Post-Linear Formalism for Gravitating Cosmic Strings

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Abstract

Linear and post-linear formalisms are generalized to incorporate gravitating Nambu and superconducting cosmic strings. The collision of two straight non-parallel strings is analised. When strings scatter at a sufficiently small angle the point of their minimal separation can move with a faster-than-light velocity, and a Cherenkov-like radiation can be anticipated. However, it is shown that for Nambu strings in the post-linear order the gravitational reaction is precisely zero. It is argued that any "faster-than-light" crossed colliding strings configuration is essentially equivalent to the parallel one which is described by the 1+2 gravity while any "slower-than-light" configuration may be reduced to some static distribution of matter. Thus in both cases we have no gravitational radiation. But we can obtain a powerful source of a Cherenkov-like electromagnetic radiation if at least one of the strings is a superconducting one. The same approach can be used in many other applications. It is shown, that this formalism provides a way to calculate vacuum polarization and to consider the problem of topological self-action of a classical charged particle in a multiconical space-time.

1 Introduction

A Complete understanding of different mechanisms of gravitational and electromagnetic radiation by cosmic strings is important to incorporate them into a realistic cosmological scenario [1]. The hypothesis of scaling solution [2] is intimately related to the conversion of the string network energy into gravitational radiation. Electromagnetic radiation seems to be one of the most important mechanisms of energy losses by superconducting strings [3]. In most cases radiation emitted by excited strings may be computed using the linearized theory. For example, the important case of radiation produced by oscillating loops falls into this category. In the case of long straight strings gravitational interaction seems to play a more important role because straight strings freely moving in the Minkowski space-time do not radiate.

We shall develop here a formalism that will allow to calculate electromagnetic and gravitational radiation from strings in situations in which relevant accelerations are due to their dynamical gravitational interaction. In such cases the linearized theory describes mutual gravitational interaction of the strings and the produced electromagnetic radiation (if one of the strings is a superconducting one), while gravitational radiation appears to be the effect of the second post-linear order.

The Poincare-covariant perturbation theory developed here for gravitating cosmic strings is similar to the scheme used some time ago to treat gravitational and electromagnetic radiation from gravitationally interacting point particles [4-7]. Our primary goal in reanimating this approach was to examine the problem of collision of non-parallel straight strings. When strings are parallel the problem reduces to 1+2 gravity interacting with point particles. In this theory there is no room for gravitational waves at all and the space-time is flat everywhere outside the sources [8]. (Note, that we can not use the same arguments in the case of electromagnetic radiation, when superconducting strings are introduced in the string network, because of the existence of electromagnetic waves in 1+2 -dimensional space-time!) For non-parallel strings the situation seems to be much more interesting. In fact, when two such strings collide (but not intersect) at a sufficiently small angle, the point of there minimal separation can move faster-than-light. Since it is around this point gravitational stresses are most significant, Cherenkov-like radiation can be anticipated. To check whether it is indeed the case we perform explicit calculations. It turns out that in the case of two Nambu strings the corresponding post-linear amplitudes vanish for both independent polarization states of emitted gravitational wave. This result clearly indicates the presence of some

hidden symmetry of the Einstein-Nambu action. It may be shown [9], that crossed strings configuration can be transformed into parallel one provided the above Cherenkov condition holds. There is no gravitational radiation in the case of two parallel Nambu strings. But if one of the strings is a current-carrying one, we can obtain a powerful source of Cherenkov-like electromagnetic radiation [10]. If the point of minimal separation moves with a slower-than-light velocity the same symmetry enables one to transform the non-parallel strings configuration into a static distribution of matter, charges and currents [11]. Thus one can not obtained both gravitational and electromagnetic radiation in this case.

The smallness of the angle deficit parameter for GUT strings ($G\mu \sim 10^{-6}$) enables one to use the same perturbation scheme in many other applications involving gravitating infinitely long strings. In particular, this formalism provides a way to calculate vacuum polarization in a multistring space-time and to consider self-action of a point charged particle. These two effects are quite different from the first point of view, but nevertheless have the same explanation and may be treated in a parallel way. Indeed quantum and classical fields are both sensitive to the global structure of the space-time. Thus non-trivial boundary conditions alter both zero-point fluctuations of a quantum field and a classical electrostatic field of a charged particle leading to the effects mentioned above.

2 Einstein - Nambu - Nielsen - Olesen action

Consider the system of infinitely thin strings interacting through their gravitational and electromagnetic fields. It is described by the action (we omit indexes numerating the strings)

$$S = S_{gr} + S_{em} + \sum S_N + \sum S_{NO} + \sum S_{int}, \tag{1}$$

where $S_{gr} = -(16\pi G)^{-1} \int R\sqrt{-g}\, d^4x$ - is the Einstein action, $S_{em} = -(16\pi)^{-1} \int F^2 \sqrt{-g}\, d^4x$. Each of ordinary strings is described by the action (Nambu action)

$$S_N = -\frac{\mu}{2} \int \sqrt{-\gamma}\gamma^{AB} \frac{\partial x^\mu}{\partial \zeta^A} \frac{\partial x^\nu}{\partial \zeta^B} g_{\mu\nu}[x(\zeta)]\, d^2\zeta. \tag{2}$$

For a current- carrying (superconducting) string we choose the action, which was proposed by Nielsen and Olesen [12]

$$S_{NO} = \int \sqrt{-\gamma}\gamma^{AB}(\zeta)[-\frac{\mu}{2}\frac{\partial x^\mu}{\partial\zeta^A}\frac{\partial x^\nu}{\partial\zeta^B}g_{\mu\nu}[x(\zeta)] + \frac{1}{2}\frac{\partial\phi}{\partial\zeta^A}\frac{\partial\phi}{\partial\zeta^B}]\, d^2\zeta, \quad (3)$$

where ϕ is a scalar field on the world-sheet of superconducting string. This field determines electromagnetic properties of the string. In particular, interaction of the string with an external electromagnetic field is described by the term

$$S_{int} = -e\int \sqrt{-\gamma}\varepsilon^{AB}\frac{\partial x^\mu}{\partial\zeta^A}\frac{\partial\phi}{\partial\zeta^B}A_\mu[x(\zeta)]\, d^2\zeta, \quad (4)$$

where $\varepsilon^{AB} = \frac{e^{AB}}{\sqrt{-\gamma}}$ - two-dimensional antisymmetric tensor ($e^{01} = -e^{10} = -1$). In the expressions above A^μ is the 4-potential of the electromagnetic field, $g_{\mu\nu}$ is the space-time metric generated by the strings, R is the scalar curvature, G is the Newton constant, μ is the string tension parameter, γ_{AB} is the metric on the two-dimensional world-sheet of the string. From the constraints one obtains that for Nambu string this metric is the induced one

$$\gamma_{AB} = \frac{\partial x^\mu}{\partial\zeta^A}\frac{\partial x^\nu}{\partial\zeta^B}g_{\mu\nu}. \quad (5)$$

While in the case of superconducting string we must use some more complicated expression

$$\mu g_{\mu\nu}\partial_A x^\mu\partial_B x^\nu - \partial_A\phi\partial_B\phi - \frac{1}{2}\gamma_{AB}\gamma^{CD}(\mu g_{\alpha\beta}\partial_C x^\alpha\partial_D x^\beta - \partial_C\phi\partial_D\phi) = 0. \quad (6)$$

The action (1)-(4) is invariant under an arbitrary reparametrizations of the world-sheets, under diffeomorphisms of space-time and under gauge transformations of A^μ.

The variation of the action with respect to the space-time metric $g_{\mu\nu}$ gives the Einstein equations with the energy-momentum tensor

$$T^{\mu\nu} = \sum \mu\int \gamma^{AB}\frac{\partial x^\mu}{\partial\zeta^A}\frac{\partial x^\nu}{\partial\zeta^B}\frac{\delta^4(x - x(\zeta)}{\sqrt{-g}}\sqrt{-\gamma}d^2\zeta + T^{\mu\nu}_{em}. \quad (7)$$

The Bianchi identities reproduce the equations of motion. In the conformal gauge $g_{\mu\nu}\dot{x}^\mu x'^\nu = 0, \quad g_{\mu\nu}(\dot{x}^\mu\dot{x}^\nu + x'^\mu x'^\nu) = 0$ (coordinates on the

world-sheets are specified as usual $\zeta^0 = \tau, \zeta^1 = \sigma$ the and corresponding derivatives are denoted by a dot and a prime respectively) they read

$$\ddot{x}^\mu - x''^\mu + \Gamma^\mu_{\nu\lambda}(\dot{x}^\nu\dot{x}^\lambda - x'^\nu x'^\lambda) = \frac{e}{\mu}F^{\mu\nu}e^{AB}\partial_A x_\nu\partial_B\phi, \quad (8)$$

where e is the charge of current carriers. In the case of Nambu string the right-hand side of the equation is equal to zero. For the scalar fields we obtain the equations

$$\ddot{\phi} - \phi'' = -\frac{e}{2}F_{\mu\nu}e^{AB}\partial_A x^\mu\partial_B x^\nu. \quad (9)$$

Eqs. (8) and (9) must be solved together with the Maxwell equations with the current

$$j^\mu(x) = \sum e\int \sqrt{-\gamma}\varepsilon^{AB}\partial_A x^\mu\partial_B\phi\frac{\delta^4(x - x(\zeta))}{\sqrt{-g}}d^2\zeta. \quad (10)$$

Some exact solutions of the coupled Einstein-Nambu system for an arbitrary number of parallel Nambu strings are known [13-14]. To construct a perturbative scheme that would allow to analyze more general situations we break the general covariance of the Einstein-Nambu-Nielsen-Olesen system and cast it into Poincare-covariant form. So we write

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad (11)$$

where $\eta_{\mu\nu} = diag(1,-1,-1,-1)$ is a flat-space metric in Cartesian coordinates and $h_{\mu\nu}$ is a symmetric Minkowski-space tensor. Hereon the raising and lowering of indices will be performed with $\eta_{\mu\nu}$. It should be emphasized that neither $h_{\mu\nu}$ is supposed to be small, nor the Minkowski metric has to be considered as a metric of the physical space-time.

The next step is to impose the flat space-time de-Donder gauge

$$\partial_\nu\psi^{\mu\nu} = 0, \quad (12)$$

where $\psi_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}h\eta_{\mu\nu}, \quad h = h_{\alpha\beta}\eta^{\alpha\beta}$, and substitute the Eq.(11) into the Einstein tensor. This gives

$$\eta^{\alpha\beta}\partial_\alpha\partial_\beta\psi^{\mu\nu} = -16\pi G\tau^{\mu\nu}, \quad (13)$$

where an effective stress-energy tensor consists of a material part and a gravitational part

$$\tau^{\mu\nu} = T^{\mu\nu} + S^{\mu\nu}. \tag{14}$$

In Eq.(14) $S_{\mu\nu}$ denotes all non-linear terms coming from the Einstein tensor. Maxwell equations can be rewritten in the same form

$$\eta^{\alpha\beta}\partial_\alpha\partial_\beta A^\mu = 4\pi i^\mu, \; i^\mu = \sqrt{-g}(j^\mu + S^\mu), \tag{15}$$

where

$$S^\mu = \frac{1}{4\pi\sqrt{-g}}\partial_\nu[(\sqrt{-g}g^{\mu\lambda}g^{\nu\sigma} - \eta^{\mu\lambda}\eta^{\nu\sigma})F_{\lambda\sigma}]. \tag{16}$$

Recall again, that in spite of a linearized-like appearance, (13) and (15) are still exact Einstein and Maxwell equations written in a Minkowski-covariant form.

3 Iterative scheme

To proceed further we expend $h_{\mu\nu}$ and A^λ in terms of gravitational constant G

$$h_{\mu\nu} = \sum_{l=1}^{\infty} h^{(l)}_{\mu\nu}, \quad A^\lambda = \sum_{l=0}^{\infty} A^{(l)\lambda}. \tag{17}$$

In actual calculations involving strings dimensionless parameters in the expansions turn out to be the Lorentz-enhanced conical deficit angles [9]. The left hand sides of the Eqs.(13), (15) contain all orders of the expansions (17), and in order to build a series in terms of G one has to apply an iterative scheme. This has to be done together with the expansion of the world-sheets variables

$$x^\mu = \sum_{l=0}^{\infty} x^{(l)\mu}(\zeta). \tag{18}$$

Combining both series we collect all terms up to a given order of G in the right hand sides of Eqs. (13) and (15) get the expansions in terms of the gravitational constant

$$\tau_{\mu\nu} = \sum_{l=0}^{\infty} \tau^{(l)}_{\mu\nu}, \quad i_\mu = \sum_{l=0}^{\infty} i^{(l)}_\mu. \tag{19}$$

Using this expansions we now rewrite the Einstein equations as an infinite system of flat-space d'Alambert equations

$$\eta^{\alpha\beta}\partial_\alpha\partial_\beta\psi^{(l)}_{\mu\nu} = -16\pi G\tau^{(l-1)}_{\mu\nu}. \tag{20}$$

In the same way we can rewright the Maxwell equations

$$\eta^{\alpha\beta}\partial_\alpha\partial_\beta A^{(l)\mu} = 4\pi i^{(l)\mu}, \tag{21}$$

Non-linear terms in the strings equations of motion (8) have to be expanded according to Eqs.(17) and (18), and then by collecting all relevant products of lower order terms of the metric, the electromagnetic field and the world-sheet variables, the following set of non-homogeneous equations is obtained

$$(\partial_\tau^2 - \partial_\sigma^2)x^{(l)\mu} = f^{(l)\mu}. \tag{22}$$

In the zero order the force terms in the right hand side vanish and we get homogeneous equations for the input string world sheets which has to be solved together with the constraints.

4 Collision of two straight Nambu strings

In this section we briefly reproduce the results of our previous papers [9-11]. Let us examine a collision of two non-parallel straight Nambu strings moving with a non-zero impact parameter. For such kinematics of the motion the question arises whether a Cherenkov gravitational radiation is produced if the point of minimal separation between the strings movers faster than light in the rest frame of one of them. We start with the following input world-sheets, obviously satisfying the zero-order equations of motion (it is necessary to introduce an index specifying the strings at this point)

$$x^\mu_a = d^\mu_a + u^\mu_a\tau + \Sigma^\mu_a\sigma, \quad a = 1, 2. \tag{23}$$

Here the constant four-vectors d^μ_a, u^μ_a and Σ^μ_a are the impact parameter, four-velocity and four-orientation vector of each string. It is convenient to choose them as follows

$$u^\mu_1 = (1, 0, 0, 0), \quad \Sigma^\mu_1 = (0, 0, 0, 1),$$

$$u_2^\mu = \gamma(1, 0, -v\cos\alpha, v\sin\alpha), \quad \Sigma_2^\mu = (0, 0, sin\alpha, cos\alpha), \tag{24}$$

where Cartesian coordinates t, x, y, z are understood and $\gamma = (1 - v^2)^{-1/2}$. In this frame the velocity of the point of minimal separation can be written as follows

$$v_P = \frac{v}{sin\alpha}. \tag{25}$$

It is also convenient to chose the impact parameters satisfying the orthogonality conditions

$$(d_a u_a) = 0 = (d_a \Sigma_a). \tag{26}$$

For the Fourier-transform of $h_a^{(1)\mu\nu}$ from the Eq.(20) we obtain

$$h_a^{(1)\mu\nu}(q) = \frac{(4\pi)^3 G\mu_a}{q^2} \pi_a^{\mu\nu} e^{i(qd_a)} \delta(qu_a)\delta(q\Sigma_a), \tag{27}$$

where $(qd_a), (qu_a), (q\Sigma_a)$ are Minkowskian scalar products and

$$\pi_a^{\mu\nu} = u_a^\mu u_a^\nu - \Sigma_a^\mu \Sigma_a^\nu - \frac{1}{2}\eta^{\mu\nu}(u_a^2 - \Sigma_a^2). \tag{28}$$

We see that delta-functions in the Eq.(27) shift the momentum q from the pole $q^2 = 0$. Physically this means that the amplitude of the gravitational radiation in the first order of the perturbation theory is equal to zero.

The next step is to find corrections to the world-sheets of strings due to there gravitational interaction. Substituting (27) into (22) we obtain the following expression

$$x_a^{(1)\mu} = \int \frac{F_a^\mu(q)}{(q\Sigma_a)^2 - (qu_a)^2} e^{-i(qd_a + qu_a\tau + q\Sigma_a\sigma)} \frac{d^4q}{(2\pi)^4}, \tag{29}$$

where

$$F_1^\mu(q) = \frac{32\pi^3 iG\mu_2}{q^2} e^{i(qd_a)} (2\pi_2^{\mu\alpha} q^\beta - \pi_2^{\alpha\beta} q^\mu)(u_{1\alpha}u_{1\beta} - \Sigma_{1\alpha}\Sigma_{1\beta})\delta(qu_2)\delta(q\Sigma_2) \tag{30}$$

and similarly for another string.

First order corrections to the strings world-sheets are used to build the Fourier-transforms of the material contributions to the post-linear stress tensor. To build the gravitational stress-energy tensor we must contract two first-order Fourier-transforms (27) coming from different strings on the graviton mass-shell and obtain the expression for $S^{\mu\nu}$. We can present the post-linear tensor amplitude in the form

$$\tau^{\mu\nu}(k) = 16\pi G\mu_1\mu_2 \int \left(\frac{\theta_1^{\mu\nu}}{q^2} + \frac{\theta_2^{\mu\nu}}{(q-k)^2}\right) e^{iqd_2 - iq(q-k)d_1} D(q) d^4q. \tag{31}$$

Here $D(q)$ is the product of delta-functions

$$D(q) = \delta((k-q)u_1)\delta((k-q)\Sigma_1)\delta(qu_2)\delta(q\Sigma_2), \tag{32}$$

and $\theta_a^{\mu\nu}$ are effective contributions from two strings. One can find the explicit expression for $\theta_a^{\mu\nu}$ in our paper [9].

Now we can show how Cherenkov condition arises. Indeed, from the conservation lows (32) one obtains following relation

$$k^0 = k^z v/sin\alpha. \tag{33}$$

Obviously it can be satisfied if and only if the following inequality holds

$$v_P = v/sin\alpha > 1. \tag{34}$$

Clearly it is just the Cherenkov condition for the angle at which gravitons are emitted by a faster-than-light source.

We proceed as follows. First of all we perform three of the four integrations in (31) by means of the delta-functions (32). After that the last integration on the x component of the momentum q is performed by closing the contour in the upper half-plane of the complex variable q^x.

The calculation of the 4-momentum flux at the infinity associated with the second order gravitational potentials $\psi_{\mu\nu}^{(2)}$ leads to the following expression for the total 4-momentum loss due to gravitational radiation

$$\Delta P^\mu = \frac{G}{\pi^2} \int k^\mu \theta(k^0)\delta(k^2)(\tau_{\alpha\beta}(k)\tau^{\alpha\beta *}(k) - \frac{1}{2}\tau(k)\tau^*(k))d^4k, \tag{35}$$

where $\tau_{\alpha\beta}(k)$ is the Fourier-transform of the second order effective stress-energy tensor (19). The fourth delta-function which selects the Cherenkov condition (34) will appear squared in the integrand in (35). An extra one must be converted to the normalization length L_z

$$\delta^2(k^z - \frac{\omega}{v_P}) \to \frac{L_z}{2\pi}\delta(k^z - \frac{\omega}{v_P}), \tag{36}$$

21 Заказ 636

and omitted giving the radiation loss per unit length of the string at rest. After that only algebraic manipulations is needed to get the final result. We calculated the projections of tensor amplitudes (31) onto two independent polarization states of the emitted gravitational wave. The result was an explicit zero [9].

This result makes us suspect the presence of some symmetry of the Einstein-Nambu action which reduces the system of colliding non-parallel strings to the parallel ones which is described by 1+2 gravitational theory. In fact, our system is symmetric under the Poincare transformations of the embedding space as well as under independent reparametrizations of the string world-sheets. From these symmetries one can construct some special transformation which takes a crossed string configuration into a parallel one provided the faster-than-light condition (34) holds. The proof is based on the fact that one can use any transformations which i) leave invariant our Poincare-covariant system (13) and (22), and ii) transform the input configuration (23) into the parallel configuration. Since the solution is supposed to be constructed up to an arbitrary order of the gravitational constant starting from this input configuration, the existence of such a transformation can be regarded as a proof of the above statement. The explicit form of this transformation one can find in our paper [9].

We see that for the slower-than-light motion of the point of minimal separation gravitational radiation was forbidden kinematically. This result has a very simple explanation based on the same symmetry of the system considered [11]. In this case one can perform the Lorentz transformation from the initial frame to the frame moving along z-axis with the speed equal to v_P. After that, using the symmetry of the system under reparametrizations of the world-sheet, we obtain that in the new Lorentz frame we have a static distribution of matter. If background geometry is a static one, corresponding retarded, advanced and radiative Green functions depend on the observer's time t and the time of radiation t' through the difference $(t-t')$. Therefor the retarded potentials of any static source are independent of time. But the energy emitted is the function of the time derivatives of the radiative potentials, and thus vanishes.

Thus the absence of gravitational bremsstrahlung under collision of straight Nambu strings is an exact result for any their relative orientation.

5 Cherenkov radiation of superconducting cosmic strings

If one of the strings is a superconducting one the calculation can be made by the same way. In the absence of gravitational interaction the world surfaces of the strings are parametrized by the equations $x^\mu_a = d^\mu_a + U^\mu_a\zeta^0 + \Sigma^\mu_a\zeta^1$. It is convenient to perform the calculations in the rest frame of the ordinary (Nambu) string $(a=1)$, so

$$U^\mu_1 = (1,0,0,0), \quad \Sigma^\mu_1 = (0,0,0,1).$$

And for the world-sheet of the superconducting string $(a=2)$ we obtain

$$U^\mu_2 = \gamma(1,\ v\cos\alpha, 0, v\sin\alpha), \quad \Sigma^\mu_2 = \varepsilon(0, -sin\alpha, 0\cos\alpha), \tag{37}$$

where ε can be found from the equation (6). These values are used as the initial data for the next step of the iteration procedure. At first we must calculate the retarded solutions for potentials $\psi^{(}_{\mu\nu}1)$ and $A^{(}_\lambda 0)$. At this step the tensions S in the right parts of (13) and (15) are ignored. Then one must calculate the supplementary deformation of the world surface of the second (current-carrying) string and the corresponding perturbations of the current i^μ.

Four-momentum loss due to electromagnetic radiation is calculated with the help of the formula

$$\Delta P^\mu = \frac{1}{2\pi^2}\sum_{\lambda=\theta,\phi}\int d^4k\ k^\mu\Theta(k^0)\delta(k^2)\mid \vec{e}^{(\lambda)}(k)\vec{i}(k)\mid^2, \tag{38}$$

where λ - polarization index, $\vec{i}(k)$ - is the Fourier-transform of the first order effective current. It may be proved, that $\vec{i}(k)$ nonvanishes if the Cherenkov condition (34) holds only. In the rest frame of the Nambu string photons wave vectors form the cone withe the angle θ:

$$\cos\theta = \frac{\sin\alpha}{v} \tag{39}$$

The total energy loss due to electromagnetic radiation with the polarization $\lambda = \theta, \phi$ is the following

$$\frac{dE^{(\theta)}}{d\omega} = (4\pi G\mu_1 I_2)^2\ \gamma^3 v^2\cos^3\alpha\ \frac{e^{-\frac{2\omega d}{\gamma v}}}{\omega},$$

$$\frac{dE^{(\phi)}}{d\omega} = (\frac{tg\ \alpha}{\gamma v})^2 \frac{dE^{(\theta)}}{d\omega} , \tag{40}$$

where I is invariant current amplitude, d is the impact parameter. Formulae (40) describe the case of the space-like current, for the time-like current one have to exchange the polarization indexes. This may be connected with the symmetries of the action, in the same way as in the case of nonsuperconducting strings [9]. The logarithmic infrared divergence in the spectra can be eliminated by introducing length parameter R. It is a distance, where collective gravitational effects of the string network begin playing role. As a result for the full energy loss per unit length of the ordinary string we obtain the expression

$$\Delta E = (4\pi G\mu_1 I_2)^2 (v^2\gamma^3 \cos^3\alpha + \gamma \cos\alpha \sin^2\alpha) \ln\frac{\gamma v R}{2d} . \tag{41}$$

If $\alpha = 0$ the Cherenkov condition is fulfilled for all v. In this case the problem turns out to be 1+2 electrodynamics, so the nontrivial relation between these two theories is found. Using the symmetries of the action one can prove, that 1+2 interpretation can be expanded for the case of nonparallel strings too [9].

When two parallel bosonic strings at $v \simeq 1$ are taken into account, then

$$\frac{\Delta E}{\mu_2 \gamma} \simeq 10^{-11} \left(\frac{\gamma I}{I_{cr}}\right)^2 \ln(\frac{\gamma R}{2d}) , \tag{42}$$

where $I_{cr} = e\sqrt{\mu} \simeq 10^{22} A$ is the critical current. So at high speed the energy loss may be comparable with the string energy even at $I < I_{cr}$.

If $\alpha > 0$, the stationary source appears. In this case the intensity of radiation is proportional to the velocity of the point of minimal separation $v_P = v/sin\alpha$, as it must be expected,and we obtain

$$\frac{dE}{dt} \simeq 10^{41}\ (\frac{v_p}{c}) \left(\frac{\gamma I}{I_{cr}}\right)^2 \gamma \ln(\frac{\gamma R}{2d}) \quad \text{[erg/sec]}. \tag{43}$$

We think that this new mechanism of radiation from cosmic strings is important to incorporate superconducting strings into cosmological scenario. Moreover, it seems that electromagnetic radiation produced by current-carrying strings can be used in a search for the observable manifestations of superconducting strings.

6 Topological self-action and vacuum polarization in multiconical space-time

The aim of this section is to determine linear and post-linear corrections to the Euclidian Green function for a massless scalar field with arbitrary coupling in the space-time of N parallel cosmic strings. The coincidence limit of this terms and their derivatives enables one to calculate the vacuum expectation values $< \phi^2 >_{vac}$ and $< T_{00} >_{vac}$ and to consider the problem of topological self-action of the point charged particle in the space under consideration.

Let us consider the n -dimensional generalization of the multistring space-time [13] (in this section it is more convenient to make choice of the metric with the signature $(-,+,+,+)$)

$$ds^2 = -dt^2 + dx_{n-1}^2 + \ldots + dx_3^2 + e^{-\Omega(x_c)}\delta_{ab}dx_a dx_b , \quad a,b,c,... = 1,2, \tag{44}$$

where

$$\Omega(x_c) = \sum_{i=1}^{N} 8G\mu_i \ln r_i , \tag{45}$$

$r_i = [(x_1-\alpha_i)^2 + (x_2-\beta_i)^2]^{\frac{1}{2}}$ and μ_i - is a dimensionless parameter. If $n = 4$ μ_i become the masses per unit length of the strings. Our space-time has the structure $M_{n-2}\times V_2$, where M_{n-2} is the $(n-2)$ - dimensional Minkowski space and V_2 is the two-dimensional locally flat Riemanian space with N conical singularities at the points (α_i, β_i). We will consider the case $G\mu_i << 1$ because for the real cosmic strings $G\mu_i$ is about 10^{-6}.

Because of the certain advantages in working within the Euclidian approach let us replace t by $-ix_n$ in the line element (44). The manifold is now described by the Riemanian metric

$$ds^2 = dx_n^2 + dx_{n-1}^2 + \ldots + e^{-\Omega(x_c)}\delta_{ab}dx^a dx^b . \tag{46}$$

Euclidian Green function is the fundamental solution of the Poisson equation in the space-time (46). For our choice of coordinates it takes the form

$$\Delta_0^n G_E(x,x') = -\delta^n(x-x') - VG_E(x-x'), \tag{47}$$

where $\Delta_0^n = \delta_{\mu\nu}\partial^\mu\partial^\nu$, $\mu,\nu,\ldots = 1,2,\ldots,n$ - is the Laplacian operator in the n - dimensional Euclidian space, and we introduce operator V

$$V = -(1 - e^{-\Omega(x_c)}) \sum_{\mu=3}^{n} \frac{\partial^2}{\partial x_\mu^2} . \quad (48)$$

If all $G\mu_i << 1$ this operator may be considered as a small perturbation and one can write

$$G_E = G_E^0 + G_E^0 V G_E^0 + G_E^0 V G_E^0 V G_E^0 + \ldots , \quad (49)$$

Now we are in a position to give the approximate expression for the Euclidian Green function. Substituting (48) into (49) we obtain the first order correction to the Green function

$$G_E^{(1)}(x,x') = \int \frac{d^n q}{(2\pi)^2} \prod_{i=3}^{n} \delta(q_i)\, e^{i(qx)} \Omega(q_c)\, I^{(n)}(q) , \quad (50)$$

where

$$I^{(n)}(q) = \int \frac{d^n p}{(2\pi)^n} \frac{\sum_{i=3}^{n} p_i^2}{p^2 (p-q)^2} e^{-ip(x-x')} . \quad (51)$$

To obtain the regularized value of $G_E^{(1)}$ in the coincidence limit $x' \to x$ it is convenient to use the method of dimensional regularization. Using this method one obtains that

$$G_E(x,x) = -\sum_{i=1}^{N} \frac{2G\mu_i \ln r_i}{\pi} \qquad n = 2 \quad (52)$$

and

$$G_E(x,x) = \frac{2}{\pi^{n/2}} \frac{\Gamma^3(n/2)}{(n-2)\,\Gamma(n)} \sum_{i=1}^{N} \frac{G\mu_i}{r_i^{n-2}} \qquad n \geq 3. \quad (53)$$

We begin our treatment of classical and quantum effects in the multiconical space-time with a study of the electrostatic field of a classical point charged particle. Both classical and quantum fields are sensitive to the global structure of the manifold. This means that a regularized value of a self-energy of a charged point particle in the curved background must be a

function of its coordinates. Indeed, the self-energy of a test charge is given by the expression

$$U(x) = 2\pi e^2\, G_E(x,x) . \quad (54)$$

Where G_E is given by (53) Thus in 4-dimensional space-time the electrostatic self-energy has the form

$$U_{reg}(x) = \sum_{i=1}^{N} \frac{\pi e^2 G\mu_i}{4 r_i} . \quad (55)$$

We see that in the first order of the perturbation theory the point charged particle interacts with any string as it was a charge $e\pi G\mu/4$ at the distance r in the Euclidian space. Of cause this interpretation of the result is valid in the lowest order only. If $N = 1$ our result coincides with that of the papers [15].

It is well known that nontrivial boundary conditions alter the zero-point fluctuations of quantum fields leading to the existence of a vacuum polarization. The multiconical boundary conditions must lead to similar effects. Expression of the Euclidian Green function enables us to evaluate the vacuum average $< \phi^2 >_{vac}$ in the case in which for all i $G\mu_i << 1$

$$< \Phi^2 >_{vac} = \lim_{x' \to x} G_E^{reg}(x,x') . \quad (56)$$

Substituting (53) into (56), we obtain $n = 4$)

$$< \Phi^2 >_{vac} = \sum_{i=1}^{N} \frac{G\mu_i}{6\,\pi^2\, r_i^2} . \quad (57)$$

Given the Euclidian Green function on the manifold under consideration the vacuum energy-momentum tensor may be determined by

$$< T_\mu^\nu >_{vac} = \lim_{x' \to x} D_\mu^{\nu'} G_E^{reg}(x,x') \quad (58)$$

where operator $D_\mu^{\nu'}$ takes the form

$$D_\mu^{\nu'} = [(1-2\xi)\nabla_\mu^{\nu'} - 2\xi\nabla_\mu^\nu + (2\xi - \frac{1}{2})\, \delta_\mu^\nu \nabla_\lambda^{\lambda'}], \quad (59)$$

The coincidence limits of the various derivatives of the function $G_E(x,x')$ in the expressions (58) can be obtained in the same manner as in determination of $G_E^{reg}(x,x)$. One obtains

$$< T_{00} >_{vac} = -\frac{\Gamma^3(\frac{n}{2})}{\pi^{n/2}\Gamma(n)}\left\{\frac{n}{2(n+1)} + (n-2)(2\xi - \frac{1}{2})\right\} \sum_{i=1}^{N} \frac{G\mu_i}{r_i^n} \ . \quad (60)$$

If we consider only one string ($N = 1$) our result coincides with that of the papers [16].

All the effects considered before are the effects of the first order in gravitational constant G. In this order of the perturbation theory contributions from different strings add to each other, and superposition principle takes place. If one goes to the second postlinear order some new effects are revealed. Vacuum interaction of straight parallel cosmic strings is one of them. Indeed it may be shown that postlinear contribution to $G_E(x,x)$ and its derivatives consists of contributions of two different kinds. Contributions of the first kind are proportional to μ_i^2 and are simply the small corrections to our previous results. Another terms are proportional to the products $\mu_i\mu_k$, $i \neq k$ and are the functions of the distance between the strings. To show it we start from the expression for the total vacuum energy per unit length of the strings

$$E_{int} = -\int dx_1 dx_2 \sqrt{g^{(2)}} \ < T_0^0(x) >_{vac} \ . \quad (61)$$

In the postlinear order

$$e^{-\Omega(x_c)} < T_0^0(x) >_{vac} = \lim_{x' \to x} \{\partial_n \partial^{n'} [G_E^{(2)}(x',x) - \Omega(x_c) G_E^{(1)}(x',x)] +$$

$$(2\xi - \frac{1}{2})[\partial_\mu \partial^{\mu'} G_E^{(2)} - \Omega(x_c)\partial_\mu \partial^{\mu'} G_E^{(1)} + \Omega(x_c)\partial_a \partial^{a'} G_E^{(1)}]\} \quad (62)$$

where $G_E^{(1)}$ and $G_E^{(2)}$ are the corrections of the first and the second order to the Euclidian Green function, and one must take into account the terms which are proportional to the products $\mu_i\mu_k$ only. The function $G_E^{(1)}$ is given by the expression (53), $G_E^{(2)}$ may be determined by the same way. We obtain

$$E_{int} = -\frac{8}{\pi^{n/2-2}} \frac{n\Gamma(n/2)\Gamma(2+n/2)\Gamma(n/2-1)}{\Gamma(3+n)} \frac{G^2\mu_1\mu_2}{a^{n-2}} \ , \quad (63)$$

where $a = [(\alpha_1 - \alpha_2)^2 + (\beta_1 - \beta_2)^2]^{1/2}$ is the distance between the strings. If $n = 4$ (63) dives us the force per unit length between the strings. In the case of the four-dimensional space-time we obtain

$$E_{int} = -\frac{4}{15\pi} \frac{G^2\mu_1\mu_2}{a^2} \ . \quad (64)$$

We see that the multiconical boundary condition alter the zero point vacuum fluctuations of a scalar field leading to the existence of an attractive force by the same way as in the case of two parallel plates in the wellknown Casimir effect. In the 4-dimensional space-time this force decreases with a distance between the strings as a^{-3}.

7 Conclusion

The generalization of the fast-motion approximation scheme is given for the cosmic strings that accounts for their gravitational interactions up to the second order of the gravitational constant and allows to calculate the gravitational and electromagnetic radiation (if current-carrying strings are taken into account) in cases where the relevant accelerations are due to the gravitational interaction. It has been shown that the space-time of a network consisting of straight Nambu strings was flat outside the strings and no gravitational radiation was produced. But if at least one of the strings is a superconducting one, a powerful source of electromagnetic radiation arises. We think, that this effect may be of great importance from the cosmological point of view.

The same approach can be used to determine linear and post-linear corrections to the Euclidian Green function. The expression for G_E enables one to calculate the vacuum expectation values $< \phi^2 >_{vac}$ and $< T_{00} >_{vac}$ and to consider the problem of the topological self-action of a point charged particle. It is shown that multiconical boundary conditions lead to the existence of an attractive force between Nambu strings.

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