Hyperspherical approach to the bound state properties of exotic systems

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Introduction

There is common belief among Physicists that the three-body problem is generally too hard to deal with, so that our detailed understanding of its features can only be achieved via extensive numerical calculation [1]. A new model potential [2] for van der Waals atomic pair is proposed and shown to be in excellent agreement with ab-initio calculation & experimental potentials. The model allows the potential energy of a given pair to be estimated with good accuracy from ab-initio calculations. Neglecting the internal structure of the atoms, we consider three body systems as consisting of three atoms for example $^4He_2$-^6Li. We search for the existence of weakly bound three-body systems. The Schrödinger's equation for three interacting atoms is expressed by the hyper-spherical coordinates and we have solved this Schrödinger's equation for the case of total angular momentum quantum number, J=0. [3]. Once the hyper-spherical effective potentials are obtained we can search for the bound state properties supported by each curve by solving the three-body Schrödinger's equation.

Quantum symmetry is also known to play an essential role in determining the stability of the three-body system and the binding energies. The diagonal coupling term has a larger influence on the three body system this leads to the bigger relative gap between the lower and upper bounds for the three-body system. The tri-atomic three-body systems have been attracting considerable interest due to possibility of observing very weakly bound states. Inert–alkali tri-atomic molecular systems [4] are studied using the adiabatic hyper-spherical representation. By adopting the best pair-wise M-M and M-N interaction potentials, we search for weakly bound states of M_2-N systems. Where M= He, Ne, Ar, Xe etc. And N= Li, Na, K, Rb, Cs etc. We consider only total angular momentum J=0 for this preliminary work. We calculate the bound state energies of these molecular species and discuss the essential features of the wave functions associated with these bound state. H_2- alkali & H_2-inert three-body systems can also be studied in the similar manner. The total potential will be taken as sum of three pair potentials. The three-body wave function will be expanded in the complete set of hyper-spherical harmonics (HH). Substitution of the HH expansion in the Schrodinger equation and the use of orthogonality of HH lead to an infinite set of coupled differential equations (CDE) which will be solved by the renormalized numerov method to get the bound state energy and wave function [5]. The method has also been applied to study the ground state of few single-$\wedge$ & double-$\wedge$ hyper nuclei. The details of the methodology & the potential parameters for the $\wedge\wedge$ and core-$\wedge$ pairs can be found in [6]. For a general three body system consisting of three unequal mass particles, we label the core as particle number '1' and the two valence particles '2' and '3' respectively as shown in Fig.1.

Table I. Parameters of the ∧-core potential and corresponding ∧ separation energy in the core-∧ subsystems.

Systems	∧-core potential parameters		B_Λ(MeV)	
			Ground state	
	V_0 (MeV)	X (fm)	Expt.	Calc
$^{5}_{\Lambda}$He	-43.61	1.41	3.12±0.02	3.12
$^{7}_{\Lambda}$He	-39.63	1.41	5.23±0.00	5.23
$^{7}_{\Lambda}$Be	-39.37	1.41	5.16±0.08	5.16

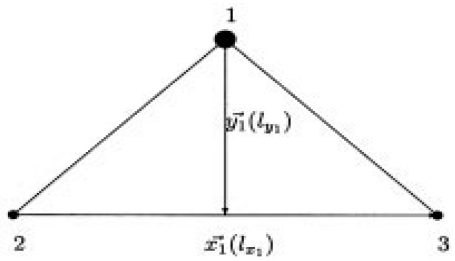

Fig.1. Choice of Jacobi coordinates for the partition '1'.

Now for a given partition '1', we taking here (i, j, k = 1, 2, 3 cyclic) the Jacobi co-ordinates are defined as: $\boldsymbol{x_i} = a_{jk}\left(\boldsymbol{r_j} - \boldsymbol{r_k}\right)$, $\boldsymbol{y_i} = a_{(jk)i}(\boldsymbol{r_i} - \frac{m_j \boldsymbol{r_j} + m_k \boldsymbol{r_k}}{m_j + m_k})$, $R = \frac{1}{M}(m_i r_i + m_j r_j + m_k r_k)$

Where a_{jk} and $a_{(jk)i}$ are coefficients of Jacobi coordinates. And m_i, $\mathbf{r_i}$ are the mass and position of the i^{th} particle, $M = (m_i + m_j + m_k)$ is the total mass and **R** is the centre of mass of the system. The hyperspherical coordinates are defined as $x_i = \rho cos\varphi$, $y_i = \rho sin\varphi$. Therefore the Schrödinger equation in terms of hyperspherical coordinates becomes

$$\left[-\frac{\hbar^2}{2\mu}\left\{\frac{1}{\rho^5}\frac{\partial}{\partial\rho}\left(\rho^5\frac{\partial}{\partial\rho}\right) - \frac{\widehat{\boldsymbol{k}}^2(\Omega_i)}{\rho^2}\right\} + V(\rho,\Omega_i) - E\right]\psi(\rho,\Omega_i) = 0$$

Where, $V(\rho,\Omega_i)$ is the total interaction potential and $\widehat{\boldsymbol{k}}^2(\Omega_i)$ is the square of hyper angular momentum operator.

Table II. Two-∧ separation energy in the ground and excited states of different double-∧ hypernuclei at K_{max} = 20

Hypernuclei	Bound states J^π	$B_{\Lambda\Lambda}$(MeV)		$\Delta B_{\Lambda\Lambda}$(MeV)	
		Expt.	Calc.	Expt.	Calc.
$^{6}_{\Lambda\Lambda}$He	0^+	7.25±0.19	7.2501	1.01	1.0101
$^{8}_{\Lambda\Lambda}$He	0^+		11.5815		1.1215
$^{8}_{\Lambda\Lambda}$Be	0^+		11.4344		1.1144

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