Graviton emission from simply rotating Kerr black holes: Transverse traceless tensor graviton modes

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Abstract

In this article we present greybody factors and Hawking radiation for tensor graviton modes (in seven dimensions and greater, $n \geq 3$) from simply rotating $(n+4)$-dimensional Kerr black holes.

1 Introduction

The reduction of the graviton perturbation equations into master variable equations for higher-dimensional rotating black holes has been one of the great challenges in recent years,[1, 2]. The method based upon the gauge invariant formalism, developed in reference [3] has allowed for the separation of the tensor mode decomposition of simply rotating Myers-Perry-(A)dS black holes [1, 4, 5], which has recently been used for a stability analysis of Kerr-AdS black holes [6, 7]. In this work we shall focus on a simply rotating black hole in $(n+4)$-dimensional Kerr spacetime.

The wave equation for the tensorial mode of the gravitational perturbation for $n \geq 3$ is equivalent to the wave equation of a massless free scalar field [1, 6] where the determinant is given by the product of the base metric [6] and higher-dimensional spherical harmonics [10]. The separation of the wave equation is implemented by making the ansatz:

$$\Phi = e^{i\omega t - im\varphi} R(r) S_{jlm}(\theta) Y_{j,i_1,i_2,\ldots,i_{n-1}}(\theta_{n-1},\phi) \ , \tag{1}$$

where $Y_{j,i_1,i_2,\ldots,i_{n-1}}(\theta_{n-1},\phi)$ are the hyperspherical harmonics on the n-sphere with eigenvalues $-j(j+n-1)$. This separation ansatz leads to a generalized hyper-spheroidal equation for the $S_{jlm}(\theta)$ functions and an equation for $R(r)$ with separation constant A_{ljm}. These equations are coupled via ω. The restrictions on m, j and l [8] are:$l > j + |m|$; $\frac{l-j+|m|}{2} \in \{0, 1, 2 \ldots\}$, with $j = 2, 3, \ldots, l$ and $|m| = 0, 1, \ldots, l-j$. The degeneracy for a traceless symmetric tensor on an n-sphere [9] is given by:

$$D_j^T = \frac{(n+1)(n-2)(j+n)(j-1)(2j+n-1)(j+n-3)!}{2(n-1)!(j+1)!} \ . \tag{2}$$

For dimensions $n \geq 3$ we can parameterize the black hole mass, M, in terms of the horizon radius r_h: $2M = r_h^{n-1}(r_h^2 + a^2)(1 - \lambda r_h^2)$. By defining the transform: $R(r) = r^{-n/2}(r^2+a^2)^{-1/2}\Phi(r)$, and tortoise coordinates [6]:$dy = \frac{r^2+a^2}{\Delta_r} dr$, where $\frac{\Delta_r}{r^2+a^2} = 1 - \frac{2M}{(r^2+a^2)r^{n-1}} - \frac{2\Lambda}{(n+2)(n+3)} r^2$. After defining the dimensionless variables: $x = r/r_h$, $\omega_\star = \omega r_h$, $y_\star = y/r_h$, $\Delta_\star = \Delta/r_h^2$, $a_\star = a/r_h$, $\lambda_\star = \lambda r_h^2$ and $\Lambda_\star = \Lambda r_h^2$, the radial equation takes the WKB form [6]:

$$\frac{d^2\Phi}{dy_\star^2} + Q(x)\Phi = 0 \ , \tag{3}$$

The CFM for the flat case was implemented using the input parameters described in reference [8].

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In the near the horizon limit, $x \to 1$, the radial solution Φ has the following form (for *IN* modes [6, 10]):

$$\Phi_{\rm NH} \;=\; A_{\rm in}^{(H)} e^{-i\tilde{\omega}_\star x_\star} + A_{\rm out}^{(H)} e^{-i\tilde{\omega}_\star x_\star} \;, \tag{4}$$

where $\tilde{\omega}_\star = \omega_\star - m\Omega_\star$, $\Omega_\star = \frac{a_\star(1-\lambda_\star)}{(1+a_\star^2)}$. Imposing that there are no outgoing modes at the black hole horizon $A_{\rm out}^{(H)} = 0$, we obtain the IVP:

$$\Phi_{\rm NH}(x_0) = 1 \;, \quad \Phi'_{\rm NH}(x_0) = -i\tilde{\omega}_\star \frac{x_0^2 + a_\star^2}{\Delta_\star(x_0)} \;, \tag{5}$$

where $x_0 = 1 + \epsilon$ with $\epsilon \sim 10^{-5}$. The greybody factor can be determined numerically with the above horizon IVP matched onto the appropriate far-field form. The solutions have a far field (FF) form at spatial infinity, where $x_\star \to x$ (for $x \to \infty$):

$$\Phi_{\rm FF} \approx x^{-\frac{n+1}{2}} \left(A_{\rm in}^{(\infty)} e^{-i\omega_\star x} + A_{\rm out}^{(\infty)} e^{i\omega_\star x} \right) \;. \tag{6}$$

The NH solution can then be matched onto the FF equation (6) [11], where the reflection coefficient is then defined as the ratio $|\mathcal{R}_{ljm}|^2 = |A_{\rm out}^{(\infty)}|^2/|A_{\rm in}^{(\infty)}|^2$, and the relationship between the absorption and reflection coefficient is:

$$|\mathcal{A}_{ljmn}|^2 = 1 - |\mathcal{R}_{ljmn}|^2 = 1 - \left| \frac{A_{\rm out}^{(\infty)}}{A_{\rm in}^{(\infty)}} \right|^2 \;. \tag{7}$$

Some typical examples of the absorption probability as a function of $\omega_\star$ ($=\omega r_h^2$) in the asymptotically flat limit are shown in the top two panels of Fig. 1. Solutions with charge or rotation undergo super-radiance

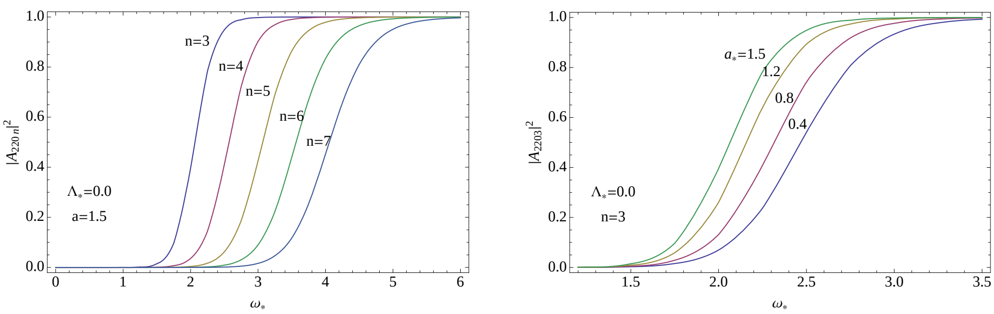
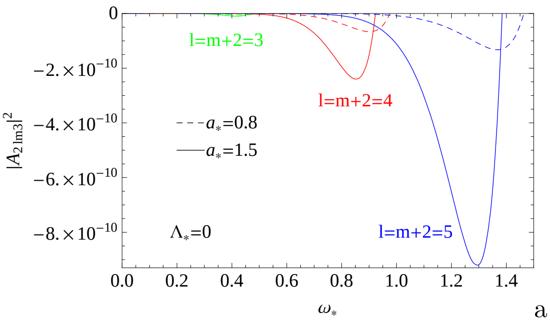


Figure 1: *Various plots of absorption probabilities in the asymptotically flat case, where unless stated all plots are for $n = 3$. Note that on the scale of these plots superradiance is too small to discern.*

[12], where the condition for super-radiance to occur is that the absorption probability becomes negative. Some plots of the superradiance regime are shown in Fig. 2. An interesting feature of black holes in Kerr-dS spacetimes is that the superradiance effect is enhanced by the strength of the cosmological constant, this can be seen from Fig. 2.

2 Conclusion

This note is based on our more expanded paper [13] which was the first time that the Hawking emission of these perturbations was been calculated. Some of the results can be seen in Figs. 3. The results are consistent with those of other works [10], where they considered the bulk emission of scalar spin-0 fields on the Kerr-AF background. An important difference is that because the modes start from $j = 2, 3, 4, \ldots$ the spectrum is shifted to the right (larger j corresponds to larger scattering energies ω). For spin-0 fields the sums start from $j = 0$, which implies lower energy emissions. The lack of $j = 0, 1$ modes has another effect, which can be seen from Fig. 4.



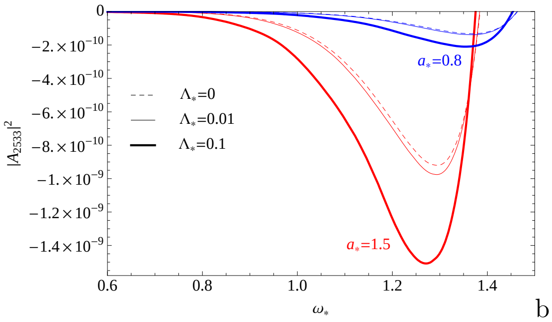


Figure 2: *Absorption probability plots in the superradiance regime for the asymptotically flat case. In (b) superradiance plots for various choices of the cosmological constant (Λr_h^2), for the rotations $a_\star = 0.8$ (blue) and $a_\star = 1.5$ (red).*

Perhaps the most interesting result from our investigation of the Kerr-dS case is the effect that the cosmological constant has on enhancing superradiance a larger cosmological constant leads to more superradiance and hence will cause the black hole to spin down more quickly. Hopefully, within this decade, a separable set of Master equations for all the graviton perturbations will be obtained.

References

[1] H. Kodama, Lect. Notes Phys. **769**, 427 (2009) [arXiv:0712.2703 [hep-th]].

[2] H. Kodama and A. Ishibashi, Prog. Theor. Phys. **110**, 701 (2003) [arXiv:hep-th/0305147].

[3] H. Kodama, A. Ishibashi and O. Seto, Phys. Rev. D **62**, 064022 (2000) [arXiv:hep-th/0004160].

[4] H. Kodama, Prog. Theor. Phys. Suppl. **172**, 11 (2008) [arXiv:0711.4184 [hep-th]].

[5] R. C. Myers and M. J. Perry, Annals Phys. **172**, 304 (1986).

[6] H. Kodama, R. A. Konoplya and A. Zhidenko, Phys. Rev. D **79**, 044003 (2009) [arXiv:0812.0445 [hep-th]].

[7] H. Kodama, R. A. Konoplya and A. Zhidenko, arXiv:0904.2154 [gr-qc].

[8] E. Berti, V. Cardoso and M. Casals, Phys. Rev. D **73**, 024013 (2006) [Erratum-ibid. D **73**, 109902 (2006)] [arXiv:gr-qc/0511111].

[9] M. A. Rubin and C. R. Ordonez, J. Math. Phys. **26**, 65 (1985); M. A. Rubin and C. R. Ordonez, J. Math. Phys. **25**, 2888 (1984), URL http://link.aip.org/link/?JMP/25/2888/1

[10] M. Casals, S. R. Dolan, P. Kanti and E. Winstanley, JHEP **0806**, 071 (2008) [arXiv:0801.4910 [hep-th]].

[11] D. Ida, K. y. Oda and S. C. Park, Phys. Rev. D **71**, 124039 (2005) [arXiv:hep-th/0503052].

[12] B. S. DeWitt, Phys. Rept. **19**, 295 (1975).

[13] Doukas, Jason and Cho, H. T. and Cornell, A. S. and Naylor, Wade, Phys. Rev.D80(2009) 045021.

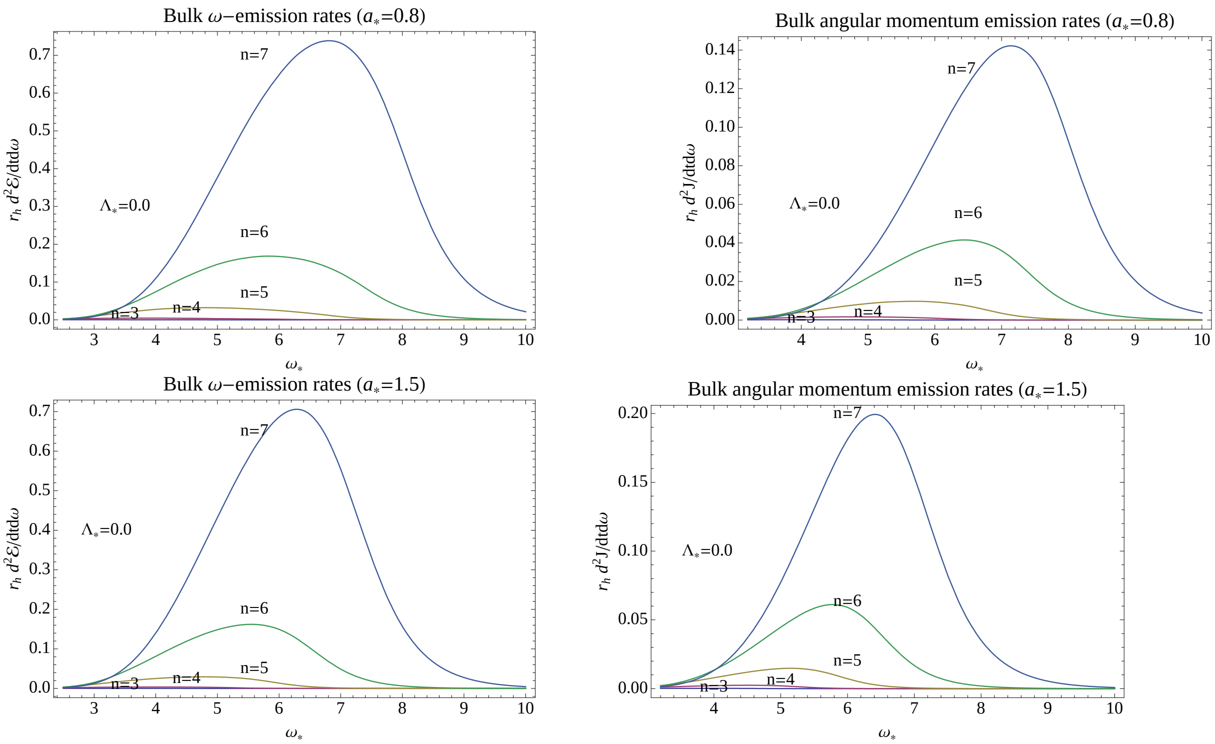


Figure 3: *Energy and angular momentum emissions in asymptotically flat space* ($\Lambda = 0$) *for different dimensions* $n + 4$ *with* $a/r_h = 0.8$ *(top) and with* $a/r_h = 1.5$ *(bottom).*

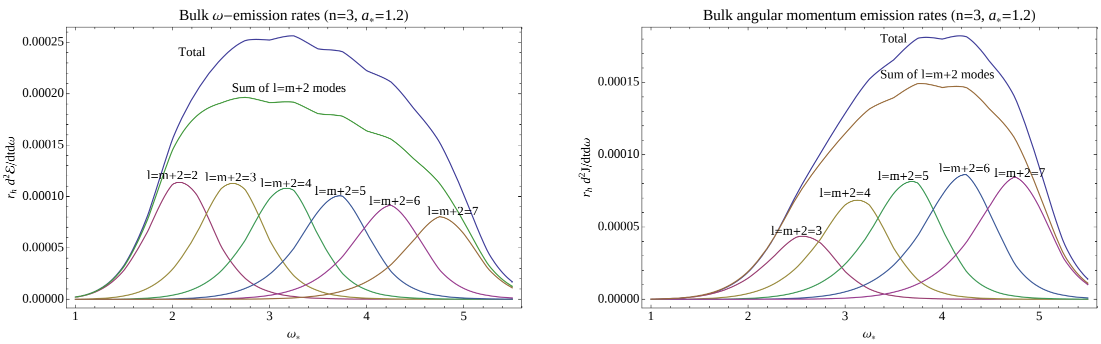


Figure 4: *Contribution to energy and angular momentum emissions from the dominant* $l = m+2 = 3, \ldots$ *modes for* $n = 3$ *and* $a_\star = 1.2$ *(with* $\Lambda = 0$*).*