NLO CALCULATION OF ϵ'/ϵ IN QCD AND QED WITHIN THE LATTICE QCD FRAMEWORK.

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Abstract

We present[1] the results of our recent calculation of ϵ'/ϵ at the Next-to-Leading (NLO) order in QCD and QED, in the framework of $\Delta S=1$ Effective Hamiltonians. The operator matrix elements are taken from lattice QCD, at a scale $\mu = 2$ GeV. NLO corrections seem to lower the value of ϵ'/ϵ, favouring the experimental result of E731. Contributions from different operators are analyzed and dependence on m_t, μ and Λ_{QCD} discussed.

[1]Work done in collaboration with M. Ciuchini, E. Franco and G. Martinelli

Introduction

We have calculated the Effective Hamiltonian for $\Delta S = 1$ decays at NLO in QCD and QED [8, 9]. The Wilson coefficients of the $\Delta S = 1$ Effective Hamiltonian have been derived from the (10×10) anomalous dimension matrix, which includes corrections at orders α_s, α_s^2, α_e, $\alpha_s\alpha_e$. The matrix elements for the corresponding operators have been taken from lattice QCD .

The knowledege of the $\Delta S = 1$ Effective Hamiltonian at NLO in QCD and QED is important for several reasons: 1) heavy mass (like m_t) effects are indeed a NLO effect; 2) the stability of the perturbative calculation can be checked and a limit on its reliability can be fixed; 3) the Λ_{QCD} parameter can be used in a proper way, taking it from different experiments; 4) the effect of different operators in the OPE (QCD-penguins, QED-penguins, etc.) can be better determined.

We finally present our results for the ϵ'/ϵ parameter of direct CP-violation in K-decays. The effect of QCD and QED corrections at NLO seems to lower the value of ϵ'/ϵ, favouring the experimental value of E731 [11].

$\Delta S = 1$ Effective Hamiltonians at NLO: general results

The Effective Hamiltonian for $\Delta S = 1$ decays is given by:

$$\begin{aligned} \mathcal{H}_{eff}^{\Delta S=1} &= \lambda_u \frac{G_F}{\sqrt{2}} \Big[(1-\tau)\Big(C_1(\mu)\,(O_1(\mu) - O_1^c(\mu)) + C_2(\mu)\,(O_2(\mu) - O_2^c(\mu)) \Big) \\ &+ \tau \sum_{i=3,\dots,10} O_i(\mu) C_i(\mu) \Big] \end{aligned} \quad (1)$$

where $\lambda_u = V_{ud}V_{us}^*$ and similarly we can define λ_c and λ_t. $\tau = -\lambda_t/\lambda_u$ and V_{ij} is one of the elements of the CKM mixing matrix.

We have used the following complete basis of operators when QCD and QED corrections are taken into account:

$$\begin{array}{lll} \text{Vertex-type} & \rightarrow & \left\{ \begin{array}{l} O_1 = (\bar{s}_\alpha d_\alpha)_{(V-A)}(\bar{u}_\beta u_\beta)_{(V-A)} \\ O_2 = (\bar{s}_\alpha d_\beta)_{(V-A)}(\bar{u}_\beta u_\alpha)_{(V-A)} \\ O_1^c = (\bar{s}_\alpha d_\alpha)_{(V-A)}(\bar{c}_\beta c_\beta)_{(V-A)} \\ O_2^c = (\bar{s}_\alpha d_\beta)_{(V-A)}(\bar{c}_\beta c_\alpha)_{(V-A)} \end{array} \right. \\ \text{QCD-Penguins} & \rightarrow & \left\{ \begin{array}{l} O_{3,5} = (\bar{s}_\alpha d_\alpha)_{(V-A)} \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_{(V\mp A)} \\ O_{4,6} = (\bar{s}_\alpha d_\beta)_{(V-A)} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{(V\mp A)} \end{array} \right. \\ \text{QED-Penguins} & \rightarrow & \left\{ \begin{array}{l} O_{7,9} = \frac{3}{2} (\bar{s}_\alpha d_\alpha)_{(V-A)} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\beta)_{(V\pm A)} \\ O_{8,10} = \frac{3}{2} (\bar{s}_\alpha d_\beta)_{(V-A)} \sum_q e_q (\bar{q}_\beta q_\alpha)_{(V\pm A)} \end{array} \right. \end{array} \quad (2)$$

where the subscript $(V \pm A)$ indicates the chiral structure and α and β are colour indices.

The operators $O_i(\mu)$ are renormalized at the scale $\mu < M_W$ in $\overline{MS}$ in some given regularization scheme (e.g. HV ('t Hooft-Veltman) or NDR (Naive Dimensional Reduction)). The corresponding coefficients, C_i, are also scheme dependent.

We have matched the full and the effective theory at $\mu = M_W$. This gives the initial conditions for the evolution of the coefficient functions. It's here that the main dependence on the heavy top mass enters. Then, at a generic scale $\mu < M_W$, the NLO evolved coefficient function can be expressed as [8, 9, 2, 4, 5]:

$$\vec{C}(\mu) = \hat{W}[\mu, M_W]\vec{C}(M_W) \tag{3}$$

where $\vec{C}(\mu)$ is a vector, whose components are the corresponding Wilson coefficients. $\vec{C}(M_W)$ is the vector of the initial conditions at $\mu = M_W$ and $\hat{W}[\mu, M_W]$ the renormalization group evolution matrix. The matrix $\hat{W}[\mu, M_W]$ depends on the one-loop and two-loop coefficients of the Anomalous Dimension Matrix (ADM) $\hat{\gamma}$ for the operator basis in (2):

$$\hat{\gamma} = \frac{\alpha_s}{4\pi}\hat{\gamma}_s^{(0)} + \frac{\alpha_e}{4\pi}\hat{\gamma}_e^{(0)} + \frac{\alpha_s^2}{(4\pi)^2}\hat{\gamma}_s^{(1)} + \frac{\alpha_s}{4\pi}\frac{\alpha_e}{4\pi}\hat{\gamma}_e^{(1)} \tag{4}$$

Both $\vec{C}(M_W)$ and $\hat{W}[\mu, M_W]$ are regularization scheme dependent. We have computed $\vec{C}(M_W)$, $\hat{\gamma}_s^{(1)}$ and $\hat{\gamma}_e^{(1)}$ both in HV and NDR scheme and verified the scheme independence of the final result (1). Moreover, we have checked our results both at the matrix level and at the diagram by diagram level.

In ref.[9] we discuss all the technical details of our calculation and the differences between our results and those obtained in ref.[2, 4, 5] by the Munich group.

Results for ϵ'/ϵ

In the expression for ϵ' ($e^{i\delta_i}A_i = \langle \pi\pi(I=i)|\mathcal{H}_{eff}^{|\Delta S|=1}|K\rangle$):

$$\epsilon' = \frac{e^{i\pi/4}}{\sqrt{2}}\frac{\omega}{\mathrm{Re}A_0}\left[\omega^{-1}(\mathrm{Im}A_2)' - (1-\Omega_{IB})\,\mathrm{Im}A_0\right] \tag{5}$$

the real parts $\mathrm{Re}A_0$ and $\mathrm{Re}A_2$ are taken from experiments ($\omega = \mathrm{Re}A_0/\mathrm{Re}A_2 = 0.045$), while the imaginary parts $\mathrm{Im}A_0$ and $(\mathrm{Im}A_2)'$ can be derived from $\mathcal{H}_{eff}^{\Delta S=1}$ in the following form:

$$\begin{aligned}\mathrm{Im}A_0 \;=\; & -\frac{G_F}{\sqrt{2}}\mathrm{Im}\,(V_{ts}^*V_{td})\Big\{-\Big(C_6B_6 + \frac{1}{3}C_5B_5\Big)Z + \Big(C_4B_4 + \frac{1}{3}C_3B_3\Big)X + \\ & C_7B_7^{1/2}\Big(\frac{2Y}{3} + \frac{Z}{6} - \frac{X}{2}\Big) + C_8B_8^{1/2}\Big(2Y + \frac{Z}{2} + \frac{X}{6}\Big) - \\ & C_9B_9^{1/2}\frac{X}{3} + \Big(\frac{C_1B_1^c}{3} + C_2B_2^c\Big)X\Big\}\end{aligned} \tag{6}$$

and

$$\begin{aligned}(\mathrm{Im}A_2)' \;=\; & -\frac{G_F}{\sqrt{2}}\mathrm{Im}\,(V_{ts}^*V_{td})\Big\{C_7B_7^{3/2}\Big(\frac{Y}{3} - \frac{X}{2}\Big) + \\ & C_8B_8^{3/2}\Big(Y - \frac{X}{6}\Big) + C_9B_9^{3/2}\frac{2X}{3}\Big\}\end{aligned} \tag{7}$$

where we have introduced $(\mathrm{Im}A_2)'$ defined as:

$$\mathrm{Im}A_2 = (\mathrm{Im}A_2)' + \Omega_{IB}(\omega\,\mathrm{Im}A_0) \tag{8}$$

$\Omega_{IB} = 0.25 \pm 0.10$ represents the isospin breaking contribution, see for example ref. [3].

The Wilson coefficients have been calculated at the NLO in QCD and QED as explained before. The matrix elements of the corrisponding operators have been expressed in terms of

$B_K, B_9^{(3/2)}$	B_{1-2}^c	$B_{3,4}$	$B_{5,6}$	$B_{7-8-9}^{(1/2)}$	$B_{7-8}^{(3/2)}$
0.8 ± 0.2	$0 - 0.15^{(*)}$	$1 - 6^{(*)}$	1.0 ± 0.2	$1^{(*)}$	1.0 ± 0.2

Table 1: Values of the B-parameters. Entries with a $^{(*)}$ are educated guesses; the others are taken from lattice QCD calculations.

quantities X, Y and Z (see [7, 8]) and B-parameters taken (whenever possible) from lattice QCD. We recall that the B-parameter for a given operator is defined as the ratio of its matrix element to the same matrix element evaluated in the VIA (Vacuum Insertion Approximation). The values of the still missing B-parameters are guessed on the basis of some reasonable arguments. In particular, $O^{1/2}_{7,8,9}$ turn out to have negligible coefficients; while $O^c_{1,2}$ and $O_{3,4}$ have very large coefficients. Thus we have fixed $B^{1/2}_{7,8,9}$ to their VIA value (= 1); while we have allowed $B^c_{1,2}$ and $B_{3,4}$ to vary in a quite large range around their VIA value (0 and 1 respectively). For a more detailed discussion and full references on recent lattice calculations see ref. [7, 8]. We report the values used for the B-parameters in Table 1.

Writing ϵ'/ϵ in terms of *relative* contributions of different operators with respect to the O_6 penguin operator, i.e.:

$$\epsilon'/\epsilon \sim R \times C_6 B_6 \Big(1 - \sum_i \Omega_i\Big) \tag{9}$$

where $\Omega_i = C_i B_i / C_6 B_6$, we get that terms Ω_2^c, Ω_4, $\Omega^{3/2}_{7,8,9}$ give the main contributions. A detailed discussion of the phenomenological analysis performed is presented in ref. [8].

Our results can be summarized as follows:

- Fixing m_t = 140 GeV, μ = 2 GeV and allowing B-parameters, m_s, Λ_{QCD}, Ω_{IB}, CKM parameters, etc. to vary around their central values (see Table 2), we get an idea of the theoretical incertainty and of the influence of NLO corrections. The main observation is that, the sums $\Omega_2^c + \Omega_4$ and $\Omega_7^{3/2} + \Omega_8^{3/2} + \Omega_9^{3/2}$ (despite significant individual variations from LO to NLO in the last case) are almost *stable* with respect to NLO corrections. Therefore the behaviour of the central value of ϵ'/ϵ is still governed by the contribution of O_6 and the decreasing of C_6 with NLO corrections lowers the central value of ϵ'/ϵ, favouring the E731 result (see Fig.(2)).

- Varying m_t between 100 GeV and 200 GeV, we find that NLO corrections are much more important for higher values of m_t. Thus the central value of ϵ'/ϵ decreases with increasing m_t (see Figs.((2)-(3)).

- Varying μ and Λ_{QCD} (see Table 2), we note that below $\mu \sim$ 1 GeV the perturbative approach is not reliable anymore; while the behaviour of the Wilson coefficients is quite stable for higher values of μ (see Fig.(1)). This observation greatly supports the use of lattice results, which allows to match matrix elements and Wilson coefficients at a scale $\mu \sim$ 2 GeV.

parameter	*value*
Λ_{QCD}	340 ± 120 GeV
$m_s(2\,\text{GeV})$	(170 ± 30) MeV
$m_c(2\,\text{GeV})$	1.5 GeV
$m_b(2\,\text{GeV})$	4.5 GeV
$A\lambda^2$	0.047 ± 0.004
$\sqrt{\rho^2+\eta^2} = V_{ub}/(\lambda V_{cb})$	0.50 ± 0.14
ϵ_{exp}	$2.28 \cdot 10^{-3}$
$\text{Re}A_0$	$2.7 \cdot 10^{-7}\text{GeV}$

Table 2: Values of experimental parameters used in this work.

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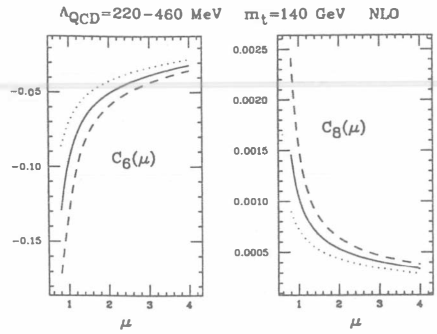

Figure 1: C_6 and C_8 as a function of μ for Λ_{QCD} = 220 (dotted),340 (solid) and 460 (dashed) MeV.

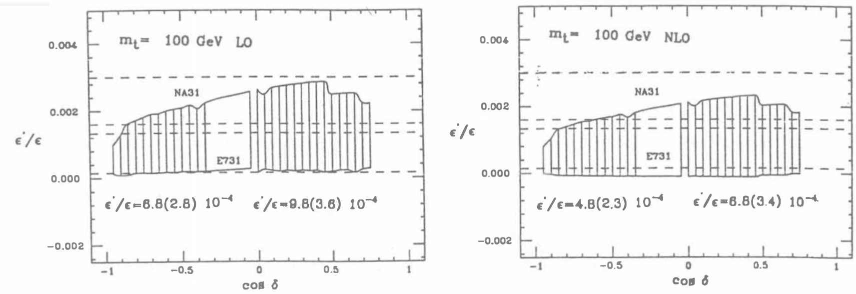

Figure 2: Band of allowed values for ϵ'/ϵ at m_t = 100 GeV at LO and NLO. The dashed lines represent the experimental results of NA31, $(2.3 \pm 0.7) \cdot 10^{-3}$ [10] and E731, $(0.74 \pm 0.59) \cdot 10^{-3}$ [11].

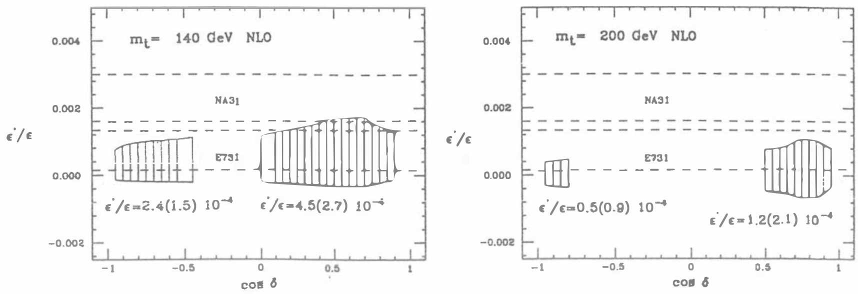

Figure 3: Same as in Fig.(2), only at NLO, for m_t = 140 and m_t = 200 GeV.