Cosmological Tachyon Condensation

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We consider the dynamics of the open string tachyon condensation in a framework of the cubic fermionic String Field Theory including a non-minimal coupling with closed string massless modes, the graviton and the dilaton. We show explicitly that the influence of the dilaton on the tachyon condensation is essential and provides a significant effect: oscillations of the Hubble parameter and the state parameter become of a cosmological scale. We give an estimation for the period of these oscillations $(0.1-1)$ Gyr and note a good agreement of this period with the observed oscillations with a period $(0.15-0.65)$ Gyr in a distribution of quasar spectra.

1 Introduction and Setup

Contemporary cosmological observational data [2] strongly support that the present Universe exhibits an accelerated expansion providing thereby an evidence for a dominating Dark Energy (DE) component. Recent results of WMAP together with the data on Ia supernovae give the following bounds for the DE state parameter $w_{\rm DE} = -1^{+0.14}_{-0.11}$. This range of w includes quintessence models, $w > -1$, containing an extra light scalar field which is not in the Standard Model set of fields, the cosmological constant, $w = -1$, and "phantom" models, $w < -1$, which can be described by a scalar field with a ghost (phantom) kinetic term. In this case all natural energy conditions are violated and there are problems of instability both at the classical and quantum levels. Models with a crossing of the $w = -1$ barrier are also a subject of recent studies. Simplest ones include two scalar fields (one phantom and one usual field. General κ-essence models can have both $w < -1$ and $w \geqslant -1$ but a dynamical transition between these domains is forbidden under general assumptions. Some projects are directly aimed at exploring whether w varies with the time or is a constant. Varying w obviously corresponds to a dynamical model of the DE which generally speaking includes a scalar field. Modified models of General Relativity also generate an effective scalar field.

We consider cosmological models coming from the open String Field Theory (SFT) tachyon dynamics, namely from the cubic SFT formulation of the open fermionic NSR string with the GSO$-$ sector (see [3] for a review). A non-minimal coupling of the open string tachyon and the dilaton motivated by an investigation of the linear dilaton background in the flat space-time is of main concern here. Analyzing cosmological consequences of our model we demonstrate that there is a possibility to find oscillations in the Hubble parameter and w of cosmological scales.

We work in $1+3$ dimensions,the coordinates are denoted by x^μ with Greek indices running

[a] Based on [1], see also refs. therein.

from 0 to 3. Action motivated by the closed string field theory reads

$$S_c = \int d^4x\sqrt{-g}\frac{e^{-\Phi}}{2\kappa^2}\left(R + \partial_\mu\Phi\partial^\mu\Phi - \frac{\partial_\mu T\partial^\mu T}{2} + \frac{T^2}{\alpha'} - \frac{1}{\alpha'}V(\tilde{T})\right). \tag{1}$$

Here κ is the gravitational coupling constant $\kappa^2 = 8\pi G = \frac{1}{M_P^2}$. G is the Newton's constant, M_P is the Planck mass, α' is the string length squared, g is a metric, $\tilde{T} = \mathcal{G}_c(\alpha'\Box)T$ with $\Box = D^\mu\partial_\mu$ and D_μ being a covariant derivative, Φ is the dilaton field and T is the closed string tachyon. Fields are dimensionless. $\mathcal{G}_c$ is supposed to be an analytic function of the argument. $V(\tilde{T})$ is a closed string tachyon potential. Factor $1/\alpha'$ in front of the tachyon potential looks unusual and can be easily removed by a rescaling of fields. For our purposes it is more convenient keeping all the fields dimensionless. Action motivated by the open string field theory reads

$$S_o = \int d^4x\sqrt{-g}\frac{1}{g_o^2}\left(e^{-\Phi/2}\left(-\frac{1}{2}g^{\mu\nu}\partial_\mu\tau\partial_\nu\tau + \frac{1}{2\alpha'}\tau^2\right) - \frac{1}{\alpha'}\widehat{e^{-\Phi/2}v(\tilde{\tau})}\right). \tag{2}$$

Here g_o is the open string coupling, $[g_o] = \text{length}$, $\tilde{\tau} = \mathcal{G}_o(\alpha'\Box)\tau$, hat denotes an operator inverse to the tilde such that $\hat{\tilde{\varphi}} = \varphi$ and τ is the open string tachyon, it is dimensionless. Both $\mathcal{G}_o$ and its inverse are supposed to be analytic functions of the argument. $v(\tilde{\tau})$ is an open string tachyon potential. A peculiar coupling of the tachyon potential with the dilaton involving an action of a non-local operator hat on the dilaton exponent is supported considering the linear dilaton CFT. We study a minimal gravitational coupling of lightest open and closed string modes using the action

$$S = S_c + S_o. \tag{3}$$

2 Tachyons Around Vacuum without Dilaton Field

The open string tachyon when all massive states are integrated out by means of equations of motion acquires a non-trivial potential with a non-perturbative minimum. Rolling of the tachyon from the unstable perturbative extremum towards this minimum describes, according to the Sen's conjecture, a transition of an unstable D-brane to a true vacuum where no perturbative states of the open string are present.

We consider the tachyons near their true vacuum and linearize equations of motion using $T = T_0 + Z$ and $\tau = \tau_0 - \zeta$. Let us focus on the open string tachyon field τ. A linearized equation becomes

$$\Box\zeta - g^{\mu\nu}\frac{\partial_\mu\Phi\partial_\nu\zeta}{2} + \frac{\zeta}{\alpha'} - \frac{v''(\tau_0)}{\alpha'}e^{\Phi/2}\left(\widetilde{\widehat{e^{-\Phi/2}\tilde{\zeta}}}\right) = 0. \tag{4}$$

If the dilaton field is a constant (or at least one can neglect its variation compared with the tachyon field) one gets the following equation

$$\Box\hat{\hat{\zeta}} + \frac{\hat{\hat{\zeta}}}{\alpha'} - \frac{v''(\tau_0)}{\alpha'}\zeta = 0. \tag{5}$$

A way of solving such an equation is based on the fact that an eigenfunction of the $\Box$ operator with an eigenvalue $\omega^2 = \bar{\omega}^2/\alpha'$ solves the latter equation if $\bar{\omega}^2$ solves the transcendental equation

$$\left(\bar{\omega}^2 + 1\right)\mathcal{G}_o^{-2}(\bar{\omega}^2) = v''(\tau_0). \tag{6}$$

Using a level truncated cubic fermionic string field theory we have $\mathcal{G}_o^{-2}(\bar{\omega}^2) = e^{\beta\bar{\omega}^2}$. A solution for $\bar{\omega}^2$ becomes

$$\bar{\omega}^2 = \frac{W(\gamma) - \beta}{\beta}, \quad \gamma = v''(\tau_0)\beta e^{\beta}, \tag{7}$$

where W is Lambert W-function which solves an equation $xe^x = y$ w.r.t. x. All $\bar{\omega}$, β and γ are dimensionless. Notice, that for $v''(\tau_0) = 1$ we have $\omega^2 = 0$ irrespectively of β. For the tachyon at large times one has for a constant Hubble parameter

$$\zeta = e^{-\frac{3}{2}Ht}\mathrm{Re}\left(\zeta_- e^{\frac{1}{2}t\sqrt{9H^2-4\omega^2}} + \zeta_- e^{-\frac{1}{2}t\sqrt{9H^2-4\omega^2}}\right). \tag{8}$$

ω^2 given by equation (7) is in general a complex number.

3 Tachyons Around Vacuum in Dilaton Background

Scales related to the dilaton should be of cosmological magnitudes. For instance, the present Hubble parameter is related to the Planck mass as $H_{\mathrm{our}} \approx 10^{-60}M_P$. The same level of smallness is applicable to a rate of change of the dilaton field. Indeed, in the dilaton gravity $\dot{\Phi}$ determines a speed of change of the Newton's constant. Thus $\dot{\Phi}$ should be extremely small. For instance, assuming that Newton's constant changes at most e times during the life-time of the universe we conclude that in a model with a linear dilaton one should have $\Phi \approx \pm H_{\mathrm{our}}t$. It looks counterintuitive that such small quantities can affect somehow processes related to the tachyon condensation. The reason is that all ingredients related to string excitations have α' as a scale. α' is the string length squared and can be written as $1/M_s^2$ where M_s is the string mass. This string mass in any case cannot be less than 1 TeV to be compatible with present experiments. Moreover, M_s is often associated with the Planck mass M_P.

Examining more carefully equation (4) and the succeeding formulae we observe, however, that parameters γ and β introduced in the previous Section play a crucial role and new interesting phenomena can emerge. Let us make an approximation which produces a very useful toy equation

$$\Box\zeta + \frac{\zeta}{\alpha'} - (1+\epsilon)\frac{v''(\tau_0)}{\alpha'}\tilde{\zeta} = 0. \tag{9}$$

Compared to equation (4) we have dropped a term proportional to the speed of the dilaton[b] and assumed that exponents of the dilaton with all non-local operators acting on them can be accounted as a constant ϵ. This latter constant is positive for non-local operators coming from the SFT. Similar to (7)

$$\omega_\epsilon^2 = \frac{W((1+\epsilon)\gamma) - \beta}{\beta\alpha'}, \quad \gamma = v''(\tau_0)\beta e^{\beta} \tag{10}$$

and we see that we have shifted the parameter γ and consequently ω^2. For a linear dilaton $\Phi = 2V_0 t$ this shift ϵ can be computed in the SFT to be of order $\alpha' V_0^2$, $[V_0] = \mathrm{length}^{-1}$. Accounting the above discussion about cosmological scales we see that $\epsilon \sim \alpha' H^2$. For these scales a constant H approximation is easily justified and the dynamics of a linearized tachyon $\tau = \tau_0 + \zeta_\epsilon$ is given by an analog of (8)

$$\zeta_\epsilon = e^{-\frac{3}{2}Ht}\mathrm{Re}\left(\zeta_+ e^{\frac{1}{2}t\sqrt{9H^2-4\omega_\epsilon^2}} + \zeta_- e^{-\frac{1}{2}t\sqrt{9H^2-4\omega_\epsilon^2}}\right). \tag{11}$$

A distinguished case which may have a signature in the late cosmology is that an unshifted $\omega^2 = 0$. To have $\omega^2 = 0$ one has to require $v''(\tau_0) = 1$ irrespectively of β. This means that the full tachyon potential including the mass term should have zero second derivative in the minimum. Parameter γ becomes $\gamma = \beta e^{\beta}$, $\gamma \geq -\frac{1}{e}$ for real β, $\gamma = -\frac{1}{e}$ for $\beta = -1$.

[b] Strictly speaking this is done to make an analytic analysis possible. Further one can demonstrate numerically that restoring this term does not change the main effect described in this Section.

4 Cosmological Signature

To get an insight in numbers we derive let us take $V_0 = \sigma H_{\rm our}$ meaning that during the evolution of the universe the Newton's constant has changed $e^{2\sigma}$ times, σ is dimensionless. This gives a shift $\epsilon = \sigma^2\alpha' H_{\rm our}^2 = \sigma^2 10^{-120} M_P^2/M_s^2 = \sigma^2 10^{-120+2n}$ where we put $M_s = 10^{-n}M_P$ with $0 \leq n \leq 16$. The upper bound on n gives strings of TeV mass which is a minimal string mass compatible with current experiments.

The most interesting case is $\gamma > -1/e$, i.e. $\beta \neq -1$. In this case $\omega_\epsilon^2 \approx \frac{\epsilon}{(1+\beta)\alpha'}$ and for the estimated $\epsilon \sim \sigma^2\alpha' H_{\rm our}^2$ oscillations come out if $\sigma > 1.036$. Taking $\sigma = 1.1$ we get the frequency of oscillations in ζ_ϵ is approximately $H_{\rm our}/2$. This frequency is doubled and is equal to $H_{\rm our}$ in the Hubble parameter and the state parameter. This produces oscillations with a period of order 10 Gyr which is of order of the universe age. Taking other value of β which can result from an inclusion of the higher massive modes in the SFT analysis we may get smaller values of the period of the oscillations. Taking $\beta = -0.95$ and $\sigma = 1.1$ the period of oscillations of the Hubble parameter and the state parameter becomes of order 1 Gyr. We note a good agreement of this estimated period with the observed oscillations in the z-distribution of quasar spectra [4]. The reported value is $(0.15 - 0.65)$ Gyr. The closer β to -1 from above the shorter period is. The higher the dilaton speed the shorter period is as well. Thus we see that in the case $\gamma > -1/\epsilon$ one can have an effect of oscillations in the Hubble parameter and the state parameter w accessible for observations.

In the case $\gamma = -1/e$ (i.e. $\beta = -1$) one has $\alpha'\omega_\epsilon^2 \approx -I\sqrt{2\epsilon}$, this term most likely dominates under the square root in (11) and the resulting frequency becomes of order $\sqrt{\sigma}10^{+30-n/2}H_{\rm our}$. In order to make the period of oscillations of order 1 Gyr we have to assume $\sigma \sim 10^{-46}$ for the TeV string mass with $n = 16$. This means that the dilaton changes very slowly, but it is difficult to figure out an appearance of a new small parameter $V_0 \sim 10^{-106}M_P$. The closed string tachyon can be considered analogously and will give the same effect provided its behavior is similar to the open string tachyon.

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References

1. I.Ya. Aref'eva, A.S. Koshelev, [arXiv:0804.3570].
2. E. Komatsu, [arXiv:0803.0547].
3. I.Ya. Aref'eva, D.M. Belov, A.A. Giryavets, A.S. Koshelev, P.B. Medvedev, [hep-th/0111208].
4. A.I. Ryabinkov, A.D. Kaminker, D.A. Varshalovich, *Mon. Not. Roy. Astron. Soc.* **376**, 1838 (2007), [astro-ph/0703277].