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ON THE PROBLEM OF PRODUCTION OF RELATIVISTIC LEPTON BOUND STATES IN THE DECAYS OF LIGHT MESONS

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I. Introduction

The study of electrodynamic bound states such as positronium (e^+e^-) and muonium ($\mu^+\mu^-$) is of great importance in recent years for experiments [1] on quantum electrodynamics (QED). The probability for producing a bound e^+e^- pair in the $\pi^0 \to \gamma(e^+e^-)_{s=1}$ decay was experimentally measured to be [2] (spin $S = 1$)

$$\rho\left[\pi^0 \to \gamma(e^+e^-)_{s=1}\right] \equiv \Gamma\left[\pi^0 \to \gamma(e^+e^-)_{s=1}\right]/\Gamma(\pi^0 \to \gamma\gamma)$$
$$= (0.35 \div 0.71)\,\alpha^4 , \tag{1}$$

where Γ is the width of the relevant decay.

The estimates of ρ for the $\pi^0, \eta^0 \to \gamma(e^+e^-)_{s=1}$ and $\eta^0 \to \gamma(\mu^+\mu^-)_{s=1}$ decays were made in ref. [3]. In this case, a nonrelativistic wave function of a hydrogen atom at the coordinate origin was used that has been summed over all principal quantum numbers. The radiative corrections $O(\alpha)$ to the width of the $\pi^0 \to \gamma(e^+e^-)_{s=1}$ decay, calculated in ref. [4], are mainly due to the polarization operator contribution and the one-photon exchange in the (e^+e^-) pair. A general contribution of radiative corrections amounts to $26\alpha/(9\pi)$.

The most consistent representation of lepton bound states is possible in terms of three-dimensional wave function (w.f.) satisfying a relativistic two-body equation, for instance, the Bethe-Salpeter equation (B-S).

In this paper we use the method, developed in ref. [5], that allows one to obtain a relativistic wave function in the "lowest approximation" in the systematic perturbation approach to the B-S theory in QED. The "exact" w.f. is composed of the one mentioned above and a series of decreasing correction terms. The calculations of $\rho[\pi^0, \eta^0 \to \gamma(\bar{\ell}\ell)_{s=1}]$ were made in the model of quark triangular loop taking account of the mass of constituent quarks and using a relativistic wave function of lepton bound states. The obtained results are compared with the experimental data on the $\pi^0 \to \gamma(e^+e^-)_{s=1}$ decay [2].

2. Green Function. Wave Function

The whole information on lepton bound states with masses m_n is provided by the two-particle propagator $G(P; p_2, p_1)$ with the poles at the energies

$$P^0 = \omega_n = \sqrt{m_n^2 + \vec{P}^2}\,, \qquad P = (P^0, \vec{P})\,. \tag{2}$$

Near these poles $G(P; p_2, p_1)$ has the following form [6]

$$G(P; p_2, p_1) \xrightarrow[P^0 = \omega_n]{} \frac{i}{2\omega_n} \cdot \frac{\sum_j \Psi_{nj}(p_2)\, \bar{\Psi}_{nj}(p_1)}{(P^0 - \omega_n + i\varepsilon)}\,, \tag{3}$$

where p_1 and p_2 are relative momenta of incoming and outcoming pairs of particles, $\Psi_n(p)\,(\bar{\Psi}_n(p))$ is the B-S wave function (its conjugate).

The normalization condition for the B-S w.f. is derived from the basic equation

$$G^{-1}(P)\; G(P) = 1\,. \tag{4}$$

Having rewritten (4) taking account its expansion over the degrees of the bound state energy

$$\left[G^{-1}(P_n) + (P^0 - \omega_n) \frac{\partial}{\partial P^0} G^{-1}(P^0)\Big|_{P = P_n} + \dots \right] \times$$
$$\times \left[(i/2\omega_n) \cdot \frac{\sum_j \Psi_{nj}\, \bar{\Psi}_{nj}}{(P^0 - \omega_n + i\varepsilon)} + \Phi + \dots \right] = 1\,, \tag{5}$$

where $P = (P^0, \vec{P})$, $P_n = (\omega_n, \vec{P})$, comparing the right-hand side (5) with the left-hand side and using the linear independence of $\Psi_n(p)$, we get two equations

$$G^{-1}(P_n)\; \Psi_{nj}(p) = 0\,, \tag{6}$$

$$\bar{\Psi}_{nj}(p)\; G^{-1}(P_n) = 0\,. \tag{7}$$

Calculating the coefficients for $(P^0 - \omega_n)$ in (5)

$$G^{-1}(P_n)\,\Phi + \frac{\partial}{\partial P^0} G^{-1}(P^0)\Big|_{P = P_n} \cdot \frac{i}{2\omega_n} \sum_j \Psi_{nj}\, \bar{\Psi}_{nj} = 1\,, \tag{8}$$

and multiplying (8) from the left by $\bar{\Psi}_{nj}$ taking into account (7), we get the following normalisation condition:

$$\bar{\Psi}_{ni} \left\{ i \frac{\partial}{\partial P^0} G^{-1}(P_n) \right\}\Big|_{P = P_n} \Psi_{nj} = 2\omega_n\, \delta_{ij}\,, \tag{9}$$

where

$$G^{-1}(P_n) = S^{-1}(P_n) - K(P_n)\,, \tag{10}$$

S^{-1} is the inverse operator of the product of two total single-particle propagators and K is the kernel of the B-S equation [7]. If the kernel K is independent of energy, we get the normalisation condition for Ψ_n in the form

$$\int \frac{d^3\vec{p}}{(2\pi)^3} \cdot \frac{d^3\vec{q}}{(2\pi)^3}\, \bar{\Psi}_n(\vec{p})\, i \frac{\partial}{\partial P^0} S^{-1}(P^0)\Big|_{P = P_n} \Psi_n(\vec{q}) = 2\omega_n\,, \tag{11}$$

where $\vec{p}$ and $\vec{q}$ are the particle momenta of the initial and final states in the c.m.s.

Note that the normalisation condition (11) can also be applied to the three-dimensional w.f. satisfying the nonrelativistic Schroedinger equation.

Representing S^{-1} in the form

$$S^{-1}(P^0; \vec{p}, \vec{q}) \to i\,(2\pi)^3\, \delta^{(3)}(\vec{p} - \vec{q}) \left\{ m_1 + m_2 - P^0 + \frac{p^2 (m_1 + m_2)}{2\, m_1 \cdot m_2} \right\}, \tag{12}$$

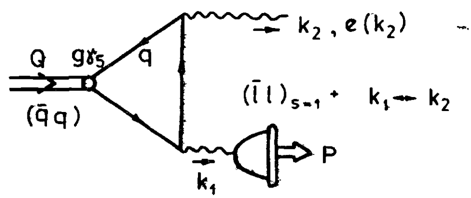
we get the normalisation condition in the usual form

$$\int \frac{d^3\vec{p}}{(2\pi)^3}\, \bar{\Psi}_n(\vec{p})\, \Psi_n(\vec{p}) = 2 m_n\,. \tag{13}$$

3. Decay probability of $(\bar{q}q) \to \gamma\,(\bar{\ell}\ell)_{s=1}$

Expression for the quarkonium decay matrix element $(\bar{q}q) \to \gamma\,(\bar{\ell}\ell)_{s=1}$ (see the figure) in the zero perturbation order in the coupling constant α_s has the following form (for definiteness, consider a quarkonium consisting of u- and d- quarks):

$$M\left[(\bar{q}q)\to\gamma\,(\bar{\ell}\ell)_{s=1}\right]=N_c\sum_{f=u,d}(\lambda_f\cdot e_f^2)(4\pi\alpha)^{3/2}\cdot\mathcal{F}_s(Q^2,P^2)\times$$
$$\times\,\varepsilon^{\alpha\mu\beta\nu}(k_2)_\alpha\,Q_\beta\,e_\mu(k_2)\,\gamma_\nu\,\frac{-i}{k_1^2}\int\frac{d^4p}{(2\pi)^4}\,\Psi_n(p)\;+\;(1\leftrightarrow 2), \tag{14}$$



Figure

where N_c is the number of colours, e_f are quark charges of the f-th flavour, $\lambda_f : \lambda_{u(d)} = 1(-1)$, $\mathcal{F}_s(Q^2, P^2)$ is the function of a "strong" part of a quark triangle, $\Psi_n(p)$ is the normalised B-S w.f. of the lepton bound state, and n is the principal quantum number.

The function $\mathcal{F}_s(Q^2, P^2)$ is the transitional form factor and its explicit form depends on calculation of the "quark" triangle integral

$$I(Q^2,P^2;m)=\int\frac{d^4q}{(2\pi)^4}\cdot\frac{1}{[q^2-m_i^2+i\varepsilon][(q+k_2)^2-m_j^2+i\varepsilon][(q+Q)^2-m_k^2+i\varepsilon]}, \tag{15}$$

where $m_{i,j,k}$ are quark masses.

Considering the case with equal quark masses m_q, we find the following expression for $\mathcal{F}_s(Q^2, P^2; m_q)$:

$$\mathcal{F}_s(Q^2,P^2;m_q)=\frac{4g\,m_q}{\pi^2(Q^2-P^2)}\left\{\arcsin^2\left(\frac{\sqrt{P^2}}{2m_q}\right)-\arcsin^2\left(\frac{\sqrt{Q^2}}{2m_q}\right)\right\}, \tag{16}$$

where g is the pseudoscalar-quark-antiquark coupling constant. The result (16) is valid at $\sqrt{Q^2} < 2m_q$, *i.e.* m_q is large at physical values of meson masses $\sqrt{Q^2}$.

At small values of the lepton bound state mass $\sqrt{P^2}$, from (16) we get

$$\mathcal{F}_s(Q^2,P^2;m_q)\cong\frac{-4g\,m_q}{\pi^2\cdot Q^2}\times$$
$$\times\left\{\left(1+P^2/Q^2\right)\arcsin^2\left(\frac{\sqrt{Q^2}}{2m_q}\right)-\frac{P^2}{(2m_q)^2}\cdot\left[1+P^2\left(\frac{1}{Q^2}+\frac{1}{12m_q^2}\right)\right]\right\}. \tag{17}$$

To compare, we give an expression for $\mathcal{F}_s(Q^2, P^2; m_q)$ in the massless $Q^2 = 0$ limit for small $\sqrt{P^2}$

$$\mathcal{F}_s(0,P^2;m_q)\cong\frac{-g}{\pi^2\cdot m_q}\cdot\left(1+\frac{P^2}{12\cdot m_q^2}+\cdots\right). \tag{18}$$

It is expedient to compare the above formulae for $\mathcal{F}_s$ with the result obtained within the vector dominance model (VDM)

$$\mathcal{F}^{(VDM)}(P^2;m_{\rho,\omega,\varphi})\cong g_{\gamma\gamma}^{(VDM)}\cdot\left\{1+\frac{P^2}{m^2_{\rho,\omega,\varphi}}+\cdots\right\}, \tag{19}$$

where $g_{\gamma\gamma}^{(VDM)}$ is the normalisation constant, $m_{\rho,\omega,\varphi}$ are the masses of vector (ρ-, ω-, φ-) mesons. The numerical estimates [8], performed within the quantum chromodynamics, indicate that the contribution of higher states to the transitional form factor for the Dalitz decays of pseudoscalars $\to \gamma\,\bar{\ell}\ell$ does not exceed 5-10%. Therefore, the VDM can describe the Dalitz decays and the decays with production of lepton bound pairs with good accuracy.

It is seen from the comparison of formulae (18) and (19) that owing to the Q^2-duality we should use in numerical calculations not current quark masses but masses of constituent quarks (m_q = 300-400 MeV) rather than current masses of quarks. Assuming in (16) $P^2 = 0$ we immediately get the normalisation constant of $(\bar{q}q)\to\gamma\gamma$ decay [9]:

$$f_{\gamma\gamma}(Q^2)=\frac{-4g\,m_q}{\pi^2\cdot Q^2}\cdot\arcsin^2\left(\frac{\sqrt{Q^2}}{2m_q}\right), \tag{20}$$

which in the massless limit $Q^2 = 0$ transforms into

$$f_{\gamma\gamma}(0)=\frac{-g}{\pi^2\cdot m_q}. \tag{21}$$

Formula (21) is a well-known relation for the "triangle anomaly". Certainly, one should take into account s quarks for an exact calculation of the production probability of positronium and muonium in the $\eta^0 \to \gamma(\bar{\ell}\ell)_{s=1}$ - decay η^0-meson being represented as a quark superposition $\eta^0 \sim (1/\sqrt{3})\{\bar{u}u + \bar{d}d - \bar{s}s\}$. In what follows we shall use the value of the $\eta - \eta'$ meson mixing angle $\Theta = -19^\circ$ /10/.

The matrix element $\eta^0 \to \gamma(\bar{\ell}\ell)_{s=1}$ is of the following form

$$M\left[\eta^0 \to \gamma(\bar{\ell}\ell)_{s=1}\right] = \frac{(4\pi\alpha)^{3/2}\cdot g}{\sqrt{3}\,\pi^2 (m_\eta^2 - M^2)\cdot M^2} \times$$

$$\times \left\{ (|\bar{u}u + \bar{d}d>\left(\sqrt{\tfrac{1}{2}}\cos\Theta + \sin\Theta\right)\mathcal{F}_{\bar{u}u} + |\bar{s}s>(-\sqrt{2}\cos\Theta + \sin\Theta)\mathcal{F}_{\bar{s}s}\right\} \times \tag{22}$$

$$\times\, \mathcal{E}^{\alpha\mu\beta 0}(k_2)_\alpha\, Q_\beta\, e_\mu(k_2)\,\gamma_0 \int \frac{d^4 p}{(2\pi)^4}\, \Psi_n(p) + (1 \leftrightarrow 2),$$

where

$$\mathcal{F}_{\bar{q}q} = m_q \cdot \left\{ \arcsin^2\left(\frac{M}{2m_q}\right) - \arcsin^2\left(\frac{m_\eta}{2m_q}\right)\right\}, \tag{23}$$

$\bar{q}q:\ \bar{u}u,\ \bar{s}s$; m_η is the mass of η^0 -meson, M is the mass of lepton bound state and Θ is the angle of $\eta - \eta'$ mixing.

4. Relativistic wave function of the lepton bound state

For concrete calculations by formula (14) and (22) one should approximate the B-S w.f. $\Psi_n(p)$ by the Barbieri-Remiddi (B-R) wave function $\Phi_n(p)$ /5/ which though less complex is a nontrivial function of a physical charge e. In the B-S equation /7/

$$G = S + S K G \tag{24}$$

we choose propagator S and kernel K corresponding to a weakly bound system of leptons: S is chosen as a product of two free fermion propagators $(S(P;q,p) = (2\pi)^4\,\delta(p-q)\cdot S(P;p))$

$$S(P;p) = \left(\frac{i}{\frac{1}{2}\hat{P} + \hat{p} - m}\right)^{(1)} \cdot \left(\frac{i}{-\frac{1}{2}\hat{P} + \hat{p} - m}\right)^{T(2)} \tag{25}$$

and for K the following expression is used $(P = (m_p, \vec{0}) = (2\omega, \vec{0}))$:

$$K(P;q,p) = \gamma_0^{(1)}\lambda(\vec{p})T(\vec{p})(-i)V[(\vec{p}-\vec{q})^2]\,T(\vec{q})\,\tau(\vec{q})(-\gamma_0)^{T(2)}, \tag{26}$$

where

$$\lambda(\vec{p}) = \left(\frac{2\varepsilon_p}{\varepsilon_p + m}\right)\cdot\left[\Lambda_+(\vec{p})\tfrac{1}{2}(1+\gamma_0)\right]^{(1)}\cdot\left[\tfrac{1}{2}(1-\gamma_0)\cdot\Lambda_-(\vec{p})\right]^{T(2)},$$

$$\tau(\vec{q}) = \left(\frac{2\varepsilon_q}{\varepsilon_q + m}\right)\cdot\left[\tfrac{1}{2}(1+\gamma_0)\Lambda_+(\vec{q})\right]^{(1)}\cdot\left[\Lambda_-(\vec{q})\tfrac{1}{2}(1-\gamma_0)\right]^{T(2)},$$

$$\varepsilon_p = \sqrt{\vec{p}^2 + m^2}, \qquad \varepsilon_q = \sqrt{\vec{q}^2 + m^2},$$

$$T(\vec{p}) = \left(\frac{2m}{\varepsilon_p + \omega}\right)^{1/2}, \qquad T(\vec{q}) = \left(\frac{2m}{\varepsilon_q + \omega}\right)^{1/2},$$

$$\Lambda_\pm(\vec{p}) = \frac{1}{2\varepsilon_p}\cdot\left[\varepsilon_p \pm (m - \vec{\gamma}\cdot\vec{p})\gamma_0\right],$$

$$\Lambda_\pm(\vec{q}) = \frac{1}{2\varepsilon_q}\cdot\left[\varepsilon_q \pm (m - \vec{\gamma}\cdot\vec{q})\gamma_0\right],$$

$$V[(\vec{p}-\vec{q})^2] = \frac{-4\pi\alpha}{(\vec{p}-\vec{q})^2}.$$

In the limit, when $|\vec{q}|, |\vec{p}| \ll m$ (m is a lepton mass), the kernel (26) can be transformed in the following simple form:

$$\frac{1}{2}(1+\gamma_0)^{(1)}\,\frac{1}{2}(1-\gamma_0)^{T(2)}\,\frac{4\pi\alpha i}{(\vec{p}-\vec{q})^2}. \tag{27}$$

The solution of eq. (24) is represented as a sum of three terms /11/

$$G(P;q,p) = S(P;q,p) + \tag{28}$$

$$+ \frac{(-\gamma_0)^{T(2)}\lambda(\vec{p})T(\vec{p})\left[i4\pi\alpha/(\vec{p}-\vec{q})^2 + W(\vec{p},\vec{q})\right]T(\vec{q})\,\tau(\vec{q})\,(\gamma_0)^{(1)}}{\left[p_0^2 - (\varepsilon_p - \omega - i\varepsilon)^2\right]\cdot\left[q_0^2 - (\varepsilon_q - \omega - i\varepsilon)^2\right]},$$

where

$$W(\vec{p},\vec{q}) \doteq (i\,4\pi\alpha^2)\,2m\,\varepsilon_0 \int_0^1 dx\, \frac{x^{-\nu}}{[4x(\vec{p}-\vec{q})^2\varepsilon_0^2 + (1-x)^2(\vec{p}^2+\varepsilon_0^2)(\vec{q}^2+\varepsilon_0^2)]}, \quad (29)$$

$$\varepsilon_0 = \sqrt{m^2-\omega^2}, \qquad \nu = m\cdot\alpha/(2\cdot\varepsilon_0).$$

Using the term "potential" we should like to note that the function (28) contains: a) the zero potential term which is a free two-particle propagator, b) the "one-potential" term corresponding to the one-photon exchange and c) the multipotential term corresponding to the two- and more photon exchanges. The Green function has poles at the energy values

$$\omega_0 = m\left(1-\frac{\alpha^2}{4n^2}\right)^{1/2} \cong m\left(1-\frac{\alpha^2}{8n^2}\right) + O(\alpha^4), \quad (30)$$

$$n = 1, 2, \ldots$$

The wave function $\Phi(p)$ is determined in a usual way (in the rest frame of a bound state)

$$G\left[(p^0,\vec{0});q,p\right] \underset{p^0=2\omega_0}{\longrightarrow} \frac{i}{4\omega_0}\cdot\frac{\sum_j \Phi_j(p)\,\bar{\Phi}_j(q)}{(p^0-2\omega_0+i\varepsilon)} \quad (31)$$

and satisfies the following homogeneous equation

$$\Phi(p) = i\,S(P;p)\int\frac{d^4p'}{(2\pi)^4}\,K(P;p,p')\cdot\Phi(p'), \quad (32)$$

whereas its conjugate function satisfies the equation

$$\bar{\Phi}(q) = i\int\frac{d^4q'}{(2\pi)^4}\,\bar{\Phi}(q')\,K(P;q,q')\cdot S(P;q). \quad (33)$$

For the states in which the orbital quantum number $\ell \neq 0$, the w.f. tends to zero at the values $n\cdot p \to 0$ (n is the principal quantum number, $p \equiv |\vec{p}|$ is momentum of a particle in a bound state).

In what follows we shall consider lepton bound pairs produced in the ground state $n=1$, $\ell = 0$ and the wave functions will be used for these quantum numbers. In calculating the integrals

in the matrix elements (14) and (22) we use instead of the B-S wave function the $\Phi_{n=1}(p)$ w.f. of the form (see ref./5/)

$$\Phi_{n=1}(p) \cong 2i\,A(p)\cdot(\varepsilon_p - m_0)\cdot\varphi_{n=1}(\vec{p}), \quad (34)$$

satisfying eq. (32) with the kernel (26). In formula (34)

$$A(p) = \left[p_0^2 - (\varepsilon_p - m_0 - i\varepsilon)^2\right]^{-1}, \quad (35)$$

$$\varepsilon_p = \sqrt{\vec{p}^2+m^2}, \qquad m_0 = m\cdot\sqrt{1-\alpha^2/4}, \quad (36)$$

$$\varphi_{n=1}(\vec{p}) = \sqrt{2m\beta^3/\pi}\cdot\frac{8\pi\beta}{(\vec{p}^2+\beta^2)^2}, \quad (37)$$

$$\beta = m\alpha/2. \quad (38)$$

The normalisation condition for the three-dimensional wave function (37) is

$$\int\frac{d^3\vec{p}}{(2\pi)^3}\,|\varphi_{n=1}(\vec{p})|^2 = 2m. \quad (39)$$

5. The results

Using the wave function (34) we get the following formulae for the production probability of $(e^+e^-)_{s=1}$ and $(\mu^+\mu^-)_{s=1}$ in the $\pi^0, \eta^0 \to \gamma(\ell\ell)_{s=1}$ decays in a weak coupling limit of a lepton bound state

$$\rho\left[\pi^0\to\gamma(e^+e^-)_{s=1}\right] = \frac{\alpha^4}{2}\left(1-\frac{4m_e^2}{m_\pi^2}\right)\cdot\left[1-\frac{\arcsin^2\left(\frac{m_e}{m_{u,d}}\right)}{\arcsin^2\left(\frac{m_\pi}{2m_{u,d}}\right)}\right], \quad (40)$$

where m_e, m_π and $m_{u,d}$ are the masses of an electron, π^0 - meson and u-(d-) quarks, respectively:

$$\rho\left[\eta^0\to\gamma(\bar\ell\ell)_{s=1}\right]=\frac{\alpha^4}{2}\left(1-\frac{4m_\ell^2}{m_\eta^2}\right)\times \tag{41}$$

$$\times\frac{\left\{(|\bar uu\rangle+|\bar dd\rangle)(\sqrt{1/2}\cos\Theta+\sin\Theta)\mathcal{F}_{\bar uu}+|\bar ss\rangle(-\sqrt{2}\cos\Theta+\sin\Theta)\,\mathcal{F}_{\bar ss}\right\}^2}{\left\{(|\bar uu\rangle+|\bar dd\rangle)(\sqrt{1/2}\cos\Theta+\sin\Theta)f_{\bar uu}+|\bar ss\rangle(-\sqrt{2}\cos\Theta+\sin\Theta)\,f_{\bar ss}\right\}^2},$$

where

$$f_{\bar qq}=m_q\cdot\arcsin^2(m_\eta/2m_q), \tag{42}$$

$\bar qq:\ \bar uu,\ \bar ss$; $m_q:\ m_u, m_s$; m_ℓ is the lepton mass and m_s is the s quark mass.

Using the values of masses of constituent u-, d-, s-quarks typical of the model of quark triangle loops (cf. with the results obtained in ref. /12/)

$$m_{u,d}\cong\frac{1}{2}\,m_{\rho,\omega}\cong 380\ \mathrm{MeV}, \tag{43}$$

$$m_{u,d}+\delta m_{u,d}\cong m_s\simeq\frac{1}{2}\,m_\varphi=510\ \mathrm{MeV},$$

$$\delta m_{u,d}/m_{u,d}\simeq 1/3,$$

we get the following numerical values for ρ :

$$\rho\left[\pi^0;\eta^0\to\gamma(e^+e^-)_{s=1}\right]=\alpha^4/2; \tag{44}$$

$$\rho^{\Theta=-19^\circ}\left[\eta^0\to\gamma(\mu^+\mu^-)_{s=1}\right]=0.348\,\alpha^4; \tag{45}$$

$$\rho^{\Theta=0^\circ}\left[\eta^0\to\gamma(\mu^+\mu^-)_{s=1}\right]=0.332\,\alpha^4. \tag{46}$$

It is seen from (45) and (46) that the result for the $\eta^0\to\gamma(\mu^+\mu^-)_{s=1}$ decay depends weakly on a chosen value of the mixing angle of η-η'-mesons. At the same time, the inclusion of s quarks (with their concrete masses) increases ρ by 11% in comparison with a naive quark model disregarding in η^0 meson the differences between the masses of nonstrange and strange quarks.

The probability of producing a $(\mu^+\mu^-)_{s=1}$ bound state in comparison with $(e^+e^-)_{s=1}$ state in decay of η^0 meson amounts to about 62%.

The transitional form factor with the inclusion of masses of constituent quarks in formula (41) gives a 23% contribution to the total yield of $(\mu^+\mu^-)_{s=1}$ bound states.

6. Conclusion

We have calculated the probabilities for producing $(e^+e^-)_{s=1}$ and $(\mu^+\mu^-)_{s=1}$ bound states in the decay of π^0 and η^0 mesons with the use of the relativistic w.f. of the lepton bound state $\Phi_{n=1}(p)$ (34). A "strong" part of the decay has been calculated within the model of quark triangle loop with the point-like $g\gamma_5$ interaction between a meson and constituent quarks entering into this meson. As the quark triangle loop was a success in the calculations of the decay widths of mesons $\to\gamma\gamma$ /9,13,12/ , we had no doubts that the use of these loops may lead to admissible results for other electromagnetic decays. The results of the calculations of $\rho[\pi^0,\eta^0\to\gamma(\bar\ell\ell)_{s=1}]$ may be treated as a test in expediency of using the above described quark model in calculating the basic characteristics of the decays of light mesons with the production of electrodynamic composite systems described by the relativistic wave functions. The numerical value of $\rho[\pi^0\to\gamma(e^+e^-)]=\frac{\alpha^4}{2}$ is in good agreement with the experimental data /2/. This model is almost insensitive to the effect of transitional form factors in the $\pi^0,\eta^0\to\gamma(e^+e^-)_{s=1}$ decays. For comparison we should like to note that in the vector dominance model the effect of the form factor on the spectrum of effective masses of e^+e^- pairs in the $\pi^0\to\gamma e^+e^-$ decay amounts to 2%. Radiative corrections in the $\pi^0,\eta^0\to\gamma(e^+e^-)_{s=1}$ decays are small in comparison with analogous corrections of the Dalitz $\pi^0,\eta^0\to\gamma e^+e^-$ - decays proportional to $\alpha\cdot\ln^2(s/m^2)$, $4m^2<s<m^2_{\pi,\eta}$ (s is the square of the invariant mass of e^+e^- - pair).

New predictions have been made for $\rho[\eta^0\to\gamma(\mu^+\mu^-)_{s=1}]$ (45), (46) taking account of the masses of constituent u-(d-) and s-quarks. A weak dependence of the values of $\rho[\eta^0\to\gamma(\mu^+\mu^-)_{s=1}]$ on the value of the angle Θ has been reported.

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К вопросу о рождении связанных релятивистских лептонных состояний в распадах легких мезонов

Рассматриваются и вычисляются вероятности образования связанных лептон-антилептонных ортосостояний в распадах легких псевдоскалярных мезонов в модели, учитывающей массы конституентных кварков. В вычислениях используется релятивистская волновая функция связанного лептонного состояния. Проводится сравнение с экспериментальными данными.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Kozlov G.A. E2-87-557

On the Problem of Production of Relativistic Lepton Bound States in the Decays of Light Mesons

The probabilities for the production of lepton-antilepton bound orthostates in the decays of light pseudoscalar mesons are calculated in the model allowing for the mass of constituent quarks. The relativistic wave function of a lepton bound state is used in the calculations. The results are compared with the experimental data.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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