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Research Article

Exclusive Reconstruction of *B*-Decays with Missing Neutrals

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Often decay channels that are of theoretical interest cannot be reconstructed exclusively due to missing neutrals (such as neutrinos), or due to single-track vertices. This situation appears both in underground astrophysics experiments as well as in conventional accelerator experiments. A method to "recover" such missing particles from their kinematics and reconstruct "exclusively" the modes would benefit both domains in a number of ways. The main idea is to combine 4-momentum conservations in vertices with available geometric information in the event. The paper gives details of such methods on the $B_s^0 \to D_s^- K^+$, $D_s^- \to K^+ K^- \pi^- (\pi^0)$ prototype decay, which also encounters 2-fold ambiguities in its solutions. Such ambiguities can be lifted and the paper shows how, while also addressing the potential the method has in physics analyses and detector studies.

1. Introduction

The kinematics of decay modes with a missing particle, usually an elusive neutrino, are of interest in both underground astrophysics experiments as well as in conventional accelerator experiments. Exclusive reconstruction in today's new generation experiments is important for precision physics, performant background rejections needed in rare decays, and so forth. Some modes that are of considerable theoretical interest, from the experimental point of view, pose inherent difficulties with missing neutrals and single-track vertices. While such is not an impossible situation (since the vertex must be somewhere on the track), the precise location eludes in general the reconstruction of the mode. Nonetheless, in certain cases, information from the rest of the event provides this capability—in general in the form of 4-momentum conservation in subsequent vertices. Work on recovery using dynamic information has been explored before [1, 2], not necessarily in association with vertexing. Code addressing

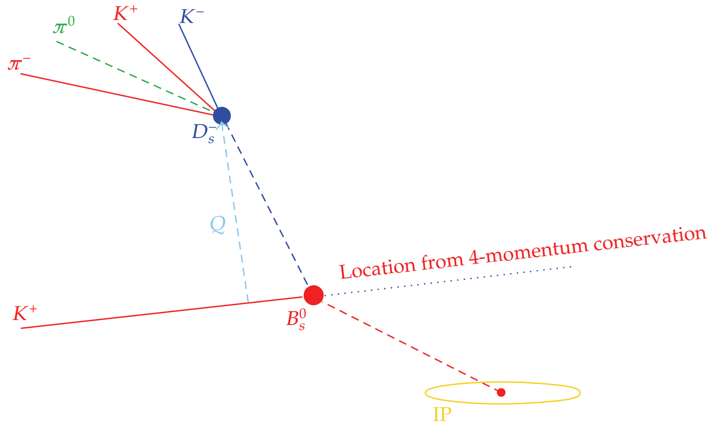

Figure 1: Topology of the mode considered in the absence of magnetic field: $B_s^0 \to D_s^-(K^+K^-\pi^-\pi^0)\ K^+$. The line $\vec{Q}$ is the perpendicular from the D-vertex onto the kaon track.

such methods stems from the days of the CLEO collaboration, KWFIT [3, 4] and of BaBar Collaboration, KinFitter [5].

In underground experiments the geometric equivalent to accelerator B-physics programs are strange-particle decays [6, 7], where kinematic imprecisions associated with the missing neutrino and a traditional lack of some form of vertexing have prevented exclusive reconstruction of the modes. At the new underground astrophysics experiments (such as LAGUNA [8–12]) tracking-vertexing facilities are being discussed in some of the studies. The methods studied could in principle be applied for all "stealth" particles (neutrino, π^0, other neutrals) decaying further downstream.

The objective in such cases is to recover if possible exclusively the event, by combining conservation equations and geometric aspects in the decays. The case made in this paper is for tracking-vertexing assemblies in underground astrophysics experiments and for more precise vertexing in accelerator-based experiments.

Consider the prototype mode $B_s^0 \to D_s^- K^+$, $D_s^- \to K^+K^-\pi^-\ (\pi^0)$, which is of interest for γ_{CKM} measurement, however, with a somewhat fair branching ratio. With π^0 in the final state the branching ratio is roughly equal to that without the π^0 [13], thus putting the two together essentially doubles the statistics. Reconstruction of the π^0 would in principle be possible in the calorimeter, however, with less resolution than using tracking.

Fortunately there is sufficient information in the event for reconstructing the mode without explicit calorimeter information on the π^0—in principle with better resolution also. This information is from 4-momentum conservation in the B- and D-vertices. A particular 2-fold ambiguity from the quadratic equations arises in this (and other) modes. It is important to be able to lift this ambiguity without imposing cuts (which due to detector resolution can eliminate signal). Figure 1 shows the topology of the event (in the absence of magnetic field—such as for the LHCb experiment at the LHC. A small track bending over the vertexing region can also be included, in the form of an $\mathcal{O}_{\mathrm{III}}$ helical correction, giving a solution also for other detectors with full-strength magnetic field in the vertexing region).

Vertexing methods reject reasonably well QCD backgrounds (originating at the interaction point (IP), with some resolution there around); however, other B-modes can constitute significant backgrounds. One such mode is the sister mode with pion in the B-vertex, roughly 4-5 times more abundant. Kinematically and topologically similar modes from B_d^0 also exist—the latter roughly 3 times more abundant than B_s^0. Although there is no exact replica of the mode in B_d^0 version, there are many modes with nonnegligible branching ratios that "losing" one track can mimic this mode. Due to the similarities in topology (displaced vertices) and kinematics (B-masses, center of mass decay energies), the method cannot distinguish between such cases—not with current detector resolution. The only remaining solution is performant Particle ID, making sure that the B-vertex track is a kaon (B_d^0 being less "kaon-productive" than B_s^0) and that there are two kaons present in the D-vertex. All other kinematic parameters offer little protection against B_d^0 modes—due to the high boost factors, mass similarities, and center of mass decay energies. A pleiad of such parameters has been studied: the discriminant of the second order equation, pointback, reconstructed invariant masses, angles between track-combinations or decay-planes, 2-3 parameters combinations of the aforementioned, and so forth. For all the detector resolution was insufficient to tell apart the current mode from its sister mode (best achievable ca. 41% rejection), or from other B_d^0 originating modes (ca. 45% rejection).

2. Dynamic Vertexing

The 4-momentum conservations in the B- and D-vertices are

$$\begin{aligned}(E_K + E_D)^2 &= M_B^2 + \left(p_D^2 + p_K^2 + 2\vec{p}_D\vec{p}_K\right),\\ (E_D - E_v)^2 &= m_\pi^2 + \left(p_D^2 + p_v^2 - 2\vec{p}_D\vec{p}_v\right),\end{aligned} \tag{2.1}$$

where v refers to the "virtual" particle having 4-momentum equal to the sum of the charged tracks in the D-vertex. These can be rewritten as

$$\begin{aligned}\vec{p}_D\vec{p}_K &= \frac{M_K^2 + M_D^2 - M_B^2}{2} + E_K E_D,\\ \vec{p}_D\vec{p}_v &= -\frac{M_D^2 + M_v^2 - m_\pi^2}{2} + E_v E_D.\end{aligned} \tag{2.2}$$

Since $\vec{p}_B = \vec{p}_D + \vec{p}_K$, the three vectors lie in the same plane. Also, the B-vertex is on the kaon track, hence $\vec{p}_D = \lambda\vec{Q} + \mu\vec{p}_k$, the two constants λ and μ being determined from (2.1) as linear functions of E_D with some coefficients:

$$\begin{aligned}\lambda &= \alpha E_D + \beta,\\ \mu &= a E_D + b.\end{aligned} \tag{2.3}$$

However, $p_D^2 = E_D^2 - M_D^2 = (\lambda\vec{Q} + \mu\vec{p}_K)^2$, resulting in a second order equation in E_D:

$$E_D^2 - M_D^2 = E_D^2\left(\alpha\vec{Q} + a\vec{p}_K\right)^2 + E_D\cdot 2\left(\alpha\vec{Q} + a\vec{p}_K\right)\left(\beta\vec{Q} + b\vec{p}_K\right) + \left(\beta\vec{Q} + b\vec{p}_K\right)^2. \tag{2.4}$$

For convenience $\vec{Q}$ has the direction of $\vec{Q}$ in the figure and the absolute value of 1 GeV/c.

The ambiguity raised by the second order equation is solved in favor of the "+" sign. What actually happens is that $(\beta\vec{Q} + b\vec{p}_k)$ is rather small $(\vec{\epsilon})$ and the equation looks like

$$E_D^2 - M_D^2 = \vec{\chi}^2 E_D^2 + \vec{\epsilon}(2\vec{\chi}E_D + \vec{\epsilon}). \tag{2.5}$$

The solution with "−" sign is therefore negative in value, hence unphysical.

Once E_D is known, the coefficients λ and μ are known and so are $\vec{p}_D$ and $\vec{p}_B$.

Having the D-vertex and $-\vec{p}_D$ pointing back at the kaon track, a B-vertex can be now inferred. The problem is formulated as follows: the B-vertex is a point inside the $1\sigma_K$ kaon track error-tube with the property that aiming along the $\vec{p}_D$ direction, the D-vertex is missed by about $1\sigma_D$, while aiming along the $-\vec{p}_B$ direction, the IP is missed by about $1\sigma_{IP}$. The kaon track is parametrised as $\vec{r}_{0K} - \lambda_K\vec{n}_K$ where $\vec{n}_K = \vec{p}_K/p_K$, $\vec{r}_{0K}$, some origin on the track, and λ_K, a running parameter describing the track. Therefore, if $\vec{B}$ is the B-vertex, the above requirements translate into

(1) kaon track tube, σ_K, from $-\vec{r}_{0K} + \lambda_K\vec{n}_K + \vec{B}$,

(2) D-vertex $\pm\ \sigma_D$, from $\vec{B} + \lambda_D\vec{p}_D$,

(3) IP $\pm\ \sigma_{IP}$, from $\vec{B} + \lambda_B\vec{p}_B$.

This implies minimising the above 3 constraints simultaneously:

$$\begin{aligned}
&\left\langle \vec{B} + \lambda_K\vec{n}_K - \vec{r}_{0K} \left|\sigma_K^{-2}\right| \vec{B} + \lambda_K\vec{n}_K - \vec{r}_{0K} \right\rangle \\
&\quad + \left\langle \vec{B} + \lambda_D\vec{p}_D - \vec{v}_D \left|\sigma_D^{-2}\right| \vec{B} + \lambda_D\vec{p}_D - \vec{v}_D \right\rangle \\
&\quad + \left\langle \vec{B} + \lambda_B\vec{p}_B - \vec{v}_{IP} \left|\sigma_{IP}^{-2}\right| \vec{B} + \lambda_B\vec{p}_B - \vec{v}_{IP} \right\rangle
\end{aligned} \tag{2.6}$$

with respect to the three running parameters λ_K (units [m]), λ_D, and λ_B (both in units of [m]/[GeV/c]) and to $\vec{B}$. Annulling the respective derivatives:

$$\begin{aligned}
\lambda_K &= \frac{\left\langle \vec{B} - \vec{r}_{0K} \middle| \sigma_K^{-2} \middle| \vec{n}_K \right\rangle}{\left\langle \vec{n}_K \middle| \sigma_K^{-2} \middle| \vec{n}_K \right\rangle}, \\
\lambda_D &= \frac{\left\langle \vec{B} - \vec{v}_D \middle| \sigma_D^{-2} \middle| \vec{n}_D \right\rangle}{\left\langle \vec{n}_D \middle| \sigma_D^{-2} \middle| \vec{n}_D \right\rangle}, \\
\lambda_B &= \frac{\left\langle \vec{B} - \vec{v}_{IP} \middle| \sigma_{IP}^{-2} \middle| \vec{n}_B \right\rangle}{\left\langle \vec{n}_B \middle| \sigma_{IP}^{-2} \middle| \vec{n}_B \right\rangle}.
\end{aligned} \tag{2.7}$$

Minimising with respect to the B-vertex $\vec{B}$, the following is obtained:

$$\left|\vec{B}\right\rangle = \left[\sum_{x=K,D,B} \sigma_x^{-2}\left(1 - \frac{|\vec{n}_x\rangle\langle\vec{n}_x|\sigma_x^{-2}}{\langle\vec{n}_x|\sigma_x^{-2}|\vec{n}_x\rangle}\right)\right]^{-1} |\vec{q}\rangle \tag{2.8}$$

with

$$\left|\vec{q}\right\rangle = \sum_{x=K,D,B} \sigma_x^{-2} \left(1 - \frac{|\vec{n}_x\rangle\langle\vec{n}_x|\sigma_x^{-2}}{\langle\vec{n}_x|\ \sigma_x^{-2}\ |\vec{n}_x\rangle} \right) |\vec{v}_x\rangle. \tag{2.9}$$

For the kaon track error tube an extended acception of error ellipsoid was used: one with infinite long-axis. In the case of a round cross-section (equal axes) the error-ellipsoid operator reduces to the perpendicular to track projector. Therefore (in $\vec{q}$) only the perpendicular to track error of $\vec{r}_{0K}$ will contribute to the B-vertex error.

Similarly, for spherical error-ellipsoids, the D-vertex and the IP would contribute only with their respective errors perpendicular to $\vec{n}_D$ and $\vec{n}_B$. This is not exactly the case, both ellipsoids being elongated, the IP along z and the D-vertex along its flight direction. This extra elongation simply translates into the addition of a parallel projector along said directions to the respective error-ellipsoid operators. For the D-vertex the elongation being exactly along $\vec{n}_D$ and greater than the spherical part of the operator, the combined effect in $\vec{q}$ is the same as having only the spherical part. For the IP the elongation is along z, which is not $\vec{n}_B$, but almost. So again, to first approximation, only the waist of the IP will play a determinant part in the B-vertex errors. The IP being very accurate waist-wise (for the LHCb-IP: $\sigma_{xy} = 20\,\mu$m versus $\sigma_z = 60\,\mu$m [14]), the main contribution will come from the D-vertex—set by smearing to a typical resolution of this detector [15].

The code was tested using a PYTHIA simulation of the LHC environment for the LHCb experiment (LHC energy $b\bar{b}$ events, "HardQCD:gg2bbbar"), for which the tracks were smeared both in momentum value and direction (like a needle deviating in direction within an error-tube). The momentum value smearing was chosen such as to give 2-body mass resolutions as those reported by this experiment in literature [16, 17] (ca. $12\,\text{MeV}/c^2$), while the error-tube diameter (ca. $15\,\mu$m) was chosen such as to give the vertex locator resolutions reported in the literature [17] (ca. $150\,\mu$m longitudinally and $25\,\mu$m transversally). Once the particle tracks and vertices are obtained in a form simulating detector response, they are fed to the code for reconstruction. Figure 2 shows the reconstructed B-vertex resolution in terms of proper time, determined from $\tau_{\text{proper}} = d_{\text{flight}} M_B / c p_B$. The physically relevant directions are parallel to flight direction and transverse to this. The method uses 3 constraints combined, hence a rudimentary approximation is to allot a gaussian in the resolution fit for each. The top part of the figure shows the residuals with respect to flight-direction (which is almost z-aligned, fitted with 3 Gaussians). The combined resolution is on the order of 32 fs. The bottom part shows residuals transverse to the flight direction (approximately the xy-plane) fitted with 3 half-Gaussians centered at zero, resolution on the order of 5.4 fs. It is evident that the resolution is dominated by the boost direction (flight direction elongated error-ellipsoid). This is to be expected due to the high boosts, where the shallow opening angles among tracks translate pixel detector errors into large longitudinal ones.

The obtained resolution is similar to usual resolutions in B-reconstructions achieved in 4 track decays [18] (40 fs). The obvious advantage here is that although particles are missing, their kinematics helps "recover" them as if they were present and this is an exclusive reconstruction.

The method can also be used for detector studies, such as calibrating Particle ID, or calorimetry, in the lepton (electron/muon) sector. This implies in general using semileptonic modes. Although the traditional leptonic decay of J/ψ is standard, it yields two hard tracks, quite close to each other (which in the electron case makes difficult estimating the individual

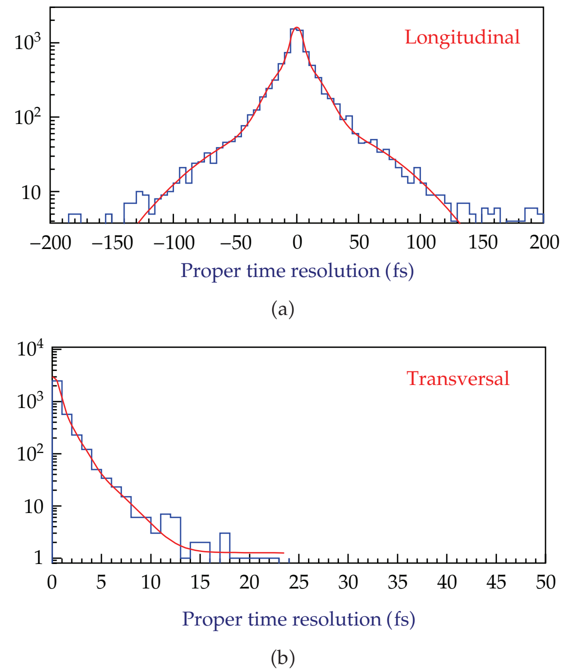

Figure 2: Residuals of the reconstructed B-vertex with respect to the Monte Carlo position: (a) projected onto the B-flight direction, (b) transverse to the B-flight direction. The dimensions are proper time in fs. Three Gaussians were used in the fits (in the case of the transverse centered at zero), the global resolutions being ca. 32 fs (longitudinally) and 5.4 fs (transversally)—details are in the text.

radiative losses). Also Particle ID performs lesser on tracks very close to one another, the real performance (used in B-physics) being actually better. Therefore, the larger variety given by semileptonic modes would be preferable, if only a pure enough (better than Particle ID miss-ID rates) sample could be selected. Consider, for instance, the mode $B_d^0 \rightarrow D^{*-} (K^-\pi^+ l^- \overline{\nu}_l)$ $\pi^+\pi^-\pi^+$. The B-vertex provides 3 "clean" pion tracks useful for Particle-ID studies in the hadron sector. The D-vertex would also be good, if only labels could be attached to the tracks. Owing to the large mass difference between kaon/pion and electron, the equations would prevent a vertexable solution if the masses were wrongly attached. However, in the case $l = \mu$, the pion and lepton tracks are similar in mass. Still, they are quite different in dynamics, one coming from a lepton line, united to the baryon half of the reaction via a W^-, while the other from a continuous quark line. The imbalance in this case would come from the different momentum properties (putting the wrong momentum in $M_{l\nu_l}$, for instance, versus other dynamic quantities in the reaction). In this sense the method can act as a track-labeler in the D-vertex, offering a quite clean sample of B-origin leptons for Particle-ID studies—which is rare. A study was performed for kaon/pion separation power of the method. The mode with no π^0 in the final state was chosen. Of the existing tracks in the D-vertex one was purposely reported "lost" to the program and its mass fed to the code. If the kaon track is reported "lost," but pion mass is given for it to the code, the inner kinematics of the reactions will work out wrong, further influencing the (highly nonlinear) vertexing procedure described aforehand. The vertexing produces now new momenta for all particles implied and an invariant mass can be recomputed for the "lost" particle's mass (different, but close, to the one inputted). This mass error versus pointback is a criterion that gives a ca. 45% kaon-pion separation. As pointed above, for leptons additional handles (due to inner dynamics) exist (i.e., $-M_{l\nu_l}$) and so the rejection power should be better (especially for the electron, where the mass is also much lighter).

3. Conclusions

The method aims to recover modes otherwise nonusable, with missing neutrals lost (neutrino), or less precise (calorimetry). This is very important to both underground based experiments as well as accelerator-based experiments, where background rejection or measurement precision depends on the exclusive reconstruction of the event. The paper lifts the two-fold ambiguity arising in the second order equation involved, which otherwise relied on other criteria (pointback for instance) which cut on signal.

The potential of the method for Particle-ID source studies (lepton sector) is also discussed.

The methods studied could in principle be applied for all "stealth" particles (neutrino, π^0, other neutrals) decaying further downstream. Plans for extending the method could involve these options together with the Particle-ID studies.

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