the idea that the acceleration could be due to the presence of some scalar field dominating present day density of the universe (Ratra and Peebles 1988; Peebles and Ratra 1988). Quintessence names a scalar field that coupled to gravity. The canonical Lagrangian of a scalar field is:

$$\mathscr{L} = \frac{1}{2}(\partial_i \phi)^2 - V(\phi) \tag{81}$$

the first term being the kinetic energy ($X = \frac{1}{2}(\partial_i \phi)^2$), and the second one the potential. The stress-energy tensor has a form identical to that of an ideal fluid with pressure and density given by:

$$p = \mathscr{L} = \frac{1}{2}(\partial_i \phi)^2 - V(\phi) \tag{82}$$

$$\rho = 2\, X \partial_X \mathscr{L} - \mathscr{L} = \frac{1}{2}(\partial_i \phi)^2 + V(\phi) \tag{83}$$

For a field which is spatially homogeneous the equation of state parameter w is:

$$w = \frac{p}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \tag{84}$$

which remains ≥ -1. The equation driving the field in a FL world is:

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\mathrm{d}V}{\mathrm{d}\phi}(\phi) \tag{85}$$

The original scenario was proposed (Ratra and Peebles 1988) with a potential of the form:

$$V(\phi) = \frac{M^{4+\alpha}}{\phi^{\alpha}} \tag{86}$$

Let us suppose that we are at some late time when radiation or matter dominates. Then the expansion factor is $a(t) \propto t^n$ with $n = \frac{1}{2}, \frac{2}{3}$. A power law solution of Eq. 85 $\phi \propto t^{\beta}$ behaves as:

$$\phi \propto t^{\frac{2}{\alpha+2}} \tag{87}$$

and the ratio of the density of the scalar field to the total density is:

$$\frac{\rho_\phi}{\rho} \propto a^{\frac{4}{n(\alpha+2)}} \tag{88}$$

So that if $\alpha > 0$ the density of the scalar field will be dominant. Another important property is that solution 87 is an attractor in that it describes the late solution of a large class of initial conditions. If the kinetic term becomes small at a later epoch in 84, the field behaves like a cosmological constant. This does not necessary solve the coincidence problem, but at least alleviates it.

Dark energy has become the subject of intense theoretical efforts, which lead to investigate different forms of the Lagrangian. One first generalization consists in modifying the potential $V(\phi)$. A more radical approach is to modify the kinetic term. In $k-$essence models (Armendariz-Picon et al. 2000, 2001), inspired from $k-$inflation (Armendariz-Picon et al. 1999), the Lagrangian is written in a general form:

$$\mathscr{L} = p(\phi, X) \tag{89}$$

and contains therefore a non-canonical kinetic term.

Few constraints can be added. It can be requested that the density ρ remains positive, although there is no decisive argument against a negative cosmological constant and so this constraint might not be necessary. On the other hand, such a term would lead to a rapid big crunch, which is unwanted. Another possible constraint is that the square of the sound's speed remains positive (Armendariz-Picon et al. 1999):

$$c_s^2 = \frac{\partial_X p}{\partial_X \rho} = \frac{\mathscr{L}_{,X}}{\mathscr{L}_{,X} + 2X\mathscr{L}_{,XX}} \tag{90}$$

($,X$ stands for $\dfrac{\partial}{\partial X}$ and $,XX$ $\dfrac{\partial^2}{\partial X^2}$).

Such models easily lead to large variations of c_s (while quintessence based on (81) automatically leads to $c_s = 1$) and one can have $c_s > 1$, this is not problematic because the theory remains Lorentz invariant, so there is no violation of causality (Erickson et al. 2002; Babichev et al. 2008).[11] Within such models the equation of state parameter w could take any value (Melchiorri et al. 2003) and so one can have:

$$w < -1. \tag{91}$$

Such dark energy is referred to as phantom (ghost) energy. A simple example is obtained by changing the sign of the kinetic energy term in 81.

So from the astrophysical point of view w can be regarded as a free function which values should be determined from observations. From the fundamental point of view $V(\phi)$ or the Lagrangian itself are the quantities to be determined and therefore observations have to be used to constrain directly these quantities, rather than the parameter $w(z)$. Finally, let us mention that more details on scalar field models of dark energy can be found in the recent review (Copeland et al. 2006).

5.6 New gravity law on large scales

The existence of a large scale acceleration in the universe is a serious indication that our present knowledge of gravity is actually incomplete. Although the introduction of a new component, quintessence, might provide the explanation for the

[11] For a different point of view see Bonvin et al. (2006); Ellis et al. (2007).

acceleration, i.e., a new term in the energy momentum tensor, the historical cosmological constant introduced a new term in the other side of Einstein equation. Therefore, it might well be that the geometrical term is more complex that we used to believe. This option of modifying gravity to account for the acceleration is briefly introduced in this section. More details can be found in recent reviews on this topic (Durrer and Maartens 2008; Nojiri and Odintsov 2007).

While general relativity can be derived from the Einstein–Hilbert action:

$$\mathscr{S} = \int \mathrm{d}x^4 R \tag{92}$$

where R is the Ricci scalar, one can try a more general action, leading to the so called $f(R)$ theories:

$$\mathscr{S} = \int \mathrm{d}x^4 f(R) \tag{93}$$

where f is an unknown function. Such theories have to pass solar system tests as well as pulsar chronometry tests on small scales and cosmological tests on large scales. In practice, such theories met considerable problems, and it is far from being clear that viable models can be built in this way. An alternative procedure, is the so called Palatini formalism, in which the variation principle is modified: the metric and the connection are regarded as independent quantities, and the resulting equations are in general different but for $f(R) \propto R$. Within this formalism, Amarzguioui et al. (2006) have examined constraints on a specific model:

$$f(R) = R\left(1 + \alpha\left(\frac{-R}{H_0^2}\right)^{\beta-1}\right) \tag{94}$$

where the second term is essentially a correction term to the classical Einstein–Hilbert Lagrangian. Acceptable regions of the parameter space encompass the standard ΛCDM model, but these models differ from standard cosmological models, as the relations between Ω_k, Ω_m, q_0 and the growth rate $\frac{\mathrm{d}\ln D}{\mathrm{d}\ln a}$ are different.

A more radical way to modify gravity in our world is through the idea of higher dimensional space. Such propositions can be traced back to 1919 with the work of Kaluza (1921) who proposed an unification scheme for gravity and electromagnetism within a fifth dimensional space. Klein (1926) pointed out the interest of having a fifth compact dimension to avoid observational constraints on large additional dimension. Superstring theory and supergravity theories possess remarkable properties in higher dimensional space and have therefore deserved strong attention from theorists. Modern versions are known as braneworlds or brane cosmology (Arkani-Hamed et al. 1998; Binétruy et al. 2000a). Our $3+1$ world lies on the brane, while the remaining space is the bulk. Matter, with pressure and density, is present only in the brane, but gravity is present in all dimensions. The vacuum energy in the brane provides a tension term σ and there is an other vacuum energy in the bulk Λ_{B}. The general action now contains terms involving an equivalent of the Ricci scalar corresponding to gravity in higher dimension. The inferred equations describing the expansion on the brane, the generalized FL equations, contain more new terms which could be non-linear and which are related to the properties of the bulk (Binétruy et al. 2000b):

$$H^2 \propto \cdots \rho^2 + \cdots \Lambda_B + \frac{\mathscr{C}}{a(t)^4} - \frac{k}{a(t)} \tag{95}$$

the third additional term behaves like radiation but comes from the bulk and is sometime called the dark radiation on which BBN provides stringent constraints (Binétruy et al. 2000a,b; Flanagan et al. 2000; Iocco et al. 2009).

Direct modification of the FL equations have also been proposed through Cardassian models (Freese and Lewis 2002):

$$H^2 = A\rho + B\rho^n \tag{96}$$

which does not contain a vacuum contribution, although in models with $n = 0$ the second term of the right-hand side behaves identically to a Λ term. Such a form for the EFL equation can arise from some particular fluid properties of dark matter or from higher dimensions (Gondolo and Freese, 2003).

In DGP models, extra dimensions are infinite (Dvali et al. 2000), and the effective action contains explicitly a 4-D Einstein–Hilbert term, i.e., on the brane in addition to the 5-D term on the bulk. This introduces a scale r_c and two distinct regimes appear: on scales smaller than r_c gravity essentially results from the 4-D term and classical GR is recovered (although high precision tests might lead to some differences (Lue and Starkman 2003; Battat et al. 2008); on larger scales the gravity is "leaking" the expansion is eventually accelerated. The FL equation is replaced by (Deffayet 2001):

$$H^2 = \left(\sqrt{\frac{\rho}{3M_{Pl}^2} + \frac{1}{4r_c^2}} + \varepsilon \frac{1}{2r_c} \right)^2 \tag{97}$$

with $\varepsilon = \pm 1$. From this it is clear that at early times $\frac{\rho}{3M_{Pl}^2} \gg \frac{1}{4r_c^2}$ allows to recover the classical EFL regime, while at late times the accelerated expansion is recovered (provided that $\varepsilon = +1$). Detailed predictions of DGP models may not be identical to those of ΛCDM, and recent investigations found some tension between data and theory (Rydbeck et al. 2007; Rubin et al. 2009; Fang et al. 2008), although the predictions are not as straightforward as in the standard model. From the astrophysical point of view, an interesting aspect of these classes of models in which gravity is modified is that relations between cosmological parameters are not identical to those in the ΛCDM and the evolution of the growth factor is expected to be different (Linder 2005), with a possible dependence on scales, allowing for possible discrimination between various possible origins for the acceleration (Uzan 2007; Bertschinger and Zukin 2008).

A more radical option has emerged in recent years: given the fact that the accelerated expansion clearly needs some revision of our knowledge of the gravitational sector, could it be that the problem of dark matter itself is due to the break of standard gravity laws at finite distance? This possibility has been introduced in an empiric way by Milgrom to explain the rotation curves of galaxies without invoking dark matter (Milgrom 1983). A fully relativistic theory has been built by

Table 1 Summary of the mean values and 68% confidence intervals for the parameters constrained from CMB, SNIa and BAO for different models (θ is the ratio of sound horizon to angular diameter distance)

Parameter	Vanilla	Vanilla + Ω_k	Vanilla + w	Vanilla + $\Omega_k + w$
$\Omega_b h^2$	0.0227 ± 0.0005	0.0227 ± 0.0006	0.0228 ± 0.0006	0.0227 ± 0.0005
$\Omega_c h^2$	0.112 ± 0.003	0.109 ± 0.005	0.109 ± 0.005	0.109 ± 0.005
θ	1.042 ± 0.003	1.042 ± 0.003	1.042 ± 0.003	1.042 ± 0.003
τ	0.085 ± 0.017	0.088 ± 0.017	0.087 ± 0.017	0.088 ± 0.017
n_s	0.963 ± 0.012	0.964 ± 0.013	0.967 ± 0.014	0.964 ± 0.014
Ω_k	0	-0.005 ± 0.007	0	-0.005 ± 0.0121
w	-1	-1	-0.965 ± 0.056	-1.003 ± 0.102
Ω_λ	0.738 ± 0.015	0.735 ± 0.016	0.739 ± 0.014	0.733 ± 0.020
Age	13.7 ± 0.1	13.9 ± 0.4	13.7 ± 0.1	13.9 ± 0.6
Ω_M	0.262 ± 0.015	0.270 ± 0.019	0.261 ± 0.020	0.272 ± 0.029
σ_8	0.806 ± 0.023	0.791 ± 0.030	0.816 ± 0.014	0.788 ± 0.042
z_{re}	10.9 ± 1.4	11.0 ± 1.5	11.0 ± 1.5	11.0 ± 1.4
h	0.716 ± 0.014	0.699 ± 0.028	0.713 ± 0.015	0.698 ± 0.037

These constraints are quite tight, most of them are below 5%, and are stable when additional degrees of freedom are added to the model (w,Ω_k), adapted from Ferramacho et al. (2009)

Bekenstein (2004) which behaves like Milgrom' law in the weak field regime: the TeVeS (tensor-vector-theory) theory. Quite remarkably, it has been shown recently that this theory can reproduce the observed cosmic acceleration, large scale power spectrum (at the time of the work) and CMB with comparable success as ΛCDM (Skordis et al. 2006). The present status of this theory is not as satisfactory as with the standard concordance model: this theory did not lead to specific predictions which have been verified a posteriori, and it needs the introduction of a new type of fields in physics in the form of vector fields. However, MOND is a clear example of an alternative view, which differs drastically in its fundamental ingredients and illustrates the fact that our understanding of the gravitation sector relevant to cosmology might be more limited than commonly assumed.

6 The area of precision cosmology

After the discovery of the fluctuations of the cosmological background by COBE the perspective to achieve precision measurements of the angular power spectrum of these fluctuations appears to be within reach and two satellite experiments were designed to reach this target: WMAP and Planck. This has opened a new avenue for Cosmology to benefit from high precision measurements with well-controlled systematics. Indeed, WMAP delivered data of high accuracy, allowing for high precision estimations of the cosmological parameters (Spergel et al. 2003; Dunkley et al. 2009). Furthermore, the consistency of SNIa, BAO, and CMB data allows reliable accurate estimations by combining constraints (Komatsu et al. 2009; Kowalski et al. 2008). The possibility of reaching high precision measurements with different techniques (distant SNIa with SNAP, properties of distant X-ray clusters as proposed by Panoramix some years ago or the recent WFXT (2008), or with the SZ sample of clusters expected with Planck and other CMB experiment like SPT, full sky weak lensing surveys with DUNE, or the more recently proposed redshift surveys of hundred millions of galaxies with SPACE or ADEPT) has not only reinforced the perspective to determine cosmological parameters with high precision, but will also allow to investigate the very nature of dark energy. The Dark Energy Task Force report (Albrecht et al. 2006) presented a summary of the different levels of progress expected for various projects. Ultimate constraints will be obtained by combination of various experiments. It is interesting to summarize what present day data allow. There are many possible data to combine, but the three most currently used are SNIa, CMB and measure of the large scale distribution, either through the power spectrum or through the correlation function. A combination of these three data sets leads to tight constraints on the minimal ΛCDM model. These constraints are summarized in Table 1 adapted from Ferramacho et al. (2009). Different groups recently got similar results with approaches which slightly differ in the technical details (Komatsu et al. 2009; Kowalski et al. 2008; Sánchez et al. 2009), so these numbers can be regarded as very robust. As one can see, most of the cosmological parameters describing our Friedmann–Lemaître universe are constrained to an accuracy better than 5%. Furthermore, when the parameter space is enlarged the constraints remain essentially unchanged. This calls for some caution. Quoted uncertainties reflects statistical uncertainties. Unidentified systematics are the critical issues in this topic. Variations in published values of σ_8 from various approaches (CMB, Clusters, weak lensing) have provided an illustration that systematics uncertainties could alter preferred values beyond statistical uncertainties. On the other hand, given possible systematics which have been identified until now, it is likely that future estimations will remain within the two sigma domains. First, when combining only two probes, one already gets tight constraints which are within this range. A second argument is that when one allows for additional parameters (free equation of state parameters w, curvature, non-zero neutrino mass, tensor contribution, ...) preferred values and interval ranges are not changed by much. These are indications that we are already in the precision area of cosmology: present day estimations of cosmological parameters are likely not to change by much in the near future and investigations of the nature of dark energy will need extremely accurate control of systematics. Whether the necessary investments will be valuable for the astronomical field is a subject of debate (White 2007).

7 Conclusions

The Copernician model of the world was the first revolution of a series in the construction of modern cosmology, and the discovery of the accelerated expansion being the latest in date. Theoretical considerations have always been a source of remarkable observational investigation and Cosmology has always benefited from the confrontation of models with observations. Since the 30s, the big bang picture, the modern version of Lemaître's primeval atom has been remarkably successful, based on simple assumptions and physics laws that have been validated by accurate experimental results. Although alternative theories have been developed, these alternative were based on hypothetical unknown physics advocated to interpret cosmological observations. None of these alternative theories has produced predictions that have been comforted a posteriori. Rather new observations in agreement with predictions of the big bang picture necessitated deep revision of the unorthodox view, at the cost of rather ad hoc assumptions added to fit the new observations. The situation has evolved when the standard picture has necessitated the introduction of new ingredients, first dark matter and more recently dark energy. The very nature of these new ingredients, which are supposed to dominate the mean density of the universe has not been established by direct laboratory experiments, nor by astronomical observations, and this situation may some time lead to the question whether cosmologists have not introduced new aethers. We had the opportunity to see that the situation is not so. The introduction of—cold—non-baryonic dark matter has led to specific predictions, the amplitude of the fluctuations of the cosmological background on various angular scales, which were verified with high accuracy precision. The presence of dark energy has lead to a specific prediction, the shape of the power spectrum on large scales, which has been verified a posteriori. Although, the inclusion of a cosmological constant was concomitant to general relativity, the actual origin of dark energy remains totally unknown and the presence of dark energy in the present day universe represents probably the most fundamental and unexpected new element in modern physics.

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