RESONANCES – Experimental 2 $(B = 1, S = 0)$

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NUCLEON RESONANCES

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On the basis of the pion-nucleon phase-shift analyses of groups at Berkeley[1], CERN[2] and Saclay[3], it was possible a year ago to conclude[4,5] that there was evidence for the existence of 19 or possibly 20 resonance states of mass less than 2.2 GeV in the pion-nucleon system. The general procedure used to infer the existence of resonance states is to study the Argand diagram of the function $2qf_\ell$, where f_ℓ is the usual partial wave amplitude

$$f_\ell = \frac{1}{2iq}\left[\eta_\ell\, \ell^{2i\delta_\ell} - 1\right],$$

q being the c.m. momentum. If a resonance exists, then this function will describe a counter-clockwise circle, or an appreciable part thereof, in the Argand diagram. This is easily seen by considering the amplitude to be represented by a Breit-Wigner resonance, or as a Breit-Wigner resonance plus background. Some typical situations are shown in Fig. 1. When there is background present, the resonance circle may be distorted considerably and displaced appreciably from the symmetric position, but the amplitude still retains the general resonance features. The same situation occurs in the case of a partial-wave amplitude in an inelastic process, say, $\pi + N \rightarrow K + \Lambda$. If there is a resonance present, then the amplitude will describe an appreciable part of a counter-clockwise circle, more or less distorted according to the background. One difference to be noted in the case of an inelastic amplitude is that the imaginary part of the amplitude is no longer restricted to be positive, as it is in the elastic case, and in general there is an arbitrary phase factor involved.

This general feature of a resonance implying a counter-clockwise circle has led to the inference that a counter-clockwise circle automatically implies a resonance, and this is the criterion which has been used in the phase-shift analyses. It was known, however, that this reversal is not necessarily true; for example it is not difficult to show in multi-channel potential scattering that it is possible to produce

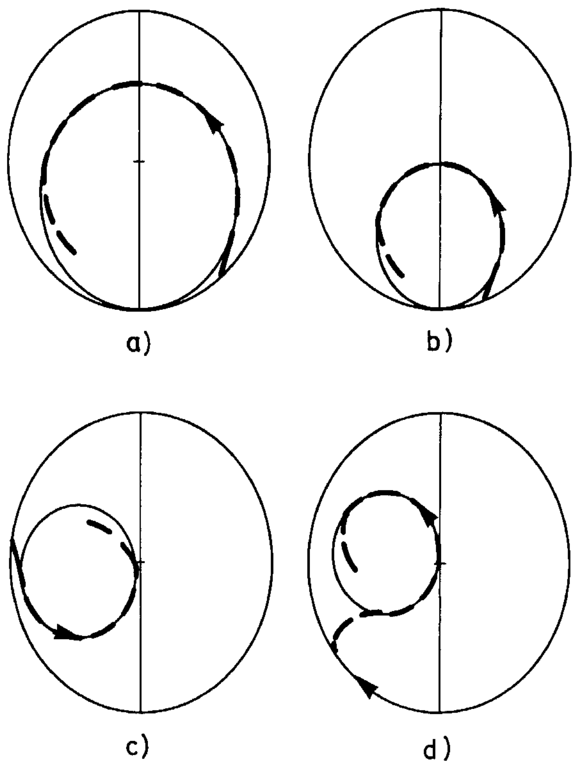

Fig. 1 Typical resonance configurations
a) pure Breit-Wigner, $\Gamma_{el} > 1/2\ \Gamma_{tot}$
b) pure Breit-Wigner, $\Gamma_{el} < 1/2\ \Gamma_{tot}$
c) Breit-Wigner with attractive background
d) Breit-Wigner with repulsive background.

resonance-type circles in any given channel without requiring that there be a resonance. In this context, it is important to recall that "resonance" is synonymous with "a pair of second sheet poles". This point has been given considerable prominence recently, following Schmid's[6] comment that the partial wave projections of Regge-pole amplitudes freely exhibit resonance-type circles, and give at least a qualitative representation of the structure observed in the phase-shift analyses, but explicitly do not have any second sheet poles. There is now a variety of opinion on this topic[7-9], but its significance, or lack of it, is still unclear. However, one point is ambiguous. The Regge-pole amplitude is a priori a smooth function of energy and cannot give rise to any peaks in total cross-sections, the various partial-wave peaks combining in just such a way as to produce this smooth result. Now both the T = 3/2 and T = 1/2 total cross-sections show considerable structure. In the former, peaks are associated with $P_{33}(1236)$, $F_{37}(1950)$ and the $S_{31}(1640)$, $P_{33}(1690)$, $D_{33}(1690)$ complex which appears as a shoulder on the lower side of the F_{37} peak. In the T = 1/2 case, peaks are associated with $D_{13}(1520)$, $P_{11}(1470)$ which appears as a shoulder on the lower side of the D_{13} peak, the $D_{15}(1680)$, $F_{15}(1690)$ pair and $G_{17}(2190)$. It is very difficult not to associate a resonance with the observed structure in these partial waves. Further, it is my conviction that all the structure observed in pion-nucleon scattering should be associated with resonances, and it is from this viewpoint that this report is written.

1. $\pi + N \to \pi + N$

The results of two new phase-shift analyses, one from Glasgow[10] and one from CERN[11] confirm most of the structure claimed by CERN I*).

Phase-shift analysis is conventionally tackled in one of two ways. Either an energy-dependent analysis is performed parametrizing the partial-wave amplitudes as functions of energy and fitting to all the data simultaneously, or an energy independent analysis is performed, searching extensively at each energy for different solutions and then using continuity between adjacent energies to select among the solutions found. The former approach is a practical one if the number of partial waves is not too great, and if the energy range is sufficiently limited, and it was applied with considerable success in the early days of pion-nucleon phase-shift analysis when these conditions were reasonably well satisfied. In the latter case, imposing continuity is a very serious problem, and sophisticated techniques have been developed to handle it. In the CERN I analysis, a complicated iterative procedure was evolved, making use of partial-wave dispersion relations[12]. In the Berkeley analysis the "shortest-path" technique was developed and used successfully[13]. This is to find the smallest value of the quantity

$$X(k_1, k_2, \ldots) = \left\{ \sum_{i,\ell,j,T} \left(j+\tfrac{1}{2}\right) \left| f_i^{(k_i)}(\ell,j,T) - f_{(i-1)}^{(k_{i-1})}(\ell,j,T) \right|^2 \right\}^{\frac{1}{2}},$$

where $f_i^{k_i}(\ell,j,T)$ is the k^{th} solution at the i^{th} energy for the partial-wave amplitude with orbital angular momentum ℓ, total angular momentum j and isospin T. This technique can be considered as a "resonance eraser", naturally preferring not to go round resonance loops if they can be possibly avoided, i.e. it gives a lower bound on the possible structure.

In the CERN II analysis, this technique has been applied to π^+p scattering, and the results are shown in Fig. 2. The π^+p shortest path is unique up to 1821 MeV (solid line) and qualitatively unique to ∿1950 MeV, the main ambiguity being in the position of the F_{37} resonance. There are still only five branches at 2025 MeV and the dashed continuation shown is the shortest of all. The dotted line in these figures is the dispersion relation fit of CERN I, and is clearly very similar to the shortest path, i.e. the purely experimental lower bound on the amount of structure, as given by the shortest path, is very close to the previous theoretically-favoured dispersion relation fit for π^+p. As in the CERN I analysis, the only structure which appears at all dubious is the D_{35} (∿1950). However, it should be noted that the existence of the $P_{33}(1688)$ structure depends on the experiments at a single energy. It is present if only good solutions are accepted at 1688 MeV (double dot-dashed line), but is absent if the shortest path is allowed to pass through a solution with $P(\chi^2) < 0.001$ (solid line). More π^+p experiments in this region would be valuable. In some solutions there is some evidence for a possible P_{33}(∿2030), but it is not conclusive.

*) To avoid confusion, we shall denote the 1967 CERN analysis[2] by CERN I and the 1968 CERN analysis[11] by CERN II.

The method of search used in the Glasgow analysis was a hybrid one, in which the energy range was broken down into intervals of about 100 MeV and a quadratic energy dependence assumed for the phases and elasticities in each range. The fits were overlapped from range to range, each successive solution, being tied to the last, or the second last point of the previous one. The energy ranges chosen are sufficiently small for the restriction to a quadratic approximation not to be a serious handicap. The amplitudes so obtained, which still contain some awkward corners, were then smoothed further by fitting energy dependent forms to them, the choice being a multichannel Breit-Wigner resonance[14] superimposed on a smooth background. Two results were obtained by this procedure, which are qualitatively very similar not only to each other but also to CERN I. The main differences are that the Glasgow results do not have the D_{35}(∿1950), D_{13}(∿1700), D_{13}(∿2030) and F_{17}(∿1980). In fact they change the parity of this last one, and make it G_{17}(1906). CERN I also found some structure in G_{17} at about this mass, but put it down to bad charge-exchange data. This appears to be the most probable explanation and this ambiguous F_{17}/G_{17} structure should not be considered seriously for the present. The evidence of structure obtained in the various phase-shift analyses is summarized in Table 1. From this, it is clear that D_{35}(1950), D_{13}(1730), F_{17}(1900) and D_{13}(2030) are in a precarious state, at least as far as the phase-shift analyses are concerned. Of these four, there is no real evidence elsewhere for D_{35}(1950) and F_{17}(1980), and consequently these two should be rejected, at least for the present. There is some evidence elsewhere for both D_{13}(1730) and D_{13}(2030), particularly the former, so we shall retain both.

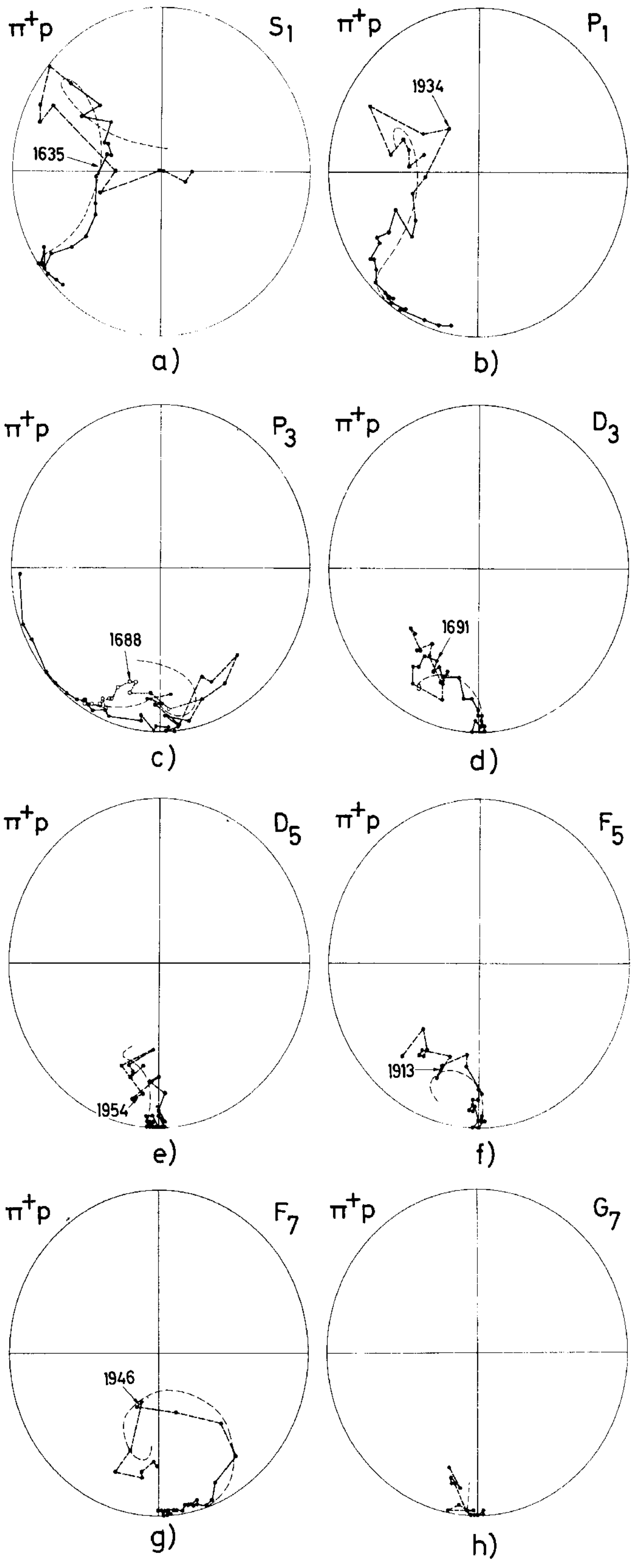

Fig. 2 T = 3/2 pion-nucleon amplitudes. Comparison of CERN I[2] dispersion fit (smooth dot-dash line) with CERN II[11] shortest path.

The proposed structure for this year is shown in Table 2, with the masses, widths and elasticities taken from CERN I and Glasgow (A) and (B) solutions. The variation in the resonance parameters quoted gives some indication of the uncertainties inherent in extracting these parameters from Argand diagrams. This point seems little known, and it is worth stressing. There are three basic causes of these variations. Firstly, different methods of analysis produce somewhat different sets of phase shifts. Secondly, the amplitudes obtained are not smooth with energy, and some smoothing procedure must be applied before extracting the resonance parameters. There is no "correct" way of doing this, and different choices of smoothing functions will produce somewhat different smoothed results, even starting with the same set of experimental amplitudes. Thirdly, having obtained a smooth plot it is necessary to specify some criterion to separate resonance from background, and different criteria produce, once again, somewhat different results. It is notable that the Argand diagrams of the three solutions quoted differ by much less than would be expected from the differences of their resonance parameters.

All the phase-shift analyses reported above support, to a greater or lesser extent, the structure proposed by CERN I. However, there is the possibility that a qualitatively different solution has been found at Berkeley[15] which contains very little of this structure. This result is still highly provisional and there are some serious objections to it, particularly with respect to the renormalization of some of the experiments (which is allowed in the fit), which in some cases appears to be excessive and with respect to the structure in the solution, which in some amplitudes is extremely rapid, being of the

TABLE 1

Conjectured pion-nucleon resonance assignments below 2.2 GeV mass, with the status of the corresponding structure observed in the five most recent phase-shift analyses.

Possible resonances	Berkeley[1]	CERN I[2]	Saclay[3]	Glasgow[10]	CERN II[11]
$P_{33}(1236)$		We will not argue about this one			
$S_{31}(1640)$	Definite	Definite	Definite	Definite	Definite
$D_{33}(1690)$	Possible	Possible	Ambiguous	Definite	Definite
$P_{33}(1690)$	Probable	Probable	Ambiguous	Possible	Definite
$F_{35}(1910)$	Probable	Probable	Ambiguous	Definite	Definite
$P_{31}(1930)$	Probable	Probable	Ambiguous	Definite	Definite
$F_{37}(1950)$	Definite	Definite	Definite	Definite	Definite
$D_{35}(1950)$	Doubtful	Doubtful	Ambiguous	No	Possible
$P_{11}(1470)$	Definite	Definite	Definite	Definite	-
$D_{13}(1520)$	Definite	Definite	Definite	Definite	-
$S_{11}(1550)$	Definite	Definite	Definite	Definite	-
$D_{15}(1680)$	Definite	Definite	Definite	Definite	-
$F_{15}(1690)$	Definite	Definite	Definite	Definite	-
$S_{11}(1710)$	Definite	Definite	Definite	Definite	-
$D_{13}(\sim 1730)$	No	Use imagination	No	No	-
$P_{11}(1750)$	No	Possible	No	Definite	-
$P_{13}(1860)$	No	Possible	No	Definite	-
$F_{17}(1980)$	No	Doubtful	No	Transferred to G_{17}	-
$D_{13}(\sim 2030)$	No	Probable	No	No	-
$G_{17}(2190)$	Ambiguous	Definite	-	-	-

"hairpin" variety. It has to be studied in much more detail before it can be entertained as a serious alternative.

Experimentally there has been little to offer in elastic pion-nucleon scattering, the only results presented at the Conference being 12 different $\pi^- p$ cross-sections in the mass range 1500 to 1770 MeV.

There are still some serious experimental shortages in the mass range being considered, i.e. below 2.2 GeV. $\pi^+ p$ and $\pi^- p$ differential cross-sections are adequate almost everywhere, except in the 1690 MeV region as noted above, and so is $\pi^- p$ polarization. $\pi^+ p$ polarization data are rather thinly spread and it would be desirable to have more. The information

TABLE 2

Resonances observed in pion-nucleon scattering with a mass of less than 2.2 GeV. The masses, widths and elasticities conjectured in the CERN I[2]) analysis and the two results of the Glasgow[10]) analysis are shown, together with the "average". In forming this "average", the two Glasgow results were first combined together, and then taken with the CERN I analysis. The differences in the resonance parameters give some guide to the uncertainty in extracting these numbers from Argand diagrams.

Partial wave	CERN I			Glasgow (A)			Glasgow (B)			Composite		
	Mass	Γ_{tot}	Γ_{el}/Γ_{tot}	Mass	Γ_{tot}	Γ_{el}/Γ_{tot}	Mass	Γ_{tot}	Γ_{el}/Γ_{tot}	Mass	Γ_{tot}	Γ_{el}/Γ_{tot}
P_{33}	1236	125	1.00	1238	120	1.00	1238	120	1.00	1237	122.5	1.00
S_{31}	1640	177	0.28	1617	141	0.28	1623	140	0.25	1630	160	0.27
D_{33}	1690	269	0.14	1649	188	0.12	1650	174	0.13	1670	225	0.13
P_{33}	1690	281	0.10	-	-	-	-	-	-	1690	280	0.10
F_{35}	1910	350	0.16	1841	136	0.20	1852	150	0.19	1880	250	0.18
P_{31}	1930	339	0.30	1914	290	0.18	1843	231	0.24	1905	300	0.25
F_{37}	1950	221	0.39	1935	196	0.51	1935	212	0.39	1940	210	0.42
P_{11}	1470	211	0.66	1462	391	0.49	1436	224	0.46	1460	260	0.57
D_{13}	1520	114	0.57	1512	106	0.45	1512	125	0.49	1515	115	0.52
S_{11}	1550	116	0.33	1502	36	0.36	1499	53	0.35	1525	80	0.34
D_{15}	1680	173	0.39	1669	115	0.50	1667	115	0.43	1675	145	0.43
F_{15}	1690	132	0.68	1685	104	0.54	1684	123	0.54	1690	125	0.61
S_{11}	1710	300	0.79	1766	404	0.56	1671	121	0.51	1715	280	0.66
D_{13}	1730?	?	?	-	-	-	-	-	-	1730?	?	?
P_{11}	1750	327	0.32	1770	445	0.43	1867	525	0.30	1785	405	0.34
P_{13}	1860	296	0.21	1844	449	0.40	1854	307	0.26	1855	335	0.27
D_{13}	2030?	290	0.26	-	-	-	-	-	-	2030?	290?	0.26?
G_{17}	2190	300	0.35	-	-	-	-	-	-	2190	300	0.35

available on $\pi^- p$ charge exchange is poor, and very scarce above 1900 MeV, and the information on $\pi^- p$ charge-exchange polarization is nil. This latter process will be the most important one in the immediate future, since it is a very sensitive function of the partial-wave amplitudes. The most important test would be provided by the Wolfenstein parameters R and A, which explicitly contain extra information, although presumably this will remain a theoretician's dream for some years to come. Finally, it would be nice to have two measurements of total cross-sections which agree with each other.

2. $\pi + N \to \eta + N$

The important role which inelastic channels can play in helping to elucidate the pion-nucleon resonance structure is clearly evinced by the analyses of the reaction $\pi^- + p \to \eta + n$ which gave the first tangible proof that the structure observed in the S_{11} pion-nucleon amplitude in the vicinity of the η production threshold is indeed due to a resonance, and is not merely a threshold effect. There have been several such analyses, and they are typified by that of Davies and Moorhouse[16], where earlier references may be found. The experimental angular distributions show considerable anisotropy not far above threshold and the nearness of the $D_{13}(1515)$ resonance posed the question of whether the η production proceeds mainly through this resonance, the large S-wave being a scattering length effect only, or whether an S-wave resonance really exists. Davies and Moorhouse, using a rather general multichannel effective range formalism, showed that the latter situation was indeed the case, demonstrating explicitly that their solution has second sheet poles. They also found that the S-wave term is so large that the anisotropy can easily be explained by this interfering with a very small D-wave, the $D_{13}(1515)$ having a partial decay width to ηN of less than 1 MeV.

An interesting energy-dependent analysis of all η-production data below 1900 MeV has been carried out by Deans, Holladay and Rush[17]. They assume the existence of all CERN I T = 1/2 resonances and represent the background by the direct and crossed nucleon pole terms. The resonance and pole couplings are varied to fit the data, but no attempt is made to vary the resonance positions or widths, the level of data not justifying it. Their results are in accord with those of Davies and Moorhouse, with a strong $S_{11}(1525)$ and a small, but non-zero contribution from $D_{13}(1515)$, which cannot seriously be separated from background. They also find non-negligible contributions from $S_{11}(1785)$, which of course, do not contribute close to threshold. All other resonance couplings are consistent with zero.

Some measurements of the reaction $\pi^+ + n \to \eta + p$ are now available[18] near threshold. The variation in cross-section and the production angular distributions are quite in accord with the charge symmetric reaction $\pi^- + p \to \eta + n$.

3. $\pi^- + p \to K^0 + \Lambda^0$

Deans, Holladay and Rush[17] have also analysed this reaction in terms of the CERN I T = 1/2 resonances. In this case the background was represented by the direct channel nucleon pole, the crossed channel Σ pole and a t-channel K^* pole, with vector and tensor coupling. Their analysis takes in all data below 1850 MeV mass, and shows clearly the dominance of the $S_{11}(1710)$ and the D_{11} partial wave, this being shared out between $P_{11}(1460)$ and $P_{11}(1785)$. There are also possible contributions from $P_{13}(1855)$ and the D_{13} partial wave.

Lovelace, Wagner and Iliopoulos[19] have also looked at this process, applying the random search plus shortest path technique to all data below 2025 MeV. Some use was made of the pion-nucleon phase-shift analysis, but only in two respects. Firstly, it was used to determine which partial waves were elastic and could be neglected in this analysis. This turned out to be G_{19} and the H-waves up to 2025 MeV, and F_{17} and G_{17} below 1800 MeV. Because of the waves were retained in the analysis at 1617 MeV, and only S-, P- and D-waves below 1640 MeV. Secondly, the phase-shift analysis was used to find the phase of phase-shift analysis was used to find the phase of those waves containing a Breit-Wigner resonance, i.e. D_{15}, F_{15}. As was mentioned in the Introduction, all partial waves in an inelastic channel can be multiplied by an arbitrary phase factor. However, for a pure Breit-Wigner resonance, the amplitude in any inelastic channel is a real multiple of the amplitude and, since both $D_{15}(1675)$ and $F_{15}(1685)$ have a very regular Breit-Wigner shape in the elastic analysis, it was assumed that the $K\Lambda$ amplitudes for these two waves would be real multiples of the πp up to 1800 MeV. The same assumption was made for G_{17} at higher energies. The real proportionality factors involved were varied in the fit independently at each energy. This is quite different from the analysis of Deans et al.[17], where for each Breit-Wigner the same constant of proportionality was used at all energies. In the case of Lovelace et al.[19], the effect is to reduce the phase ambiguity to a sign ambiguity only. This sign ambiguity was not resolved, and a complete change of sign is possible in one of their solutions. The same is true of the results of Deans et al.[17].

Lovelace et al.[19] find four acceptable solutions, which are shown in Fig. 3. Clearly, the main effect is dominance by S_{11} and P_{11} resonances. There is also some evidence for a smaller resonant contribution from P_{13} or D_{13}, but this is much less clear-cut. In all solutions, the contributions from the D_{15} and F_{15} resonances is very small.

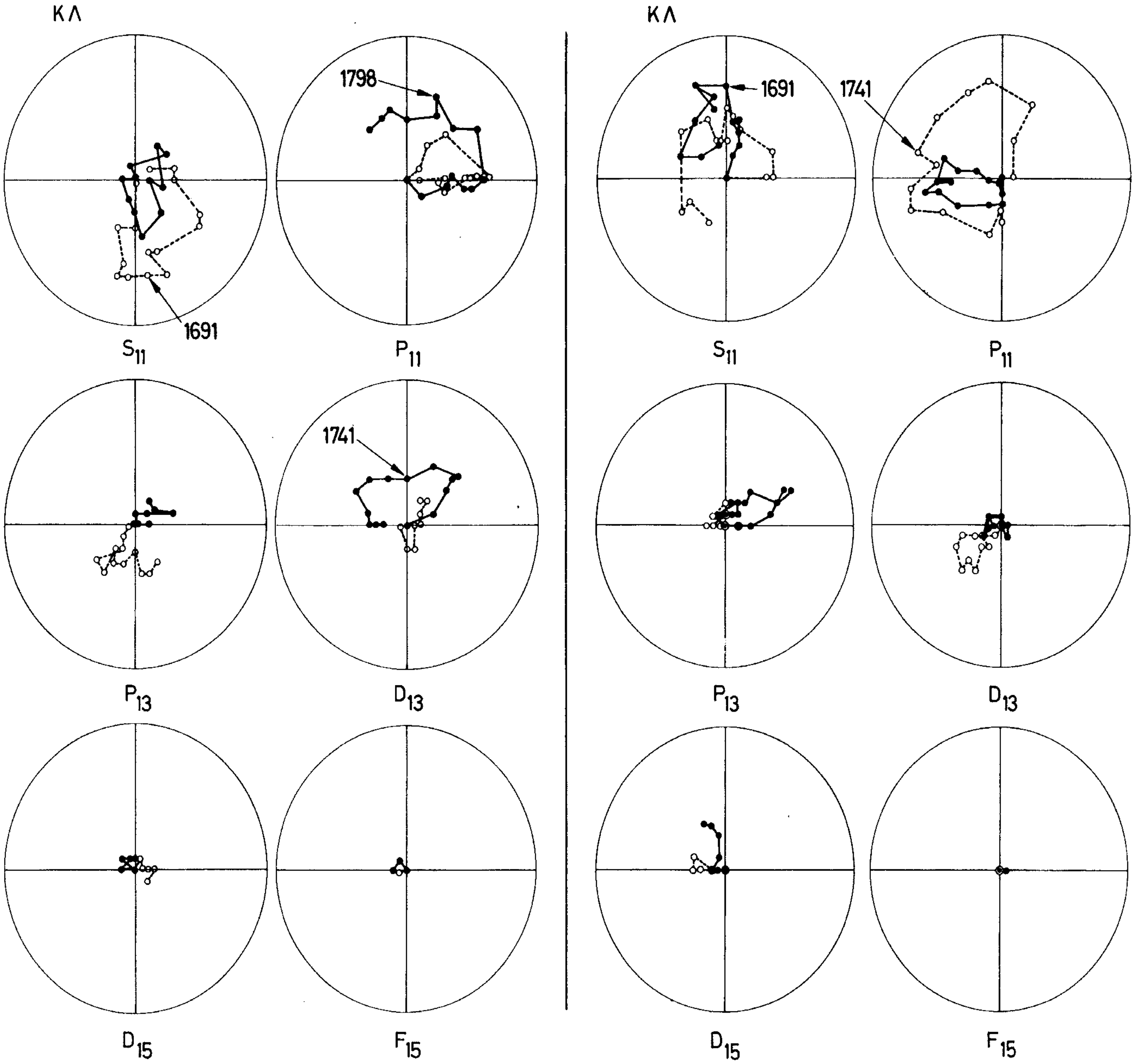

Fig. 3 The four best results of the CERN $K\Lambda$ analysis. The radius of the circle is half the radius of the unitary circle.

The S_{11} resonance should clearly be identified with the S_{11}(1715), and the P_{11} with the P_{11}(1785). It is also tempting to identify the D_{13}, which appears to have a mass of about 1750 MeV, with the D_{13}(1730), but there is no natural partner for the possible P_{13}, which in KΛ, if it exists, has a mass of about 1750 MeV, but in πp it has a mass of about 1860 MeV.

The results of Deans et al.[17] and Lovelace et al.[19] are qualitatively very similar, i.e. dominance by the S_{11} and P_{11} partial waves, with some contribution from D_{13}, but there is one apparent difference which should be noted. Deans et al.[17] associate the S_{11} amplitude more with S_{11}(1525) than S_{11}(1715) and the P_{11} amplitude more with P_{11}(1460) than with P_{11}(1785), although both the lower mass resonances are well below the KΛ threshold. This can be explained by the fact that, for the resonances, they used a standard Breit-Wigner form, which has a notoriously long tail -- much longer than observed resonances appear to have. The KΛ threshold is sufficiently far above the P_{11}(1460) and S_{11}(1525) for this tail to look just like part of the background, which is arbitrary, and to say that they make a large contribution is synonymous with saying that there is a lot of background in KΛ in the vicinity of the S_{11}(1715) and P_{11}(1785) resonances. It is obvious by looking at Fig. 3 that there is considerable background. If there were none, the S_{11} and P_{11} resonance circles would be symmetric about the vertical, and they clearly are not. This is yet another example of the resonance/background problem discussed in connection with elastic scattering, and illustrates again the necessity for displaying amplitudes as the result of phase-shift analysis, rather than resonance parameters.

4. $\pi + p \to K + \Sigma$

Lovelace et al.[19] have also studied the preconference data on $\pi^+ + p \to K^+ + \Sigma^+$ up to 1859 MeV, and concluded that the data were not good enough to prove anything.

A measurement of polarization effects in the reaction $\pi^- p \to \Sigma^- K^+$ at 1742 MeV has been made by Edginton et al.[20] who have also attempted an analysis of the three channels $\pi^+ + p \to \Sigma^+ + K^+$, $\pi^- + p \to \Sigma^- + K^+$, $\pi^- + p \to \Sigma^0 + K^0$ up to 1763 MeV. Like Lovelace et al.[19] they found that the data are not good enough to resolve any structure, and the simple parametrization

$$A^{(T)}_{\ell,j} = a^{(T)}_{\ell,j}\, e^{i\psi^{(T)}_{\ell,j}}\, q_\Sigma^{(\ell+\frac{1}{2})}$$

for the partial wave amplitudes is adequate to fit the data.

New data are also available[21] on $\pi^+ + p \to \Sigma^+ + K^+$ at 1856, 1901 and 2016 MeV, and a preliminary analysis has been made at these energies. The angular distributions show no evidence for partial wave higher than F. Since there is a known $\pi^+ p$ resonance, the F_{37}, in this region and the total cross-section (shown in Fig. 4), peaks just in this region, a natural inference is that this will be an important amplitude. With this assumption, a study of the Legendre polynomial coefficients leads Borreani and Kalmus to try the combinations

a) S_{31}, D_{33}, F_{37}
b) S_{31}, P_{31}, D_{35}, F_{37}
c) S_{31}, P_{33}, D_{35}, F_{37}

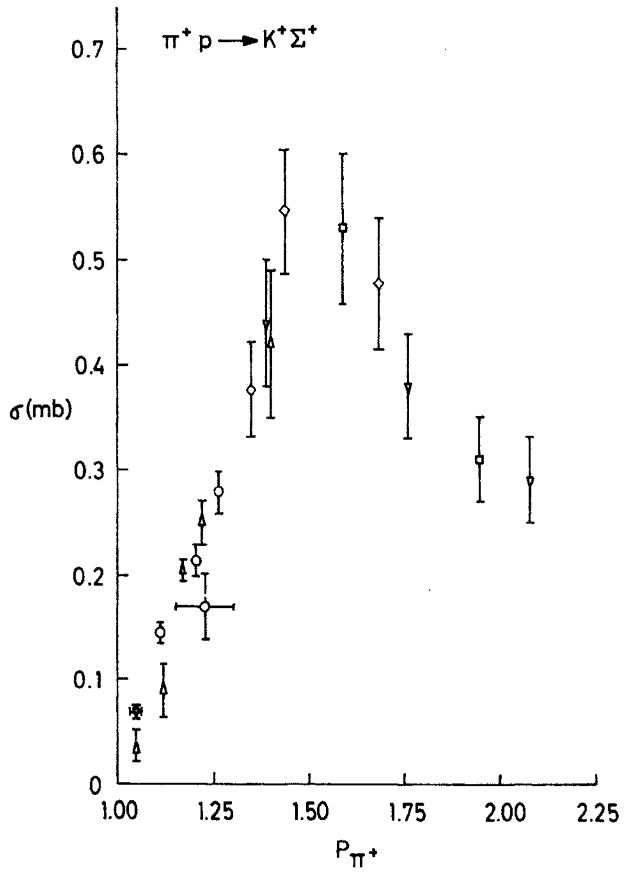

Fig. 4 $\pi^+ p \to K^+\Sigma^+$ total cross-section.

parametrizing background amplitudes by the simple form

$$T = (A + Bk) \exp \{i(c + Dk)\}$$

and using a Breit-Wigner for the resonant amplitudes. The results of the analysis are as yet inconclusive, except to confirm the presence of a strong F_{37}.

5. $\pi + N \rightarrow \pi + \pi + N$

Since this is the dominant inelastic channel throughout most of the resonance region, it is clear that a proper understanding of the resonances will not be attained without the inclusion of the three-body channels. This poses a serious problem of analysis. Exactly how should three-body states be treated and how should the results be related to what is known about the elastic amplitudes? The answer has always been to use the isobar model, sometimes blatantly, sometimes discreetly disguised, and until recently inelastic data have not been available in sufficient quantity to warrant anything more sophisticated. This was evident in the analysis of Morgan[22], who applied a generalized version of the isobar model to a study of the T = 1/2 single pion production processes in the range 1450 to 1575 MeV and found that, with the data then available, it was quite unnecessary to go beyond the confines of the isobar model. Morgan's analysis also gave some indication of the value of the information which could be hidden in the three-body data; in particular he suggested that this data required the existence of a second P_{11} resonance, decaying strongly via ρN or $\pi\Delta$ with a mass very roughly between 1500 and 1700 MeV. This suggestion was made simultaneously with, and independently of, the phase-shift analyses of the elastic data finding a second P_{11}, the $P_{11}(1785)$.

New data are now becoming available which will compare favourably in statistical quality with the elastic data, and the time is rapidly approaching when the isobar model will need an appreciable overhaul. In each of the three analyses on which I shall report one of two simplifying assumptions has been made to the isobar model: either that all the three-body events are quasi two-body with negligible background; or there is background but it is incoherent, is given by phase space, and can be removed to leave the "genuine" quasi two-body events. The two assumptions are mutually exclusive and neither is strictly correct. It is unlikely that they will affect the dominant qualitative features obtained in the analyses, since the background is generally estimated to be of the order of 20%-30% at most, but the less dominant features and detailed quantitative features are certainly more in doubt.

A partial wave analysis of the reactions

$$\pi^+ + p \rightarrow \pi^+ + p + \pi^0$$
$$\pi^+ + n + \pi^+$$

is being carried out at Saclay, and preliminary results are reported[23] at 1510, 1580, 1640 and 1690 MeV, using the isobar model, with the first of the above two assumptions, in the manner proposed by Deler and Valladas[24] and assuming also that only $\Delta(1236)$ is produced. Only the inelastic angular distributions are used in the fit, and the results are then compared with the three possible invariant mass distributions and with the results of the elastic phase-shift analyses. Agreement is acceptable in the former case, and typical fits are shown in Fig. 5, along with the fitted angular distributions.

The comparison with the elastic data is shown in Fig. 6, where the quantity $\sqrt{1 - \eta^2}$ is plotted. In computing the values from the fitted three-body data, it is necessary to neglect overlap terms, which do not vanish, although generally they appear to be small. The dashed and heavy lines are the values obtained from the CERN I[2] and Saclay[3] analyses. The agreement is acceptable except in the S_{31} amplitude, where it is poor, and it is apparent that the isobar model is breaking down in the simple form in which it has been applied, and that there is background of some kind present. No significant change in the solutions has been found as a result of including production of $P_{11}(1460)$ or the S_{11} attractive interaction in the analysis, and the effect of ρ production is now being investigated.

The same approach has been applied[23] to the reactions

$$\pi^- + p \rightarrow \pi^0 + \pi^- + p$$
$$\pi^+ + \pi^- + n$$

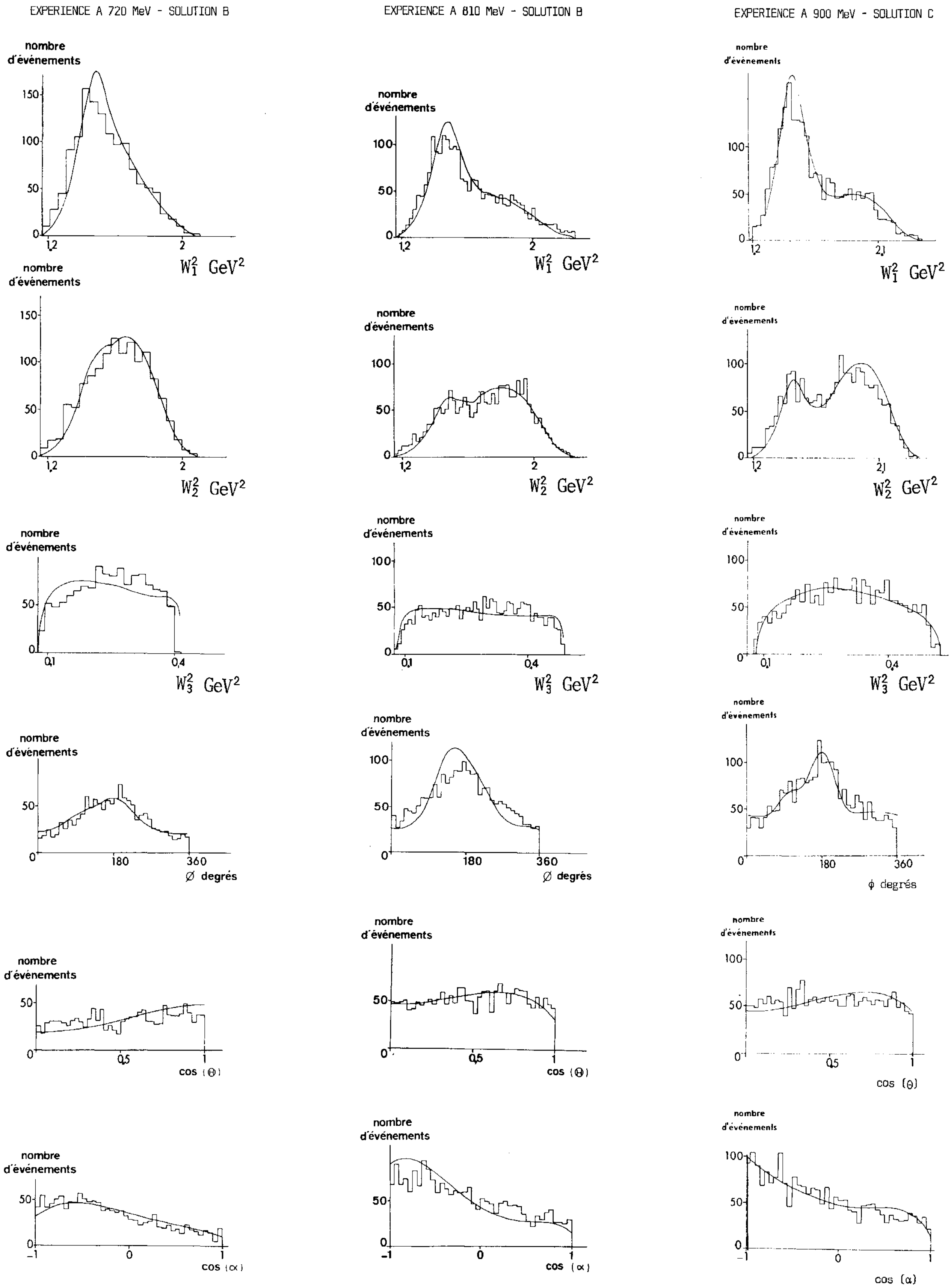

Fig. 5 Comparison of the Saclay[23] fits to $\pi^+ + p \to \pi^+ + \pi^0 + p$ with the experimental histograms at 720 MeV (Brussels), 810 MeV (Saclay) and 900 MeV (Rochester). The variables are as follows:

W_1^2 is the square of the invariant mass of the (π^+p) sub-system,
W_2^2 is the square of the invariant mass of the (π^0p) sub-system,
W_3^2 is the square of the invariant mass of the $(\pi^+\pi^0)$ sub-system,
Θ is the polar angle of the incoming π^+ in the c.m. with respect to the three-body plane,
ϕ is the azimuthal angle of the incoming π^+ in the c.m. system with respect to the three-body plane,
α is the production angle of the nucleon in the c.m. system.

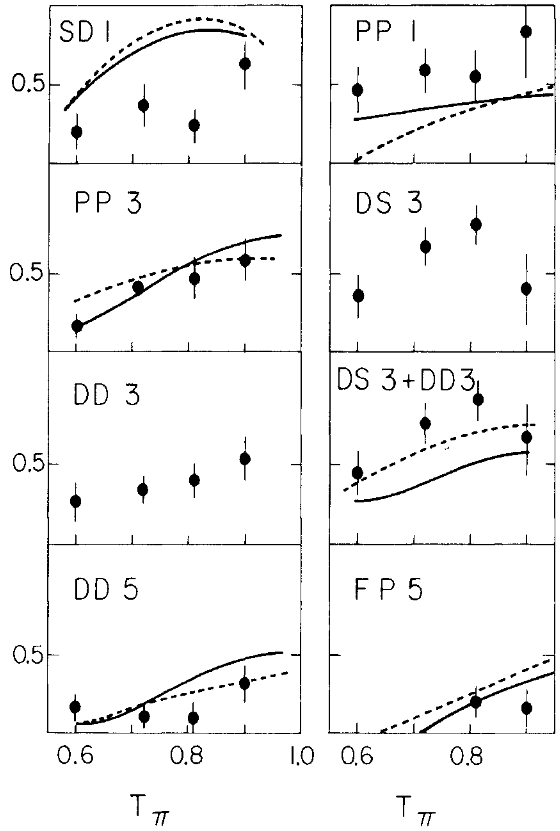

Fig. 6 Comparison of $\sqrt{1-\eta^2}$ from the Saclay[23] fits to inelastic π^+p data and the values obtained from phase-shift analysis. The dotted line is from the CERN I[2] analysis and the solid line from the Saclay analysis[3].

at 1390, 1440, 1485 and 1525 MeV, assuming that the $\pi^0\pi^-p$ final state arises primarily through the {Δ(1236) + π} channel while the $\pi^+\pi^-n$ state involves in addition the {"σ" + n} channel, and the T = 3/2 inelastic channel can be neglected. The results are again in fair agreement with the elastic analyses except for the S_{11} amplitude. The most significant conclusion that they reach is that the partial decay of the P_{11}(1460) into π + Δ(1236) is well established, this resonance decaying in approximately equal parts into πN, σN, πΔ. In common with earlier analyses in this region they find that D_{13}(1515) decays into πΔ with a large branching ratio.

The alternative approach to the isobar model, namely that of making a specific separation of events into quasi two-body and background, has been applied by Brody et al.[25] and Sun Yiu Fung et al.[26].

Brody et al. have studied π^-p interactions at 12 energies between 1500 and 1770 MeV. Their elastic events were used to obtain absolute normalization, normalizing their data with the counter data in the region $-0.8 < \cos\theta < 0.7$, where the elastic cross-sections are rather flat, and checking the result by using the (normalised) forward point. Agreement with the counter data is good. The angular distributions also agree well, and should be a useful piece of extra information for the phase-shift analyses. The reaction $\pi^-p \to \pi^-\pi^+n$ was chosen for detailed analysis since it is dominated by strong $\Delta^-(1236)\pi^+$. This latter channel was separated out explicitly by assuming an incoherent superposition of phase space, together with the channels $\pi^+\Delta^-$, $\pi^-\Delta^+$ and ρ^0n. The best fits to the mass spectra are shown in Fig. 7, which have contributions of 57.6% $\pi^+\Delta^-$, 7.2% $\pi^-\Delta^+$, 0.3% ρ^0n and the rest three-body phase space. The total cross-section for the $\pi^+\Delta^-$ channel is shown in Fig. 8, and it clearly shows an enhancement in the

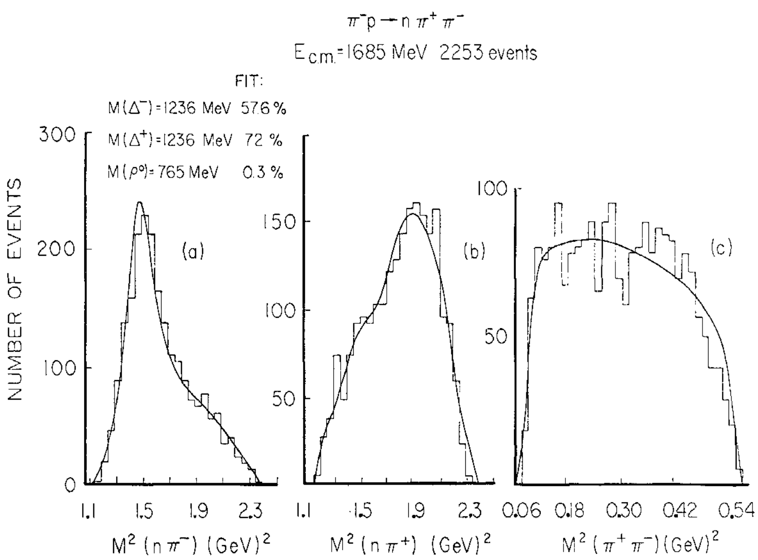

Fig. 7 Maximum likelihood fits to the $\pi^-p \to n\pi^+\pi^-$ data at 1685 MeV.

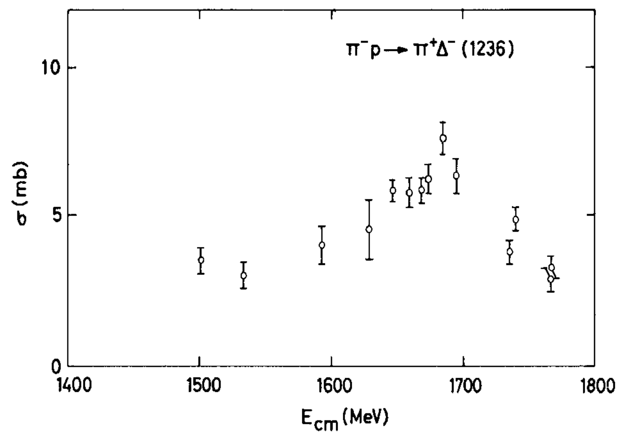

Fig. 8 Total cross-section for $\pi^+p \to \pi^+\Delta^-(1236)$.

region of 1680 MeV, which is presumably associated with $D_{15}(1675)$ and $F_{15}(1690)$. The production angular distributions of the $\pi^+\Delta^-$ channel are shown in Fig. 9 and the corresponding Legendre polynomial expansion coefficients in Fig. 10. The behaviour of the coefficients between 1600 and 1700 is consistent with the presence of the two spin 5/2 resonances, the absense of any dramatic variation of A_1 and A_3 suggesting that the phase difference between these two amplitudes changes very slowly, as is indeed the case in the elastic-scattering data. The behaviour of A_1 and A_2 between 1500 and 1600 MeV, strongly negative but going to zero as the energy increases, implies that the $D_{13}(1515)$ couples in with a negative relative sign to D_{15} and F_{15}. With this as input information, Brody et al. found it possible to obtain a good fit to the data, using the three resonances $D_{13}(1515)$, $D_{15}(1675)$ and $F_{15}(1690)$, parametrized by Breit-Wigner forms, with the addition of a simple

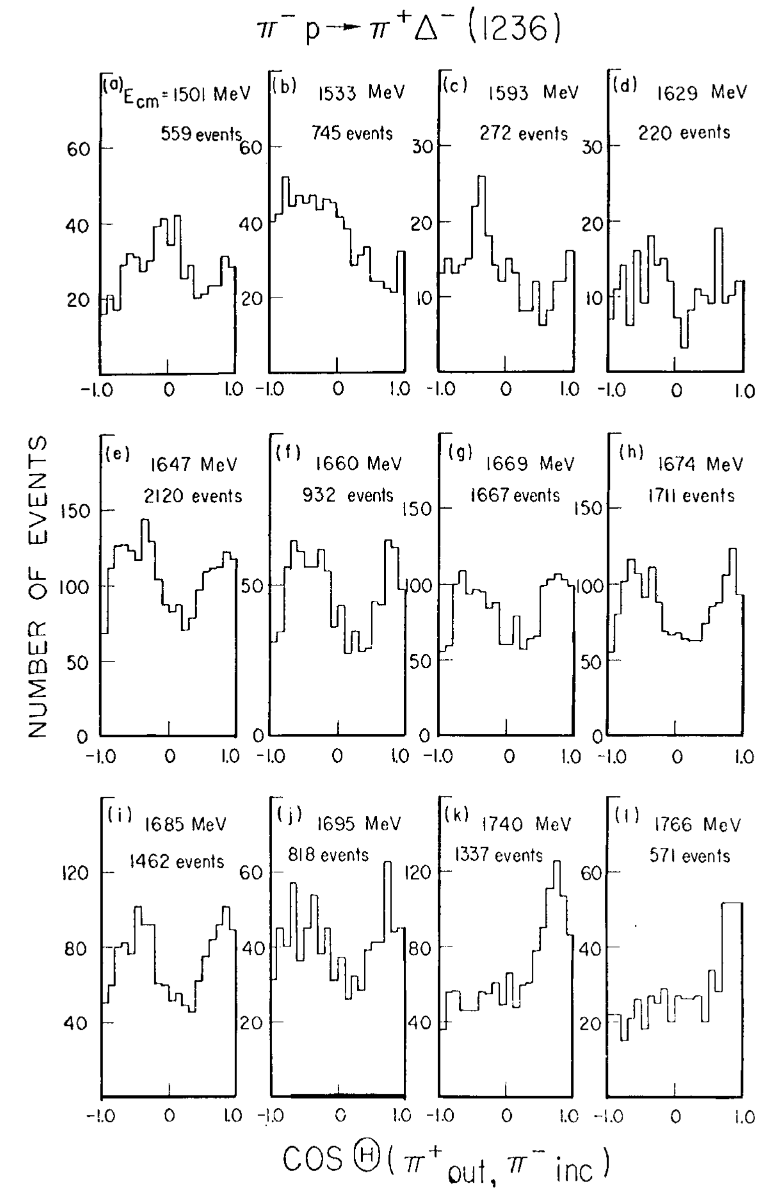

Fig. 9 Centre-of-mass angular distributions for $\pi^-p \to \pi^+\Delta^-(1236)$.

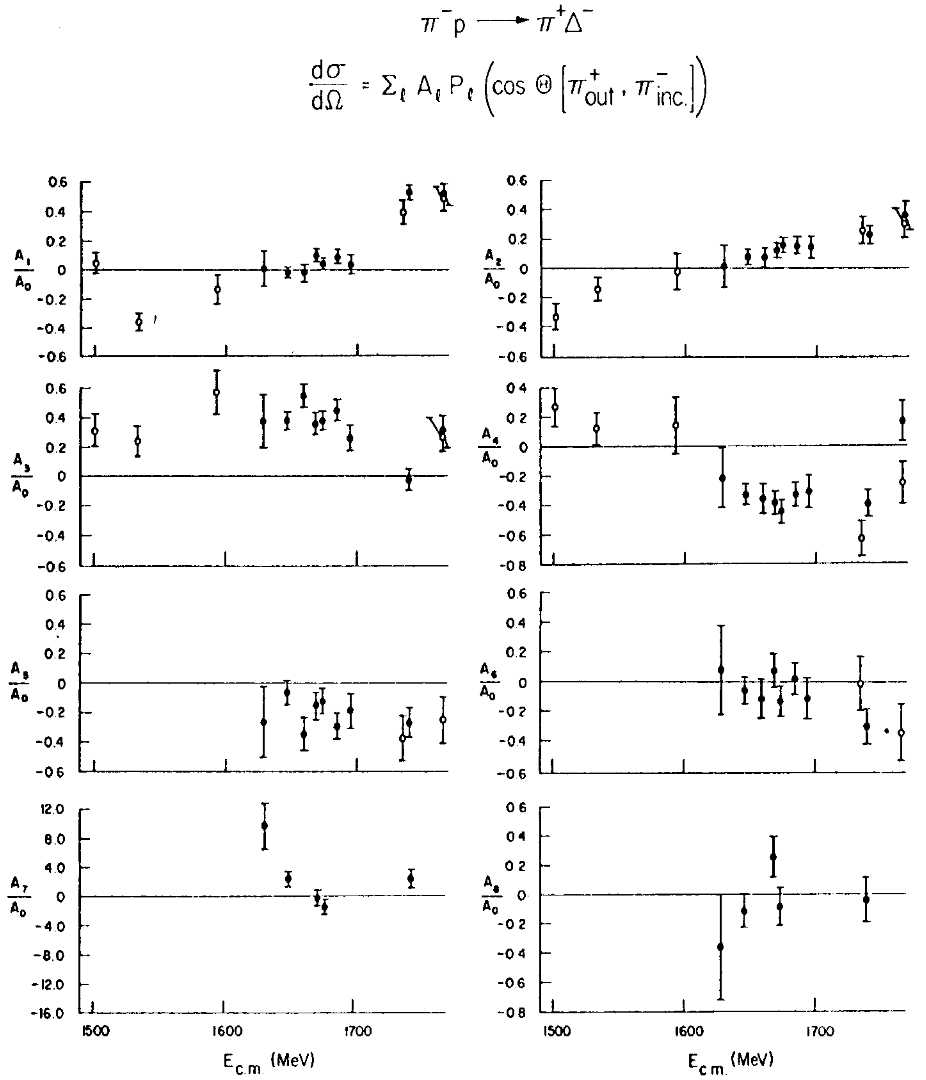

Fig. 10 Coefficients of Legendre polynomial expansion of the c.m. angular distributions of Fig. 9.

linear S-, P-, D- and F-wave background. Using the elasticities of CERN I[2)], they obtain the branching fractions

$$\frac{D_{13}(1520)\to\pi\Delta}{\text{All}} \sim 12\%, \qquad \frac{D_{15}(1680)\to\pi\Delta}{\text{All}} \sim 15\%,$$

$$\frac{F_{15}(1680)\to\pi\Delta}{\text{All}} \sim 20\% \cdot$$

Alternative parametrizations are being studied and the fits should not be considered final in any sense. They are, however, a very good guide as to what one may ultimately hope to obtain from these channels.

Sun Yin Fung et al.[26)] have adopted a similar approach to the study of the reaction

$$\pi^+ + p \to \pi^+ + p + \pi^0$$

at 1850, 1890, 2010 MeV, and to isolate the channel

$$\pi^+ + p \to \Delta^{++}(1236) + \pi^0$$

using an incoherent superposition of the channel with $\rho^0 + p$, $\Delta^+ + \pi^+$ and three-body phase space. The angular distributions for the $\pi^0\Delta^{++}$ channel are shown in Fig. 11 and the corresponding Legendre polynomial

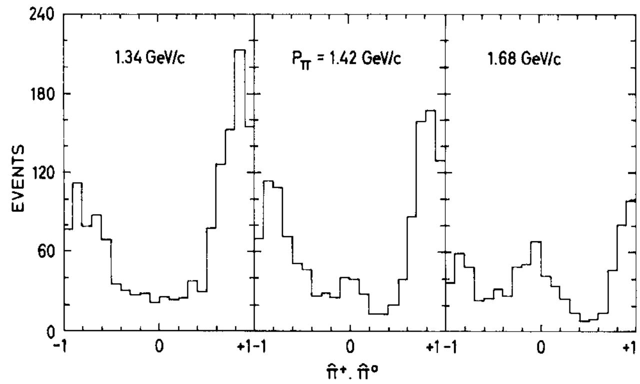

Fig. 11 Centre-of-mass angular distributions for $\pi^+p \to \pi^0\Delta^{++}(1236)$.

TABLE 3

Coefficients A_n/A_0 obtained by fitting the angular distributions in Fig. 11 with the expansion

$$\frac{d\sigma}{d\Omega} = \lambda^2 \, \Sigma \, A_n \, P_n \, (\hat{\pi}^+ \cdot \hat{\pi}^0).$$

Momentum (GeV/c)	A_1/A_0	A_2/A_0	A_3/A_0	A_4/A_0	A_5/A_0	A_6/A_0	A_7/A_0
1.34	1.11 ± 0.25	1.35 ± 0.07	0.18 ± 0.10	-0.52 ± 0.09	-0.42 ± 0.13	-0.79 ± 0.11	-0.33 ± 0.13
1.42	1.19 ± 0.26	1.81 ± 0.07	0.40 ± 0.10	-0.65 ± 0.10	-0.43 ± 0.14	-1.74 ± 0.12	-0.27 ± 0.13
1.68	-0.29 ± 0.34	0.57 ± 0.09	0.90 ± 0.14	0.95 ± 0.12	0.22 ± 0.16	-0.86 ± 0.15	0.23 ± 0.17

expansion coefficients are given in Table 3. The large negative A_6 indicates, not surprisingly, that $F_{37}(1940)$ is contributing strongly at all energies. At the present it is difficult to say what other partial waves are contributing appreciably, although D_{35} appears the most likely candidate.

6. $\gamma + N \to \pi + N$

The information on pion-nucleon resonances in photoproduction comes almost entirely from detailed multipole analysis. To date there have been five such analyses, by Schmidt, Schwiderski and Wunder[27], by Chan, Dombey and Moorhouse[28], by Engels, Schmidt and Schwiderski[29], by Walker[30], and by Berends and Donnachie[31]. Analysis of pion-photoproduction is a less objective and a less exact exercise than analysis of elastic scattering, since the data are both less precise and less varied, polarization data in particular being very scarce, and in addition there are many more multipoles than there are partial waves. Consequently, it is not possible to determine weak structural effects, which simply get lost in the background terms, and quantitative precision is lacking. The situation in fact is much the same as in the inelastic channels in πp interactions, and only dominant structure can be seen unambiguously. Resonances in this category are the $P_{33}(1236)$, $S_{11}(1525)$, $D_{13}(1515)$, $F_{15}(1690)$ and $F_{37}(1940)$. The $S_{11}(1525)$ also shows clearly in η-photoproduction[32,33]. The situation with respect to one of the more interesting ones, the $P_{11}(1460)$, is ambiguous. There is some evidence for it, although it is not clear cut and is certainly background-dependent. For example, Berends and Donnachie[31] find two possible solutions, in one of which the transition amplitude to P_{11} is very small and barely distinguishable from zero, and in the other it is quite strong, comparable in magnitude to that of Chan et al.[28], but with the opposite sign. Even at best the production of P_{11} is rather weak, the amplitude, at resonance, for production of P_{11} being not more than one-eighth of the amplitude, at resonance, for production of $P_{33}(1236)$. There is some additional evidence for photoproduction of $P_{11}(1460)$ in a preliminary analysis of $\gamma + p \to p + \pi^+ + \pi^-$ below 1650 MeV by Diambrini Palazzi et al.[34], who conclude that some P_{11} contribution is essential to explain the data.

7. PRODUCTION EXPERIMENTS

Since production experiments are being treated fully in a supplementary report by Rushbrooke[35], only two points which are of special relevance to the N* story will be selected.

The first of these is the continuing accummulation of evidence for the $\pi\Delta$ decay mode of the $P_{11}(1460)$. This is reported in $K^+p \to K^+\pi^+\pi^-p$ at 5.5 GeV/c by Antich et al.[36], in $\pi^+p \to \pi^+p\pi^-\pi^+$ at 16 GeV/c by Ballam et al.[37], in $pp \to pp\pi^+\pi^-$ at 25 GeV/c by Ehrlich et al.[38], in $pp \to pp\pi^+\pi^-$ at 22 GeV/c by Jespersen et al.[39] and in $pp \to pp\pi^+\pi^-$ at 16 GeV/c by Rushbrooke et al.[40]. The second point is further evidence for an enhancement in the vicinity of 1.73 GeV, being seen in the $\pi\Delta$ mode by Rushbrooke et al.[40] and by Ballam et al.[37]. Both of these points are conveniently illustrated in the analysis of Rushbrooke et al.[40], and their $(\pi^-\Delta^{++})$ mass plot is shown in Fig. 12. The cuts in Δ^2 remove Deck background and

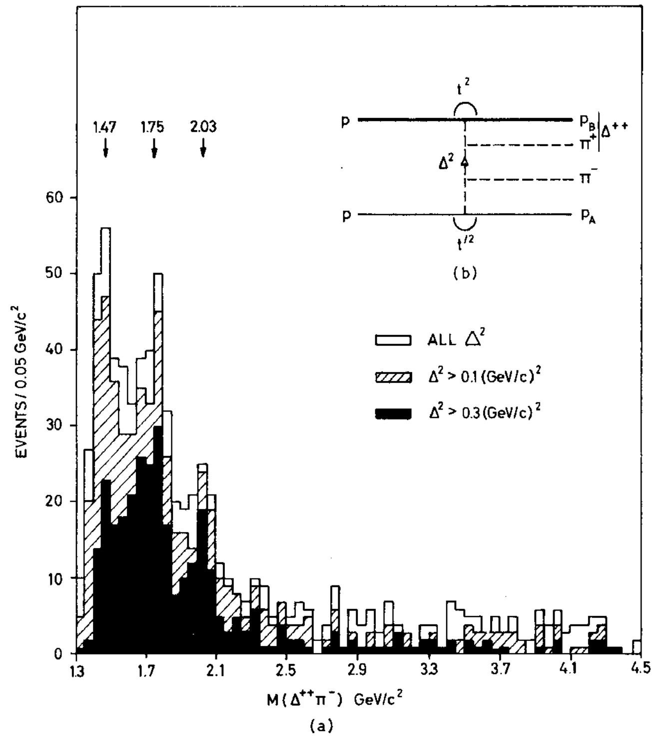

Fig. 12 a) The $m(\Delta^{++}\pi^-)$ spectrum for various Δ^2 cuts in $pp \to pp\pi^+\pi^-$ at 16 GeV/c.
b) Diagram for $pp \to pp\pi^+\pi^-$.

show up the peaks at 1.73 and 2.03 GeV even more clearly. The apparent shoulder on the lower side of the 1.73 enhancement could be associated with the 1.69 D_{15}/F_{15} pair, encouraging the belief that the 1.73 GeV enhancement is indeed something quite different. The peak observed between 1450 and 1500 MeV should be associated with $P_{11}(1460)$, since it is narrower and lower than that indicated by a Deck calculation. The angular distributions in the three peak regions are shown in Fig. 13. That in the $P_{11}(1460)$ region is consistent with isotropy, and the apparent forward peak in the 1.73 GeV enhancement (dotted line) is removed by the cut $\Delta^2 > 0.1$, i.e. by removing the Deck background, leaving a distribution which is symmetrical (and non-isotropic). Thus it is very tempting to associate this peak with $D_{13}(1730)$. A possible candidate for the enhancement at 2.03 GeV is the $D_{13}(2030)$.

The evidence for the pion-nucleon resonances, apart from the phase-shift analyses, is shown in Table 4. None of it by itself is conclusive, but the over-all effect is quite impressive. All the inelastic data are consistent with the resonance assignments of the elastic phase-shift analyses, and in some cases, for example $\pi p \to K\Lambda$, $\gamma p \to \pi p$, many of the resonances are required by the data. The experimental situation is still far from satisfactory, and since it is evident that much information on the pion-nucleon resonances can be gleaned from the inelastic channels, particularly since most of these channels appear to be dominated by small subsets of the isobars, it is to be hoped that the required experimental effort will be forthcoming.

From this, three points stand out. These are the near definite evidence for a strong $\pi\Delta$ decay mode of $P_{11}(1460)$, the accumulating evidence for $P_{11}(1785)$ and, to a somewhat lesser extent, the accumulating evidence for $D_{13}(1730)$.

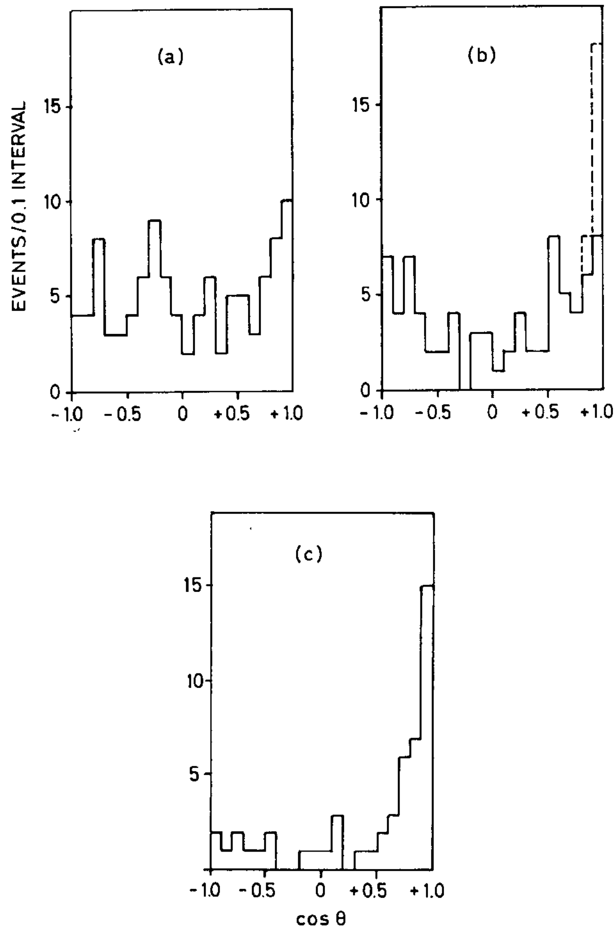

Fig. 13 Angular distributions for the decay of 3-body resonances into a Δ^{++} and π^-.
a) 1.425 GeV/c² < $m(\Delta^{++}\pi^-)$ < 1.525 GeV/c².
b) 1.70 GeV/c² < $m(\Delta^{++}\pi^-)$ < 1.80 GeV/c².
c) 1.80 GeV/c² < $m(\Delta^{++}\pi^-)$ < 1.90 GeV/c².

Finally, it is impossible to resist commenting on the remarkable success attained by the symmetric quark model in classifying the resonances, a point which is discussed in detail by Harari[41]. The situation with respect to the nucleon resonances is shown in Table 5. The only resonance missing in the lowest status is a third P_{33}, and this may well be the possible $P_{33}(2030)$ indicated in the CERN II analysis[11].

TABLE 4

The status of the decay modes of the pion-nucleon resonances observed, other than into pion-nucleon, and the evidence from production experiments. The $P_{11}(1460)$ has a strong σN mode also.

	ηN	$K^0\Lambda^0$	$K^+\Sigma^+$	$\pi\Delta$	ρN	γN	Production
$P_{33}(1237)$						Definite	Definite
$S_{31}(1630)$				Possible			}
$D_{33}(1670)$				Possible			} Possible
$P_{33}(1690)$				Possible			}
$F_{35}(1880)$							
$P_{31}(1905)$							
$F_{37}(1940)$			Probable	Definite	Definite	Definite	
$P_{11}(1460)$				Definite		Probable	Definite
$D_{13}(1515)$				Definite		Definite	
$S_{11}(1525)$	Definite					Definite	}
$D_{15}(1675)$				Probable			} Probable
$F_{15}(1690)$				Probable		Definite	}
$S_{11}(1715)$	Possible	Probable					
$D_{13}(1730)$		Possible					Possible
$P_{11}(1785)$	Possible	Probable		Probable			
$P_{13}(1855)$							
$D_{13}(2030)$							Possible
$G_{17}(2190)$						Possible	

TABLE 5

Classification of pion-nucleon resonances in the ℓ-excitation quark model with parafermi statistics. The empirical rule that only 56, $L = (2n)^+$ and 70, $L = (2n + 1)^-$ supermultiplets are required is obvious. A possible classification of the three resonances in higher levels is:

$$\begin{array}{ll} P_{11}(1785) & \ell = 4,\ 56,\ L = 0^+ \\ \left.\begin{array}{l} D_{13}(2030) \\ G_{17}(2190) \end{array}\right\} & \ell = 3,\ 70,\ L = 3^- \end{array}$$

and the unobserved P_{33} may be filled by the possible $P_{33}(2030)$ reported by the CERN II[11] analysis.

$(1s)^3$	$(1s)^2\ (1p)$	$(1s)^2\ (2s)$, $(1s)^2\ (1d)$, $(1s)\ (1p)^2$		Higher levels
56, $L = 0^+$	70, $L = 1^-$	56, $L = 0^+$	56, $L = 2^+$	
$8^{1/2}$ $P_{11}(939)$				
$10^{3/2}$ $P_{33}(1237)$				
		$8^{1/2}$ $P_{11}(1460)$		
	$8^{1/2}$ { $D_{13}(1516)$, $S_{11}(1525)$ }			
	$10^{1/2}$ { $S_{31}(1630)$, $D_{33}(1670)$ }			
	$8^{3/2}$ { $D_{15}(1674)$, $S_{11}(1714)$ }			
			$8^{1/2}$ { $F_{15}(1687)$, $P_{13}(1855)$ }	
		$10^{3/2}$ $P_{33}(1690)$		
	$D_{13}(1730?)$			
				$P_{11}(1785)$
			$10^{3/2}$ { $F_{35}(1878)$, $P_{31}(1904)$, P_{33}(unobserved), $F_{37}(1942)$ }	
				$D_{13}(2030)$
				$G_{17}(2190)$

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DISCUSSION

GREENBERG: I would like to make some remarks about Donnachie's classification of the nucleonic resonances using the parafermi quark model. It is hard to understand why the N(1470), which Donnachie places in a two-quantum excitation, should be lower than the one-quantum excitations. Nelson and I showed that the 3-triplet model can account for all the $J^P = 1/2^+$ and $3/2^+$ states, except for the $P_{33}(1690)$ about which doubt has been raised. Our mass formula gives agreement to within 15 MeV.

SCHLEIN: I would like to comment on a dynamical matter that is of serious concern to those analyses in which one attempts to extract information on the existence of resonances in diffraction dissociation produced systems. In a UCLA-LRL Collaboration result reported to this Conference, we show in a detailed analysis of $pp \to \Delta^{++}p\pi^-$ at 6.6 GeV/c (Colton et al.) that, for low momentum transfers to Δ^{++} and for high $p\pi^-$ mass (that is in the diffraction scattering region), both the shape and absolute magnitude of the $p\pi^-$ spectrum are in quantitative agreement with one-pion-exchange expectations, despite the fact that the $\pi\Delta$ spectrum displays the now well-known features of the $\pi\Delta$ interaction, namely strong N*(1470) production. Our predictions of the $p\pi^-$ mass spectrum are also in agreement with the BNL data on $pp \to p\pi^-\Delta^{++}$ at 28.5 GeV/c. One can thus conclude that it seems incorrect to think in terms of the $\Delta\pi$ spectrum as containing contributions from two separate processes, namely diffraction dissociation production of N*(1470) and a "Deck background". The entire $\Delta\pi$ cross-section is in agreement with OPE (or, if you like, the Deck cross-section), although the shape of the $\Delta\pi$ spectrum reflects the details of the $\Delta\pi$ interaction. The correspondence between the $\Delta\pi$ interaction and the observed $\Delta\pi$ mass spectrum is expected to depend, of course, on the s and $t_{p,\Delta}$ values of the sample considered. These experimental remarks support, therefore, the point of view taken by Chew and Pignotti in their discussion of the Dolen-Horn-Schmidt duality. Corresponding tests of A_1 and Q-bump production in the final states $\rho\pi p$ and $K^*\pi p$ can also be made.

LEFRANÇOIS: You have stated that the P_{11} resonance has now been seen in photoproduction. At one time the story was that it could not be seen in photoproduction on protons but should be seen in photoproduction on neutrons. Could you comment on the present status?

DONNACHIE: The particular statement originally made was based in fact on theoretical calculations which were not entirely certain. In fact, the statement that the P_{11} is seen only very weakly from photoproduction on protons has been confirmed, but the mechanism of the production is quite different from the one assumed in the early calculation. While the statement concerning photoproduction on protons is certainly correct, the statement on neutrons is now open to question. Experimentally it is seen weakly, if at all.

LEITH: You have mentioned the existence of a D_{13} (1730) in $\Delta\pi$. How sure are you of this assignment? In the $\pi^- p \to \Delta^- \pi^+$ formation experiment, we see evidence of $\Delta\pi$ excitation at 1730 MeV in P_{11}. Also in 16 GeV/c $\pi^+ p$ we find a $\Delta\pi$ enhancement at 1730 MeV, which is compatible with P_{11} assignment.

DONNACHIE: The assignment of the quantum numbers to the D_{13} and P_{11} in the πN system are quite definite. The assignments deduced from other channels are doubtful. One cannot make any serious comment about this. It is quite possible to have one or the other, or a mixture of both.

KAMAL: There was a contribution by Tokyo University to this Conference on polarized $\gamma n \to \pi^- p$ around P_{11}. They find no evidence of P_{11} contribution in this channel.

APPENDIX

NUCLEON RESONANCES IN PRODUCTION PROCESSES

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1. INTRODUCTION

As the subject of nucleon resonances in formation experiments is discussed in Dr. Donnachie's talk[1], I shall be dealing with these resonances in relation to production experiments. The dozen or so papers on the subject, submitted to this Conference, make it clear that it is still too early to pull the results together, and look for patterns and trends in cross-sections and production mechanisms. Rather I shall be reporting on some promising methods of analysis likely to be useful for the high statistics experiments which, need it be said, are required in the future.

The $\Delta(1236)$ is produced very strongly at high energies: for example, 50-70% of events in the reaction $pp \to pp\pi^+\pi^-$ involve Δ^{++} production though the exact fraction is difficult to determine, since the production of low-mass $p\pi^+\pi^-$ resonances would generate kinematically a low-mass peak in the $p\pi^+$ system even if they never decayed into $\Delta^{++}\pi^-$. I shall return to this problem in discussing inelastic decay modes of resonances. I shall be discussing the $\Delta(1236)$ only in this connection. Most papers focus attention on the P_{11} Roper resonance in a context where Deck background is possible, and I shall begin here.

2. KINEMATIC EFFECTS

Previously we had the problem of Deck background; now we have duality to contend with instead. Chew and Pignotti[2] have extended to multiperipheral processes a suggestion made by Dolen, Horn and Schmid[3] that direct channel resonances are already contained, in the sense of a local average, in the cross-channel Regge amplitudes. Thus in a multi-particle production process, say $pp \to p\Delta^{++}\pi^-$, any $\Delta^{++}\pi^-$ enhancement (a Deck peak) predicted by a t-channel model is, therefore, a prediction of resonance(s), and to attempt a