WEAK AND STRONG MAGNETIC FIELDS EFFECT ON THE NON-EXTENSIVE THERMODYNAMICS

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This study is an attempt to investigate the applicability of the non-extensive statistics involving the effects of magnetic fields on the thermodynamics of quantum chromodynamics (QCD). The non-extensive statistics are controlled by the entropic index, $q \neq 1$. In the case of $q = 1$, the Boltzmann–Gibbs statistics (extensive) are recovered. The thermodynamics such as pressure, entropy, and magnetization are determined for zero and non-zero magnetic fields. Therefore, the magnetic field is divided into strong, $eB = 0.2$, 0.3 GeV^2, and weak $eB = 0.002$, 0.003, 0.005 GeV^2 magnetic fields. The magnetic field effect is caused by adding a vacuum contribution to the free energy alongside the thermal contribution. The theoretical results are confronted with the lattice results which show overestimation especially at high temperatures and with higher entropic index q. It is concluded that QCD matter is considered to be a para-magnetic matter. Nevertheless, the non-extensive statistics might not be a favorable tool for describing the strongly coupled media responses to the external magnetic fields.

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1. Introduction

How the Universe was created is one of the most intriguing topics that scientists are attempting to answer. The Big Bang theory, which holds that the Universe was created as a result of a tremendous explosion that resulted in a fireball, may be the starting point for the solution. When

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the fireball expanded and cooled down, the elementary particles started to recombine. At that point, the fireball was made of dense and hot quarks and gluons plasma (QGP). The quarks and gluons united to form ordinary matter as a result of freezing of the QGP, starting with the fundamental components of the hydrogen atom — protons and neutrons — before being classified as hadrons. Further temperature reduction led to the formation of nuclei, which then joined to form common matter [1]. The phase transition between hadronic matter and QGP has been the subject of theoretical and experimental research, starting with a collision of two accelerating hadrons or heavy ions, such as the *pp* collision at the Large Hadron Collider (LHC), the Pb–Pb collision at the Relativistic Heavy Ion Collider, or the Au–Au collision at the latter (RHIC) [2]. Unfortunately, the QGP matter can only last for a very little amount of time (span) before expanding, which causes quarks and gluons to recombine and form hadronic matter once more at a critical temperature $T_{\rm c}$ [3–5]. Thermal freeze-out and chemical freeze-out are two steps in the freezing-out process. The point at which inelastic collisions cease and no new species form is known as the chemical freeze-out. When the elastic collisions stop and the particles are free to fly to the detectors, the thermal freeze-out stage starts [6, 7]. For more details about the freeze-out conditions and the thermal parameters that characterize these stages, see [8–10].

The hadrons are strongly interacting and can be described by the quantum chromodynamic (QCD) [11, 12]. QCD can be described non-perturbatively where the coupling constant approaches unity [13, 14] — this non-perturbative method is called lattice QCD (LQCD) [15–20]. The properties of the hot hadronic matter are best described as the properties of a statistical system, from which we can extract the thermodynamical quantities depending on the Hadron Resonance Gas Model (HRG) [21] and HRG studied at the finite chemical potential [22–24]. HRG is a powerful tool for the description of the thermal properties of the QCD matter and to reproduce the LQCD results at low temperatures [25]. However, it is in disagreement with LQCD at high temperatures, which might be a consequence of the negligence of the interaction among the particles in which the interaction is significantly effective at high temperatures [26, 27]. The HRG model has been used to study the effect of the magnetic field on the QCD matter in a series of papers *e.g.*, [28–32], a good review of the effect of a magnetic field in vacuum can be found in [33] and in the lattice QCD in [34–36]. An external magnetic field B is generated by the spectator's particles (particles do not contribute to the interaction) in the non-central heavy-ion collision. According to the illustration of the non-central heavy-ion collision, the induced magnetic field can be estimated [37, 38].

The Au–Au collision at RHIC at $\sqrt{S} = 200$ GeV produces a magnetic field of $\sim (10^{18}$–$10^{19})$ Gauss and for Pb–Pb at the LHC at $\sqrt{S} = 2.76$ TeV of $\sim 10^{20}$ Gauss $\sim 10\ m_\pi^2$ GeV^2 [39–41][1]. The phenomenon of increased quark condensate at very low temperatures, *i.e.*, below the critical temperature $T_{\rm c}$, is known as a magnetic catalysis (MC). It is attributed to the valence and the sea contributions to the quark condensate, both of which are increasing as a function of the magnetic field at $T = 0$. Actually, the magnetic field affects the motion (catalysing effect) and restricts the motion of the charged particles to move in a direction perpendicular to the magnetic field, regardless of the type of charges (valence or sea). In other words, it creates a "dimensional reduction" [42] that is discussed mathematically in a number of publications, including this one.

In contrast, it is discovered that the MC effect changed to inverse magnetic catalysis (IMC) around $T_{\rm c}$ and at $eB < 1$ GeV^2, particularly for increasing the pion mass [43]. The latter means that the quark condensate decreases with the magnetic field [44]. MC and IMC have been also investigated by the lattice simulations of QCD [35, 45, 46].

In general, the strong magnetic field plays a significant role in physical systems, including cosmology, phase transitions, and explanations of the early cosmos [34]. Additionally, the magnetic field can be produced in the peripheral heavy-ion collisions at the Large Hadron Collider (LHC) or the Relativistic Heavy Ion Collider (RHIC) [38, 47]. The extensive and non-extensive statistics have been applied in the analysis of the particle production at different energies [48, 49] and for the thermodynamics of the black hole [50]. For up to three decades, the non-extensive statistics have been applied to many physical systems and showed reasonable success in the description of QCD matter [51], neutron stars [52], and the cosmology studies [53]. In the present work, we tackle to explore the applicability of the non-extensive statistics to study the thermodynamics of particles at vanishing and non-vanishing magnetic fields scanned over masses up to 10 GeV.

The primary goal of the current effort is to improve our understanding of the non-extensive statistics, as well as the impact of magnetic catalysis (MC) and/or inverse magnetic catalysis (IMC) mechanisms on the thermodynamics of QCD matter. In addition, the most recent lattice results have fascinating qualities and traits that contrast with them. This paper is organized as follows: Section 2 is devoted to studying the non-extensive statistics in detail. The results and discussion are presented in Section 3. Finally, the concluding remarks are introduced in Section 4.

[1] The pion mass m_π^2 in GeV^2 is taken as the unit of eB, where e is the electron charge and $m_\pi \approx 140$ MeV, where 1 MeV$^2 = e \times 1.6904 \times 10^{14}$ Gauss with $\hbar = c = 1$.

2. Formulation

The most remarkable point in the Boltzmann–Gibbs (BG) statistics is the additive property, where a system A is composed of two subsystems A_1 and A_2. Thus, the total entropy can be written as

$$S_{\mathrm{BG}}(A) = S_{\mathrm{BG}}(A_1) + S_{\mathrm{BG}}(A_2)\,. \tag{1}$$

In 1988, Tsallis suggested the non-extensive statistics [54]. In other words, the additive property of entropy is no longer satisfied by Eq. (1). Therefore, the non-extensive entropy is represented as [55]

$$S_q(A_1 + A_2) = S_q(A_1) + S_q(A_2) + (1-q)S_q(A_1)S_q(A_2)\,, \tag{2}$$

where q is the non-extensive parameter or the entropic index. In the limit of $q \to 1$, one recovers the BG-statistics. Additionally, q should be greater than one and the value of q is discussed in [56–59]. The non-extensive statistics are based on q-exponential and q-logarithmic functions that are defined as follows:

— the q-exponential

$$\begin{aligned} e_q^{(+)}(x) &= [1+(q-1)x]^{\frac{1}{q-1}}\,, \quad x \geqslant 0\,, \\ e_q^{(-)}(x) &= [1+(1-q)x]^{\frac{1}{1-q}}\,, \quad x < 0\,, \end{aligned} \tag{3}$$

— the q-logarithmic

$$\begin{aligned} \log_q^{(+)}(x) &= \frac{x^{q-1}-1}{q-1}\,, \quad x \geqslant 1\,, \\ \log_q^{(-)}(x) &= \frac{x^{1-q}-1}{1-q}\,, \quad x < 1\,. \end{aligned} \tag{4}$$

According to Eqs. (3) and (4), the grand partition function and the free energy can be defined at zero and non-zero magnetic fields.

2.1. Thermal free energy at zero magnetic field

The grand-canonical partition function is defined as [60, 61]

$$\log \Xi_q(V,T,\mu) = -\xi V \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \sum_{r=\pm} \Theta(rx) \log_q^{(-r)}\left(\frac{e_q^{(r)}(x) - \xi}{e_q^{(r)}(x)}\right)\,, \tag{5}$$

where $x = \beta(E_i(\boldsymbol{p}) - \mu)$ and $\beta = \frac{1}{T}$ with $\hbar = c = k_B = 1$. The particle energy $E_i(\boldsymbol{p}) = \sqrt{\boldsymbol{p}^2 + m_i^2}$, the chemical potential μ, $\Theta(rx)$ is the step function, and $\xi = \pm 1$ refers to bosons and fermions respectively.

Equation (5) categorizes two sectors of particles: bosons and fermions in the limit of $q \to 1$. The two sectors can be represented by the step function $\Theta(rx)$, in which x can be $+/-$ which reduces again to bosons and fermions. The first sector is $\mu \leq m$, in which the partition function is defined for bosons, *i.e.* for $r = +$, and the second sector is $r = -$ for fermions, where $\mu > m$.

Moreover, the free energy is directly related to the partition function as $F_{\mathrm{thermal}}(V,T) = -\beta^{-1} \log \Xi_q(V,T,\mu)$

$$-p = \left(\frac{\partial F}{\partial V}\right), \qquad -S = \left(\frac{\partial F}{\partial T}\right), \qquad -m_B = \frac{-M_B}{V} = \frac{1}{V}\left(\frac{\partial F}{\partial B}\right). \tag{6}$$

Equation (6) can be re-written specifically

— the pressure

$$P_i = -\xi T \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \sum_{r=\pm} \Theta(rx) \log_q^{(-r)} \left(\frac{e_q^{(r)}(x) - \xi}{e_q^{(r)}(x)}\right), \tag{7}$$

— the entropy

$$s = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \sum_{r=\pm} \Theta(rx) \times \left[-[B(x)]^{\tilde{q}} \log_q^{(-r)} B(x) + \xi[1+\xi B(x)]^{\tilde{q}} \log_q^{(-r)} (1+\xi B(x))\right], \tag{8}$$

where $B(x) \equiv \left[n_q^{(r)}(x)\right]^{1/\tilde{q}}$

$$n_q^{(r)}(x) = \left(\frac{1}{e_q^{(r)}(x) - \xi}\right)^{\tilde{q}}, \quad \text{with} \quad \tilde{q} = \begin{cases} q\,, & x \geqslant 0\,, \\ 2-q\,, & x < 0\,. \end{cases} \tag{9}$$

2.2. Thermal free energy at non-zero magnetic field

In the presence of the magnetic field, the partition function can be modified. In the case of $B \neq 0$, the triple integral in Eq. (2) follows the definition [62]

$$\int \mathrm{d}^3 p \to 2\pi |Q_i| e B_z \sum_k \sum_{s_z} \int \mathrm{d}p_z\,, \tag{10}$$

where $q_i = Q_i\, e$ is the charge of the particle i with the electric charge e, and $s_z = -s, \ldots, s$ is the z-component of the particle spin (s). The spin now has two sectors: bosons with integer spin, and fermions with half-integer spin. Accordingly, the relativistic total particle energy is re-defined as [63]

$$E(\boldsymbol{p}) \longrightarrow E_i(p_z, k, s_z) = \sqrt{p_z^2 + m_{\rm eff}^2}\,. \tag{11}$$

The effective mass reads, $m_{\rm eff}^2 = m_i^2 + 2q\, B_z\,(k - s_z + 0.5)$, where m_i is the mass of a charged particle i, k is the Landau level index which depends on the magnetic field strength:

- — Strong magnetic field ($eB \gg T^2$)
  which is called the lowest Landau level (LLL) *i.e.* ($k = 0$). The dynamics of (LLL) level is discussed in detail in [64] which showed that the magnetic field in general is considered as a catalyst in spontaneous chiral symmetry breaking, in particular the (LLL) level.
- — Weak magnetic field ($eB \ll T^2$)
  $k = 0, 1, 2, \ldots, \infty$.

The next step is to modify the non-extensive partition function due to the magnetic field. Thus, Eq. (5) is re-defined as

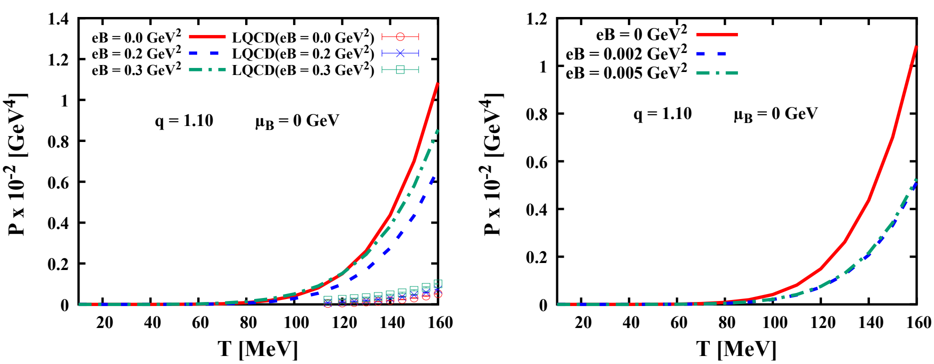
$$\begin{aligned}&\log \varXi_q^{B_z}(V, T, \mu)\\ &= \frac{-\xi V}{2\pi^2}|Q|eB_z \sum_k \sum_{s_z} \int_0^\infty \mathrm{d}p_z \sum_{r=\pm} \Theta(rx) \log_q^{(-r)} \left(\frac{e_q^{(r)}(x) - \xi}{e_q^{(r)}(x)}\right)\,.\end{aligned} \tag{12}$$

In a similar way, the thermodynamics can be obtained at the non-zero magnetic field using Eq. (12). A vacuum contribution term has been added to all calculations due to the presence of the magnetic field and regularization of the dimensions. This can be calculated easily at $T = 0$, for more details, see [29].

3. Results and discussion

The impact of the magnetic field on the hot QCD thermodynamics is presented. The effect of the weak and strong magnetic fields which lie in the range of $eB = 0.002, 0.003, 0.005$, and $eB = 0.2, 0.3$ in GeV^2, respectively, and at zero baryon chemical potential, $\mu_B = 0$ is studied. We have calculated the thermodynamics for the particles scanned from pion mass up to 10 GeV by the Particle Data Group (PDG) [65] for spin sectors (0, $\frac{1}{2}$, 1). In these spin sectors, the magnetic field should be valid due to the condition

$0 < qB < \frac{m_i^2}{2|s|-1}$ [66]. We initiate the discussion with the effect of strong and weak magnetic fields on the non-extensive pressure. In addition, the non-extensive parameter is also implemented at higher and lower values. The pressure *versus* the temperature is presented in Fig. 1 at $q = 1.1$ (higher entropic index). Figures 1 (a) and 1 (b) show a strong magnetic field and a weak magnetic field, respectively. We compare our results with the available lattice data [35, 45]. It is noticed that at $eB = 0$, the pressure is greater than at $eB \neq 0$. The pressure *versus* temperature is repeated again in Figs. 2 (a) and 2(b) at $q = 1.005$ (lower entropic index). Figure 2 (a) exhibits an increase in the pressure with increasing the magnetic field that is in agreement with the lattice results, at least qualitatively, while at weak magnetic field, Fig. 2 (b) shows the same behavior as appeared in Fig. 1 (b). It is worth to shed light here on the significant role of the entropic index q which determines the degree of non-extensivity of the medium. The non-extensive parameter q is first postulated in order to generalize the Boltzmann–Gibbs (BG) statistics in which the entropy is defined as $S_1 \equiv \lim_{q\to 1} = S_q = S_{\mathrm{BG}}$. A bias in the probabilities in the systems is represented by the parameter q. We define $\mathcal{P}_i$ for probability for the extensive systems, and $\mathcal{P}_i^q$ for the non-extensive systems. In general, the probability lies between $(0 < \mathcal{P}_i < 1)\forall i$. Then for $q < 1$, one has $\mathcal{P}_i^q > \mathcal{P}_i$ (rare events in which the probability tends to zero), while for $q > 1$, one has $\mathcal{P}_i^q < \mathcal{P}_i$ (frequent events) in which the probability tends to unity.

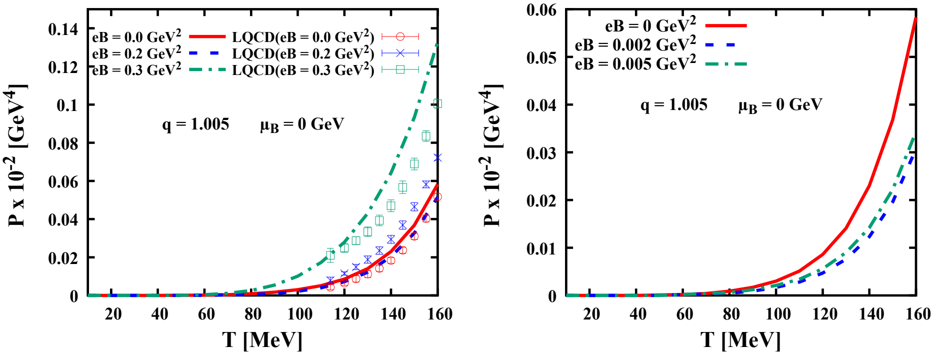


(a) strong magnetic field

(b) weak magnetic field

Fig. 1. The pressure *versus* temperature for the strong magnetic field (a) and the weak magnetic field (b), $q = 1.1$.

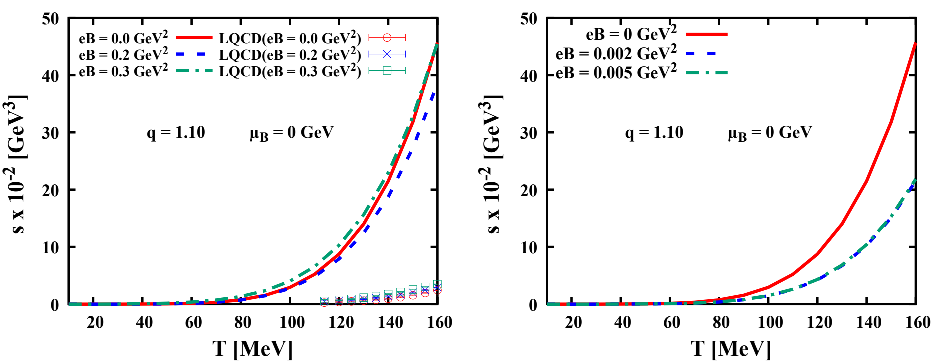
We use $q = 1.005$ which is a rough value to reveal the deviation from $q = 1$ (where BG statistics are recovered). The choice of q is worth discussing here. The thermodynamic results calculated with q deviate slightly from unity reproducing the lattice results, while the one which has larger q than



(a) strong magnetic field (b) weak magnetic field

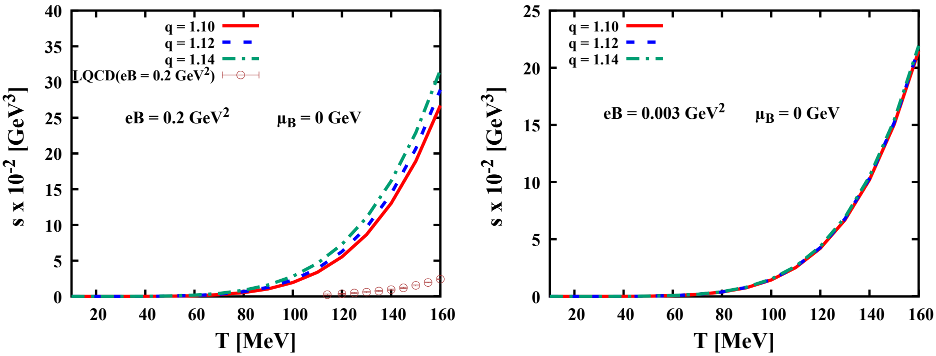
Fig. 2. The pressure *versus* temperature for the strong magnetic field (a) and the weak magnetic field (b), $q = 1.005$.

unity is inconsistent with the lattice results. This is shown in Figs. 1 (a) and 2 (a). Both figures were calculated under the same conditions except for the q value. We can notice the good agreement with the lattice in Fig. 2 (a) ($q = 1.005$) and the deviation from the lattice in Fig. 1 (a) in which the latter is calculated at $q = 1.1$. Based on the role of q, the entropy *versus* temperature is presented in Figs. 3 (a) and 3 (b) at $q = 1.1$ for weak and strong magnetic fields. Figures 4 (a) and 4 (b) show the entropy calculated for various values $q = 1.1, 1.12, 1.14$. Slight changes can be observed for the strong magnetic field. This proves the effect of the degree of non-extensivity toward non-equilibrium. This, in turn, overcomes the role of the magnetic field. Additionally, the magnetization *versus* temperature is presented in Figs. 5 (a) and 5 (b). It exhibits a positive value which confirms that the hadronic matter is a paramagnetic material.



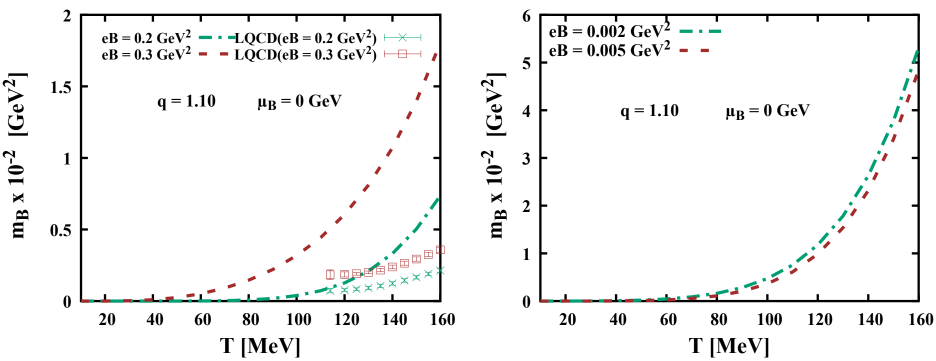
(a) strong magnetic field (b) weak magnetic field

Fig. 3. The entropy *versus* temperature at $q = 1.10$.



(a) strong magnetic field (b) weak magnetic field

Fig. 4. The entropy *versus* temperature at $eB = 0.003$ for the weak magnetic field, and at $eB = 0.2$ for the strong magnetic field.



(a) strong magnetic field (b) weak magnetic field

Fig. 5. The magnetization *versus* temperature at $q = 1.10$.

4. Conclusion

Investigation of the QCD thermodynamics (pressure, entropy, and magnetization) is presented. Thermodynamics can be conducted by using different statistics. In this paper, we have used the non-extensive statistics and studied the applicability of applying the magnetic field. The results were compared with the lattice data. It is noticed and concluded that the entropic index, q, plays a crucial role in the magnetic field. At higher q, the thermodynamics deviate from the lattice results, however, it behaves well with the magnetic field at least qualitatively. On the other hand, at lower q (*i.e.* $q \to 1$), where BG statistics can be retained, the magnetic field dominates. In addition, the magnetic field proved that it is a catalyst in chiral symmetry breaking, in which the latter acts as the magnetization process. Finally, QCD matter exhibits the paramagentic property.

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