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Neutrino masses in a two Higgs doublet model with a U(1) gauge symmetry

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ABSTRACT: General Two Higgs Doublet Models (2HDM) are popular Standard Model extensions but feature flavor changing interactions and lack neutrino masses. We discuss a 2HDM where neutrino masses are generated via type I seesaw and propose an extension where neutrino masses are generated via a type II seesaw mechanism with flavor changing interactions being absent via the presence of a U(1) gauge symmetry. After considering a variety of bounds such as those rising from collider and electroweak precision we show that our proposal stands as a UV complete 2HDM with a dark photon where neutrino masses and flavor changing interactions are addressed. A possible dark matter realization is also discussed.

KEYWORDS: Beyond Standard Model, Gauge Symmetry, Neutrino Physics

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1 Introduction

The Standard Model (SM) is the most accurate description of nature to the electroweak and strong interactions [1–3]. The discovery of a 125 GeV spin-0 state at CERN was the last piece of the puzzle in the SM [4, 5] and established the existence of, as far as we

know, an elementary scalar particle in nature. However, elementary scalar particles are common figures in many beyond the SM adventures, and among those Two-Higgs-Doublet stand out [6]. The ρ parameter, $\rho = m_W^2/(m_Z^2 \cos^2 \theta_W)$ is a powerful probe to those models featuring multiple scalar particles because they may contribute to the gauge boson masses [7] and, therefore, alter the SM prediction. The current value from global fits point to $\rho = 1.00039 \pm 0.00019$ [3]. The gauge boson masses arise from the kinetic terms of the scalars, thus the ρ parameter can be parametrized at tree level as,

$$\rho = \frac{\sum_{i=1}^n [I_i(I_i + 1) - \frac{1}{4} Y_i^2] v_i}{\sum_{i=1}^n \frac{1}{2} Y_i^2 v_i}, \tag{1.1}$$

where I_i , Y_i and v_i are the isospins and hypercharges and vacuum expectation values of the scalars. From eq. (1.1) we can see that scalar doublets ($I = 1/2$) with $Y = \pm 1$ and scalar singlets ($I = 0$) with $Y = 0$ do not contribute to the ρ parameter, and for this reason are desired extensions of the SM.

Two-Higgs-Doublet Models (2HDM) have indeed proven to be interesting models featuring a rich phenomenology concerning collider physics [8–10], axion models [11, 12], baryogenesis [13–15], flavor physics [16–20], among others [21, 22]. Several versions of 2HDM have been proposed in the literature trying to improve the original proposal in some theoretical aspects, via the inclusion of dark matter [23–32] and neutrino masses [33–38]. A proposal to explain neutrino masses in the context of 2HDM has already been put forth with no connection to gauge symmetries and absence of flavor changing neutral interactions (FCNI) [39–42]. A common feature in these studies is the presence of an ad-hoc discrete symmetry where one of the scalar doublets is odd under, which is added to avoid FCNI. It would be theoretically elegant if all these problems that general 2HDMs face could be solved in connection to gauge symmetries. Nevertheless, some proposals to extend the 2HDM via the presence of gauge symmetries have been put forth [43–45]. Some were triggered by anomalies in flavor and collider physics [46–48] and others devoted to explain neutrino masses via type I seesaw mechanism and absence of flavor changing interactions [49–52].

In this work, we propose a new model, still within the scope of 2HDM but different from previous studies. We explain the absence of FCNI via an abelian gauge symmetry $U(1)_X$ and neutrino masses via type II seesaw mechanism [53–55]. The presence of a new abelian gauge symmetry gives rise to gauge anomalies which, in order to be canceled out, impose restrictions over the SM fermion charges under the new symmetry. Moreover, this abelian group induces the presence of a Z' gauge boson. The scalar doublets develop vacuum expectation value (VEV) at the electroweak scale and the VEV of the scalar triplet cannot be large due to bounds stemming from the ρ parameter, hence the Z' is necessarily light, with masses below the weak scale. This fact has important experimental implications which will be investigated. We will also briefly discuss how one could potentially accommodate a dark matter candidate in our model.

Our work is structured as follows: in section 2, we introduce the 2HDM- $U(1)$ model and revisit how neutrino masses could be easily generated via a type I seesaw mechanism;

in section 3 we show how to implement a type II seesaw mechanism and obtain the mass spectrum of the model; in section 4 we discuss some phenomenological constraints; in section 6 we draw our conclusions. At the end we left three sections in the appendix where details of the anomaly cancellation and spontaneous symmetry breaking mechanism are shown.

2 Type I seesaw in the 2HDM-U(1)

As aforementioned, general 2HDM suffer from severe bounds rising from flavor physics. The problem of FCNI at tree level can be elegantly handled by the introduction of an extra abelian gauge symmetry. This new gauge symmetry is certainly more theoretically appealing than the usually ad-hoc Z_2 discrete symmetry which must be explicitly broken in order to not generate domain walls [56]. We will now revisit how the addition of an abelian gauge symmetry benefits 2HDM and generates neutrino masses via type I seesaw mechanism [57–60]. In this model, we have two scalar doublets $\Phi_i \sim (1, 2, 1, Q_{X_i})$ with the same hypercharge $Y = 1$ where,

$$\Phi_i = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \phi^0 = \frac{\rho_i + v_i + i\eta_i}{\sqrt{2}}, \quad i = 1, 2, \quad (2.1)$$

with $Q_{X_1} \neq Q_{X_2}$. The fact that $Q_{X_1} \neq Q_{X_2}$ leads to the scalar potential,

$$V_d = m_1^2 \Phi_1^\dagger \Phi_1 + m_2^2 \Phi_2^\dagger \Phi_2 + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1). \quad (2.2)$$

Since the scalar doublets have different charges under $U(1)_X$ only one of them will couple to SM fermions, and we arbitrarily choose Φ_2 . In this way, we get the Yukawa Lagrangian,

$$-\mathcal{L}_{Y_{2\text{HDM-I}}} = y_2^d \bar{Q}_L \Phi_2 d_R + y_2^u \bar{Q}_L \tilde{\Phi}_2 u_R + y_2^e \bar{L}_L \Phi_2 e_R + h.c. \quad (2.3)$$

Thus far our model is nearly identical to the usual type I 2HDM model proposed previously in the literature. The key difference lies in the introduction of a gauge symmetry which naturally explain the origin of the Z_2 symmetry. Since we also want to accommodate neutrino masses via type I seesaw mechanism we need to add three right-handed neutrinos as follows,

$$-\mathcal{L}_{Y_{N_R}} = y_2^D \bar{L}_L \tilde{\Phi}_2 N_R + y^M \bar{N}_R^c \Phi_s N_R + h.c. \quad (2.4)$$

where we now included a singlet scalar Φ_s , charged under $U(1)_X$, to build a majorana mass term, which features a scalar potential,

$$V_s = m_s^2 \Phi_s^\dagger \Phi_s + \frac{\lambda_s}{2} (\Phi_s^\dagger \Phi_s)^2 + \mu (\Phi_1^\dagger \Phi_2 \Phi_s + h.c.) + \lambda_{s1} (\Phi_1^\dagger \Phi_1) (\Phi_s^\dagger \Phi_s) + \lambda_{s2} (\Phi_2^\dagger \Phi_2) (\Phi_s^\dagger \Phi_s). \quad (2.5)$$

In the appendix A we show the vacuum stability bounds for the potential $V = V_d + V_s$ given by eqs. (2.2) and (2.5). The perturbative unitarity bounds, as well as the vacuum

stability bounds, for a similar potential containing two Higgs doublets plus a scalar singlet are presented in ref. [61].

With these Lagrangians we can simultaneously explain neutrino masses and the absence of FCNI. However, there is still one point that needs to be addressed which is the presence of gauge anomalies arising from the introduction of a new gauge symmetry. Since the SM fermions can be charged under the new gauge group and we have introduced new chiral fermions (right-handed neutrinos), the anomaly cancellation procedure becomes non-trivial. Generally the Φ_2 charge under $U(1)_X$ is different from zero, and therefore the SM fermions should also be charged under $U(1)_X$. The anomaly cancellation procedure is described in appendix B. We highlight that we can in principle keep the model anomaly free without the addition of right-handed neutrinos. However, without them we would not be able to explain neutrino masses the way we wished for. Anyways, with their presence we can derive the anomaly cancellation requirements that preserve our Lagrangians as follows,

$$\begin{aligned}
 q &= \frac{1}{2}(u + d), & l &= -\frac{3}{2}(u + d), & e &= -(2u + d), & n &= -(u + 2d), \\
 Q_{X_1} &= \frac{1}{2}(5u + 7d), & Q_{X_2} &= \frac{1}{2}(u - d), & q_X &= 2u + 4d,
 \end{aligned}
 \tag{2.6}$$

where u and d are the $U(1)_X$ charges of the up and down quarks respectively, q (l) the charge of the quark (lepton) doublet, e (n) the charge of the right-handed charged leptons (neutrinos), and lastly Q_{X_i} (q_X) the $U(1)_X$ charge of the scalar doublet (singlet).

The scalar singlet Φ_s is responsible for generating a majorana mass term for the right-handed neutrinos and breaking the $U(1)_X$ gauge symmetry that yields a massive Z' gauge boson. A natural question that rises to this conclusion is: is there a way to explain neutrino masses without adding a singlet scalar and right-handed neutrinos within the scope of 2HDM? Motivated by this question we will describe hereafter how one could accomplish that.

3 Type II seesaw in the 2HDM-U(1)

A popular mechanism to explain the active neutrino masses without the presence of right-handed neutrinos is the so called type II seesaw mechanism [62, 63]. In order to implement this mechanism within the scope of 2HDM the fermion charges under the gauge $U(1)_X$ symmetry need to be tied to one another to cancel out the triangle anomalies. We have seen above that one of the anomaly cancellation conditions is $n = -(u + 2d)$, which comes from the $U(1)^3$ triangle anomaly as shown in the appendix B, where n is the right-handed neutrino charge under $U(1)_X$. Therefore, if there are no right-handed neutrinos we must set $u = -2d$ to be free from gauge anomalies. Compared to the type I seesaw scenario, instead of having two independent charges (u and d), we now have only one, say d . That implies into,

$$\begin{aligned}
 q &= -\frac{d}{2}, & l &= \frac{3d}{2}, & e &= 3d, \\
 u &= -2d, & Q_{X_2} &= -\frac{3d}{2}.
 \end{aligned}
 \tag{3.1}$$

Charges in Type II seesaw 2HDMs free from FCNI

Fields	u_R	d_R	Q_L	L_L	e_R	Δ	Φ_2	Φ_1
Charges	$-2d$	d	$-d/2$	$3d/2$	$3d$	$-3d$	$-3d/2$	$\neq Q_{X_2}$
$U(1)_N$	0	0	0	0	0	0	0	$\neq Q_{X_2}$
$U(1)_{Y'}$	$4/3$	$-2/3$	$1/3$	-1	-2	2	1	$\neq Q_{X_2}$

Table 1. The table shows anomaly free Type I 2HDM where neutrino masses are generated via a type II seesaw mechanism. The first row shows the generic charges as functions of the d_R quark charge, d . Two particular cases are shown for $d = 0$ and $d = -2/3$, which correspond to sequential Z' and dark photon models, respectively. Notice that to prevent FCNI the scalar doublets have different charges under the $U(1)_X$ gauge symmetry.

The charge of the first doublet is free, as long as $Q_{X_1} \neq Q_{X_2}$, in order to recover the Yukawa Lagrangian (2.3) and keep the model free from FCNI. As shown in the table 1, there is essentially only two different possibilities. One where the SM fermions are neutral and the other where they are charged under $U(1)_X$. If a particular nonzero value is chosen for d , any other multiple of this value would produce a physically equivalent model, because a change in d can be balanced by a rescaling on the gauge coupling constant g_X , so that the $U(1)_X$ interaction remains the same. In particular, taking $d = -2/3$ we notice that the charges of the SM fermions under $U(1)_X$ are similar to the SM weak hypercharge. In this way, it is clear that a type II seesaw realization in the 2HDM- $U(1)$ gives rise either to a fermiophobic or a sequential Z' boson.

The implementation of type II seesaw mechanism requires an $SU(2)_L$ scalar triplet $\Delta \sim (1, 3, 2, q_{X_t})$, where the quantum numbers refers to the transformation properties under the symmetry group $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X$. The field Δ can be parameterized as,

$$\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}, \tag{3.2}$$

with,

$$\Delta^0 = \frac{\rho_t + v_t + i\eta_t}{\sqrt{2}}. \tag{3.3}$$

The $SU(2)_L$ symmetry allows the introduction of an interaction between Δ and the leptons via,

$$- \mathcal{L}_{Y_t} = y_L \bar{L}_L^c i\sigma^2 \Delta L_L + h.c. \tag{3.4}$$

which requires Δ to have hypercharge $Y_t = 2$ and lepton number $L_t = 2$, automatically forbidding interactions to quarks. The inclusion of eq. (3.4) implies,

$$2l + q_{X_t} = 0, \tag{3.5}$$

and using eq. (3.1) we get,

$$q_{X_t} = -3d, \tag{3.6}$$

explaining the Δ charge shown in table 1.

Since Δ carries lepton number, when the neutral scalar Δ^0 develops a VEV, v_t , lepton number is violated, and from eq. (3.4) we can easily see that it generates a majorana mass term for the neutrinos with,

$$m_\nu = \sqrt{2}g_L v_t. \tag{3.7}$$

Thus, v_t has to be very small in order to accommodate neutrino masses in the sub-eV range. In summary, with the presence of a $U(1)_X$ gauge symmetry, we can explain the absence of FCNI and accommodate neutrino masses via a type II seesaw without extra fermions, which is the main idea of this work. However, we need also to study the phenomenological implications of such proposal before concluding whether we have a feasible theoretical model. We start studying the mass spectrum of the model.

3.1 Mass spectrum — scalars

Our goal in this section is to study the spontaneous symmetry breaking pattern to find the physical scalars, gauge bosons, and neutrino masses. The SM charged lepton masses are the same as in the SM. That said, we begin our reasoning with the scalar sector.

The scalar sector is described by the Lagrangian,

$$\mathcal{L}_{\text{scalar}} = (D_\mu \Phi_i)^\dagger (D^\mu \Phi_i) + \text{Tr}[(D_\mu \Delta)^\dagger (D^\mu \Delta)] - V(\Phi_1, \Phi_2, \Delta), \tag{3.8}$$

where the covariant derivatives of the scalar doublets and the triplet read,

$$D_\mu \Phi_i = \partial_\mu \Phi_i + ig\tau^a W_\mu^a + ig' \frac{Y}{2} \hat{B}_\mu \Phi_i + ig_X \frac{Q_{X_i}}{2} \hat{X}_\mu \Phi_i, \tag{3.9}$$

$$D_\mu \Delta = \partial_\mu \Delta + ig[\tau^a W_\mu^a, \Delta] + ig' \frac{Y_t}{2} \hat{B}_\mu \Delta + ig_X \frac{q_{X_t}}{2} \hat{X}_\mu \Delta, \tag{3.10}$$

where τ^a are the generators of the $SU(2)_L$ group. The scalar potential in eq. (3.8), invariant under all gauge symmetries is given by

$$\begin{aligned} V(\Phi_1, \Phi_2, \Delta) = & m_1^2 \Phi_1^\dagger \Phi_1 + m_2^2 \Phi_2^\dagger \Phi_2 + m_t^2 \text{Tr}(\Delta^\dagger \Delta) + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 \\ & + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \lambda_{t1} (\Phi_1^\dagger \Phi_1) \text{Tr}(\Delta^\dagger \Delta) \\ & + \lambda_{t2} (\Phi_2^\dagger \Phi_2) \text{Tr}(\Delta^\dagger \Delta) + \lambda_{tt1} \Phi_1^\dagger \Delta \Delta^\dagger \Phi_1 + \lambda_{tt2} \Phi_2^\dagger \Delta \Delta^\dagger \Phi_2 \\ & + \lambda_t [\text{Tr}(\Delta^\dagger \Delta)]^2 + \lambda_{tt} \text{Tr}(\Delta^\dagger \Delta)^2 + \mu_{t2} (\Phi_2^T i\sigma^2 \Delta^\dagger \Phi_2 + \text{h.c.}). \end{aligned} \tag{3.11}$$

The necessary conditions for having vacuum stability with this potential are given in appendix A. Due the presence of the scalar triplet, the unitarity bounds are much more involved than the potential with the scalar singlet in section 2. We postpone an complete analysis of the unitarity and vacuum stability bounds to another work.

Observe in the potential above that the terms $\Phi_1^T i\sigma^2 \Delta^\dagger \Phi_2$ and $\Phi_1^T i\sigma^2 \Delta^\dagger \Phi_1$ are forbidden by the $U(1)_X$ symmetry, as we require $Q_{X1} \neq Q_{X2}$. There is only one non-hermitian term $\Phi_2^T i\sigma^2 \Delta^\dagger \Phi_2$, which breaks lepton number in two units. Such lepton number violation is a common feature in seesaw type II models. It is important to note that neutrino masses are generated when Δ^0 develops a vacuum expectation value as shown in eq. (3.7) and that would be related to lepton number violation since the scalar triplet carries lepton number. However, notice that the non-hermitian term in eq. (3.11) already explicitly

violates lepton number, thus lepton number had been violated even before Δ^0 develops a non-trivial vacuum expect value. We checked that without this non-hermitian term in the scalar potential the pseudoscalar from the scalar triplet field would remain massless, i.e. a majoron field [64].

Anyway, substituting the VEVs,

$$\langle \phi_i^0 \rangle = \frac{v_i}{\sqrt{2}}, \quad \langle \Delta^0 \rangle = \frac{v_t}{\sqrt{2}}, \quad (3.12)$$

in order to break spontaneously the gauge symmetries, we have the following constraint equations for a minimal point of the potential,

$$m_1^2 + \frac{1}{2} [2\lambda_1 v_1^2 + (\lambda_3 + \lambda_4)v_2^2 + (\lambda_{t1} + \lambda_{tt1})v_t^2] = 0, \quad (3.13)$$

$$m_2^2 + \frac{1}{2} [(\lambda_3 + \lambda_4)v_1^2 + 2\lambda_2 v_2^2 + (\lambda_{t2} + \lambda_{tt2})v_t^2 - 2\sqrt{2}\mu_{t2}v_t] = 0, \quad (3.14)$$

$$\frac{v_t}{2} [2m_t^2 + (\lambda_{t1} + \lambda_{tt1})v_1^2 + (\lambda_{t2} + \lambda_{tt2})v_2^2 + 2(\lambda_t + \lambda_{tt})v_t^2] - \frac{\mu_{t2}v_2^2}{\sqrt{2}} = 0. \quad (3.15)$$

In the Standard Model, the symmetry is spontaneously broken when the mass term flips sign. In the case of a type II seesaw, the mass term of the scalar triplet in the scalar potential does not need to flip sign to break the symmetry. We emphasize that the gauge symmetry is broken when the scalar doublets acquire a non-zero vacuum expectation value. See [65] for a detailed discussion about the type II seesaw vacuum. Moreover, we will see later on that the mass term of the scalar triplet should be positive in order to generate a pseudoscalar with positive mass.

Assuming that $2m_t^2$ is the dominant term between the brackets in the constraint equation (3.15), we have a seesaw relation,

$$v_t \simeq \frac{\mu_{t2}v_2^2}{\sqrt{2}m_t^2}, \quad (3.16)$$

which leads to a naturally dwindled v_t for $|m_t^2| \gg |\mu_{t2}v_2|$. In this way, a small v_t can be understood as a simply consequence of having the coefficient of the bilinear term in Δ to be comparatively large with respect to the other energy scales of the scalar potential. Note that from eq. (3.16) we conclude that m_t^2 and μ_{t2} should have the same sign.

In the scalar sector, Φ_i and Δ render the existence of seven physical fields: 3 CP-even scalars, h , H and H_t ; one CP-odd, A ; two singly charged H^+ , H_t^+ and one doubly charged H^{++} . The other scalar degrees of freedom are absorbed as longitudinal components by the gauge bosons, W^\pm , Z and Z' , making them massive.

In the basis (ρ_1, ρ_2, ρ_t) the neutral scalars mix according to the following mass matrix,

$$M_{\text{CPeven}}^2 = \begin{pmatrix} 2\lambda_1 v_1^2 & (\lambda_3 + \lambda_4)v_1 v_2 & (\lambda_{t1} + \lambda_{tt1})v_1 v_t \\ (\lambda_3 + \lambda_4)v_1 v_2 & 2\lambda_2 v_2^2 & (\lambda_{t2} + \lambda_{tt2})v_2 v_t - \sqrt{2}\mu_{t2}v_2 \\ (\lambda_{t1} + \lambda_{tt1})v_1 v_t & (\lambda_{t2} + \lambda_{tt2})v_2 v_t - \sqrt{2}\mu_{t2}v_2 & 2(\lambda_t + \lambda_{tt})v_t^2 + \frac{\mu_{t2}v_2^2}{\sqrt{2}v_t} \end{pmatrix}. \quad (3.17)$$

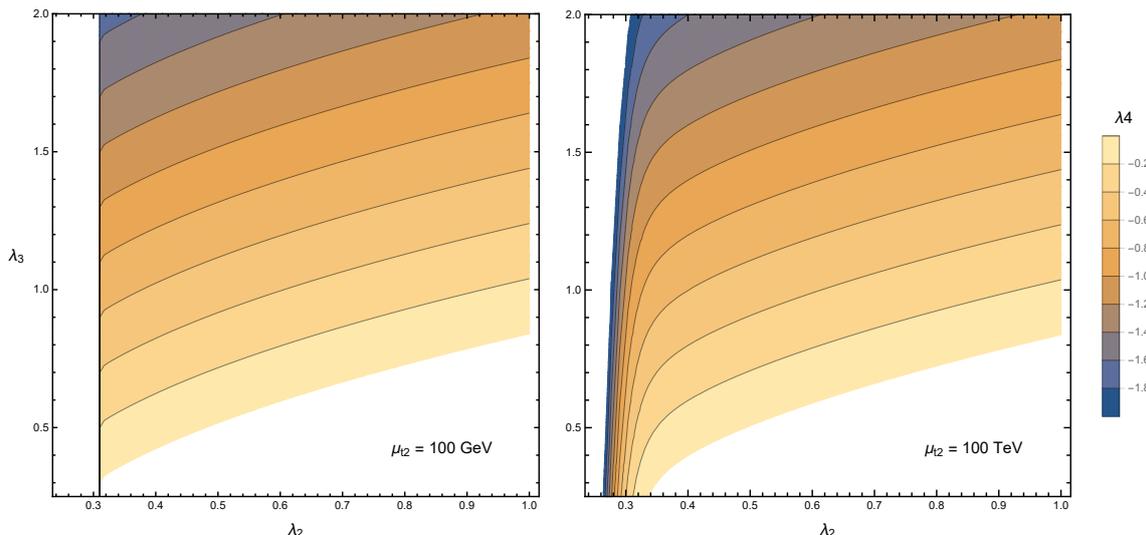


Figure 1. Region of parameter space that leads to a 125 GeV SM-like Higgs boson for $v_2 = 200$ GeV, $v_t = 1$ MeV and $\lambda_1 = 0.6$. In the left panel $\mu_{t2} = 100$ GeV and in the right panel, $\mu_{t2} = 100$ TeV.

From the diagonalization procedure of this mass matrix we will get three physical scalars, h , H and H_t . We can parametrize this diagonalization in terms of three mixing angles α , α_1 and α_2 ,

$$\begin{pmatrix} h \\ H \\ H_t \end{pmatrix} = \begin{pmatrix} c_\alpha & s_\alpha & 0 \\ -s_\alpha & c_\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{\alpha_1} & 0 & s_{\alpha_1} \\ 0 & 1 & 0 \\ -s_{\alpha_1} & 0 & c_{\alpha_1} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{\alpha_2} & s_{\alpha_2} \\ 0 & -s_{\alpha_2} & c_{\alpha_2} \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_t \end{pmatrix}, \quad (3.18)$$

where $s_{\alpha,\alpha_1,\alpha_2}$ and $c_{\alpha,\alpha_1,\alpha_2}$ are sine and cosine functions. We choose h to denote the 125 GeV SM-like Higgs found in the LHC [4, 5]. The angles are determined by the parameters of the potential and the scalar VEVs. Fully analytic expressions for the masses and eigenvectors are complicated but we can obtain approximate results. As shown in the appendix C.1, in the limit $v_t \ll v_i$, the masses of the CP-even scalars are approximately,

$$m_h^2 = \lambda_1 v_1^2 + \lambda_2 v_2^2 - \sqrt{(\lambda_1 v_1^2 - \lambda_2 v_2^2)^2 + (\lambda_3 + \lambda_4)^2 v_1^2 v_2^2} - 2\sqrt{2} \sin^2 \alpha \mu_{t2} v_t \quad (3.19)$$

$$m_H^2 = \lambda_1 v_1^2 + \lambda_2 v_2^2 + \sqrt{(\lambda_1 v_1^2 - \lambda_2 v_2^2)^2 + (\lambda_3 + \lambda_4)^2 v_1^2 v_2^2} - 2\sqrt{2} \cos^2 \alpha \mu_{t2} v_t \quad (3.20)$$

$$m_{H_t}^2 = \frac{\mu_{t2} v_2^2}{\sqrt{2} v_t}. \quad (3.21)$$

This limit $v_t \ll v_i$ that will be assumed throughout this work yields a higgs boson, h , with the correct mass is shown in figure 1. It is straightforward to see that we can easily find a higgs with the correct mass for couplings of order one and μ_{t2} either at the weak or multi-TeV scale.

As for the pseudoscalars, in the basis (η_1, η_2, η_t) the mass matrix is given by,

$$M_{\text{CPodd}}^2 = \sqrt{2} \mu_{t2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2v_t & -v_2 \\ 0 & -v_2 & \frac{v_2^2}{2v_t} \end{pmatrix}. \quad (3.22)$$

Note that η_1 is decoupled and massless. Thus it can be immediately recognized as a Goldstone boson, G_1 . After the diagonalization procedure we found another Goldstone boson, G_2 . These two massless pseudoscalars represent the degrees of freedom needed to generate the Z and Z' masses. In the diagonalization procedure we find the rotation matrix,

$$\begin{pmatrix} G_1 \\ G_2 \\ A \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{\beta'} & s_{\beta'} \\ 0 & -s_{\beta'} & c_{\beta'} \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_t \end{pmatrix}, \quad (3.23)$$

where,

$$\tan \beta' = \frac{2v_t}{v_2}, \quad (3.24)$$

which gives rise to two massless fields as aforementioned and a massive pseudoscalar, A , with mass,

$$m_A^2 = \frac{\mu_{t2}(v_2^2 + 4v_t^2)}{\sqrt{2}v_t}. \quad (3.25)$$

Observe that v_t and μ_{t2} must have the same sign in order to have $m_A^2 > 0$. We had concluded previously from eq. (3.16) that μ_{t2} and m_t^2 should have the same sign to keep v_t positive definite, thus from eq. (3.25) μ_{t2} must be positive to generate a positive squared mass for the pseudoscalar A . Hence, both μ_{t2} and m_t^2 are strictly positive.

It is important to stress that even with the introduction of a new gauge symmetry, the pseudoscalar A , which is a common figure in 2HDM, remains in the spectrum. Under the assumption that v_t is smaller than μ_{t2} the pseudoscalar can have a mass sufficiently large to evade existing bounds, as we shall discuss further.

The charged scalars mass matrix in the basis $(\phi_1^+, \phi_2^+, \Delta^+)$ is,

$$M_{\text{Charged}}^2 = \frac{1}{2} \begin{pmatrix} -\lambda_4 v_2^2 - \lambda_{tt1} v_t^2 & \lambda_4 v_1 v_2 & \lambda_{tt1} v_1 v_t / \sqrt{2} \\ \lambda_4 v_1 v_2 & -\lambda_4 v_1^2 - \lambda_{tt2} v_t^2 + 2\sqrt{2}\mu_{t2} v_t & \frac{1}{2}(\sqrt{2}\lambda_{tt2} v_t - 4\mu_{t2})v_2 \\ v_1 v_t \lambda_{tt1} / \sqrt{2} & \frac{1}{2}(\sqrt{2}\lambda_{tt2} v_t - 4\mu_{t2})v_2 & \frac{\sqrt{2}\mu_{t2} v_2^2}{v_t} - \frac{1}{2}(\lambda_{tt1} v_1^2 + \lambda_{tt2} v_2^2) \end{pmatrix}. \quad (3.26)$$

The physical fields are given by performing the following rotation,

$$\begin{pmatrix} G^+ \\ H^+ \\ H_t^+ \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta & 0 \\ -s_\beta & c_\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{\beta_1} & 0 & s_{\beta_1} \\ 0 & 1 & 0 \\ -s_{\beta_1} & 0 & c_{\beta_1} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{\beta_2} & s_{\beta_2} \\ 0 & -s_{\beta_2} & c_{\beta_2} \end{pmatrix} \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \\ \Delta^+ \end{pmatrix}. \quad (3.27)$$

The Goldstone boson G^+ is absorbed by W^+ , and the physical states H^+ and H_t^+ have masses,

$$m_{H^+}^2 = \frac{1}{8}(A - \sqrt{A^2 - B}), \quad (3.28)$$

$$m_{H_t^+}^2 = \frac{1}{8}(A + \sqrt{A^2 - B}), \quad (3.29)$$

where,

$$\begin{aligned}
 A &= -2\lambda_4 (v_1^2 + v_2^2) - \lambda_{tt1} (v_1^2 + 2v_t^2) - \lambda_{tt2} (v_2^2 + 2v_t^2) + 2\sqrt{2}\frac{\mu_{t2}}{v_t} (v_2^2 + 2v_t^2), \\
 B &= 8(v_1^2 + v_2^2 + 2v_t^2) \left[\lambda_4 (\lambda_{tt1}v_1^2 + \lambda_{tt2}v_2^2) + \lambda_{tt1}\lambda_{tt2}v_t^2 - 2\sqrt{2}\frac{\mu_{t2}}{v_t} (\lambda_4v_2^2 + \lambda_{tt1}v_t^2) \right].
 \end{aligned}$$

The doubly charged scalar Δ^{++} does not mix any other field. This mass eigenstate, which we will denote henceforth by H^{++} , has a mass given by,

$$m_{H^{\pm\pm}}^2 = \frac{\mu_{t2}v_2^2}{\sqrt{2}v_t} - \frac{1}{2}(\lambda_{tt1}v_1^2 + \lambda_{tt2}v_2^2 + 2\lambda_{tt}v_t^2). \quad (3.30)$$

In summary, the scalar mass spectrum is largely controlled by the relative sizes of v_t , μ_{t2} and v_i . As v_i is fixed to be ~ 100 GeV and, as we will see later on, v_t is constrained to be $\lesssim O(1)$ GeV, we will always take $v_t \ll v_i$. In this limit, the masses of h , H and H^+ are rather insensitive to v_t and μ_{t2} , while the masses of H_t , A , H_t^+ and H^{++} strongly depend on them. As μ_{t2} , in principle, remains as a free parameter, we can distinguish three different regimes according to its size:

- $\mu_{t2} \sim v_t \ll v_i$: in this case, μ_{t2} has little influence on the masses of h and H . H remains always heavier than h , with a mass around the 100–300 GeV range, for λ 's of order ~ 1 . The masses of H_t , A , H_t^+ and H^{++} are controlled by the ratio μ_{t2}/v_t , with H_t and A nearly mass degenerate. In particular, for $\mu_{t2} = v_t$, the masses are around 200 GeV. Such low masses can be dangerous in light of existing bounds [66–74].
- $\mu_{t2} \sim v_i$: in this scenario the spectrum is shifted up and the scalar masses can be significantly larger than 100 GeV. A , H_t , H_t^+ and H^{++} are mass degenerate and may reach masses in the TeV domain. For example, taking $\mu_{t2} = v_2 = 100$ GeV and $v_t = 100$ MeV, we obtain $m_h = 125$ GeV, $m_H = 404$ GeV, $m_{H^+} = 507$ GeV, $m_{H_t} \simeq m_A \simeq m_{H_t^+} \simeq m_{H^{++}} \simeq 2.65$ TeV. We have adopted $\lambda_1 = 1.6$, $\lambda_2 = 0.9$, $\lambda_3 = 7.7$, $\lambda_4 = -8.5$, $\lambda_{ti} = \lambda_{tti} = 0.5$.
- $\mu_{t2} \gg v_i$: this case can be recognized as the canonical type II seesaw scenario, in which m_t and μ_{t2} come from new physics at very high energy scale, like Grand Unification scale. In this case, only h , H and H^+ remain in the weak scale, while H_t , A , H_t^+ and H^{++} decouple and are still degenerate, getting very high masses of order $\sim v\sqrt{\mu_{t2}/v_t}$.

As stressed above, the masses of H_t , A , H_t^+ and H^{++} are always close to each other because their masses follow $m_{H_i}^2 = m_t^2 + O(v, v_t)$, with $m_t^2 \simeq \mu_{t2}v_2^2/\sqrt{2}v_t$. Thus, for small v_t , the m_t^2 term is the dominant one, so that the masses are all approximately given by $m_{H_i}^2 \simeq m_t^2$. The mass splittings are controlled by the scalar VEVs. At leading order,

$$\begin{aligned}
 m_{H_t}^2 - m_A^2 &\simeq O(v_t^2), \\
 m_A^2 - m_{H_t^+}^2 &\simeq m_{H_t^+}^2 - m_{H^{++}}^2 \simeq \frac{1}{4}(\lambda_{tt1}v_1^2 + \lambda_{tt2}v_2^2).
 \end{aligned} \quad (3.31)$$

These mass splittings are noticeable only when the masses are small, i.e., for small μ_{t2} . For $\mu_{t2} \gtrsim v_i$, they are basically mass degenerate.

As aforementioned, the masses of h , H and H^+ are less sensitive to v_t and μ_{t2} , and depend mostly on the VEVs v_i and the λ_i 's. Therefore, they naturally lie at the weak scale. As for H^+ , we find $m_{H^+}^2 \simeq -\frac{1}{2}\lambda_4 v^2$ (see eq. (C.36) in the appendix C.2), which requires λ_4 to be negative. If we took $|\lambda_4| > 1$, we would have charged scalar masses above 500 GeV, which can easily evade existing limits [75–82]. In order to have scalar masses above 500 GeV we need couplings larger than the unit. This typically true in seesaw type II models.

Returning to the mass expressions of the CP-even scalars, eqs. (3.19) and (3.20), we see that if there were just the two scalar doublets, the neutral scalar masses would be given by these expressions with v_t, μ_{t2} set to zero. However, the scalar triplet generate negative correction terms proportional to $\mu_{t2}v_t$, so that these scalars become lighter than they would be if there was not the triplet. However, as shown in figure 1, the parameter space allows to fit a mass $m_h = 125$ GeV for the Higgs boson h . Furthermore, the eqs. (3.19) and (3.20) imply an upper bound in the combination $\mu_{t2}v_t$: a large value of μ_{t2} must be balanced by a small value of v_t in order to avoid negative squared masses for h and H . Looking at these equations, we conclude that in order to preserve the masses positive, v_t must satisfy,

$$v_t \lesssim \frac{(\lambda v)^2}{\mu_{t2}}, \tag{3.32}$$

where we assume $\lambda v \sim 100$ GeV. For instance, $\mu_{t2} \sim 10^{14}$ GeV implies $v_t \lesssim 10^{-1}$ eV. Thus, one may naturally generate small v_t taking μ_{t2} at a Grand Unification scale [83]. Notice that eq. (3.32) is another kind of seesaw relation between v_t and μ_{t2} valid in the limit $v_t \ll \mu_{t2}$, which is independent of the relation in eq. (3.16).

Now that we have finished with the scalar sector, we will derive the masses of the gauge bosons.

3.2 Mass spectrum — gauge bosons

The Lagrangian for the kinetic terms of the gauge fields associated to the hypercharge $U(1)_Y$ and the $U(1)_X$ symmetry is given by,

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}\hat{B}_{\mu\nu}\hat{B}^{\mu\nu} + \frac{\epsilon}{2\cos\theta_W}\hat{X}_{\mu\nu}\hat{B}^{\mu\nu} - \frac{1}{4}\hat{X}_{\mu\nu}\hat{X}^{\mu\nu}, \tag{3.33}$$

where ϵ is the kinetic mixing parameter.

A canonical gauge kinetic Lagrangian is obtained through a $GL(2, R)$ rotation on the fields \hat{B}_μ and \hat{X}_μ ,

$$\begin{aligned} \hat{X}_\mu &\simeq X_\mu \\ \hat{B}_\mu &\simeq B_\mu + \frac{\epsilon}{\cos\theta_W}X_\mu, \end{aligned} \tag{3.34}$$

so that the covariant derivatives (3.9) and (3.10) become,

$$D_\mu \Phi_i = \partial_\mu \Phi_i + ig\tau^a W_\mu^a + ig' \frac{Y}{2} B_\mu \Phi_i + \frac{i}{2} G_{X_i} X_\mu \Phi_i, \quad (3.35)$$

$$D_\mu \Delta = \partial_\mu \Delta + ig[T^a W_\mu^a, \Delta] + ig' \frac{Y_t}{2} B_\mu \Delta + \frac{i}{2} G_{X_t} X_\mu \Delta, \quad (3.36)$$

where $G_{X_i} = g' \frac{\epsilon Y_i}{\cos \theta_W} + g_X Q_{X_i}$ and $G_{X_t} = g' \frac{\epsilon Y_t}{\cos \theta_W} + g_X q_{X_t}$.

After spontaneous symmetry breaking and performing the electroweak rotation,

$$\begin{aligned} B_\mu &= \cos \theta_W A_\mu - \sin \theta_W Z_\mu^0, \\ W_\mu^3 &= \sin \theta_W A_\mu + \cos \theta_W Z_\mu^0, \end{aligned}$$

the spectrum of vector bosons turns out to be comprised of: the charged W_μ^\pm ; the photon, A_μ ; two neutral states, Z_μ^0 and X_μ , mixing to each other. They have the following mass Lagrangian

$$\mathcal{L}_{\text{mass}} = m_W^2 W_\mu^- W^{+\mu} + \frac{1}{2} m_{Z^0 X}^2 Z_\mu^0 Z^{0\mu} - m_{Z^0 X}^2 Z_\mu^0 X^\mu + \frac{1}{2} m_X^2 X_\mu X^\mu, \quad (3.37)$$

where,

$$m_W^2 = \frac{1}{4} g^2 (v^2 + 2v_t^2), \quad (3.38)$$

$$m_{Z^0}^2 = \frac{1}{4} g_Z^2 (v^2 + 4v_t^2), \quad (3.39)$$

$$m_{Z^0 X}^2 = \frac{1}{4} g_Z (G_{X1} v_1^2 + G_{X2} v_2^2 + 2G_{X_t} v_t^2), \quad (3.40)$$

$$m_X^2 = \frac{1}{4} (v_1^2 G_{X1}^2 + v_2^2 G_{X2}^2 + G_{X_t}^2 v_t^2), \quad (3.41)$$

with $g_Z^2 = g^2 + g'^2 = g^2 / \cos^2 \theta_W$, $v^2 = v_1^2 + v_2^2$ and $v^2 + 2v_t^2 = (246 \text{ GeV})^2$.

We see that the W_μ^\pm bosons are already the mass-eigenstates with mass m_W . The Z and Z' gauge bosons on the other hand mix and lead to the following mass matrix,

$$M_{Z'Z}^2 = \begin{pmatrix} m_{Z^0}^2 & -m_{Z^0 X}^2 \\ -m_{Z^0 X}^2 & m_X^2 \end{pmatrix}. \quad (3.42)$$

The diagonalization leads to,

$$\begin{aligned} m_Z^2 &= \frac{1}{2} \left[m_{Z^0}^2 + m_X^2 + \sqrt{(m_{Z^0}^2 - m_X^2)^2 + 4(m_{Z^0 X}^2)^2} \right], \\ m_{Z'}^2 &= \frac{1}{2} \left[m_{Z^0}^2 + m_X^2 - \sqrt{(m_{Z^0}^2 - m_X^2)^2 + 4(m_{Z^0 X}^2)^2} \right]. \end{aligned} \quad (3.43)$$

where,

$$\begin{pmatrix} Z_\mu \\ Z'_\mu \end{pmatrix} = \begin{pmatrix} \cos \xi & -\sin \xi \\ \sin \xi & \cos \xi \end{pmatrix} \begin{pmatrix} Z_\mu^0 \\ X_\mu \end{pmatrix}, \quad (3.44)$$

with ξ given by,

$$\tan 2\xi = \frac{2m_{Z^0 X}^2}{m_{Z^0}^2 - m_X^2}. \quad (3.45)$$

This mixing angle is constrained to be very small by the LEP electroweak precision measurements on the Z boson pole [84]. Thus,

$$\xi \simeq \frac{m_{Z^0 X}^2}{m_{Z^0}^2 - m_X^2}. \quad (3.46)$$

Also, as we are interested in a light Z' , we will assume the limit $m_{Z^0}^2 \gg m_X^2$ (which implies $m_{Z^0}^2 \gg m_{Z^0 X}^2$ as well). In this limit,

$$\xi \simeq \frac{m_{Z^0 X}^2}{m_{Z^0}^2}, \quad (3.47)$$

and we can write approximate expressions for the masses of Z and Z' as

$$\begin{aligned} m_{Z,Z'}^2 &= \frac{1}{2} \left\{ m_{Z^0}^2 + m_X^2 \pm (m_{Z^0}^2 - m_X^2) \left[1 + \frac{4(m_{Z^0 X}^2)^2}{(m_{Z^0}^2 - m_X^2)^2} \right]^{\frac{1}{2}} \right\} \\ &\simeq \frac{1}{2} \left[m_{Z^0}^2 + m_X^2 \pm \left(m_{Z^0}^2 - m_X^2 + \frac{2(m_{Z^0 X}^2)^2}{m_{Z^0}^2} \right) \right]. \end{aligned}$$

For Z , we have

$$m_Z^2 \simeq m_{Z^0}^2 + \frac{(m_{Z^0 X}^2)^2}{m_{Z^0}^2},$$

and, at leading order, $m_Z^2 \simeq m_{Z^0}^2$,

$$m_Z^2 \simeq \frac{1}{4} g_Z^2 (v^2 + 4v_t^2). \quad (3.48)$$

For Z' ,

$$\begin{aligned} m_{Z'}^2 &\simeq m_X^2 - \frac{(m_{Z^0 X}^2)^2}{m_{Z^0}^2} \\ &\simeq \frac{g_X^2}{4} (Q_{X_1} - Q_{X_2})^2 \frac{v_1^2 v_2^2}{v^2} \left(1 - \frac{4v_t^2}{v^2} \right). \end{aligned}$$

In terms of β , defined by $\tan \beta = v_2/v_1$ (see appendix C.2),

$$m_{Z'}^2 \simeq \frac{g_X^2}{4} (Q_{X_1} - Q_{X_2})^2 v^2 \sin^2 \beta \cos^2 \beta \left(1 - \frac{4v_t^2}{v^2} \right). \quad (3.49)$$

Note that the presence of the triplet induces only a tiny correction proportional to $(v_t/v)^2$, so that the addition of a triplet scalar cannot generate a heavy Z' , as opposed to the singlet case [52]. Thus, the Z' mass lies below the electroweak scale, being controlled by the value of g_X . For instance, taking $\tan \beta = 10$ and $Q_{X_1} - Q_{X_2} = 1$, $m_{Z'}$ varies from 1 MeV – 1 GeV, for g_X in the range of $10^{-3} - 10^{-1}$, regardless of the value of v_t , as long as $v_t < 2$ GeV.

In summary, we have proposed a type II seesaw mechanism for neutrino masses within the scope of 2HDM which prevent FCNI via gauge symmetries. Having discussed the mass spectrum of the model, we now will pay attention to some phenomenological constraints.

4 Phenomenological constraints

4.1 Electroweak precision

The ρ parameter,

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2 \theta_W}, \tag{4.1}$$

which measures the relative intensity between the neutral and charged currents, is very accurately determined experimentally, $\rho = 1.00039 \pm 0.00019$ [3] at 1σ level. In the SM, the ρ parameter is equal to 1 at tree level, and its good agreement with the experimental value poses tight constraints on new physics models with extended scalar sector. In our model, the ρ parameter places an upper bound on the VEV of the triplet scalar because it contributes to the masses of Z and W^\pm bosons according to eqs. (3.38) and (3.48), translating into

$$\rho = \frac{v^2 + 2v_t^2}{v^2 + 4v_t^2}. \tag{4.2}$$

Hence at 3σ we obtain

$$v_t \leq 2.3 \text{ GeV}, \tag{4.3}$$

where we used $v^2 + 2v_t^2 = 246^2 \text{ GeV}^2$. As we are interested in a small v_t for the generation of tiny neutrino masses, this constraint can be easily satisfied in our model. Notice that as v_t becomes very small the scalar masses increase as can be seen, for instance in eq. (3.25) and eq. (3.30).

4.2 Collider bounds

4.2.1 LHC — Z'

The $U(1)_X$ symmetry is spontaneously broken by the VEV of the doublets and the triplet, which also contributed to the mass generation of the Z' vector boson. As v_t is small and v is at the electroweak scale, the Z' mass will be at the electroweak scale or below, depending on the value of g_X and the other parameter such as $\tan \beta$. Such a light Z' is subject to a variety of experimental constraints. Notice that we have two possible Z' models ($U(1)_N$ or $U(1)_{Y'}$), one which resembles the sequential Z' model, and other the dark photon model. Concerning the latter, LHC bounds are weakened because the $Z' - Z$ mixing is necessarily small, and that would suppress its production cross section at the LHC [85–87].

As for the $U(1)_{Y'}$ model, we do not have much freedom since the SM fermions are charged under $U(1)_{Y'}$ the production cross section is much larger. In this scenario the LHC bounds are rather restrictive. Assuming $g_X = 1$ the LHC severely rules Z' masses below 3 TeV [88]. In our model the Z' mass is set by g_X . In order to have Z' masses around 100 GeV, g_X should be around 0.1, which is not sufficiently small to evade LHC limits [89]. If we adopt $g_X = 0.01$ we will get $m_{Z'} = 1 \text{ GeV}$, and for such small coupling we can easily evade LHC limits [89]. We have used eq. (3.49) and assumed Q_{X1} of the same order of Q_{X2} to find the corresponding Z' mass. We point out that the kinetic mixing parameter ϵ while not relevant for the Z' mass it is important to determine the Z' interactions with SM fermions. The conclusions drawn above are valid for sufficiently small kinetic mixing.

LHC 13TeV — $\mathcal{L} = 12.9 fb^{-1}$	$m_{H^{++}} > 760 \text{ GeV}$
LHC 13TeV — $\mathcal{L} = 36 fb^{-1}$	$m_{H^{++}} > 980 \text{ GeV}$
High-Luminosity LHC — $\mathcal{L} = 1000 fb^{-1}$	$m_{H^{++}} > 1.9 \text{ TeV}$
High-Energy LHC 27 TeV, $\mathcal{L} = 1000 fb^{-1}$	$m_{H^{++}} > 3 \text{ TeV},$

Table 2. Summary of collider bounds on the doubly charged scalar in our model using current and planned configurations. We used 13TeV of center-of-mass energy for the LHC configurations, whereas 27TeV for the high-energy upgrade. We can see that LHC and its upgrade will be paramount to probe the model up to the TeV scale.

4.2.2 LHC — doubly charged scalar

Regarding the scalar spectrum of our model, the most relevant ones come from LHC searches for heavy Higgs and triplet scalars. The cleanest signature signal is the doubly charged Higgs. We have then implemented the model in Madgraph [90, 91] and followed the recipe described in [92]. Assuming no hierarchy in the Yukawa couplings the doubly charged scalar decays essentially, with equal branching ratios, into charged leptons. That said, we found the current LHC bound with $\mathcal{L} = 36 fb^{-1}$ of integrated luminosity and performed future projects for the High Luminosity and High Energy LHC setups as summarized in the table 2.

In the light of current bounds our model is in agreement with existing bounds if we take $\mu_{t2} \leq v_i$ which predicts masses at the TeV scale as discussed previously. We highlight that we need couplings larger than one to find charged scalar masses above the TeV scale. Therefore, LHC and its planned upgrade will be important because it will probe a large portion of the model. The presence of a doubly charged scalar is the key signature of the type II seesaw mechanism.

We highlight that these bounds rely on lepton flavor violation channels with a degenerate neutrino mass spectrum with absolute mass around 0.1 eV [92]. The consideration of different mass hierarchies will not bring much impact to our paper which focuses on the proposition of a new 2HDM model. Anyways, strictly speaking, one should keep in mind that different mass hierarchies are subject to different collider bounds as pointed out [93]. Anyway, if we find ourselves in a situation where both doubly charged scalar and Z' fields are observed, our model stands as a potential well motivated environment for them.

4.3 LHC — Higgs

Now the higgs decays to SM fermions and gauge bosons fermions have been constrained [94–96], one can use Higgs data to place important limits on the model.

The couplings of the Higgs-like scalar h with the SM fermions and gauge bosons are given by,

$$\mathcal{C}_{h\bar{f}f} = \frac{(s_\alpha c_{\alpha_2} - c_\alpha s_{\alpha_1} s_{\alpha_2})}{s_\beta} \mathcal{C}_{h\bar{f}f}^{\text{SM}} \quad (4.4)$$

$$\mathcal{C}_{hWW} = (c_\alpha c_{\alpha_1} c_\beta + s_\alpha c_{\alpha_2} s_\beta - c_\alpha s_{\alpha_1} s_{\alpha_2} s_\beta) \mathcal{C}_{hWW}^{\text{SM}} \quad (4.5)$$

$$\mathcal{C}_{hZZ} = (c_\alpha c_{\alpha_1} c_\beta + s_\alpha c_{\alpha_2} s_\beta - c_\alpha s_{\alpha_1} s_{\alpha_2} s_\beta) \mathcal{C}_{hZZ}^{\text{SM}}, \quad (4.6)$$

where $\mathcal{C}_{h\bar{f}f}^{\text{SM}} = \frac{m_f}{v}$, $\mathcal{C}_{hWW}^{\text{SM}} = \frac{1}{2}g^2v$ and $\mathcal{C}_{hZZ}^{\text{SM}} = \frac{1}{2}\frac{g^2v}{\cos^2\theta_W}$. In the expressions for the gauge bosons we have neglected small terms proportional to ϵ and $\sin\xi$. As shown in the appendix C.1, the angles α_1 and α_2 (and also β_1 and β_2) are suppressed by v_t/v_2 . Then taking $\alpha_1, \alpha_2 \rightarrow 0$ in the above expressions, we get,

$$\mathcal{C}_{h\bar{f}f} = \frac{s_\alpha}{s_\beta} \mathcal{C}_{h\bar{f}f}^{\text{SM}} \tag{4.7}$$

$$\mathcal{C}_{hWW} = c_{\beta-\alpha} \mathcal{C}_{hWW}^{\text{SM}} \tag{4.8}$$

$$\mathcal{C}_{hZZ} = c_{\beta-\alpha} \mathcal{C}_{hZZ}^{\text{SM}}, \tag{4.9}$$

with $c_{\beta-\alpha} \equiv \cos(\beta-\alpha)$. When $\alpha = \beta$, we fall in the alignment limit [97, 98]. In this regime, h couples to the SM particles identically to the SM Higgs. Conversely, the couplings of the heavier Higgses H and H_t , which are proportional to $s_{\beta-\alpha}$ and s_{α_i} , respectively, vanish in this limit.

For $c_{\beta-\alpha} \sim 1$, $\tan\beta$ can take on essentially any value, as long as the Higgs-like decays are concerned (see, e.g., figure 3 of ref. [52]¹). Regarding the charged Higgs H^+ , its coupling to fermions is suppressed by a factor of $\tan\beta$. Therefore, large values of $\tan\beta$ weakens the LHC limits. In summary, our model can be made fully consistent in the alignment limit with no prejudice.

4.4 LHC — heavy Higgs

An interesting signature of our model is the decay of the heavy Higgs, H , into heavy gauge bosons [50],

$$\Gamma(H \rightarrow Z'Z') = \frac{g^2}{128\pi} \frac{m_H^2}{m_{Z'}^2} (\delta \tan\beta)^4 \left(\frac{\cos^3\beta \cos\alpha - \sin^3\beta \sin\alpha}{\cos\beta \sin\beta} \right)^2 \tag{4.10}$$

where,

$$\delta = g_X \frac{\cos\theta_W m_Z}{g m_{Z'}} (Q_{X1} \cos^2\beta + Q_{X2} \sin^2\beta). \tag{4.11}$$

This decay is kinematically available because the Z' gauge boson is very light. Depending on the magnitude of g_X , Z' might decay inside the detector. Thus, the possible signature of this heavy scalar is the four lepton channel [99, 100]. We plan to investigate the LHC discovery reach of this decay mode in the foreseeable future. A detail phenomenology is out of the scope of the current paper, but it is important to stress that in the aforementioned alignment limit, this decay channels closes and the bounds stemming from heavy Higgs weaken [49].

¹Care must be taken when comparing our results with the ones in ref. [52], because the physical scalars and mixing angle α are defined following different conventions (see eq. (4.13) from that paper and compare with eq. (3.18) in section 3.1). For this reason, $c_{\beta-\alpha} = 0$ in figure3 of that paper, is equivalent to $c_{\beta-\alpha} = 1$ in ours.

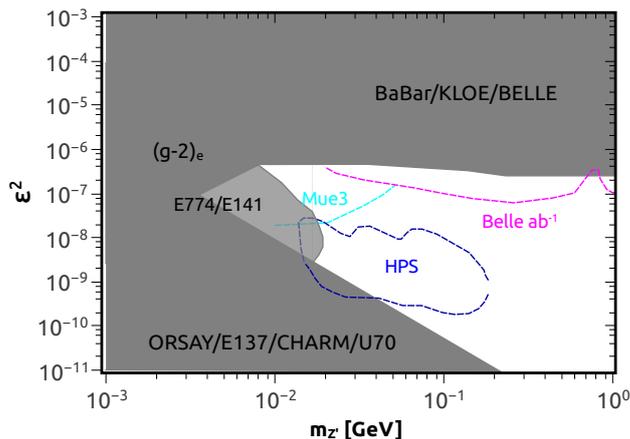


Figure 2. Summary of experimental limits applicable to the model $U(1)_N$ and to the model $U(1)_{Y'}$ assuming $g_X \ll \epsilon$. Current limits are in gray while projected ones in color.

4.4.1 Belle-II and KLOE2

Belle and KLOE collaborations represent e^+e^- colliders searching for light gauge bosons with the $\epsilon/2F^{\mu\nu}F'_{\mu\nu}$. The two models proposed here feature a similar term. In the $U(1)_N$ model SM fermions are uncharged under $U(1)_N$, thus the Z' will couple to SM fermions only via its mixing with the Z boson generated by the presence of the kinetic mixing. In this case, our model would be a UV complete version of the simplified dark photon model [101, 102]. This scenario for heavy Z' masses was investigated in [103]. For the $U(1)_{Y'}$ model, where the SM fermions are charged under the gauge symmetry, if we take $g_X \ll 1$ and $g_X < \epsilon$, again the model falls back to the dark photon model because the Z' interactions to SM via the kinetic mixing would be more pronounced, the experimental limits on dark photon become applicable to our study.

In summary, the experimental limits derived for dark photon models apply here, except in the case where $g_X \gg \epsilon$ and $m_{Z'} \gg 1$ GeV. Experimental collaborations usually display their bounds in terms of ϵ^2 . In figure 2 we display a summary of the existing (gray) and planned (color) constraints. For $m_{Z'} \sim 10 - 30$ MeV, current bounds impose $\epsilon < 10^{-4}$, limiting the region of parameter space of our model. Anyhow, we emphasize that we can still obey such bounds by taking g_X and ϵ to be very small as it is usually assumed in dark photon models.

4.5 Accelerators

There are several accelerators using electron/positron or hadronic beams which search for bremsstrahlung of dark photons or its appearance in meson decays. These bounds are inside the gray region in figure 2. It is important to point out the future sensitivity on the flavor violating decay $\mu \rightarrow 3e$ [104], in case of no signal, will give rise to the upper limit in cyan. Moreover, The Heavy Photon Search Experiment (HPS) which was already installed at SLAC collides highly energetic electrons into a tungsten target, and in the process electrons may radiate dark photons. The experimental sensitivity of HPS is shown in blue.

4.6 Low energy probes

The muon anomalous magnetic moment (g-2) [105], neutrino-electron scattering [106] and atomic parity violation [52] provide complementary but subdominant limits to our model. For instance, neutrino-electron scattering rules out $\epsilon > 10^{-5}$ [106]. One cannot accommodate g-2 with the $U(1)_{Y'}$ model because the electrons are charged under the gauge symmetry, and these couplings are subject to tight constraints [107–109]. In the $U(1)_N$ model, where our model resembles the dark photon one, the favored region to explain g-2 has already been excluded [105].

4.7 Dark matter possibility

In the two models described here, there are no dark matter candidates. One could simply add a vector-like fermion charged under the gauge symmetry while preserving the gauge anomaly cancellation. The dark matter relic abundance, direct detection and indirect detection signals would be governed the kinetic mixing term and the g_X gauge coupling. For concreteness, if we take the $U(1)_N$ model, where SM fermions are not charged under the new gauge symmetry, the dark matter phenomenology would be similar to the dark photon portal investigated recently in the literature [110]. It has been shown that in this setup if the dark matter mass is smaller than the Z' one, only s-channel processes would be relevant, and this case is nearly ruled out by current data for dark matter masses above 10 MeV. There is tiny region for $\epsilon \sim 10^{-5}$ and $m_{Z'} \sim 10$ MeV which obeys on existing limits and reproduce the correct relic density. If the dark matter particle is heavier than the Z' then the secluded dark matter setup arises [111], scenario which is much less restricted by data. In the near future we plan to carry out a detailed dark matter phenomenology in both models taking into account the particularities such as the presence of both mass mixing and kinetic mixing terms as well as the existence of many other scalars which might alter the overall predictions.

5 Discussion

There are important things to be stressed about the two models we proposed to explain neutrino masses and the absence of flavor changing interactions:

- The gauge symmetry imposed to distinguish Φ_1 from Φ_2 and then allow just one scalar doublet to couple to SM fermions gives rise to two very different type of models. In the $U(1)_N$ model, the SM fermions are uncharged under the gauge symmetry, and the corresponding massive Z' only couples to SM fermions via its mixing with the Z boson. In the $U(1)_{Y'}$ setup, the Z' gauge boson will have a neutral current with SM fermions determined by the $U(1)_{Y'}$ SM fermion charges, leading to a sequential Z' model [112].
- Since we have added a triplet scalar to explain neutrino masses via a type II seesaw mechanism, nothing prohibits from one to consider off-diagonal yukawa couplings, involving for instance the scalar triplet, to be non-vanishing. This will lead again

to flavor changing interactions and give rise to $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$, $\mu - e$ conversion processes which are rather restricted by data. In particular, the product of the yukawa terms are limited to be smaller than 10^{-7} [105]. Anyways, this feature is common in the models which extend the SM scalar sector and, thus, we can set to zero off-diagonal couplings involving the extra scalars without prejudice.

- The additional gauge symmetry allow us to easily introduce a dark matter candidate, a vector-like fermion, without spoiling the anomaly cancellation requirements. In summary, we argue that the addition of a gauge symmetry and a triplet scalar is well-motivated since it adds nice features to the original 2HDM proposal.
- The bounds discussed previously can be safely satisfied by taking g_X and ϵ to be sufficiently small, below 10^{-3} similarly to dark photon models.

6 Conclusions

Two Higgs Doublet Models represent interesting extensions of the Standard Model. However they lack neutrino masses and a first principle explanation for the absence of flavor changing neutral interactions. Typically, a Z_2 symmetry is invoked where one scalar doublet is charged under, to prevent the other to couple to Standard Model fermions.

In this work, we proposed two models where neutrino masses are explained within the type II seesaw mechanism via the addition of a scalar triplet and a gauge symmetry that allows only one scalar doublet to couple to fermions. In this way can simultaneously explain neutrino masses and avoid flavor changing neutral interactions.

We have investigated several constraints coming from low energy probes, electroweak precision and collider. In particular, we derived collider bounds on the mass of the doubly charged scalar using current and planned LHC reach with high-luminosity and high-energy configurations. We discussed which regions of parameter space are consistent with current data to conclude that both models stand as viable alternatives to the original Two Higgs Doublet Model proposal.

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A Vacuum stability bounds

In this appendix we give the vacuum stability bounds, also known as bounded from below conditions, for the scalar potentials in the sections 2 and 3. General discussions concerning vacuum stability bounds for a scalar potential of few scalar fields can be found, for example, in refs. [113, 114]. In order to determine the vacuum stability bounds it is sufficient to consider only the quartic terms in the potential, once they turn out to be the dominant contribution for large values of the fields.

A.1 Vacuum stability bounds for the 2HDM-U(1) with type I seesaw mechanism

The vacuum stability bounds for the model with the two scalar doublets and a complex scalar singlet of the model with type I seesaw mechanism in section 2 can be found following the same steps of ref. [61], where it was treated the case with a real singlet. The analysis will be also useful next when dealing with the potential of the type II seesaw mechanism.

It is convenient to parameterize the fields according to

$$\begin{aligned}
 r^2 &= \Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 + \Phi_s^\dagger \Phi_s, \\
 \Phi_1^\dagger \Phi_1 &= r^2 \sin^2 \theta \cos^2 \phi, \\
 \Phi_2^\dagger \Phi_2 &= r^2 \sin^2 \theta \sin^2 \phi, \\
 \Phi_s^\dagger \Phi_s &= r^2 \cos^2 \theta,
 \end{aligned}
 \tag{A.1}$$

in which $0 \leq r \leq \infty$, $0 \leq \theta \leq \frac{\pi}{2}$ and $0 \leq \phi \leq \frac{\pi}{2}$. We also define the ratio

$$\frac{\Phi_1^\dagger \Phi_2}{|\Phi_1| |\Phi_2|} = c e^{i\alpha},
 \tag{A.2}$$

with $0 \leq c \leq 1$ and $\alpha \in \mathfrak{R}$. Thus, the potential of the quartic terms, V_4 , from eqs. (2.2) and (2.5) is such that

$$\begin{aligned}
 V_4/r^4 &= \left\{ \frac{\lambda_1}{2} (1-x)^2 + \frac{\lambda_2}{2} x^2 + (\lambda_3 + \lambda_4 c^2) x (1-x) \right\} y^2 \\
 &\quad + \frac{\lambda_s}{2} (1-y)^2 + \{\lambda_{s1} (1-x) + \lambda_{s2} x\} y (1-y) \\
 &= A_x y^2 + B_x (1-y)^2 + C_x y (1-y),
 \end{aligned}
 \tag{A.3}$$

where we define $x = \sin^2 \phi$, $y = \sin^2 \theta$ and the function A_x , B_x , C_x which can be read from the first and second lines of eq. (A.3). The condition $V_4 > 0$ implies that

$$A_x > 0, \quad B_x > 0, \quad \text{and} \quad 2\sqrt{A_x B_x} + C_x > 0.
 \tag{A.4}$$

The first condition above,

$$A_x = \frac{\lambda_1}{2} (1-x)^2 + \frac{\lambda_2}{2} x^2 + (\lambda_3 + \lambda_4 c^2) x (1-x) > 0,
 \tag{A.5}$$

is the requirement that the potential is positive along the $y = 1$ direction for large of the fields. It has the same form of eq. (A.3) so that the directions $x = 0, 1$ give

$$\lambda_1 > 0, \quad \lambda_2 > 0. \quad (\text{A.6})$$

Analogously, a condition similar to the last one in eq. (A.5) gives, for $c = 0, 1$,

$$\lambda_3 + \sqrt{\lambda_1 \lambda_2} > 0, \quad \lambda_3 + \lambda_4 + \sqrt{\lambda_1 \lambda_2} > 0. \quad (\text{A.7})$$

The second condition in eq. (A.4) is the requirement that the potential is positive along the $y = 0$ direction for large of the fields and gives

$$\lambda_s > 0. \quad (\text{A.8})$$

The last inequality in eq. (A.4) arises from requiring that the discriminant of eq. (A.3), as a polynomial in y , is negative, which implies that $4A_x B_x - C_x^2 > 0$. There are two cases to consider for C_x . One is the case in which $\lambda_{s1} > 0$ and $\lambda_{s2} > 0$ resulting in $C_x > 0$ that automatically satisfies eq. (A.4). The other case is for the couplings $\lambda_{s1} < 0$ or/and $\lambda_{s2} < 0$ so that we must have from the discriminant that

$$(\lambda_1 \lambda_s - \lambda_{s1}^2) (1-x)^2 + (\lambda_2 \lambda_s - \lambda_{s2}^2) x^2 + 2 \{ (\lambda_3 + \lambda_4 c^2) \lambda_s - \lambda_{s1} \lambda_{s2} \} x (1-x) > 0. \quad (\text{A.9})$$

In the same way that follows from eq. (A.5), this leads to the conditions

$$\begin{aligned} \lambda_{s1} + \sqrt{\lambda_1 \lambda_s} &> 0, & \lambda_{s2} + \sqrt{\lambda_2 \lambda_s} &> 0, \\ 2(\lambda_3 \lambda_s - \lambda_{s1} \lambda_{s2}) + \sqrt{(\lambda_1 \lambda_s - \lambda_{s1}^2)(\lambda_2 \lambda_s - \lambda_{s2}^2)} &> 0, & (\text{A.10}) \\ 2(\lambda_3 \lambda_s + \lambda_4 \lambda_s - \lambda_{s1} \lambda_{s2}) + \sqrt{(\lambda_1 \lambda_s - \lambda_{s1}^2)(\lambda_2 \lambda_s - \lambda_{s2}^2)} &> 0. \end{aligned}$$

A.2 Vacuum stability bounds for the 2HDM-U(1) with type II seesaw mechanism

In order to obtain the vacuum stability bounds for the potential with two scalar doublets plus a scalar triplet of the 2HDM-U(1) with type II seesaw mechanism in section 3, we use the same sort of parameterization as in A.1

$$\begin{aligned} r^2 &= \Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 + \text{Tr}(\Delta^\dagger \Delta), \\ \Phi_1^\dagger \Phi_1 &= r^2 \sin^2 \theta \cos^2 \phi = r^2 y (1-x), \\ \Phi_2^\dagger \Phi_2 &= r^2 \sin^2 \theta \sin^2 \phi = r^2 y x, \\ \text{Tr}(\Delta^\dagger \Delta) &= r^2 \cos^2 \theta = r^2 (1-y), \end{aligned} \quad (\text{A.11})$$

in which $0 \leq r \leq \infty$, $0 \leq \theta \leq \frac{\pi}{2}$ and $0 \leq \phi \leq \frac{\pi}{2}$. Now we define the ratios

$$\frac{\Phi_1^\dagger \Phi_2}{|\Phi_1| |\Phi_2|} = c e^{i\alpha}, \quad \frac{\Phi_i^\dagger \Delta \Delta^\dagger \Phi_i}{\Phi_i^\dagger \Phi_i \text{Tr}(\Delta^\dagger \Delta)} = \xi_i, \quad \frac{\text{Tr}(\Delta^\dagger \Delta)^2}{[\text{Tr}(\Delta^\dagger \Delta)]^2} = \zeta, \quad (\text{A.12})$$

with $0 \leq c \leq 1$, $\alpha \in \mathfrak{R}$, $0 \leq \xi_{1,2} \leq 1$ and $\frac{1}{2} \leq \zeta \leq 1$.² Thus, the potential of the quartic terms, V_{4t} , from eq. (3.11) is such that

$$V_{4t}/r^4 = A_x y^2 + B_x (1-y)^2 + C_x y(1-y). \quad (\text{A.13})$$

where

$$\begin{aligned} A_x &= \frac{\lambda_1}{2} (1-x)^2 + \frac{\lambda_2}{2} x^2 + (\lambda_3 + \lambda_4 c^2) x(1-x), \\ B_x &= \frac{1}{2} \overline{\lambda}_t(\zeta) = \frac{1}{2} (\lambda_t + \lambda_{tt} \zeta), \\ C_x &= \overline{\lambda}_{t1}(\xi_1) (1-x) + \overline{\lambda}_{t2}(\xi_2) x = (\lambda_{t1} + \lambda_{tt1} \xi_1) (1-x) + (\lambda_{t2} + \lambda_{tt2} \xi_2) x, \end{aligned} \quad (\text{A.14})$$

with $\overline{\lambda}_t(\zeta)$, $\overline{\lambda}_{t1}(\xi_1)$ and $\overline{\lambda}_{t2}(\xi_2)$ defined just for convenience.

The direction $y = 1$ implies that $A_x > 0$ and we have the same conditions of eqs. (A.6) and (A.7), i. e.,

$$\begin{aligned} \lambda_1 &> 0, & \lambda_2 &> 0, \\ \lambda_3 + \sqrt{\lambda_1 \lambda_2} &> 0, & \lambda_3 + \lambda_4 + \sqrt{\lambda_1 \lambda_2} &> 0. \end{aligned} \quad (\text{A.15})$$

The direction $y = 0$ in eq. (A.13) implies that $B_x > 0$ and this gives the conditions, for the values $\zeta = 0, \frac{1}{2}$,

$$\lambda_t > 0, \quad \lambda_t + \frac{\lambda_{tt}}{2} > 0. \quad (\text{A.16})$$

It remains to analyse the inequality $C_x + 2\sqrt{A_x B_x} > 0$, which guarantees that $V_{4t} > 0$ given that $A_x > 0$ and $B_x > 0$. In the same way as discussed in the subsection A.1, there are two cases to consider for C_x . The case in which $\overline{\lambda}_{t1}(\xi_1) > 0$ and $\overline{\lambda}_{t2}(\xi_2) > 0$ gives $C_x > 0$, so that the inequality is automatically satisfied once $A_x B_x > 0$. The other case is the one in which $\overline{\lambda}_{t1}(\xi_1) < 0$ or/and $\overline{\lambda}_{t2}(\xi_2) < 0$, and the condition from the negative discriminant from eq. (A.13) is

$$\begin{aligned} & \left(\lambda_1 \overline{\lambda}_t(\zeta) - \overline{\lambda}_{t1}^2(\xi_1) \right) (1-x)^2 + \left(\lambda_2 \overline{\lambda}_t(\zeta) - \overline{\lambda}_{t2}^2(\xi_2) \right) x^2 \\ & + 2 \left((\lambda_3 + \lambda_4 c^2) \overline{\lambda}_t(\zeta) - \overline{\lambda}_{t1}(\xi_1) \overline{\lambda}_{t2}(\xi_2) \right) x(1-x) > 0. \end{aligned} \quad (\text{A.17})$$

This implies, from $x = 0, 1$,

$$\sqrt{\lambda_1 \overline{\lambda}_t(\zeta)} + \overline{\lambda}_{t1}(\xi_1) > 0, \quad \sqrt{\lambda_2 \overline{\lambda}_t(\zeta)} + \overline{\lambda}_{t2}(\xi_2) > 0 \quad (\text{A.18})$$

and

$$\begin{aligned} & 2 \{ (\lambda_3 + \lambda_4 c^2) \overline{\lambda}_t(\zeta) - \overline{\lambda}_{t1}(\xi_1) \overline{\lambda}_{t2}(\xi_2) \} \\ & + \sqrt{\left(\lambda_1 \overline{\lambda}_t(\zeta) - \overline{\lambda}_{t1}^2(\xi_1) \right) \left(\lambda_2 \overline{\lambda}_t(\zeta) - \overline{\lambda}_{t2}^2(\xi_2) \right)} > 0. \end{aligned} \quad (\text{A.19})$$

²The range of variation of ξ_i is defined by the Cauchy-Schwarz inequality, while the one for ζ is obtained through minimization of $\frac{\text{Tr}(\Delta^\dagger \Delta)^2}{[\text{Tr}(\Delta^\dagger \Delta)]^2} = \frac{1+\beta^2}{(1+\beta)^2}$.

For the extremum values of ζ and $\xi_{1,2}$ we have from the inequalities in (A.18) the conditions:

$$\begin{aligned} \sqrt{\lambda_1 \lambda_t} + \lambda_{t1} &> 0, & \sqrt{\lambda_1 \lambda_t} + \lambda_{t1} + \lambda_{tt1} &> 0, \\ \sqrt{\lambda_1 \left(\lambda_t + \frac{\lambda_{tt}}{2} \right)} + \lambda_{t1} &> 0, & \sqrt{\lambda_1 \left(\lambda_t + \frac{\lambda_{tt}}{2} \right)} + \lambda_{t1} + \lambda_{tt1} &> 0; \end{aligned} \quad (\text{A.20})$$

and

$$\begin{aligned} \sqrt{\lambda_2 \lambda_t} + \lambda_{t2} &> 0, & \sqrt{\lambda_2 \lambda_t} + \lambda_{t2} + \lambda_{tt2} &> 0, \\ \sqrt{\lambda_2 \left(\lambda_t + \frac{\lambda_{tt}}{2} \right)} + \lambda_{t2} &> 0, & \sqrt{\lambda_2 \left(\lambda_t + \frac{\lambda_{tt}}{2} \right)} + \lambda_{t2} + \lambda_{tt2} &> 0. \end{aligned} \quad (\text{A.21})$$

In the same way, the inequality in (A.19) leads to the conditions:

$$2(\lambda_3 \lambda_t - \lambda_{t1} \lambda_{t2}) + \sqrt{(\lambda_1 \lambda_t - \lambda_{t1}^2)(\lambda_2 \lambda_t - \lambda_{t2}^2)} > 0, \quad (\text{A.22})$$

$$2(\lambda_3 \lambda_t - \lambda_{t1}(\lambda_{t2} + \lambda_{tt2})) + \sqrt{(\lambda_1 \lambda_t - \lambda_{t1}^2)(\lambda_2 \lambda_t - (\lambda_{t2} + \lambda_{tt2})^2)} > 0, \quad (\text{A.23})$$

$$2(\lambda_3 \lambda_t - (\lambda_{t1} + \lambda_{tt1})\lambda_{t2}) + \sqrt{(\lambda_1 \lambda_t - (\lambda_{t1} + \lambda_{tt1})^2)(\lambda_2 \lambda_t - \lambda_{t2}^2)} > 0, \quad (\text{A.24})$$

$$\begin{aligned} &2(\lambda_3 \lambda_t - (\lambda_{t1} + \lambda_{tt1})(\lambda_{t2} + \lambda_{tt2})) \\ &+ \sqrt{(\lambda_1 \lambda_t - (\lambda_{t1} + \lambda_{tt1})^2)(\lambda_2 \lambda_t - (\lambda_{t2} + \lambda_{tt2})^2)} > 0, \end{aligned} \quad (\text{A.25})$$

$$\begin{aligned} &2\left(\lambda_3 \left(\lambda_t + \frac{\lambda_{tt}}{2}\right) - \lambda_{t1} \lambda_{t2}\right) \\ &+ \sqrt{\left(\lambda_1 \left(\lambda_t + \frac{\lambda_{tt}}{2}\right) - \lambda_{t1}^2\right) \left(\lambda_2 \left(\lambda_t + \frac{\lambda_{tt}}{2}\right) - \lambda_{t2}^2\right)} > 0, \end{aligned} \quad (\text{A.26})$$

$$\begin{aligned} &2\left(\lambda_3 \left(\lambda_t + \frac{\lambda_{tt}}{2}\right) - \lambda_{t1}(\lambda_{t2} + \lambda_{tt2})\right) \\ &+ \sqrt{\left(\lambda_1 \left(\lambda_t + \frac{\lambda_{tt}}{2}\right) - \lambda_{t1}^2\right) \left(\lambda_2 \left(\lambda_t + \frac{\lambda_{tt}}{2}\right) - (\lambda_{t2} + \lambda_{tt2})^2\right)} > 0, \end{aligned} \quad (\text{A.27})$$

$$\begin{aligned} &2\left(\lambda_3 \left(\lambda_t + \frac{\lambda_{tt}}{2}\right) - (\lambda_{t1} + \lambda_{tt1})\lambda_{t2}\right) \\ &+ \sqrt{\left(\lambda_1 \left(\lambda_t + \frac{\lambda_{tt}}{2}\right) - (\lambda_{t1} + \lambda_{tt1})^2\right) \left(\lambda_2 \left(\lambda_t + \frac{\lambda_{tt}}{2}\right) - \lambda_{t2}^2\right)} > 0, \end{aligned} \quad (\text{A.28})$$

$$\begin{aligned} &2\left(\lambda_3 \left(\lambda_t + \frac{\lambda_{tt}}{2}\right) - (\lambda_{t1} + \lambda_{tt1})(\lambda_{t2} + \lambda_{tt2})\right) \\ &+ \sqrt{\left(\lambda_1 \left(\lambda_t + \frac{\lambda_{tt}}{2}\right) - (\lambda_{t1} + \lambda_{tt1})^2\right) \left(\lambda_2 \left(\lambda_t + \frac{\lambda_{tt}}{2}\right) - (\lambda_{t2} + \lambda_{tt2})^2\right)} > 0, \end{aligned} \quad (\text{A.29})$$

$$2((\lambda_3 + \lambda_4)\lambda_t - \lambda_{t1} \lambda_{t2}) + \sqrt{(\lambda_1 \lambda_t - \lambda_{t1}^2)(\lambda_2 \lambda_t - \lambda_{t2}^2)} > 0, \quad (\text{A.30})$$

$$2((\lambda_3 + \lambda_4)\lambda_t - \lambda_{t1}(\lambda_{t2} + \lambda_{tt2})) + \sqrt{(\lambda_1 \lambda_t - \lambda_{t1}^2)(\lambda_2 \lambda_t - (\lambda_{t2} + \lambda_{tt2})^2)} > 0, \quad (\text{A.31})$$

$$2((\lambda_3 + \lambda_4)\lambda_t - (\lambda_{t1} + \lambda_{tt1})\lambda_{t2}) + \sqrt{(\lambda_1 \lambda_t - (\lambda_{t1} + \lambda_{tt1})^2)(\lambda_2 \lambda_t - \lambda_{t2}^2)} > 0, \quad (\text{A.32})$$

$$2((\lambda_3 + \lambda_4)\lambda_t - (\lambda_{t1} + \lambda_{tt1})(\lambda_{t2} + \lambda_{tt2})) + \sqrt{(\lambda_1\lambda_t - (\lambda_{t1} + \lambda_{tt1})^2)(\lambda_2\lambda_t - (\lambda_{t2} + \lambda_{tt2})^2)} > 0, \quad (\text{A.33})$$

$$2((\lambda_3 + \lambda_4)\left(\lambda_t + \frac{\lambda_{tt}}{2}\right) - \lambda_{t1}\lambda_{t2}) + \sqrt{\left(\lambda_1\left(\lambda_t + \frac{\lambda_{tt}}{2}\right) - \lambda_{t1}^2\right)\left(\lambda_2\left(\lambda_t + \frac{\lambda_{tt}}{2}\right) - \lambda_{t2}^2\right)} > 0, \quad (\text{A.34})$$

$$2((\lambda_3 + \lambda_4)\left(\lambda_t + \frac{\lambda_{tt}}{2}\right) - \lambda_{t1}(\lambda_{t2} + \lambda_{tt2})) + \sqrt{\left(\lambda_1\left(\lambda_t + \frac{\lambda_{tt}}{2}\right) - \lambda_{t1}^2\right)\left(\lambda_2\left(\lambda_t + \frac{\lambda_{tt}}{2}\right) - (\lambda_{t2} + \lambda_{tt2})^2\right)} > 0, \quad (\text{A.35})$$

$$2((\lambda_3 + \lambda_4)\left(\lambda_t + \frac{\lambda_{tt}}{2}\right) - (\lambda_{t1} + \lambda_{tt1})\lambda_{t2}) + \sqrt{\left(\lambda_1\left(\lambda_t + \frac{\lambda_{tt}}{2}\right) - (\lambda_{t1} + \lambda_{tt1})^2\right)\left(\lambda_2\left(\lambda_t + \frac{\lambda_{tt}}{2}\right) - \lambda_{t2}^2\right)} > 0, \quad (\text{A.36})$$

$$2((\lambda_3 + \lambda_4)\left(\lambda_t + \frac{\lambda_{tt}}{2}\right) - (\lambda_{t1} + \lambda_{tt1})(\lambda_{t2} + \lambda_{tt2})) + \sqrt{\left(\lambda_1\left(\lambda_t + \frac{\lambda_{tt}}{2}\right) - (\lambda_{t1} + \lambda_{tt1})^2\right)\left(\lambda_2\left(\lambda_t + \frac{\lambda_{tt}}{2}\right) - (\lambda_{t2} + \lambda_{tt2})^2\right)} > 0. \quad (\text{A.37})$$

B Anomaly cancellation

In this section we will describe how the gauge anomalies can be cancelled in our model which simply adds an abelian gauge group to the SM. Taking the SM fermion charges under the new weak hypercharge, Y' , to be l (left-handed leptons), q (left-handed quarks), e (right-handed leptons), u (right-handed quarks with positive isospin), d (right-handed quarks with positive isospin). Let us consider each relevant gauge anomaly individually,

$[\text{SU}(3)_c]^2 \text{U}(1)_X$:

$$\begin{aligned} \mathcal{A} &= \text{Tr} \left[\left\{ \frac{\lambda^a}{2}, \frac{\lambda^b}{2} \right\} Y'_R \right] - \text{Tr} \left[\left\{ \frac{\lambda^a}{2}, \frac{\lambda^b}{2} \right\} Y'_L \right] \\ \mathcal{A} &\propto \sum_{\text{quarks}} Y'_R - \sum_{\text{quarks}} Y'_L = [3u + 3d] - [3 \cdot 2q] = 0. \end{aligned}$$

Hence,

$$u + d - 2q = 0. \quad (\text{B.1})$$

$[\text{SU}(2)_L]^2 \text{U}(1)_X$:

$$\mathcal{A} = -\text{Tr} \left[\left\{ \frac{\sigma^a}{2}, \frac{\sigma^b}{2} \right\} Y'_L \right] \propto -\sum Y_L = -[2l + 3 \cdot 2q] = 0.$$

Thus,

$$l = -3q. \quad (\text{B.2})$$

$[\text{U}(1)_Y]^2 \text{U}(1)_X :$

$$\begin{aligned} \mathcal{A} &= \text{Tr} [\{Y_R, Y_R\} Y'_R] - \text{Tr} [\{Y_L, Y_L\} Y'_L] \propto \sum Y_R^2 Y'_R - \sum Y_L^2 Y'_L \\ \mathcal{A} &\propto \left[(-2)^2 e + 3 \left(\frac{4}{3}\right)^2 u + 3 \left(-\frac{2}{3}\right)^2 d \right] - \left[2(-1)^2 l + 3 \cdot 2 \left(\frac{1}{3}\right)^2 q \right] = 0. \end{aligned}$$

We conclude that,

$$6e + 8u + 2d - 3l - q = 0. \tag{B.3}$$

$\text{U}(1)_Y [\text{U}(1)_X]^2 :$

$$\begin{aligned} \mathcal{A} &= \text{Tr} [\{Y'_R, Y'_R\} Y_R] - \text{Tr} [\{Y'_L, Y'_L\} Y_L] \propto \sum Y_R Y_R'^2 - \sum Y_L Y_L'^2 \\ \mathcal{A} &\propto \left[(-2) e^2 + 3 \left(\frac{4}{3}\right) u^2 + 3 \left(-\frac{2}{3}\right) d^2 \right] - \left[2(-1) l^2 + 3 \cdot 2 \left(\frac{1}{3}\right) q^2 \right] = 0. \end{aligned}$$

That implies,

$$-e^2 + 2u^2 - d^2 + l^2 - q^2 = 0. \tag{B.4}$$

$[\text{U}(1)_X]^3 :$

$$\begin{aligned} \mathcal{A} &= \text{Tr} [\{Y'_R, Y'_R\} Y'_R] - \text{Tr} [\{Y'_L, Y'_L\} Y'_L] \propto \sum Y_R'^3 - \sum Y_L'^3 \\ \mathcal{A} &\propto [e^3 + 3u^3 + 3d^3] - [2l^3 + 3 \cdot 2q^3] = 0. \end{aligned}$$

Consequently we get,

$$e^3 + 3u^3 + 3d^3 - 2l^3 - 6q^3 = 0. \tag{B.5}$$

These relations between the charges under $\text{U}(1)$ are general, since we have not added any condition concerning the Lagrangians of the model. If we want to accommodate neutrino masses either via type I or type II seesaw mechanisms and explain the absence of flavor changing interactions, further relations between hypercharges under the new gauge symmetry $\text{U}(1)$ arise as summarized in eq. (2.6) and eq. (3.1).

C Scalar masses and mixings

In this appendix we show in details how to obtain approximate expressions for the masses and mixing angles for the physical scalars in the relevant regime adopted in this paper, $v_i \sim 100 \text{ GeV}$, $v_t \ll v_i$. In the first section we treat the CP-even scalars, and in the next we treat the charged scalars.

C.1 CP-even scalars

In general, it is not possible to obtain analytic expressions for the diagonalization of the mass matrix $M_{\text{CP-even}}^2$, given in eq. (3.17). The masses of the neutral scalars (eigenvalues of that matrix) are given implicitly as the solutions of the polynomial equation,

$$ax^3 + bx^2 + cx + d = 0, \tag{C.1}$$

with a , b , c and d given by,

$$a = 8v_1v_2v_t, \tag{C.2}$$

$$b = -16v_1v_2v_t[\lambda_1v_1^2 + \lambda_2v_2^2 + v_t^2(\lambda_t + \lambda_{tt})] - 4\sqrt{2}\mu_{t2}(v_1^2v_2^2 + v_1^2v_t^2 + v_2^2v_t^2) \tag{C.3}$$

$$c = 8v_1v_2v_t[4\lambda_1\lambda_2v_1^2v_2^2 - (\lambda_3 + \lambda_4)^2v_1^2v_2^2 + (4\lambda_1\lambda_t - \lambda_{t1}^2)v_1^2v_t^2 + (4\lambda_2\lambda_t - \lambda_{t2}^2)v_2^2v_t^2 + 4\lambda_{tt}v_t^2(\lambda_1v_1^2 + \lambda_2v_2^2) - 2v_t^2(\lambda_{t1}\lambda_{tt1}v_1^2 + \lambda_{t2}\lambda_{tt2}v_2^2) - v_t^2(\lambda_{tt1}^2v_1^2 + \lambda_{tt2}^2v_2^2)] + 8\sqrt{2}\mu_{t2}[\lambda_1v_1^4(v_2^2 + v_t^2) + \lambda_2v_2^4(v_1^2 + v_t^2) + \lambda_tv_t^4(v_1^2 + v_2^2) + \lambda_{tt}v_t^4(v_1^2 + v_2^2) + (\lambda_3 + \lambda_4)v_1^2v_2^2v_t^2 + (\lambda_{t1} + \lambda_{t2})v_1^2v_2^2v_t^2 + (\lambda_{tt1} + \lambda_{tt2})v_1^2v_2^2v_t^2] \tag{C.4}$$

$$d = 16v_1^3v_2^3v_t^3\{\lambda_1(\lambda_{t2} + \lambda_{tt2})^2 + \lambda_2(\lambda_{t1} + \lambda_{tt1})^2 + (\lambda_t + \lambda_{tt})[(\lambda_3 + \lambda_4)^2 - 4\lambda_1\lambda_2] - (\lambda_3 + \lambda_4)(\lambda_{t1} + \lambda_{tt1})(\lambda_{t2} + \lambda_{tt2})\} - 4\sqrt{2}\mu_{t2}\{[4\lambda_1\lambda_2 - (\lambda_3 + \lambda_4)^2]v_1^4v_2^4 + [4\lambda_2(\lambda_t + \lambda_{tt}) - (\lambda_{t2} + \lambda_{tt2})^2]v_2^4v_t^4 + [4\lambda_1(\lambda_t + \lambda_{tt}) - (\lambda_{t1} + \lambda_{tt1})^2]v_1^4v_t^4\} + 8\sqrt{2}\mu_{t2}v_1^2v_2^2v_t^2\{[(\lambda_{t1} + \lambda_{tt1})(\lambda_{t2} + \lambda_{tt2}) - 2(\lambda_3 + \lambda_4)(\lambda_t + \lambda_{tt})]v_t^2 + [(\lambda_3 + \lambda_4)(\lambda_{t1} + \lambda_{tt1}) - 2\lambda_1(\lambda_{t2} + \lambda_{tt2})]v_1^2 + [(\lambda_3 + \lambda_4)(\lambda_{t2} + \lambda_{tt2}) - 2\lambda_2(\lambda_{t1} + \lambda_{tt1})]v_2^2\} - 16\mu_{t2}^2v_1v_2v_t[(\lambda_3 + \lambda_4)v_1^2v_2^2 + (\lambda_{t1} + \lambda_{tt1})v_1^2v_t^2 + (\lambda_{t2} + \lambda_{tt2})v_2^2v_t^2] + 8\sqrt{2}\mu_{t2}^3v_1^2v_2^2v_t^2, \tag{C.5}$$

This equation can be solved numerically once the set of parameters is fixed. For the mixing angles, it is very difficult even to furnish an equation that determine them in terms of the parameters of the potential, because of the difficulty in computing the eigenvectors of M_{CPeven}^2 .

There are some limits, however, in which these expressions are calculable. The idea is to take advantage of the different energy scales involved and decompose the original matrix into matrices whose entries belong to the same scale.

Let us decompose M_{CPeven}^2 ,

$$M_{\text{CPeven}}^2 = \begin{pmatrix} 2\lambda_1v_1^2 & (\lambda_3 + \lambda_4)v_1v_2 & (\lambda_{t1} + \lambda_{tt1})v_1v_t \\ (\lambda_3 + \lambda_4)v_1v_2 & 2\lambda_2v_2^2 & (\lambda_{t2} + \lambda_{tt2})v_2v_t - \sqrt{2}\mu_{t2}v_2 \\ (\lambda_{t1} + \lambda_{tt1})v_1v_t & (\lambda_{t2} + \lambda_{tt2})v_2v_t - \sqrt{2}\mu_{t2}v_2 & 2(\lambda_t + \lambda_{tt})v_t^2 + \frac{\mu_{t2}v_2^2}{\sqrt{2}v_t} \end{pmatrix},$$

in the following way,

$$M_{\text{CPeven}}^2 = M_1^2 + M_2^2 = v_1v_2 \begin{pmatrix} 2\lambda_1\frac{v_1}{v_2} & \lambda_3 + \lambda_4 & 0 \\ \lambda_3 + \lambda_4 & 2\lambda_2\frac{v_2}{v_1} & 0 \\ 0 & 0 & 0 \end{pmatrix} + \sqrt{2}\mu_{t2}v_2 \begin{pmatrix} 0 & 0 & (\lambda_{t1} + \lambda_{tt1})\frac{v_1v_t}{\sqrt{2}\mu_{t2}v_2} \\ 0 & 0 & (\lambda_{t2} + \lambda_{tt2})\frac{v_t}{\sqrt{2}\mu_{t2}} - 1 \\ (\lambda_{t1} + \lambda_{tt1})\frac{v_1v_t}{\sqrt{2}\mu_{t2}v_2} & (\lambda_{t2} + \lambda_{tt2})\frac{v_t}{\sqrt{2}\mu_{t2}} - 1 & (\lambda_t + \lambda_{tt})\frac{2v_t^2}{\sqrt{2}\mu_{t2}v_2} + \frac{v_2}{2v_t} \end{pmatrix}. \tag{C.6}$$

The matrix M_1^2 would be the mixing matrix of the neutral scalars in case there were just the two doublets, while M_2^2 account for the effects of the presence of the triplet. Let us

first consider the limit $v_t, \mu_{t2} \ll v_i$. In this case, the decomposition (C.6) makes it clear that the triplet decouples from the doublets. The matrix M_1^2 remains the same and M_2^2 reduces to $M_2^2 = \text{diag}(0, 0, \mu_{t2}v_2^2/\sqrt{2}v_t)$, from which we obtain immediately the mass of H_t . To diagonalize M_1^2 we need only one mixing angle, so that we can make $\alpha_1, \alpha_2 \rightarrow 0$ in eq. (3.18), leading to the following physical fields,

$$\begin{pmatrix} h \\ H \\ H_t \end{pmatrix} = \begin{pmatrix} c_\alpha & s_\alpha & 0 \\ -s_\alpha & c_\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_t \end{pmatrix}$$

with α given by,

$$\tan 2\alpha = \frac{(\lambda_3 + \lambda_4)v_1v_2}{\lambda_1v_1^2 - \lambda_2v_2^2}. \tag{C.7}$$

From the eigenvalues of M_1^2 and M_2^2 , we have the masses,

$$m_{h,H}'^2 = \lambda_1v_1^2 + \lambda_2v_2^2 \pm \sqrt{(\lambda_1v_1^2 - \lambda_2v_2^2)^2 + (\lambda_3 + \lambda_4)^2v_1^2v_2^2} \tag{C.8}$$

$$m_{H_t}^2 = \frac{\mu_{t2}v_2^2}{\sqrt{2}v_t}, \tag{C.9}$$

with $m_h' < m_H'$.³

Now, relaxing the condition on μ_{t2} and allowing it to increase to the same order of v_i or higher, this comparatively large value of μ_{t2} produces a sizable perturbation on the spectrum obtained above, but as we will see, the mass expressions are somewhat similar to the ones obtained in eqs. (C.8) and (C.9). In this case, is still possible to diagonalize the matrices M_1^2 and M_2^2 almost independently. First, as v_t is always taken to be small but now μ_{t2} can be large, M_2^2 can be approximated by,

$$M_2^2 = \sqrt{2}\mu_{t2}v_2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & \frac{v_2}{2v_t} \end{pmatrix}. \tag{C.10}$$

M_2^2 is diagonalized by moving to an intermediate basis (H_1, H_2, H_3) through a rotation R_{α_2} ,

$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{\alpha_2} & s_{\alpha_2} \\ 0 & -s_{\alpha_2} & c_{\alpha_2} \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_t \end{pmatrix}, \tag{C.11}$$

with,

$$\sin \alpha_2 = \frac{2v_t}{v_2}, \quad \cos \alpha_2 \simeq 1, \tag{C.12}$$

so that,

$$\begin{aligned} M_{2\text{diag}}^2 &= R_{\alpha_2} M_2^2 R_{\alpha_2}^T \\ &= \sqrt{2}\mu_{t2}v_2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\frac{2v_t}{v_2} & O\left(\frac{v_t^2}{v_2^2}\right) \\ 0 & O\left(\frac{v_t^2}{v_2^2}\right) & \frac{v_2}{2v_t} \end{pmatrix}. \end{aligned} \tag{C.13}$$

³The prime in $m_{h,H}'^2$ was inserted here to differentiate these mass expressions from the masses $m_{h,H}^2$, given in eqs. (C.19) and (C.20), which are the masses of h and H calculated in another limit.

The effect of this rotation on M_1^2 is

$$R_{\alpha_2} M_1^2 R_{\alpha_2}^T = v_1 v_2 \begin{pmatrix} 2\lambda_1 \frac{v_1}{v_2} & (\lambda_3 + \lambda_4) c_{\alpha_2} & -(\lambda_3 + \lambda_4) s_{\alpha_2} \\ (\lambda_3 + \lambda_4) c_{\alpha_2} & 2\lambda_2 \frac{v_2}{v_1} c_{\alpha_2}^2 & -2\lambda_2 \frac{v_2}{v_1} s_{\alpha_2} c_{\alpha_2} \\ -(\lambda_3 + \lambda_4) s_{\alpha_2} & -2\lambda_2 \frac{v_2}{v_1} s_{\alpha_2} c_{\alpha_2} & 2\lambda_2 \frac{v_2}{v_1} s_{\alpha_2}^2 \end{pmatrix}. \quad (\text{C.14})$$

As $\sin \alpha_2 \ll 1$, at leading order we have $R_{\alpha_2} M_1^2 R_{\alpha_2}^T \simeq M_1^2$, and the rotation R_{α_2} does not change M_1^2 . Then, rotating M_1^2 by R_α , with α given in eq. (C.7), we move to the physical basis,

$$\begin{pmatrix} h \\ H \\ H_t \end{pmatrix} = \begin{pmatrix} c_\alpha & s_\alpha & 0 \\ -s_\alpha & c_\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix}, \quad (\text{C.15})$$

such that,

$$\begin{aligned} M_{1\text{diag}}^2 &= R_\alpha M_2^2 R_\alpha^T \\ &= \begin{pmatrix} m_h'^2 & 0 & 0 \\ 0 & m_H'^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \end{aligned} \quad (\text{C.16})$$

where $m_h'^2$ and $m_H'^2$ are as given in eq. (C.8). This second rotation R_α , albeit diagonalize M_1^2 , tends to disturb the previous diagonalization of M_2^2 , by generating corrections to the diagonal elements and also off-diagonal elements in $M_{2\text{diag}}^2$,

$$M_{2\text{diag}}'^2 = R_\alpha M_{2\text{diag}}^2 R_\alpha^T = \sqrt{2} \mu_{t2} v_2 \begin{pmatrix} -2s_\alpha \frac{2v_t}{v_2} & O\left(\frac{v_t}{v_2}\right) & 0 \\ O\left(\frac{v_t}{v_2}\right) & -2c_\alpha \frac{2v_t}{v_2} & 0 \\ 0 & 0 & \frac{v_2}{2v_t} \end{pmatrix}. \quad (\text{C.17})$$

This matrix is diagonal up to $O(v_t/v_2)$ terms, which can be discarded for sufficiently small v_t . As we are interested in extracting the leading order correction to the masses of h and H , we shall keep the diagonal elements.

In summary what we have done is,

$$\begin{aligned} \boldsymbol{\rho}^T M_{\text{CPeven}}^2 \boldsymbol{\rho} &= \boldsymbol{\rho}^T (M_1^2 + M_2^2) \boldsymbol{\rho} \\ &= \mathbf{H}_1^T (R_{\alpha_2} M_1^2 R_{\alpha_2}^T + R_{\alpha_2} M_2^2 R_{\alpha_2}^T) \mathbf{H}_1 \\ &\simeq \mathbf{H}_1^T (M_1^2 + M_{2\text{diag}}^2) \mathbf{H}_1 \\ &= \mathbf{H}^T (R_\alpha M_1^2 R_\alpha^T + R_\alpha M_{2\text{diag}}^2 R_\alpha^T) \mathbf{H} \\ \boldsymbol{\rho}^T M_{\text{CPeven}}^2 \boldsymbol{\rho} &\simeq \mathbf{H}^T (M_{1\text{diag}}^2 + M_{2\text{diag}}'^2) \mathbf{H}. \end{aligned}$$

From,

$$M_{1\text{diag}}^2 + M_{2\text{diag}}'^2 = \begin{pmatrix} m_h'^2 - 2\sqrt{2} s_\alpha^2 \mu_{t2} v_t & 0 & 0 \\ 0 & m_H'^2 - 2\sqrt{2} c_\alpha^2 \mu_{t2} v_t & 0 \\ 0 & 0 & \frac{\mu_{t2} v_2^2}{\sqrt{2} v_t} \end{pmatrix}, \quad (\text{C.18})$$

we can read the scalar masses,

$$m_h^2 = \lambda_1 v_1^2 + \lambda_2 v_2^2 - \sqrt{(\lambda_1 v_1^2 - \lambda_2 v_2^2)^2 + (\lambda_3 + \lambda_4)^2 v_1^2 v_2^2} - 2\sqrt{2} \sin^2 \alpha \mu_{t2} v_t \quad (\text{C.19})$$

$$m_H^2 = \lambda_1 v_1^2 + \lambda_2 v_2^2 + \sqrt{(\lambda_1 v_1^2 - \lambda_2 v_2^2)^2 + (\lambda_3 + \lambda_4)^2 v_1^2 v_2^2} - 2\sqrt{2} \cos^2 \alpha \mu_{t2} v_t \quad (\text{C.20})$$

$$m_{H_t}^2 = \frac{\mu_{t2} v_2^2}{\sqrt{2} v_t}. \quad (\text{C.21})$$

Note that the expressions obtained agree with those given in eq. (C.8)–(C.9) if we take $\mu_{t2} \ll v_i$, as expected. The main difference in the expressions in these two limits is the presence of the correction terms $-2\sqrt{2} \sin^2 \alpha \mu_{t2} v_t$ and $-2\sqrt{2} \cos^2 \alpha \mu_{t2} v_t$ in m_h^2 and m_H^2 , respectively, which pushes down their values, making h and H lighter than it would be in the absence of the triplet. Notice also that in this approximation, we managed to perform the diagonalization using only two mixing angles, α and α_2 , instead of the three angles needed in the general case.

C.2 Charged scalars

In this section we will apply to the charged scalars mass matrix the same method used in the previous section for the neutral scalars. First note that the matrix (3.26),

$$M_{\text{Charged}}^2 = \frac{1}{2} \begin{pmatrix} -\lambda_4 v_2^2 - \lambda_{tt1} v_t^2 & \lambda_4 v_1 v_2 & \lambda_{tt1} v_1 v_t / \sqrt{2} \\ \lambda_4 v_1 v_2 & -\lambda_4 v_1^2 - \lambda_{tt2} v_t^2 + 2\sqrt{2} \mu_{t2} v_t & \frac{1}{2} (\sqrt{2} \lambda_{tt2} v_t - 4\mu_{t2}) v_2 \\ v_1 v_t \lambda_{tt1} / \sqrt{2} & \frac{1}{2} (\sqrt{2} \lambda_{tt2} v_t - 4\mu_{t2}) v_2 & \frac{\sqrt{2} \mu_{t2} v_2^2}{v_t} - \frac{1}{2} (\lambda_{tt1} v_1^2 + \lambda_{tt2} v_2^2) \end{pmatrix},$$

can be decomposed as,

$$\begin{aligned} M_{\text{Charged}}^2 &= M_{1+}^2 + M_{2+}^2 \\ &= \frac{1}{2} \begin{pmatrix} -\lambda_4 v_2^2 & \lambda_4 v_1 v_2 & 0 \\ \lambda_4 v_1 v_2 & -\lambda_4 v_1^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ &\quad + \frac{1}{2} \begin{pmatrix} -\lambda_{tt1} v_t^2 & 0 & \lambda_{tt1} v_1 v_t / \sqrt{2} \\ 0 & -\lambda_{tt2} v_t^2 + 2\sqrt{2} \mu_{t2} v_t & \frac{1}{2} (\sqrt{2} \lambda_{tt2} v_t - 4\mu_{t2}) v_2 \\ v_1 v_t \lambda_{tt1} / \sqrt{2} & \frac{1}{2} (\sqrt{2} \lambda_{tt2} v_t - 4\mu_{t2}) v_2 & \frac{\sqrt{2} \mu_{t2} v_2^2}{v_t} - \frac{1}{2} (\lambda_{tt1} v_1^2 + \lambda_{tt2} v_2^2) \end{pmatrix}. \end{aligned} \quad (\text{C.22})$$

In the matrix M_{2+}^2 are contained all the mixing effects among the doublets and the triplet. If M_{2+}^2 vanished, there would be mixing only between the two doublets, as described by M_{1+}^2 , leading to a charged Goldstone boson and a charged physical scalar, as in the usual 2HDM. Let's again consider the limit $v_t, \mu_{t2} \ll v_i$, in which M_{1+}^2 remains the same and M_{2+}^2 reduces to $M_{2+}^2 = \text{diag}(0, 0, \sqrt{2} \mu_{t2} v_2^2 / 2v_t - \lambda_{tt1} v_1^2 / 2 - \lambda_{tt2} v_2^2 / 2)$, so that the triplet completely decouples from the doublets, which still mix with themselves. In this case, it is necessary only one angle in the diagonalization and we can make $\beta_1, \beta_2 \rightarrow 0$ in eq. (3.27), leading to the following physical fields,

$$\begin{pmatrix} G^+ \\ H^+ \\ H_t^+ \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta & 0 \\ -s_\beta & c_\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \\ \Delta^+ \end{pmatrix}$$

with,

$$\tan 2\beta = \frac{2v_1v_2}{v_1^2 - v_2^2}, \quad (\text{C.23})$$

which can be put in the form $\tan 2\beta = 2 \tan \beta / (1 - \tan^2 \beta)$ by dividing numerator and denominator by v_1^2 , so that

$$\tan \beta = \frac{v_2}{v_1}. \quad (\text{C.24})$$

The masses of H^+ and H_t^+ in this approximation are,

$$m_{H^+}^2 = -\frac{1}{2}\lambda_4 v^2, \quad (\text{C.25})$$

$$m_{H_t^+}^2 = \frac{\mu_{t2}v_2^2}{\sqrt{2}v_t} - \frac{1}{4}(\lambda_{tt1}v_1^2 + \lambda_{tt2}v_2^2). \quad (\text{C.26})$$

As mentioned early, for $\mu_{t2} \simeq v_t$ the mass of H_t^+ is small and may be in tension with existing bounds, so that in this limit $\mu_{t2}/v_t > 1$ is favored.

Now, allowing μ_{t2} be large but keeping v_t small, M_{2+}^2 reduces to,

$$M_{2+}^2 = \mu_{t2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sqrt{2}v_t & -v_2 \\ 0 & -v_2 & \frac{v_2^2}{\sqrt{2}v_t} \end{pmatrix}. \quad (\text{C.27})$$

M_{2+}^2 is diagonalized by moving to an intermediate basis (H_1^+, H_2^+, H_3^+) through a rotation R_{β_2} ,

$$\begin{pmatrix} H_1^+ \\ H_2^+ \\ H_3^+ \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{\beta_2} & s_{\beta_2} \\ 0 & -s_{\beta_2} & c_{\beta_2} \end{pmatrix} \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \\ \Delta^+ \end{pmatrix}, \quad (\text{C.28})$$

so that,

$$R_{\beta_2} M_{2+}^2 R_{\beta_2}^T = M_{2+\text{diag}}^2, \quad (\text{C.29})$$

with,

$$\sin \beta_2 = \frac{\sqrt{2}v_t}{\sqrt{v_2^2 + 2v_t^2}} \simeq \frac{\sqrt{2}v_t}{v_2}, \quad (\text{C.30})$$

and,

$$\cos \beta_2 = \frac{v_2}{\sqrt{v_2^2 + 2v_t^2}} \simeq 1. \quad (\text{C.31})$$

The effect of this rotation on M_{1+}^2 is

$$R_{\beta_2} M_{1+}^2 R_{\beta_2}^T = \frac{1}{2} \begin{pmatrix} -\lambda_4 v_2^2 & \lambda_4 v_1 v_2 c_{\beta_2} & \lambda_4 v_1 v_2 s_{\beta_2} \\ \lambda_4 v_1 v_2 c_{\beta_2} & -\lambda_4 v_1^2 c_{\beta_2}^2 + \frac{\lambda_{tt1}v_1^2 + \lambda_{tt2}v_2^2}{2} s_{\beta_2}^2 & -\left(\lambda_4 v_1^2 + \frac{\lambda_{tt1}v_1^2 + \lambda_{tt2}v_2^2}{2}\right) s_{\beta_2} c_{\beta_2} \\ -\lambda_4 v_1 v_2 s_{\beta_2} & \left(\lambda_4 v_1^2 + \frac{\lambda_{tt1}v_1^2 + \lambda_{tt2}v_2^2}{2}\right) s_{\beta_2} c_{\beta_2} & -\frac{\lambda_{tt1}v_1^2 + \lambda_{tt2}v_2^2}{2} s_{\beta_2}^2 + \lambda_4 v_1^2 c_{\beta_2}^2 \end{pmatrix} \quad (\text{C.32})$$

As $\sin \beta_2 \ll 1$, at leading order we have $R_{\beta_2} M_{1+}^2 R_{\beta_2}^T \simeq M_{1+}^2$, and the rotation R_{β_2} does not change M_{1+}^2 , as we wanted. Then, rotating M_{1+}^2 by a matrix R_β , with β given by eq. (C.24), we move to the physical basis,

$$\begin{pmatrix} G_1^+ \\ H^+ \\ H_t^+ \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta & 0 \\ -s_\beta & c_\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} H_1^+ \\ H_2^+ \\ H_3^+ \end{pmatrix}. \quad (\text{C.33})$$

Note that this second rotation R_β does not disturb the diagonalization of M_{2+}^2 ,

$$R_\beta M_{2+\text{diag}}^2 R_\beta^T = \begin{pmatrix} c_\beta & s_\beta & 0 \\ -s_\beta & c_\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{\mu_{t2} v_2^2}{\sqrt{2} v_t} \end{pmatrix} \begin{pmatrix} c_\beta & -s_\beta & 0 \\ s_\beta & c_\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{\mu_{t2} v_2^2}{\sqrt{2} v_t} \end{pmatrix} = M_{2+\text{diag}}^2. \quad (\text{C.34})$$

Thus, the diagonalization of M_{1+}^2 and M_{2+}^2 leads to,

$$M_{1+\text{diag}}^2 + M_{2+\text{diag}}^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\frac{1}{2} \lambda_4 v^2 & 0 \\ 0 & 0 & \frac{\mu_{t2} v_2^2}{\sqrt{2} v_t} \end{pmatrix}, \quad (\text{C.35})$$

from which we obtain the masses for H^+ and H_t^+ ,

$$m_{H^+}^2 = -\frac{1}{2} \lambda_4 v^2, \quad (\text{C.36})$$

$$m_{H_t^+}^2 = \frac{\mu_{t2} v_2^2}{\sqrt{2} v_t}. \quad (\text{C.37})$$

As a consistency check, notice that taking the limit $v_t \ll v_i$ directly in the eqs. (3.28) and (3.29), we obtain as result the eqs. (C.36) and (C.37). Finally, note that for the diagonalization in this limit we need to use only two mixing angles, β and β_2 , instead of the three angles needed in the general case.

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