



Universal leading quantum correction to the Newton potential

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Abstract The derivation of effective quantum gravity corrections to Newton's potential is an important step in the whole effective quantum field theory approach. We hereby add new strong arguments in favor of omitting all the diagrams with internal lines of the massive sources, and we also recalculate the corrections to the Newtonian potential using functional methods in an arbitrary parametrization of the quantum fluctuations of the metric. The general proof of the gauge- and parametrization-independence within this approach is also explicitly given. On top of that, we argue that the universality of the result holds regardless of the details of the ultraviolet completion of quantum gravity theory. Indeed, it turns out that the logarithm quantum correction depends only on the low energy spectrum of the theory that is responsible for the analytic properties of loop's amplitudes.

1 Introduction

Quantum corrections to Newton's potential for the classical gravitational force is a kind of theoretical “standard candle” for the effective quantum gravity theory. The first calculation of these corrections are half a century old [1], but the explosion of interest to the particular example and the effective quantum gravity in general, started from the seminal works due to Donoghue [2, 3]. The total amount of publications (see, e.g., [4, 5] and [6] and references therein) is very large, such that we are probably not aware of all of them. However, in our opinion, there remain at least two relevant issues which were not sufficiently well discussed.

First, starting from the paper [7] there was a growing understanding that the gauge-fixing independence of the quantum corrections is an important and useful criterion for the correctness and consistent definition of these corrections.

In the mentioned paper, the simplified model of scalar electrodynamics was used to prove that disregarding some of the diagrams is inconsistent as this procedure does not provide the gauge-fixing independence of the potential. In the subsequent papers, i.e., [4, 5], all the diagrams were taken into account and the invariance was proved (see, e.g., [8, 9]). However, it turns out that there are serious reasons to take out some of these diagrams [11, 13].

The calculations in most of the mentioned (and many other) papers were done using Feynman diagrams where two massive scalar fields model the massive bodies. Then, the complete set of diagrams includes those of *L*- and *P*-type. The first, *L*-type subset consists of the graphs with internal lines of massless gravitons and massless gauge (Faddeev–Popov) ghosts. These diagrams give only a very small $\mathcal{O}(r^{-3})$ -type addition to the Newton potential between the two-point masses. Phenomenologically, these graphs may be irrelevant, but conceptually they are the most important ones since the much stronger contributions coming from the *P*-type diagrams should be disregarded because of physical reasons [13]. The point is that our final purpose is to get the quantum corrections to the force between two *macroscopic* bodies, which are modeled by the scalar field. Taking into account the *internal* lines of these scalars means that we are regarding the massive macroscopic bodies as quantum objects. For example, if the purpose is to explore the quantum effects on the motion of an asteroid around the Sun, we are quantizing this asteroid and the Sun. Or we are quantizing, e.g., the Earth and the Moon. Indeed, the Moon does not like to be quantized. Thus, we may commit a serious mistake in this way.

Two other arguments for omitting the *P*-type diagrams are as follows. The macroscopic body is made from atoms, and most of the mass of the atom nucleons is due to the quantum effects. Thus, the argument concerning the expansions in \hbar is not operational in this case. Furthermore, nowadays there are strong experimental confirmations of not quantiz-

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ing the classical source in a physically similar situation. It is good to remember the success of the theoretical description of Casimir force (see, e.g., [14]). Despite the existing controversies (see, for example, the critical discussion in [15]), there are no many doubts that the Casimir effect is experimentally detected, and we know that its theoretical description does not involve the quantization of the sources (such as conducting plates) but only the electromagnetic field. There are no reasons to assume that things should be different for the gravitational Casimir effect because the basic formalism is, in general, similar [16]. Thus, the correct procedure is to separate and use only L -type diagrams, i.e., those with only massless internal lines. From the quantum field theory viewpoint, this means that the object of our interest is the functional integral

$$e^{i\Gamma} = \int dg d\bar{c} dc \exp \{ iS_{grav}(g) + iS_{gf}(g) + iS_{gh}(g, \bar{c}, c) + iS_{sources}(g, \Phi) \}, \quad (1)$$

where $g = g_{\alpha\beta}$ is the quantum metric, $\bar{c} = \bar{c}_\alpha$ and $c = c^\beta$ are gauge ghosts, and Φ represent the external massive sources. The approach described above was pursued in important work [10] and, most recently, in [11]. Following the last reference, in the calculations described below, we choose these sources as a massive point-like mass and a light point-like test particle, but in principle, the integral (1) can be explored for arbitrary massive sources, e.g., for the dark matter (DM), finite-size stars, or interstellar gas. In all these cases, the integration over Φ is not included for the reasons described above. This means that the variables Φ play the role of external parameters, independent of the quantum variables such as the metric components.

A general observation is in order. There are relevant aspects of the semiclassical approach (when metric is a classical background while matter fields quantum), related to the possible transformations which are mixing classical and quantum variables [12]. One can try to elaborate the same idea in the *anti-semiclassical* approach, when only gravity is quantized. However, in the effective framework, in the IR only the massless fields are relevant, leaving only the effects of graviton and photon [2]. In this effective setting, our *anti-semiclassical* approach emerges naturally and the mixing of the massless degree of freedom with the ones of massive (macroscopic) degrees of freedom describing the classical sources does not look reasonable.

In practice, one can use the background field method for the useful definition and derivation of the integral (1). Then, the effective action in the *l.h.s.* is $\Gamma(g, \Phi)$ and the integral variables change accordingly. We shall discuss the explicit formulas below.

The paper is organized in the following way. Section 2 describes the derivation of the relevant part of $\Gamma(g, \Phi)$. Sec-

tion 3 aims to define the gravitational field in the Newtonian approximation, in the theory with quantum effective contributions. Section 4 describes the motion of test particles and defined the final form of the corrected Newtonian potential. Section 5 is devoted to the general proof of the gauge- and parametrization-independence of the effective quantum correction to Newton's potential. In Section 6 we elaborate on the constraints that an effective field theory should satisfy in order to be consistent in the quantum field theory framework, and after that we explain the universality of the leading correction to Newton's potential, which only depends on the fundamental theory low-energy spectrum. Finally, in Sect. 7 we draw our conclusions.

The notations include the Minkowski metric $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$, while the Riemann curvature tensor is defined as

$$R^\alpha{}_{\beta\mu\nu} = \partial_\mu \Gamma^\alpha_{\beta\nu} - \partial_\nu \Gamma^\alpha_{\beta\mu} + \Gamma^\alpha_{\mu\tau} \Gamma^\tau_{\beta\nu} - \Gamma^\alpha_{\nu\tau} \Gamma^\tau_{\beta\mu}. \quad (2)$$

The definitions of the Ricci tensor and the scalar curvature are $R_{\mu\nu} = R^\alpha{}_{\mu\alpha\nu}$ and $R = g^{\mu\nu} R_{\mu\nu}$, respectively. We also adopt the units system with $c = 1$ and $\hbar = 1$.

2 Derivation of effective action

In the effective quantum gravity [3,6], it is assumed that the low-energy effects are described by Einstein's general relativity with zero cosmological constant because this term is irrelevant at the astrophysical scale where the Newtonian limit applies. The possible contributions of higher derivative terms either decouple (see, e.g., [17] for preliminary analysis, coherent with the standard approach in particle physics [18, 19]) or regarded as small perturbations by definition [20]. The reader can consult [21] and [22] for a general discussion of different approaches. However, the net result is that we have to start from the classical action of the form

$$S = -\frac{1}{\kappa^2} \int d^4x \sqrt{-g} R + S_{sources}(g, \Phi), \quad (3)$$

where $\kappa^2 = 16\pi G$ and G is the Newton constant. As the source term, we shall consider the action of the free massive particle S_M , with coordinates y^μ

$$S_M = -M \int ds = -M \int \sqrt{g_{\mu\nu} dy^\mu dy^\nu}. \quad (4)$$

To quantize the gravitational field, we apply the background field method with the most general (at the one-loop level) parametrization of the metric, as formulated in Ref. [23] (see also [24]), namely

$$g_{\mu\nu} \longrightarrow g'_{\mu\nu} = e^{2\gamma_0\kappa\sigma} [g_{\mu\nu} + \kappa(\gamma_1\phi_{\mu\nu} + \gamma_2\phi g_{\mu\nu}) + \kappa^2(\gamma_3\phi_{\mu\rho}\phi_\nu^\rho + \gamma_4 g_{\mu\nu}\phi_{\rho\sigma}\phi^{\rho\sigma} + \gamma_5\phi\phi_{\mu\nu} + \gamma_6 g_{\mu\nu}\phi^2)]. \quad (5)$$

In this formula, $g_{\mu\nu}$ is the background field and $\sigma, \phi_{\mu\nu}$ are quantum fields. The indices are lowered and raised with the background metric $g_{\mu\nu}$. Also, $\phi = g_{\mu\nu}\phi^{\mu\nu}$. Finally, $\gamma_{0,1,\dots,6}$ are arbitrary coefficients which parameterize quantum variables.

Treating the conformal factor σ as a new field introduces an artificial conformal symmetry [25]. In order to remove the corresponding degeneracy, we implement the conformal gauge fixing

$$\sigma = \lambda\phi, \quad (6)$$

where λ is a gauge-fixing parameter. Using (6) and expanding the exponential, one can rewrite Eq. (5) in the form

$$g'_{\mu\nu} = g_{\mu\nu} + \kappa A_{\mu\nu}^{\alpha\beta} \phi_{\alpha\beta} + \kappa^2 \phi_{\lambda\tau} B_{\mu\nu}^{\lambda\tau,\rho\sigma} \phi_{\rho\sigma} + \mathcal{O}(\kappa^3), \quad (7)$$

where

$$A_{\mu\nu}^{\alpha\beta} = \gamma_1 \delta_{\mu\nu}^{\alpha\beta} + (\gamma_2 + 2\gamma_0\lambda) g_{\mu\nu} g^{\alpha\beta}, \quad (8)$$

and

$$\begin{aligned} B_{\lambda\tau}^{\mu\nu,\alpha\beta} = & \frac{\gamma_3}{4} \left(\delta_{\lambda\tau}^{\mu\alpha} g^{\nu\beta} + \delta_{\lambda\tau}^{\nu\alpha} g^{\mu\beta} + \delta_{\lambda\tau}^{\mu\beta} g^{\nu\alpha} + \delta_{\lambda\tau}^{\nu\beta} g^{\mu\alpha} \right) \\ & + \gamma_4 \delta^{\mu\nu,\alpha\beta} g_{\lambda\tau} + \frac{1}{2} (\gamma_5 + 2\gamma_0\lambda\gamma_1) \\ & (\delta_{\lambda\tau}^{\mu\nu} g^{\alpha\beta} + \delta_{\lambda\tau}^{\alpha\beta} g^{\mu\nu}) \\ & + (\gamma_6 + 2\gamma_0\lambda\gamma_2 + 2\gamma_0^2\lambda^2) g^{\mu\nu} g^{\alpha\beta} g_{\lambda\tau}, \end{aligned} \quad (9)$$

with

$$\delta_{\alpha\beta}^{\mu\nu} = \frac{1}{2} (\delta_{\alpha}^{\mu} \delta_{\beta}^{\nu} + \delta_{\beta}^{\mu} \delta_{\alpha}^{\nu}). \quad (10)$$

To fix the diffeomorphism invariance and simplify the calculations,¹ we introduce the following minimal gauge fixing action [23],

$$\begin{aligned} S_{gf} = & -\frac{1}{2} \int d^4x \sqrt{-g} \left[\gamma_1 \nabla_{\lambda} \phi_{\mu}^{\lambda} \right. \\ & \left. - \frac{1}{2} (\gamma_1 + 2\gamma_2 + 4\gamma_0\lambda) \nabla_{\mu} \phi^{\mu} \right]^2. \end{aligned} \quad (11)$$

Performing the expansion of (5) in (3) with the minimal gauge-fixing term, we arrive at the bilinear, in the quantum metric, part of the action

$$(S + S_{gf})^{(2)} = \frac{1}{2} \int d^4x \sqrt{-g} \phi_{\mu\nu} \mathbf{H}^{\mu\nu,\alpha\beta} \phi_{\alpha\beta}, \quad (12)$$

where

$$\mathbf{H}^{\mu\nu,\alpha\beta} = -(\mathbf{K}^{\mu\nu,\alpha\beta} \square + \Pi^{\mu\nu,\alpha\beta} + \mathbf{M}^{\mu\nu,\alpha\beta}), \quad (13)$$

with

$$\mathbf{K}^{\mu\nu,\alpha\beta} = A_{\lambda\tau}^{\mu\nu} K^{\lambda\tau,\rho\sigma} A_{\rho\sigma}^{\alpha\beta},$$

¹ The non-minimal gauge was considered in [11] and proved to give a gauge-independent result for the one-loop contribution to Newton's gravitational potential, so we avoid discussing it here.

$$\begin{aligned} \Pi^{\mu\nu,\alpha\beta} &= A_{\lambda\tau}^{\mu\nu} K^{\lambda\tau,\rho\sigma} A_{\rho\sigma}^{\alpha\beta} - 2B_{\lambda\tau}^{\mu\nu,\alpha\beta} G^{\mu\nu}, \\ \mathbf{M}^{\mu\nu,\alpha\beta} &= \kappa^2 \left(B_{\lambda\tau}^{\mu\nu,\alpha\beta} T^{\lambda\tau} - A_{\lambda\tau}^{\mu\nu} M^{\lambda\tau,\rho\sigma} A_{\rho\sigma}^{\alpha\beta} \right), \end{aligned} \quad (14)$$

where we used the notations:

$$\begin{aligned} G^{\mu\nu} &= R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R, \\ K^{\mu\nu,\alpha\beta} &= \frac{1}{2} (\delta^{\mu\nu,\alpha\beta} - \frac{1}{2} g^{\mu\nu} g^{\alpha\beta}), \\ \Pi^{\mu\nu,\alpha\beta} &= \frac{1}{2} (R^{\mu\alpha\nu\beta} + R^{\mu\beta\nu\alpha}) \\ &+ \frac{1}{4} (g^{\mu\alpha} R^{\nu\beta} + g^{\mu\beta} R^{\nu\alpha} + g^{\nu\alpha} R^{\mu\beta} + g^{\nu\beta} R^{\mu\alpha}) \\ &- \frac{1}{2} (g^{\mu\nu} R^{\alpha\beta} + g^{\alpha\beta} R^{\mu\nu}) - K^{\mu\nu,\alpha\beta} R. \end{aligned} \quad (15)$$

Furthermore, in the source sector,

$$T^{\mu\nu}(x) = M \int ds \delta(x - y(s)) u^{\mu} u^{\nu} \quad (16)$$

is the point-like mass energy-momentum tensor and

$$M^{\mu\nu,\alpha\beta}(x) = \frac{M}{4} \int ds \delta(x - y(s)) u^{\mu} u^{\nu} u^{\alpha} u^{\beta}. \quad (17)$$

It is worth noting that the boldface notation are used for the operators depending on the parametrization and gauge fixing. Also, in (16) and (17),

$$u^{\mu} = \frac{dy^{\mu}}{ds} \quad (18)$$

is the particle four-velocity and the delta function is introduced through the relation

$$\int d^4x \sqrt{-g(x)} \delta(x - y(s)) = 1. \quad (19)$$

The standard Schwinger-DeWitt technique for the one-loop divergences requires to reduce the Hessian (13) to the standard expression for the minimal operator, i.e., $\hat{1}\square + \hat{\mathbf{P}} - (1/6)\hat{1}R$. Therefore, we multiply (13) with the negative inverse of (14), namely

$$\mathbf{K}_{\mu\nu,\alpha\beta}^{-1} = (A^{-1})_{\mu\nu}^{\lambda\tau} K_{\lambda\tau,\rho\sigma}^{-1} (A^{-1})_{\alpha\beta}^{\rho\sigma}, \quad (20)$$

where

$$\begin{aligned} K_{\lambda\tau,\rho\sigma}^{-1} &= 2(\delta_{\lambda\tau,\rho\sigma} - \frac{1}{2} g_{\lambda\tau} g_{\rho\sigma}), \\ (A^{-1})_{\mu\nu}^{\alpha\beta} &= \frac{1}{\gamma_1} \left(\delta_{\mu\nu}^{\alpha\beta} - \frac{\gamma_2 + 2\gamma_0\lambda}{\gamma_1 + 4\gamma_2 + 8\gamma_0\lambda} g_{\mu\nu} g^{\alpha\beta} \right). \end{aligned} \quad (21)$$

Adopting the suppressed index notation of Ref. [26], we get

$$\hat{\mathbf{H}} \equiv -\mathbf{K}_{\mu\nu,\lambda\tau}^{-1} \cdot \mathbf{H}^{\lambda\tau,\alpha\beta} = \hat{1}\square + \hat{\mathbf{P}} - \frac{1}{6} \hat{1}R, \quad (22)$$

where $\hat{1} = \delta^{\alpha\beta}_{\mu\nu}$ is the identity matrix for symmetric rank-2 tensors and

$$\hat{\mathbf{P}} = \mathbf{K}^{-1}_{\mu\nu,\lambda\tau} \cdot \mathbf{\Pi}^{\lambda\tau,\alpha\beta} + \mathbf{K}^{-1}_{\mu\nu,\lambda\tau} \cdot \mathbf{M}^{\lambda\tau,\alpha\beta} + \frac{1}{6} \hat{1} R. \quad (23)$$

Performing the substitution (5) in (1), we have the one-loop contribution to the effective action

$$\bar{\Gamma}^{(1)} = \frac{i}{2} \text{Tr} \ln \hat{\mathbf{H}} - i \text{Tr} \ln \hat{\mathbf{H}}_{gh}, \quad (24)$$

where $\hat{\mathbf{H}}_{gh}$ is the bilinear part of the Faddeev–Popov ghost action. The contribution of this term to (24) is the same as in Ref. [23] hence we do not repeat it here.

The divergent part of the first term in (24) can be evaluated by the trace of the coincidence limit of the Schwinger–DeWitt expansion \hat{a}_2 coefficient [27] (we do not include here the surface terms, as they are irrelevant for our purpose)

$$\begin{aligned} \frac{i}{2} \text{Tr} \ln \hat{\mathbf{H}}|_{div} = & -\frac{\mu^{n-4}}{(4\pi)^2(n-4)} \int d^n x \sqrt{-g} \text{tr} \\ & \times \left\{ \frac{1}{180} \hat{1} (R^2_{\mu\nu\alpha\beta} - R^2_{\mu\nu}) + \frac{1}{2} \hat{\mathbf{P}}^2 + \frac{1}{12} \hat{\mathcal{R}}^2_{\mu\nu} \right\}, \end{aligned} \quad (25)$$

where

$$\hat{\mathcal{R}}_{\lambda\tau} = -\frac{1}{2} (\delta^\alpha_\mu R^\beta_{\nu\rho\sigma} + \delta^\beta_\mu R^\alpha_{\nu\rho\sigma} + \delta^\alpha_\nu R^\beta_{\mu\rho\sigma} + \delta^\beta_\nu R^\alpha_{\mu\rho\sigma}) \quad (26)$$

and μ is the renormalization group scale.

To simplify (25), we use the identity $g_{\mu\nu} u^\mu u^\nu = 1$ which implies in the following equations for the expressions (16) and (17):

$$g_{\mu\nu} M^{\mu\nu,\alpha\beta} = \frac{1}{4} T^{\alpha\beta}, \quad (27)$$

$$M_{\mu\nu,\alpha\beta} M^{\mu\nu,\alpha\beta} = \frac{1}{16} T_{\mu\nu} T^{\mu\nu}, \quad (28)$$

$$T_{\mu\nu} T^{\mu\nu} = T^2, \quad (29)$$

where

$$T \equiv T^\mu_\mu = M \int ds \delta(x - y(s)) \quad (30)$$

is the trace of the energy-momentum tensor for a massive point-like particle. There are also equations analogous to (27) for other index contractions since $M^{\mu\nu,\alpha\beta}$ is totally symmetric.

The evaluation of (25) follows the scheme explained in detail in [23]. Thus, skipping the algebra, we get, for the one-loop divergences

$$\begin{aligned} \bar{\Gamma}^{(1)}_{div} = & -\frac{\mu^{n-4}}{(4\pi)^2(n-4)} \int d^n x \sqrt{-g} \\ & \times \{ c_1 R^2_{\mu\nu\alpha\beta} + c_2 R^2_{\mu\nu} + c_3 R^2 \\ & + \kappa^2 c_4 R_{\mu\nu} T^{\mu\nu} + \kappa^2 c_5 R T + \kappa^4 c_6 T^2 \}, \end{aligned} \quad (31)$$

where the dimensionless coefficients are given by

$$\begin{aligned} c_1 = \frac{53}{45}, \quad c_2 = -\frac{361}{90} - \frac{4\xi_2}{\gamma_1^4 Z^2}, \quad c_3 = \frac{43}{36} + \frac{\xi_3}{3\gamma_1^4 Z^4}, \\ c_4 = -\frac{2\xi_4}{\gamma_1^4 Z^4}, \quad c_5 = \frac{5}{24} - \frac{\xi_5}{6\gamma_1^4 Z^4}, \quad c_6 = \frac{1}{32} - \frac{\xi_6}{4\gamma_1^4 Z^4}, \end{aligned} \quad (32)$$

where $Z = \gamma_1 + 4\gamma_2 + 8\gamma_0\lambda$, and the bulky expressions for the parametrization dependent coefficients ξ_i can be found in Appendix A.

In the limit $\gamma_1 \rightarrow 1$, $\gamma_2, \dots, \gamma_6 \rightarrow 0$ and $\gamma_0 \rightarrow 0$ for the parameters of (5), Eq. (31) reproduces the result of Ref. [11], evaluated in the standard parametrization in the minimal gauge.

Using the classical equations of motion

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{\kappa^2}{2} T_{\mu\nu} \quad (33)$$

and (29), Eq. (24) boils down to

$$\begin{aligned} \bar{\Gamma}^{(1)}_{div}|_{\text{on-shell}} = & -\frac{\mu^{n-4}}{(4\pi)^2(n-4)} \int d^n x \sqrt{-g} \\ & \times \left\{ \frac{53}{45} R^2_{\mu\nu\alpha\beta} - \frac{373}{480} \kappa^4 T^2 \right\}, \end{aligned} \quad (34)$$

independently of the choice of parametrization and gauge parameters, exactly as it should be.

To consider the Newtonian limit it is useful to rewrite Eq. (31) in a more convenient Weyl basis, using the relations

$$R^2_{\mu\nu\alpha\beta} = 2C^2 - E + \frac{1}{3} R^2, \quad R^2_{\mu\nu} = \frac{1}{2} C^2 - \frac{1}{2} E + \frac{1}{3} R^2. \quad (35)$$

Here $C^2 = R^2_{\mu\nu\alpha\beta} - 2R^2_{\mu\nu} + 1/3 R^2$ is the square of the Weyl tensor and $E = R^2_{\mu\nu\alpha\beta} - 4R^2_{\mu\nu} + R^2$ is the integrand of the (topological at $n = 4$) Gauss–Bonnet term. Discarding the non essential for us topological term, we get for (31):

$$\begin{aligned} \bar{\Gamma}^{(1)}_{div} = & -\frac{\mu^{n-4}}{n-4} \int d^n x \sqrt{-g} \left\{ \beta_2 C^2 - \frac{1}{3} \beta_0 R^2 \right. \\ & \left. - 2\kappa^2 \beta_{RT1} R_{\mu\nu} T^{\mu\nu} + \kappa^2 \beta_{RT2} R T + \kappa^4 \beta_{TT} T^2 \right\}, \end{aligned} \quad (36)$$

with the following relations between the coefficients,

$$\beta_2 = \frac{4c_1 + c_2}{2(4\pi)^2}, \quad \beta_0 = -\frac{c_1 + c_2 + 3c_3}{(4\pi)^2}, \quad (37)$$

$$\beta_{RT1} = -\frac{c_4}{2(4\pi)^2}, \quad \beta_{RT2} = \frac{c_5}{(4\pi)^2}, \quad \beta_{TT} = \frac{c_6}{(4\pi)^2}. \quad (38)$$

To reconstruct the effective action we use the rule (see, e.g., [22] for the introduction and further references, including the generalizations for the loops of massive fields)

$$\frac{\mu^{n-4}}{n-4} \longrightarrow \frac{1}{2} \ln \left(\frac{\square}{\mu^2} \right), \quad (39)$$

that is valid for the massless fields contributions such as the graviton. In this way, we finally get

$$\begin{aligned}\bar{\Gamma}^{(1)} = & - \int d^4x \sqrt{-g} \left\{ \frac{1}{2} \beta_2 C_{\mu\nu\alpha\beta} \ln\left(\frac{\square}{\mu^2}\right) C^{\mu\nu\alpha\beta} \right. \\ & - \frac{1}{6} \beta_0 R \ln\left(\frac{\square}{\mu^2}\right) R \\ & - \kappa^2 \beta_{RT1} R_{\mu\nu} \ln\left(\frac{\square}{\mu^2}\right) T^{\mu\nu} + \frac{1}{2} \kappa^2 \beta_{RT2} R \ln\left(\frac{\square}{\mu^2}\right) T \\ & \left. + \frac{1}{2} \kappa^4 \beta_{TT} T \ln\left(\frac{\square}{\mu^2}\right) T \right\}. \quad (40)\end{aligned}$$

3 Gravitational field in the Newtonian limit with quantum contributions

Here we are going to obtain the gravitational field in the Newtonian limit taking into account one-loop quantum corrections. The effective action is given by the classical action S , defined in (3), and the one-loop correction (40), i.e.,

$$\Gamma = S + \bar{\Gamma}^{(1)}. \quad (41)$$

To obtain the metric potentials with logarithm quantum corrections, we follow Refs. [29,30] (see also [31–33]). First, we consider small perturbations of the background metric around the Minkowski spacetime,

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}, \quad |\kappa h_{\mu\nu}| \ll 1 \quad (42)$$

and expand the action (41) to the relevant order in the weak-field approximation. In this way, we get

$$\begin{aligned}\Gamma = & - \frac{1}{2} \int d^4x \left\{ \frac{1}{2} h_{\mu\nu} f_2(\square) \square h^{\mu\nu} - h^{\mu\nu} f_2(\square) \partial_\mu \partial_\nu h^\lambda{}_\lambda \right. \\ & - \frac{1}{6} h [f_2(\square) + 2f_0(\square)] \square h \\ & + \frac{1}{3} h [f_2(\square) + 2f_0(\square)] \partial_\mu \partial_\nu h^{\mu\nu} \\ & \left. + \frac{1}{3} h_{\alpha\beta} [f_2(\square) - f_0(\square)] \frac{\partial^\alpha \partial^\beta \partial^\mu \partial^\nu}{\square} h_{\mu\nu} \right\} \\ & - \frac{\kappa}{2} \int d^4x \left\{ h_{\mu\nu} f_{RT1}(\square) T^{\mu\nu} + \kappa^2 \beta_{RT2} h_{\mu\nu} \right. \\ & \left. \ln\left(\frac{\square}{\mu^2}\right) (\partial^\mu \partial^\nu - \eta^{\mu\nu} \square) T \right\}, \quad (43)\end{aligned}$$

where we defined the form factors

$$f_i(\square) = 1 + \kappa^2 \beta_i \ln\left(\frac{\square}{\mu^2}\right) \square, \quad i = 2, 0, RT1. \quad (44)$$

Let us note that in (43), $\square = \eta^{\mu\nu} \partial_\mu \partial_\nu$ is the flat space d'Alembertian. Also, the term proportional to β_{TT} is discarded here because it is $\propto M^2$ and, therefore, is beyond the weak-field approximation.

Then, taking the functional derivative with respect to $h_{\mu\nu}$ we get the equations of motion

$$\begin{aligned}f_2(\square) (\square h^{\mu\nu} - \partial^\mu \partial^\lambda h^\nu{}_\lambda - \partial^\nu \partial^\lambda h^\mu{}_\lambda) \\ + \frac{1}{3} [f_2(\square) + 2f_0(\square)] [\eta^{\mu\nu} (\partial^\alpha \partial^\beta h_{\alpha\beta} - \square h) + \partial^\mu \partial^\nu h] \\ + \frac{2}{3} [f_2(\square) - f_0(\square)] \frac{\partial^\mu \partial^\nu \partial^\alpha \partial^\beta}{\square} h_{\alpha\beta} \\ = -\kappa \left[f_{RT1}(\square) T^{\mu\nu} + \kappa^2 \beta_{RT2} \ln\left(\frac{\square}{\mu^2}\right) (\partial^\mu \partial^\nu - \eta^{\mu\nu} \square) T \right]. \quad (45)\end{aligned}$$

The second step is to consider the perturbation in the isotropic Newtonian form

$$\kappa h_{00} = 2\varphi(r), \quad \kappa h_{ij} = 2\delta_{ij}\psi(r) \quad (46)$$

and choose, as a source of the gravitational potentials, a static point-like mass located at the origin. In this case, $u^\mu = \delta_0^\mu$, and Eq. (16) becomes

$$T^{\mu\nu} = \delta_0^\mu \delta_0^\nu \rho, \quad \rho = M\delta(\mathbf{r}). \quad (47)$$

The metric potentials (46) can be obtained from the 00-component and the trace of Eq. (45), which are given respectively by

$$\begin{aligned}[f_2(-\Delta) - f_0(-\Delta)]\Delta\varphi + [f_2(-\Delta) + 2f_0(-\Delta)]\Delta\psi = \\ = \frac{3\kappa^2}{4} \left[f_{RT1}(-\Delta) + \kappa^2 \beta_{RT2} \ln\left(-\frac{\Delta}{\mu^2}\right) \Delta \right] \rho, \quad (48)\end{aligned}$$

$$\begin{aligned}f_0(-\Delta)(\Delta\varphi - 2\Delta\psi) \\ = -\frac{\kappa^2}{4} \left[f_{RT1}(-\Delta) + 3\kappa^2 \beta_{RT2} \ln(-\Delta/\mu^2) \right] \rho, \quad (49)\end{aligned}$$

where we traded \square for $-\Delta$ since the metric is static.

To solve (48) and (49) we perform the loop expansion of the potentials,

$$\varphi = \varphi^{(0)} + \varphi^{(1)} + \mathcal{O}(\hbar^2), \quad (50)$$

$$\psi = \psi^{(0)} + \psi^{(1)} + \mathcal{O}(\hbar^2), \quad (51)$$

where $\varphi^{(l)}$ and $\psi^{(l)}$ are of the order $\mathcal{O}(\hbar^l)$. Since $\beta_i = \mathcal{O}(\hbar)$, we get the equations at zero and first orders in \hbar in the form

$$\Delta\varphi^{(0)} = \Delta\psi^{(0)} = \frac{\kappa^2 M}{4} \delta(\mathbf{r}), \quad (52)$$

$$\begin{aligned}\Delta\psi^{(1)} = \frac{\kappa^2 \beta_2}{3} \ln\left(-\frac{\Delta}{\mu^2}\right) \Delta^2(\varphi^{(0)} + \psi^{(0)}) \\ - \frac{\kappa^2 \beta_0}{3} \ln\left(-\frac{\Delta}{\mu^2}\right) \Delta^2(\varphi^{(0)} - 2\psi^{(0)}) \\ + \frac{\kappa^4 M}{4} (-\beta_{RT1} + \beta_{RT2}) \ln\left(-\frac{\Delta}{\mu^2}\right) \Delta\delta(\mathbf{r}), \quad (53)\end{aligned}$$

$$\begin{aligned}\Delta\varphi^{(1)} - 2\Delta\psi^{(1)} = \kappa^2 \beta_0 \ln\left(-\frac{\Delta}{\mu^2}\right) \Delta^2(\varphi^{(0)} - 2\psi^{(0)}) \\ + \frac{\kappa^4 M}{4} (\beta_{RT1} - 3\beta_{RT2}) \ln\left(-\frac{\Delta}{\mu^2}\right) \Delta\delta(\mathbf{r}). \quad (54)\end{aligned}$$

In the three-dimensional Fourier space, we obtain the following solutions for the transformed potentials:

$$\begin{aligned}\varphi^{(0)}(k) &= \psi^{(0)}(k) = -\frac{\kappa^2 M}{4k^2}, \\ \varphi^{(1)}(k) &= \frac{\kappa^4 M}{4} \left(\frac{4}{3} \beta_2 - \frac{1}{3} \beta_0 - \beta_{RT1} - \beta_{RT2} \right) \ln \left(\frac{k^2}{\mu^2} \right), \\ \psi^{(1)}(k) &= \frac{\kappa^4 M}{4} \left(\frac{2}{3} \beta_2 + \frac{1}{3} \beta_0 - \beta_{RT1} + \beta_{RT2} \right) \ln \left(\frac{k^2}{\mu^2} \right),\end{aligned}\quad (55)$$

where $k = |\mathbf{k}|$. Next, using the following inverse Fourier transforms,

$$\begin{aligned}\int \frac{d^3 k}{(2\pi)^3} e^{-i\mathbf{k}\cdot\mathbf{r}} \frac{1}{k^2} &= \frac{1}{4\pi r}, \\ \int \frac{d^3 k}{(2\pi)^3} e^{-i\mathbf{k}\cdot\mathbf{r}} \ln \left(\frac{k^2}{\mu^2} \right) &= -\frac{1}{2\pi r^3} \quad (r \neq 0),\end{aligned}\quad (56)$$

we arrive at the general results

$$\varphi(r) = -\frac{\kappa^2 M}{16\pi r} - \left(\frac{4}{3} \beta_2 - \frac{1}{3} \beta_0 - \beta_{RT1} - \beta_{RT2} \right) \frac{\kappa^4 M}{8\pi r^3}, \quad (57)$$

$$\psi(r) = -\frac{\kappa^2 M}{16\pi r} - \left(\frac{2}{3} \beta_2 + \frac{1}{3} \beta_0 - \beta_{RT1} + \beta_{RT2} \right) \frac{\kappa^4 M}{8\pi r^3}. \quad (58)$$

We do not write the explicit expression for the potentials in terms of the parameters $\gamma_1, \dots, \gamma_6$, γ_0 and λ since they are too cumbersome and not very illuminating. At the same time, it is easy to see, using Eqs. (37)–(38), that the obtained potentials depend on the parametrization and gauge-fixing parameters. On the other hand, the potentials $\varphi(r)$ and $\psi(r)$ do not depend on the parametrization of the background field (42). Indeed, the dependence on the parametrization of the quantum fields and the gauge-fixing should be expected, because the effective equations of motion are gauge and parametrization dependent [28].

In the standard $g_{\mu\nu} + \phi_{\mu\nu}$ parametrization limit, $\gamma_1 \rightarrow 1$, $\gamma_2, \dots, \gamma_6 \rightarrow 0$ and $\gamma_0 \rightarrow 0$. Then, Eqs. (57) and (58) become

$$\varphi(r) = -\frac{GM}{r} \left(1 + \frac{61}{60} \frac{G}{\pi r^2} \right), \quad (59)$$

$$\psi(r) = -\frac{GM}{r} \left(1 + \frac{23}{60} \frac{G}{\pi r^2} \right). \quad (60)$$

Formula (59) matches the result presented in [11] for the h_{00} component of the background metric evaluated in the standard parametrization with the minimal DeWitt gauge.

4 The motion of a test particle

Let us now follow the main idea of Ref. [11] that even though the loop corrections to the gravitational field generated by a point-like mass is parametrization and gauge-fixing dependent, the physical observables should be invariant. As an

example, consider the acceleration of a test particle moving in the gravitational field described in the previous section. The key observation is that the test particle also couples with the quantum metric and, as a consequence of this, its geodesic equation receives quantum corrections. Thus, the two types of quantum corrections are supposed to combine into the invariant result. Finally, in the non-relativistic limit, we expect to meet invariant quantum corrections to Newton's law, $m\mathbf{a} = \mathbf{F}$.

The classical action for a test particle is given by equation (4), but with another mass,

$$S_m = -m \int \sqrt{g_{\mu\nu} dx^\mu dx^\nu}, \quad (61)$$

and coordinates $x^\mu = (t, \mathbf{r})$. We assume that $m \ll M$, such that we can neglect contribution of the small mass m to the potentials (57) and (58). The one-loop corrections to (61) can be obtained from (40) by the substitution $T^{\mu\nu} \rightarrow T^{\mu\nu} + T_m^{\mu\nu}$ [11], where

$$T_m^{\mu\nu} = m \int ds \delta(y - x(s)) u^\mu u^\nu. \quad (62)$$

Applying this procedure yields

$$\begin{aligned}\bar{\Gamma}_m^{(1)} &= \int d^4 x \sqrt{-g} \left\{ \kappa^2 \beta_{RT1} R_{\mu\nu} \ln \left(\frac{\square}{\mu^2} \right) T_m^{\mu\nu} \right. \\ &\quad \left. - \frac{1}{2} \kappa^2 \beta_{RT2} R \ln \left(\frac{\square}{\mu^2} \right) T_m - \kappa^4 \beta_{TT} T \ln \left(\frac{\square}{\mu^2} \right) T_m \right\}.\end{aligned}\quad (63)$$

The total action for the test particle is $\Gamma_m = S_m + \bar{\Gamma}_m^{(1)}$. Taking the functional derivative with respect to x_μ , we find

$$\frac{1}{m} \frac{\delta \Gamma_m}{\delta x_\mu} = - \left(\frac{d^2 x^\mu}{ds^2} + \Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} \right) + \frac{1}{m} \frac{\delta \bar{\Gamma}_m^{(1)}}{\delta x_\mu} = 0, \quad (64)$$

where $\Gamma_{\alpha\beta}^\mu$ is the Christoffel symbol. In the non-relativistic limit, this equation boils down to

$$\mathbf{a} - \frac{1}{m} \frac{\delta \bar{\Gamma}_m^{(1)}}{\delta \mathbf{r}} = -\nabla\varphi, \quad \text{where} \quad \mathbf{a} = \frac{d^2 \mathbf{r}}{dt^2} \quad (65)$$

is the particle acceleration.

To evaluate the functional derivative in (65), all the metric functions in Eq. (63) should be taken at the order $\mathcal{O}(\hbar^0)$, owing to the one-loop approximation and the fact $\beta_i \sim \mathcal{O}(\hbar)$. On top of that, in the non-relativistic limit we can take $s = t$, $u^\mu = (1, 0)$. Thus, using Eq. (47) for the source energy-momentum tensor, we get for (63), in the weak-field approximation,

$$\begin{aligned}\bar{\Gamma}_m^{(1)} &= m \int dt \left\{ \kappa^2 \beta_{RT1} \ln \left(\frac{-\Delta}{\mu^2} \right) R_{00}^{(0)}(\mathbf{r}(t)) \right. \\ &\quad \left. - \frac{1}{2} \kappa^2 \beta_{RT2} \ln \left(\frac{-\Delta}{\mu^2} \right) R^{(0)}(\mathbf{r}(t)) \right. \\ &\quad \left. - \kappa^4 \beta_{TT} M \ln \left(\frac{-\Delta}{\mu^2} \right) \delta(\mathbf{r}(t)) \right\}.\end{aligned}\quad (66)$$

The $\mathcal{O}(\hbar^0)$ curvatures can be evaluated through the relations for the 00-component of the Ricci tensor and for the scalar curvature and also using Eq. (52),

$$R_{00}^{(0)}(\mathbf{r}(t)) = \Delta\varphi^{(0)} = \frac{\kappa^2 M}{4} \delta(\mathbf{r}(t)),$$

$$R^{(0)}(\mathbf{r}(t)) = 2(\Delta\varphi^{(0)} - 2\Delta\psi^{(0)}) = -\frac{\kappa^2 M}{2} \delta(\mathbf{r}(t)). \quad (67)$$

Finally, using the Eq. (56) in the coordinate space,

$$\ln\left(-\frac{\Delta}{\mu^2}\right)\delta(\mathbf{r}) = -\frac{1}{2\pi r^3}, \quad (68)$$

we find

$$\bar{\Gamma}_m^{(1)} = -\frac{\kappa^4 m M}{8\pi} (-\beta_{RT1} - \beta_{RT2} + 4\beta_{TT}) \int \frac{dt}{r^3(t)}, \quad (69)$$

implying that

$$\frac{1}{m} \frac{\delta \bar{\Gamma}_m^{(1)}}{\delta \mathbf{r}} = -\frac{\kappa^4 M}{8\pi} (-\beta_{RT1} - \beta_{RT2} + 4\beta_{TT}) \nabla \left(\frac{1}{r^3} \right). \quad (70)$$

Plugging this equation into (65) and using Eq. (57), we get

$$\mathbf{a} = -\nabla \left[-\frac{\kappa^2 M}{16\pi r} - \left(\frac{4}{3}\beta_2 - \frac{1}{3}\beta_0 - 2\beta_{RT1} - 2\beta_{RT2} + 4\beta_{TT} \right) \frac{\kappa^4 M}{8\pi r^3} \right]. \quad (71)$$

The quantum corrected Newtonian potential is defined through the relation [11]

$$\mathbf{a} \equiv -\nabla V. \quad (72)$$

Thus, (71) and (72) give

$$V(r) = -\frac{\kappa^2 M}{16\pi r} - \left(\frac{4}{3}\beta_2 - \frac{1}{3}\beta_0 - 2\beta_{RT1} - 2\beta_{RT2} + 4\beta_{TT} \right) \frac{\kappa^4 M}{8\pi r^3}. \quad (73)$$

Finally, using Eqs. (37)–(38) we get

$$V(r) = -\frac{GM}{r} \left(1 + \frac{17}{20} \frac{G}{\pi r^2} \right), \quad (74)$$

which does not depend on parametrization or gauge parameters. Let us note that the cancellation of the gauge and parametrization dependencies in this expression looks rather impressive, taking the (somehow scaring) form of the expressions in Appendix A.

The coefficient 17/20 does not agree with the value presented in Ref. [11]. This discordance is due to a missing factor of four in formula (35) of [11], as we explain in the Appendix B. It is remarkable that the complicated calculation in this work has only this small mistake and is otherwise correct.

5 General proof of gauge independence

We have evaluated the coefficients β_i explicitly in the most general parametrization for the quantum field and the particular case of massive point-like masses. However, given Eq. (36), we provided an expression for the Newtonian potential (73) independently on the explicit values of the β_i . Therefore, to prove that the potential (73) does not depend on the gauge-fixing and parametrization choices, it is enough to prove that the following combination,

$$\beta_{\text{inv}} = \frac{4}{3}\beta_2 - \frac{1}{3}\beta_0 - 2\beta_{RT1} - 2\beta_{RT2} + 4\beta_{TT}, \quad (75)$$

is a gauge and parametrization invariant quantity. For this, we follow [23, 34, 35], i.e., we employ the general statement about the gauge-fixing and parametrization independence of the on-shell effective action. In particular, the difference between the divergences of two versions of the one-loop effective action, evaluated using different gauge and parametrizations α_i and α_0 , is proportional to the classical equations of motion,

$$\delta \bar{\Gamma}_{div}^{(1)} = \bar{\Gamma}_{div}^{(1)}(\alpha_i) - \bar{\Gamma}_{div}^{(1)}(\alpha_0) = -\frac{\mu^{n-4}}{(4\pi)^2(n-4)} \times \int d^n x \sqrt{-g} \varepsilon^{\mu\nu} f_{\mu\nu},$$

$$\varepsilon^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R - \frac{\kappa^2}{2} T^{\mu\nu}. \quad (76)$$

As the divergences are local and covariant quantities with mass dimension four, the tensor function $f_{\mu\nu}$ has the following general structure

$$f_{\mu\nu} = b_1 R_{\mu\nu} + b_2 R g_{\mu\nu} + \kappa^2 b_3 T_{\mu\nu} + \kappa^2 b_4 T g_{\mu\nu}, \quad (77)$$

where the parameters $b_{1,2,3,4}$ depend on the choice of the gauge and parametrization parameters α_i . Thus, replacing (77) in (76) and using the definition (29) we get:

$$\delta \bar{\Gamma}_{div}^{(1)} = -\frac{\mu^{n-4}}{(4\pi)^2(n-4)} \int d^n x \sqrt{-g} \times \left\{ b_1 R_{\mu\nu}^2 - \left(\frac{1}{2} b_1 + b_2 \right) R^2 + \kappa^2 \left(b_3 - \frac{1}{2} b_1 \right) R_{\mu\nu} T^{\mu\nu} - \kappa^2 \left(\frac{1}{2} b_2 + \frac{1}{2} b_3 + b_4 \right) R T - \kappa^4 \left(\frac{1}{2} b_3 + \frac{1}{2} b_4 \right) T^2 \right\}. \quad (78)$$

So, under a gauge and/or parametrization transformation, the coefficients in (31) transform according to:

$$c_1 \rightarrow c_1, \quad c_2 \rightarrow c_2 + b_1, \quad c_3 \rightarrow c_3 - \left(\frac{b_1}{2} + b_2 \right),$$

$$c_4 \rightarrow c_4 + \left(b_3 - \frac{b_1}{2} \right),$$

$$c_5 \rightarrow c_5 - \frac{1}{2}(b_2 + b_3 + 2b_4), \quad c_6 \rightarrow c_6 - \frac{1}{2}(b_3 + b_4). \quad (79)$$

It is a simple exercise to check that, besides c_1 , there is the following gauge- and parametrization-independent combination:

$$c_{\text{inv}} = c_2 + c_3 + c_4 - 2c_5 + 4c_6. \quad (80)$$

In particular, in the notations of (36), the formula (80) implies that (75) is truly gauge and parametrization invariant. This also means the physical results derived on the basis of usual effective action and for the unique Vilkovisky–DeWitt effective action [28] will be the same. The reader can consult [26] for the first calculation in quantum gravity in this formalism and the more recent Ref. [24] for the explicit verification of the parametrization independence, including the conformal mode (5).

In fact, the generalization of the calculations done in [24] to include the massive point particles is straightforward. Without entering in the full details, the result for the one-loop divergences in the theory (3), using the Vilkovisky–DeWitt definition for the effective action and the general parametrization (5), has the same form as Eq. (31), but now with the following gauge and parametrization independent coefficients,

$$\begin{aligned} c_1 &= \frac{53}{45}, & c_2 &= -\frac{61}{90}, & c_3 &= \frac{25}{36}, & c_4 &= -\frac{3}{2}, \\ c_5 &= \frac{1}{2}, & c_6 &= -\frac{5}{32}. \end{aligned} \quad (81)$$

Using (38) and (73), it is easy to verify that (81) provides the same Newtonian potential of Eq. (74). Thus, the cancellation observed in (74) is a necessary feature of effective quantum gravity, especially in the case when only metric is a quantum field, while the macroscopic massive bodies are treated as classical sources, according to the arguments of [11, 13].

As the massive sources are regarded classical, one can certainly expect the parametrization independence to hold for other configurations of matter, including non-point mass distributions, such as, e.g., dark matter.

6 Universality of the effective field theory result

The effective field theory approach cannot be taken lightly, as we cannot write down all possible action operators without having any guiding principle. Indeed, it is well known that any higher derivative theory has a number of ghost-like states depending on the number of derivatives that catastrophically make the vacuum decay instantaneously in ghost and normal particles. This is a trivial consequence of the energy conservation since ghosts carry negative energy [41]. Therefore, EFTs make sense and are well defined only if they turn out to be the low-energy approximation, or large distance limit, of more fundamental theories consistent with unitarity (perturbative), and (super-)renormalizability or finiteness.

One example of finite and unitarity theory is string theory, but we here focus on ultraviolet complete gravitational theories in the quantum field theory framework. In the latter case, we have two classes of well-defined theories: higher derivative theories *à la* Lee–Wick [42–46] or more general higher derivative theories [47], in which unitarity can be formally achieved by means of the Anselmi–Piva prescription, which is an extension of conventional Wick rotation, operational for the higher derivative models [48–50]; or nonlocal theories without ghost-like perturbative degrees of freedom [51–55]. In Lee–Wick quantum gravity, the ghost particles may only have complex mass square [44, 45], while in general higher derivative theories [47] we also have real ghosts, but in both cases, such states do not appear as asymptotic states. Indeed, such homogeneous solutions of the linear equations of motion can be removed from the spectrum of the theory by hand and, most importantly, they are not created again in the loop amplitudes once the Anselmi–Piva prescription is implemented. This is very similar to what happens with the BRST ghosts that appear because of the quantum gauge invariance. Indeed, we can safely fix the number of external BRST ghosts to zero, because in the loop amplitudes, they are exactly canceled by the non-physical polarization states of the gauge bosons. Similarly, in higher derivative theories, we can fix the number of external ghosts to zero because they are not regenerated in the loop amplitudes at any perturbative order whether the prescription [48–50] is implemented.

Differently, in nonlocal quantum gravity [51–55], the perturbative degrees of freedom are the same as the local theory [60, 61] and the convergence of the loop amplitudes is achieved by introducing an exponential (but asymptotically polynomial [51–53]) form factor that has no poles in the whole complex plane. At the quantum level, the loop amplitudes are computed in the Euclidean signature, and afterwards, the physical amplitudes are obtained using an analytic continuation of the external energies from the Euclidean space to Minkowski spacetime [56, 57]. A unified nonlocal theory of all fundamental interactions with the aforementioned properties of the purely gravitational theories has been recently proposed in [62, 63]. In the ultraviolet regime all nonlocal gravitational theories (or local with more than ten derivatives in four dimensions) are asymptotically free [58] or finite [54, 59]. Theories with a number of derivatives from six to eight are super-renormalizable but, to prove asymptotic freedom, quantitative analysis and explicit computations are needed because there are divergences also at two and/or three loops. The last theory which deserves special attention is the four derivatives or Stelle’s theory [66].

Said this, it is very interesting to investigate the infrared properties of any of the theories described above. In particular, in this work, we are interested in the leading correction to the Newtonian potential. Such correction is strictly related to the analytic properties of the one-loop amplitude regardless

of the renormalizability or finiteness of the theory. Indeed, the analyticity only depends on the Landau singularities of the one-loop amplitude, namely when the denominators of the amplitude are zero for the same value of the external energy. This property depends only on the low-energy spectrum of the theory, and, hence, the contribution of the massive states is sub-dominant. Therefore, the main corrections to the quantum effective action at large distance come only from the massless states.

As an example we can consider a nonlocal scalar theory with an exponential form factor [64]. This theory is ultraviolet finite, but the one-loop amplitude at low energy, namely when $p^2/\Lambda^2 \lesssim 1$ (here Λ is the non-locality scale), shows up exactly the same logarithmic non-analyticity of the local theory. This result is understandable whether we identify Λ with the cut-off of the local theory.

Let us expand on the case of the scalar model. The nonlocal Lagrangian that we would like to consider reads:

$$\mathcal{L}_\phi = -\frac{1}{2}\phi e^{H(-\square/\Lambda^2)} (\square + m^2) \phi - \frac{\lambda}{3!}\phi^3, \quad (82)$$

where $H(z)$ is an analytic function without poles in the complex plane at finite distance. The propagator of the scalar field is:

$$G(k) = \frac{i e^{-H(k)}}{k^2 - m^2 + i\epsilon}. \quad (83)$$

The one-loop amplitude turns out to be:

$$\mathcal{M} = \frac{\lambda^2}{32\pi^4} \int_{-\infty}^{+\infty} d^4 k_E \frac{e^{-H(k_E)} e^{-H(k_E - p_E)}}{(k_E^2 + m^2) [(k_E + p_E)^2 + m^2]}, \quad (84)$$

where k_E and p_E are respectively the internal and external purely imaginary four-momenta. For the simplest non-locality, $H = \square = -k^2/\Lambda^2 = k_E^2/\Lambda^2$, the above amplitude (84) turns into a simple Gaussian integral. The explicit computation was done for the first time in [64] and more recently in [65]. The result after the analytic continuation from purely imaginary to real external energy reads:

$$\mathcal{M} = -\frac{\lambda^2}{16\pi^2} \int_0^{1/2} dx \operatorname{Ei} \left[-\left(\frac{m^2 - p^2 x(1-x)}{x\Lambda^2} \right) \right], \quad (85)$$

where $\operatorname{Ei}(z)$ is the exponential integral. Notice that the result is finite without need of any regularization.

We now consider the low energy limit $p^2/\Lambda^2 \lesssim 1$. In order to investigate such limit, we remind the Taylor expansion of the exponential integral $\operatorname{Ei}(z)$,

$$\operatorname{Ei}(z) = \gamma_E + \ln z + z + \frac{z^2}{4} + \dots \quad (86)$$

Using the above expansion in (85) we get:

$$\mathcal{M} \simeq -\frac{\lambda^2}{32\pi^2} \int_0^1 dx \ln \left(\frac{m^2 - p^2 x(1-x)}{\Lambda^2} \right). \quad (87)$$

Therefore, (85) at low energy resembles the result in the local quantum field theory up to proper identification of the non-locality scale with the cut-off of the local renormalizable theory, which we give here for completeness:

$$\Lambda^2 = \Lambda_{\text{cut-off}} e^{\gamma_E + 1 + \ln 2}. \quad (88)$$

As another example, let us now consider a nonlocal theory without perturbative degrees of freedom around the usual trivial background $\phi_B = 0$. The theory reads:

$$\mathcal{L}_\phi = -\frac{1}{2}\phi e^{H(-\square/\Lambda^2)} m^2 \phi - \frac{\lambda}{3!}\phi^3. \quad (89)$$

Clearly, at the zero-order in λ the perturbative solution of the free-theory equation of motion is $\phi = 0$, namely

$$e^{H(-\square/\Lambda^2)} m^2 \phi = 0 \implies \phi = 0. \quad (90)$$

Therefore, there are no perturbative degrees of freedom propagating around the trivial background. Equivalently, the propagator in momentum space, which can be obtained from (83) removing k^2 from the denominator, has no poles in the whole complex plane at finite energy,

$$G(k) = -\frac{i}{m^2} e^{-H(k^2/\Lambda^2)}, \quad (91)$$

consistently with the classical solution in (90). Now we are ready to compute one-loop bubble diagram for the theory (89). The one-loop amplitude reads:

$$\mathcal{M} = -\frac{\lambda^2}{32\pi^4} \int_{-\infty}^{+\infty} d^4 k_E \frac{i e^{-H(k_E)}}{m^2} \frac{i e^{-H(k_E - p_E)}}{m^2}. \quad (92)$$

Again for the case of a Gaussian non-locality, namely $H = \square + m^2 = (-k^2 + m^2)/\Lambda^2 = (k_E^2 + m^2)/\Lambda^2$, the amplitude (92) simplifies to:

$$\begin{aligned} \mathcal{M} &= -\frac{\lambda^2}{32\pi^4} \int_{-\infty}^{+\infty} d^4 k_E \frac{i e^{-\frac{k_E^2 + m^2}{\Lambda^2}}}{m^2} \frac{i e^{-\frac{(k_E - p_E)^2 + m^2}{\Lambda^2}}}{m^2} \\ &= \frac{\lambda^2}{128\pi^2} e^{-\frac{1}{2\Lambda^2}(-p^2 + (2m)^2)}, \end{aligned} \quad (93)$$

which is analytic in the whole complex plane according to the absence of perturbative degrees of freedom in the theory (89).

We conclude that the logarithm correction to the quantum effective action is universally related to the low energy spectrum of the theory regardless of the ultraviolet completion of the theory.

The same argument applies to a general nonlocal or higher derivative theory, being finite or (super-)renormalizable, with the graviton the only massless state in the spectrum. The action for a general local higher derivative gravitational theory reads:

$$S_{\text{HD}} = \int d^D x \sqrt{-g} \left[\kappa^{-2} R + \sum_{n=0}^N (\omega_{0,n} R \square^n R + \omega_{2,n} R_{\mu\nu} \square^n R^{\mu\nu}) \right]$$

$$+V\left(\mathcal{O}(\text{Riem}^3)\right)\Bigg], \quad (94)$$

where the last term plays the role of a potential because it is at least cubic in the Riemann tensor and, then, it does not contribute to the propagator around the Minkowski's background. Moreover, the potential contains at most $2N + 4$ derivatives in order to not spoil the super-renormalizability of the theory. However, the presence of $V(\mathcal{O}(\text{Riem}^3))$ is crucial to making the theory finite [53,54]. The particle spectrum of the higher derivative action (94) contains a massless graviton and a finite number of massive states (including complex and real ghosts) whose masses are defined in terms of the constants κ , $\omega_{0,n}$, and $\omega_{2,n}$. As said above, the theory (94) can be finite, namely all the loop amplitudes are convergent. However, at an energy scale much smaller than the mass of all the massive states, the one-loop quantum effective action has the same form as (40) for a proper identification (which means up to an overall dimensionless rescaling) of the renormalization group invariant scale μ in (40) with the mass scale that is implicitly hidden in the parameters $\omega_{0,n}$ and $\omega_{2,n}$. In other words, at low energy, the higher derivative operators play the role of a higher derivative regularization. Once again, this is not surprising because they are related to the analytic properties of the one-loop quantum effective action that only depends on the Landau singularities of the amplitude. Moreover, in the low energy limit the contribution of the massive states is sub-leading with respect to the massless graviton.

7 Conclusions and discussions

We have reconsidered the arguments for taking into account only the gravitational loops in effective quantum gravity. The physical reasons and, in particular, the analogy with the Casimir force show that the massive sources should not be quantized. Taking this into account, it is well-known that the numerical effects of quantum corrections are too small to be measured in the laboratory. However, this does not make irrelevant the consistent derivation of the quantum corrections and the discussion of their (non)universality.

The main part of the work reports on a calculation similar to the previously done in [11], but in an arbitrary parametrization of quantum perturbation of the metric. As a result, we have found that the effective quantum gravity correction to Newton's gravitational potential is completely independent of the choice of the parametrization. Together with the mentioned qualitative arguments and the gauge-fixing independence established in [11], our result ensures that the final outcome (73) is correct. In principle, this result can be generalized for more complicated distributions of mass. In this respect, the more general proof of invariance given in Sect. 5

is relevant, as it provides a good expectation to obtain sound results.

Furthermore, we discussed in Sect. 6 the universality and independence of the leading correction to Newton's potential in the effective field theories resulting in the low-energy limit of ultraviolet complete theories of quantum gravity in the quantum field theory framework. In particular, we showed that the leading quantum correction to Newton's potential is strictly related to the low-energy spectrum of the fundamental theory rather than the details of the ultraviolet complete quantum gravity theory, in accordance with the general effective quantum field theory approach. Indeed, the logarithm quantum correction is related to the Landau singularities and the analytic properties of the one-loop amplitude, which only depend on the low-energy spectrum of the classical theory. Therefore, not all the quantum effective theories produce the same result. The latter statement has been shown explicitly with simple examples.

Taking the parametrization and gauge invariance, the direct relation to the beta functions, and the fact that the quantum corrections are the same as in the usual and Vilkovisky–DeWitt versions of the effective actions, the main result for the quantum corrected potential can be compared to the invariant version of the renormalization group improved classical action [36,37]. In the present framework, when dealing with the Newtonian interaction between two massive particles, the interpretation of the renormalization group scale is straightforward. This interpretation is much more subtle and complicated in other physical situations, e.g., in Cosmology [38], where some specially designed procedures can be applied [39,40], but still do not guarantee unique interpretation of the scale. Thus, it would be interesting to compare the two ways of scale identification in the cases like the one considered above, with a well-defined procedure [11].

In conclusion, let us note that the main approximation in effective quantum gravity is supposed to hold between the Planck scale in the UV and the Hubble scale in the IR. In particular, this interval includes all the physically and astrophysically interesting high-energy gravitational phenomena related to early cosmology and black hole physics. Thus, there still is a possibility to find applications to the results based on correctly defined quantum gravity corrections to classical gravity.

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Data Availability Statement The manuscript has associated data in a data repository. [Authors' comment: All data included in this manuscript are available upon request by contacting with the corresponding author.]

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Appendix A

The explicit expressions for the coefficients in (31) were obtained using WOLFRAM MATHEMATICA [67] and the auxiliary tensor algebra package XACT [68, 69]. For the sake of completeness, we present the list of the coefficients used in Eq. (32):

$$\begin{aligned}
 \xi_2 = & 16\lambda^2\gamma_0^2\gamma_1^4 + 8\lambda\gamma_0\gamma_1^3\gamma_3 \\
 & - 128\lambda^2\gamma_0^2\gamma_3^2 - 32\lambda\gamma_0\gamma_1\gamma_3^2 - \gamma_1^2\gamma_3^2 \\
 & - 128\lambda\gamma_0\gamma_2\gamma_3^2 - 16\gamma_1\gamma_2\gamma_3^2 \\
 & - 32\gamma_2^2\gamma_3^2 + 16\lambda\gamma_0\gamma_1^3\gamma_5 + 4\gamma_1^2\gamma_3\gamma_5 \\
 & + 4\gamma_1^2\gamma_5^2, \\
 \xi_3 = & 3584\lambda^4\gamma_0^4\gamma_1^4 + 1024\lambda^3\gamma_0^3\gamma_1^5 + 64\lambda^2\gamma_0^2\gamma_1^6 \\
 & - 4\lambda\gamma_0\gamma_1^7 + 4096\lambda^3\gamma_0^3\gamma_1^4\gamma_2 + 512\lambda^2\gamma_0^2\gamma_1^5\gamma_2 \\
 & - 48\lambda\gamma_0\gamma_1^6\gamma_2 + 1024\lambda^2\gamma_0^2\gamma_1^4\gamma_2^2 \\
 & - 192\lambda\gamma_0\gamma_1^5\gamma_2^2 - 256\lambda\gamma_0\gamma_1^4\gamma_2^3 \\
 & - 55296\lambda^4\gamma_0^4\gamma_1^2\gamma_3 \\
 & - 26112\lambda^3\gamma_0^3\gamma_1^3\gamma_3 - 4736\lambda^2\gamma_0^2\gamma_1^4\gamma_3 \\
 & - 392\lambda\gamma_0\gamma_1^5\gamma_3 - 14\gamma_1^6\gamma_3 - 110592\lambda^3\gamma_0^3\gamma_1^2\gamma_2\gamma_3 \\
 & - 39936\lambda^2\gamma_0^2\gamma_1^3\gamma_2\gamma_3 \\
 & - 4928\lambda\gamma_0\gamma_1^4\gamma_2\gamma_3 - 220\gamma_1^5\gamma_2\gamma_3 \\
 & - 82944\lambda^2\gamma_0^2\gamma_1^2\gamma_2^2\gamma_3 \\
 & - 20352\lambda\gamma_0\gamma_1^3\gamma_2^2\gamma_3 - 1304\gamma_1^4\gamma_2^2\gamma_3 \\
 & - 27648\lambda\gamma_0\gamma_1^2\gamma_2^3\gamma_3 \\
 & - 3456\gamma_1^3\gamma_2^3\gamma_3 \\
 & - 3456\gamma_1^2\gamma_2^4\gamma_3 + 30720\lambda^4\gamma_0^4\gamma_3^2 \\
 & + 15360\lambda^3\gamma_0^3\gamma_1\gamma_3^2 + 3072\lambda^2\gamma_0^2\gamma_1^2\gamma_3^2 \\
 & + 288\lambda\gamma_0\gamma_1^3\gamma_3^2 \\
 & + 12\gamma_1^4\gamma_3^2 + 61440\lambda^3\gamma_0^3\gamma_2\gamma_3^2
 \end{aligned} \tag{95}$$

$$\begin{aligned}
 & + 23040\lambda^2\gamma_0^2\gamma_1\gamma_2\gamma_3^2 + 3072\lambda\gamma_0\gamma_1^2\gamma_2\gamma_3^2 \\
 & + 144\gamma_1^3\gamma_2\gamma_3^2 \\
 & + 46080\lambda^2\gamma_0^2\gamma_2^2\gamma_3^2 + 11520\lambda\gamma_0\gamma_1\gamma_2^2\gamma_3^2 \\
 & + 768\gamma_1^2\gamma_2^2\gamma_3^2 + 15360\lambda\gamma_0\gamma_2^3\gamma_3^2 \\
 & + 1920\gamma_1\gamma_2^3\gamma_3^2 \\
 & + 1920\gamma_2^4\gamma_3^2 - 221184\lambda^4\gamma_0^4\gamma_1^2\gamma_4 \\
 & - 110592\lambda^3\gamma_0^3\gamma_1^3\gamma_4 - 20480\lambda^2\gamma_0^2\gamma_1^4\gamma_4 \\
 & - 1664\lambda\gamma_0\gamma_1^5\gamma_4 \\
 & - 56\gamma_1^6\gamma_4 - 442368\lambda^3\gamma_0^3\gamma_1^2\gamma_2\gamma_4 \\
 & - 165888\lambda^2\gamma_0^2\gamma_1^3\gamma_2\gamma_4 - 20480\lambda\gamma_0\gamma_1^4\gamma_2\gamma_4 \\
 & - 880\gamma_1^5\gamma_2\gamma_4 - 331776\lambda^2\gamma_0^2\gamma_1^2\gamma_2^2\gamma_4 \\
 & - 82944\lambda\gamma_0\gamma_1^3\gamma_2^2\gamma_4 - 5216\gamma_1^4\gamma_2^2\gamma_4 + 768\lambda^2\gamma_0^2\gamma_1^2\gamma_2^2\gamma_5^2 \\
 & - 110592\lambda\gamma_0\gamma_1^2\gamma_2^3\gamma_4 - 13824\gamma_1^3\gamma_2^3\gamma_4 \\
 & - 13824\gamma_1^2\gamma_2^4\gamma_4 + 442368\lambda^4\gamma_0^4\gamma_3\gamma_4 \\
 & + 221184\lambda^3\gamma_0^3\gamma_1\gamma_3\gamma_4 + 41472\lambda^2\gamma_0^2\gamma_1^2\gamma_3\gamma_4 \\
 & + 3456\lambda\gamma_0\gamma_1^3\gamma_3\gamma_4 + 120\gamma_1^4\gamma_3\gamma_4 \\
 & + 884736\lambda^3\gamma_0^3\gamma_2\gamma_3\gamma_4 + 331776\lambda^2\gamma_0^2\gamma_1\gamma_2\gamma_3\gamma_4 \\
 & + 41472\lambda\gamma_0\gamma_1^2\gamma_2\gamma_3\gamma_4 \\
 & + 1728\gamma_1^3\gamma_2\gamma_3\gamma_4 + 663552\lambda^2\gamma_0^2\gamma_2^2\gamma_3\gamma_4 \\
 & + 165888\lambda\gamma_0\gamma_1\gamma_2^2\gamma_3\gamma_4 + 10368\gamma_1^2\gamma_2^2\gamma_3\gamma_4 \\
 & + 221184\lambda\gamma_0\gamma_2^3\gamma_3\gamma_4 \\
 & + 27648\gamma_1\gamma_2^3\gamma_3\gamma_4 + 27648\gamma_2^4\gamma_3\gamma_4 \\
 & + 884736\lambda^4\gamma_0^4\gamma_4^2 \\
 & + 82944\lambda^2\gamma_0^2\gamma_1^2\gamma_4^2 + 6912\lambda\gamma_0\gamma_1^3\gamma_4^2 + 240\gamma_1^4\gamma_4^2 \\
 & + 1769472\lambda^3\gamma_0^3\gamma_2\gamma_4^2 \\
 & + 82944\lambda\gamma_0\gamma_1^2\gamma_2\gamma_4^2 + 3456\gamma_1^3\gamma_2\gamma_4^2 \\
 & + 1327104\lambda^2\gamma_0^2\gamma_2^2\gamma_4^2 + 331776\lambda\gamma_0\gamma_1\gamma_2^2\gamma_4^2 \\
 & + 20736\gamma_1^2\gamma_2^2\gamma_4^2 + 442368\lambda\gamma_0\gamma_2^3\gamma_4^2 \\
 & + 55296\gamma_1\gamma_2^3\gamma_4^2 + 55296\gamma_2^4\gamma_4^2 \\
 & + 3072\lambda^3\gamma_0^3\gamma_1^3\gamma_5 \\
 & + 1024\lambda^2\gamma_0^2\gamma_1^4\gamma_5 + 112\lambda\gamma_0\gamma_1^5\gamma_5 - 2\gamma_1^6\gamma_5 \\
 & + 3072\lambda^2\gamma_0^2\gamma_1^3\gamma_2\gamma_5 + 640\lambda\gamma_0\gamma_1^4\gamma_2\gamma_5 \\
 & - 16\gamma_1^5\gamma_2\gamma_5 \\
 & + 768\lambda\gamma_0\gamma_1^3\gamma_2^2\gamma_5 - 32\gamma_1^4\gamma_2^2\gamma_5 \\
 & + 768\lambda^2\gamma_0^2\gamma_1^2\gamma_2\gamma_3\gamma_5 + 48\gamma_1^4\gamma_4\gamma_5 \\
 & + 192\lambda\gamma_0\gamma_1^3\gamma_3\gamma_5 + 24\gamma_1^4\gamma_3\gamma_5 \\
 & + 768\lambda\gamma_0\gamma_1^2\gamma_2\gamma_3\gamma_5 + 96\gamma_1^3\gamma_2\gamma_3\gamma_5 \\
 & + 192\gamma_1^2\gamma_2^2\gamma_3\gamma_5 \\
 & + 192\lambda\gamma_0\gamma_1^3\gamma_5^2 + 36\gamma_1^4\gamma_5^2
 \end{aligned}$$

$$\begin{aligned}
& + 768\lambda\gamma_0\gamma_1^2\gamma_2\gamma_5^2 + 96\gamma_1^3\gamma_2\gamma_5^2 \\
& + 192\gamma_1^2\gamma_2^2\gamma_5^2 + 1024\lambda^2\gamma_0^2\gamma_1^4\gamma_6 \\
& + 256\lambda\gamma_0\gamma_1^5\gamma_6 - 8\gamma_1^6\gamma_6 \\
& + 1024\lambda\gamma_0\gamma_1^4\gamma_2\gamma_6 - 64\gamma_1^5\gamma_2\gamma_6 \\
& - 128\gamma_1^4\gamma_2^2\gamma_6 + 48\gamma_1^4\gamma_3\gamma_6 \\
& + 192\gamma_1^4\gamma_4\gamma_6 + 192\gamma_1^4\gamma_5\gamma_6 \\
& + 384\gamma_1^4\gamma_6^2 + 442368\lambda^3\gamma_0^3\gamma_1\gamma_4^2 \\
& + 663552\lambda^2\gamma_0^2\gamma_1\gamma_2\gamma_4^2, \\
\xi_4 = & 2\lambda\gamma_0\gamma_1^5 - 16\lambda^2\gamma_0^2\gamma_1^4 + 8\lambda\gamma_0\gamma_1^4\gamma_2 \\
& - 32\lambda^2\gamma_0^2\gamma_1^2\gamma_3 - 20\lambda\gamma_0\gamma_1^3\gamma_3 \\
& - 32\lambda\gamma_0\gamma_1^2\gamma_2\gamma_3 \\
& - 8\gamma_1^2\gamma_2^2\gamma_3 + 256\lambda^2\gamma_0^2\gamma_3^2 \\
& + 64\lambda\gamma_0\gamma_1\gamma_3^2 + 2\gamma_1^2\gamma_3^2 + 256\lambda\gamma_0\gamma_2\gamma_3^2 \\
& + 32\gamma_1\gamma_2\gamma_3^2 + 64\gamma_2^2\gamma_3^2 \\
& - 24\lambda\gamma_0\gamma_1^3\gamma_5 \\
& + \gamma_1^4\gamma_5 + 4\gamma_1^3\gamma_2\gamma_5 \\
& - 8\gamma_1^2\gamma_3\gamma_5 - 8\gamma_1^2\gamma_5^2 \\
& - 2\gamma_1^3\gamma_2\gamma_3, \\
\xi_5 = & 2560\lambda^4\gamma_0^4\gamma_1^4 + 128\lambda^3\gamma_0^3\gamma_1^5 - 40\lambda^2\gamma_0^2\gamma_1^6 \\
& + 4\lambda\gamma_0\gamma_1^7 + 512\lambda^3\gamma_0^3\gamma_1^4\gamma_2 \\
& - 320\lambda^2\gamma_0^2\gamma_1^5\gamma_2 \\
& + 48\lambda\gamma_0\gamma_1^6\gamma_2 \\
& - 640\lambda^2\gamma_0^2\gamma_1^4\gamma_2^2 \\
& + 192\lambda\gamma_0\gamma_1^5\gamma_2^2 \\
& + 256\lambda\gamma_0\gamma_1^4\gamma_2^3 + 70656\lambda^4\gamma_0^4\gamma_1^2\gamma_3 \\
& + 37632\lambda^3\gamma_0^3\gamma_1^3\gamma_3 + 6992\lambda^2\gamma_0^2\gamma_1^4\gamma_3 \\
& + 536\lambda\gamma_0\gamma_1^5\gamma_3 + 17\gamma_1^6\gamma_3 + 141312\lambda^3\gamma_0^3\gamma_1^2\gamma_2\gamma_3 \\
& + 54912\lambda^2\gamma_0^2\gamma_1^3\gamma_2\gamma_3 \\
& + 6608\lambda\gamma_0\gamma_1^4\gamma_2\gamma_3 + 268\gamma_1^5\gamma_2\gamma_3 \\
& + 105984\lambda^2\gamma_0^2\gamma_1^2\gamma_2^2\gamma_3 \\
& + 26688\lambda\gamma_0\gamma_1^3\gamma_2^2\gamma_3 + 1604\gamma_1^4\gamma_2^2\gamma_3 \\
& + 35328\lambda\gamma_0\gamma_1^2\gamma_2^3\gamma_3 \\
& + 4320\gamma_1^3\gamma_2^3\gamma_3 \\
& + 4416\gamma_1^2\gamma_2^4\gamma_3 - 159744\lambda^4\gamma_0^4\gamma_3^2 \\
& - 79872\lambda^3\gamma_0^3\gamma_1\gamma_3^2 - 14592\lambda^2\gamma_0^2\gamma_1^2\gamma_3^2 \\
& - 1152\lambda\gamma_0\gamma_1^3\gamma_3^2 \\
& - 36\gamma_1^4\gamma_3^2 - 319488\lambda^3\gamma_0^3\gamma_2\gamma_3^2 \\
& - 119808\lambda^2\gamma_0^2\gamma_1\gamma_2\gamma_3^2 \\
& - 14592\lambda\gamma_0\gamma_1^2\gamma_2\gamma_3^2 \\
& - 576\gamma_1^3\gamma_2\gamma_3^2 - 239616\lambda^2\gamma_0^2\gamma_2^2\gamma_3^2 \\
& - 59904\lambda\gamma_0\gamma_1\gamma_2^2\gamma_3^2 \\
& - 3648\gamma_1^2\gamma_2^2\gamma_3^2 \\
& - 79872\lambda\gamma_0\gamma_2^3\gamma_3^2 - 9984\gamma_1\gamma_2^3\gamma_3^2 \\
& - 9984\gamma_2^4\gamma_3^2 + 258048\lambda^4\gamma_0^4\gamma_1^2\gamma_4 \\
& + 129024\lambda^3\gamma_0^3\gamma_1^3\gamma_4 \\
& + 23744\lambda^2\gamma_0^2\gamma_1^4\gamma_4 + 1904\lambda\gamma_0\gamma_1^5\gamma_4 \\
& + 68\gamma_1^6\gamma_4 + 516096\lambda^3\gamma_0^3\gamma_1^2\gamma_2\gamma_4 \\
& + 193536\lambda^2\gamma_0^2\gamma_1^3\gamma_2\gamma_4 + 23744\lambda\gamma_0\gamma_1^4\gamma_2\gamma_4 \\
& + 1048\gamma_1^5\gamma_2\gamma_4 + 387072\lambda^2\gamma_0^2\gamma_1^2\gamma_2^2\gamma_4 \\
& + 96768\lambda\gamma_0\gamma_1^3\gamma_2^2\gamma_4 + 6128\gamma_1^4\gamma_2^2\gamma_4 \\
& + 129024\lambda\gamma_0\gamma_1^2\gamma_2^3\gamma_4 \\
& + 16128\gamma_1^3\gamma_2^3\gamma_4 \\
& + 16128\gamma_1^2\gamma_2^4\gamma_4 - 884736\lambda^4\gamma_0^4\gamma_3\gamma_4 \\
& - 442368\lambda^3\gamma_0^3\gamma_1\gamma_3\gamma_4 \\
& - 82944\lambda^2\gamma_0^2\gamma_1^2\gamma_3\gamma_4 \\
& - 6912\lambda\gamma_0\gamma_1^3\gamma_3\gamma_4 - 240\gamma_1^4\gamma_3\gamma_4 \\
& - 1769472\lambda^3\gamma_0^3\gamma_2\gamma_3\gamma_4 \\
& - 663552\lambda^2\gamma_0^2\gamma_1\gamma_2\gamma_3\gamma_4 \\
& - 82944\lambda\gamma_0\gamma_1^2\gamma_2\gamma_3\gamma_4 \\
& - 3456\gamma_1^3\gamma_2\gamma_3\gamma_4 - 1327104\lambda^2\gamma_0^2\gamma_2^2\gamma_3\gamma_4 \\
& + 384\lambda\gamma_0\gamma_1^3\gamma_3\gamma_5 \\
& - 331776\lambda\gamma_0\gamma_1\gamma_2^2\gamma_3\gamma_4 \\
& - 20736\gamma_1^2\gamma_2^2\gamma_3\gamma_4 - 442368\lambda\gamma_0\gamma_2^3\gamma_3\gamma_4 \\
& + 1536\lambda^2\gamma_0^2\gamma_1^2\gamma_3\gamma_5 \\
& - 55296\gamma_1\gamma_2^3\gamma_3\gamma_4 \\
& - 55296\gamma_2^4\gamma_3\gamma_4 \\
& - 1769472\lambda^4\gamma_0^4\gamma_4^2 \\
& - 884736\lambda^3\gamma_0^3\gamma_1\gamma_4^2 \\
& - 165888\lambda^2\gamma_0^2\gamma_1^2\gamma_4^2 - 13824\lambda\gamma_0\gamma_1^3\gamma_4^2 \\
& - 480\gamma_1^4\gamma_4^2 \\
& - 3538944\lambda^3\gamma_0^3\gamma_2\gamma_4^2 \\
& - 1327104\lambda^2\gamma_0^2\gamma_1\gamma_2\gamma_4^2 - 165888\lambda\gamma_0\gamma_1^2\gamma_2\gamma_4^2 \\
& - 6912\gamma_1^3\gamma_2\gamma_4^2 \\
& - 2654208\lambda^2\gamma_0^2\gamma_2^2\gamma_4^2 \\
& - 663552\lambda\gamma_0\gamma_1\gamma_2^2\gamma_4^2 - 41472\gamma_1^2\gamma_2^2\gamma_4^2 \\
& - 884736\lambda\gamma_0\gamma_2^3\gamma_4^2 - 110592\gamma_1\gamma_2^3\gamma_4^2 \\
& - 110592\gamma_2^4\gamma_4^2 + 4608\lambda^3\gamma_0^3\gamma_1^3\gamma_5 \\
& + 512\lambda^2\gamma_0^2\gamma_1^4\gamma_5 - 88\lambda\gamma_0\gamma_1^5\gamma_5
\end{aligned}$$

$$\begin{aligned}
& + 2\gamma_1^6 \gamma_5 \\
& + 3840\lambda^2 \gamma_0^2 \gamma_1^3 \gamma_2 \gamma_5 - 256\lambda \gamma_0 \gamma_1^4 \gamma_2 \gamma_5 \\
& + 4\gamma_1^5 \gamma_2 \gamma_5 + 384\lambda \gamma_0 \gamma_1^3 \gamma_2^2 \gamma_5 \\
& - 64\gamma_1^4 \gamma_2^2 \gamma_5 \\
& + 1536\lambda \gamma_0 \gamma_1^2 \gamma_2 \gamma_3 \gamma_5 + 192\gamma_1^3 \gamma_2 \gamma_3 \gamma_5 \\
& + 384\gamma_1^2 \gamma_2^2 \gamma_3 \gamma_5 \\
& - 96\gamma_1^4 \gamma_4 \gamma_5 - 192\gamma_1^3 \gamma_2^3 \gamma_5 \\
& + 1536\lambda^2 \gamma_0^2 \gamma_1^2 \gamma_5^2 \\
& + 384\lambda \gamma_0 \gamma_1^3 \gamma_5^2 - 24\gamma_1^4 \gamma_5^2 \\
& + 1536\lambda \gamma_0 \gamma_1^2 \gamma_2 \gamma_5^2 + 192\gamma_1^3 \gamma_2 \gamma_5^2 \\
& + 384\gamma_1^2 \gamma_2^2 \gamma_5^2 - 1792\lambda^2 \gamma_0^2 \gamma_1^4 \gamma_6 \\
& - 448\lambda \gamma_0 \gamma_1^5 \gamma_6 + 20\gamma_1^6 \gamma_6 - 1792\lambda \gamma_0 \gamma_1^4 \gamma_2 \gamma_6 \\
& + 160\gamma_1^5 \gamma_2 \gamma_6 \\
& + 320\gamma_1^4 \gamma_2^2 \gamma_6 - 96\gamma_1^4 \gamma_3 \gamma_6 - 384\gamma_1^4 \gamma_4 \gamma_6 \\
& - 384\gamma_1^4 \gamma_5 \gamma_6 - 768\gamma_1^4 \gamma_6^2, \tag{98}
\end{aligned}$$

$$\begin{aligned}
\xi_6 = & - 384\lambda^3 \gamma_0^3 \gamma_1^5 - 88\lambda^2 \gamma_0^2 \gamma_1^6 - 4\lambda \gamma_0 \gamma_1^7 \\
& - 1536\lambda^3 \gamma_0^3 \gamma_1^4 \gamma_2 - 704\lambda^2 \gamma_0^2 \gamma_1^5 \gamma_2 \\
& - 48\lambda \gamma_0 \gamma_1^6 \gamma_2 \\
& - 1408\lambda^2 \gamma_0^2 \gamma_1^4 \gamma_2^2 - 192\lambda \gamma_0 \gamma_1^5 \gamma_2^2 \\
& - 256\lambda \gamma_0 \gamma_1^4 \gamma_2^3 + 9216\lambda^4 \gamma_0^4 \gamma_1^2 \gamma_3 \\
& + 5376\lambda^3 \gamma_0^3 \gamma_1^3 \gamma_3 \\
& + 944\lambda^2 \gamma_0^2 \gamma_1^4 \gamma_3 \\
& + 56\lambda \gamma_0 \gamma_1^5 \gamma_3 + \gamma_1^6 \gamma_3 \\
& + 18432\lambda^3 \gamma_0^3 \gamma_1^2 \gamma_2 \gamma_3 \\
& + 7296\lambda^2 \gamma_0^2 \gamma_1^3 \gamma_2 \gamma_3 \\
& + 752\lambda \gamma_0 \gamma_1^4 \gamma_2 \gamma_3 + 20\gamma_1^5 \gamma_2 \gamma_3 \\
& + 13824\lambda^2 \gamma_0^2 \gamma_1^2 \gamma_2^2 \gamma_3 + 3264\lambda \gamma_0 \gamma_1^3 \gamma_2^2 \gamma_3 \\
& + 148\gamma_1^4 \gamma_2^2 \gamma_3 \\
& + 4608\lambda \gamma_0 \gamma_1^2 \gamma_2^3 \gamma_3 \\
& + 480\gamma_1^3 \gamma_2^3 \gamma_3 + 576\gamma_1^2 \gamma_2^4 \gamma_3 - 43008\lambda^4 \gamma_0^4 \gamma_3^2 \\
& - 21504\lambda^3 \gamma_0^3 \gamma_1 \gamma_3^2 - 3840\lambda^2 \gamma_0^2 \gamma_1^2 \gamma_3^2 \\
& - 288\lambda \gamma_0 \gamma_1^3 \gamma_3^2 - 8\gamma_1^4 \gamma_3^2 \\
& - 86016\lambda^3 \gamma_0^3 \gamma_2 \gamma_3^2 \\
& - 32256\lambda^2 \gamma_0^2 \gamma_1 \gamma_2 \gamma_3^2 - 3840\lambda \gamma_0 \gamma_1^2 \gamma_2 \gamma_3^2 \\
& - 144\gamma_1^3 \gamma_2 \gamma_3^2 \\
& - 64512\lambda^2 \gamma_0^2 \gamma_2^2 \gamma_3^2 \\
& - 16128\lambda \gamma_0 \gamma_1 \gamma_2^2 \gamma_3^2 - 960\gamma_1^2 \gamma_2^2 \gamma_3^2 \\
& - 21504\lambda \gamma_0 \gamma_2^3 \gamma_3^2 - 2688\gamma_1 \gamma_2^3 \gamma_3^2 - 2688\gamma_2^4 \gamma_3^2 \\
& + 12288\lambda^4 \gamma_0^4 \gamma_1^2 \gamma_4 + 6144\lambda^3 \gamma_0^3 \gamma_1^3 \gamma_4
\end{aligned}$$

$$\begin{aligned}
& + 1088\lambda^2 \gamma_0^2 \gamma_1^4 \gamma_4 + 80\lambda \gamma_0 \gamma_1^5 \gamma_4 + 4\gamma_1^6 \gamma_4 \\
& + 24576\lambda^3 \gamma_0^3 \gamma_1^2 \gamma_2 \gamma_4 \\
& + 9216\lambda^2 \gamma_0^2 \gamma_1^3 \gamma_2 \gamma_4 + 1088\lambda \gamma_0 \gamma_1^4 \gamma_2 \gamma_4 \\
& + 56\gamma_1^5 \gamma_2 \gamma_4 \\
& + 18432\lambda^2 \gamma_0^2 \gamma_1^2 \gamma_2^2 \gamma_4 \\
& + 4608\lambda \gamma_0 \gamma_1^3 \gamma_2^2 \gamma_4 + 304\gamma_1^4 \gamma_2^2 \gamma_4 \\
& + 6144\lambda \gamma_0 \gamma_1^2 \gamma_2^3 \gamma_4 \\
& + 768\gamma_1^3 \gamma_2^3 \gamma_4 + 768\gamma_1^2 \gamma_2^4 \gamma_4 \\
& - 147456\lambda^4 \gamma_0^4 \gamma_3 \gamma_4 - 73728\lambda^3 \gamma_0^3 \gamma_1 \gamma_3 \gamma_4 \\
& - 13824\lambda^2 \gamma_0^2 \gamma_1^2 \gamma_3 \gamma_4 - 1152\lambda \gamma_0 \gamma_1^3 \gamma_3 \gamma_4 \\
& - 40\gamma_1^4 \gamma_3 \gamma_4 \\
& - 294912\lambda^3 \gamma_0^3 \gamma_2 \gamma_3 \gamma_4 \\
& - 110592\lambda^2 \gamma_0^2 \gamma_1 \gamma_2 \gamma_3 \gamma_4 \\
& - 13824\lambda \gamma_0 \gamma_1^2 \gamma_2 \gamma_3 \gamma_4 \\
& - 576\gamma_1^3 \gamma_2 \gamma_3 \gamma_4 \\
& - 221184\lambda^2 \gamma_0^2 \gamma_2^2 \gamma_3 \gamma_4 \\
& - 55296\lambda \gamma_0 \gamma_1 \gamma_2^2 \gamma_3 \gamma_4 - 3456\gamma_1^2 \gamma_2^2 \gamma_3 \gamma_4 \\
& - 73728\lambda \gamma_0 \gamma_2^3 \gamma_3 \gamma_4 \\
& - 9216\gamma_1 \gamma_2^3 \gamma_3 \gamma_4 - 9216\gamma_2^4 \gamma_3 \gamma_4 \\
& - 294912\lambda^4 \gamma_0^4 \gamma_4^2 - 147456\lambda^3 \gamma_0^3 \gamma_1 \gamma_4^2 \\
& - 27648\lambda^2 \gamma_0^2 \gamma_1^2 \gamma_4^2 \\
& - 2304\lambda \gamma_0 \gamma_1^3 \gamma_4^2 - 80\gamma_1^4 \gamma_4^2 \\
& - 589824\lambda^3 \gamma_0^3 \gamma_2 \gamma_4^2 - 221184\lambda^2 \gamma_0^2 \gamma_1 \gamma_2 \gamma_4^2 \\
& - 27648\lambda \gamma_0 \gamma_1^2 \gamma_2 \gamma_4^2 \\
& - 1152\gamma_1^3 \gamma_2 \gamma_4^2 - 442368\lambda^2 \gamma_0^2 \gamma_2^2 \gamma_4^2 \\
& - 110592\lambda \gamma_0 \gamma_1 \gamma_2^2 \gamma_4^2 \\
& - 6912\gamma_1^2 \gamma_2^2 \gamma_4^2 \\
& - 147456\lambda \gamma_0 \gamma_2^3 \gamma_4^2 - 18432\gamma_1 \gamma_2^3 \gamma_4^2 \\
& - 18432\gamma_2^4 \gamma_4^2 + 1536\lambda^3 \gamma_0^3 \gamma_1^3 \gamma_5 + 128\lambda^2 \gamma_0^2 \gamma_1^4 \gamma_5 \\
& - 40\lambda \gamma_0 \gamma_1^5 \gamma_5 - 2\gamma_1^6 \gamma_5 \\
& + 768\lambda^2 \gamma_0^2 \gamma_1^3 \gamma_2 \gamma_5 - 256\lambda \gamma_0 \gamma_1^4 \gamma_2 \gamma_5 - 28\gamma_1^5 \gamma_2 \gamma_5 \\
& - 384\lambda \gamma_0 \gamma_1^3 \gamma_2^2 \gamma_5 \\
& - 128\gamma_1^4 \gamma_2^2 \gamma_5 - 192\gamma_1^3 \gamma_2^3 \gamma_5 \\
& + 768\lambda^2 \gamma_0^2 \gamma_1^2 \gamma_3 \gamma_5 \\
& + 192\lambda \gamma_0 \gamma_1^3 \gamma_3 \gamma_5 + 8\gamma_1^4 \gamma_3 \gamma_5 \\
& + 768\lambda \gamma_0 \gamma_1^2 \gamma_2 \gamma_3 \gamma_5 + 96\gamma_1^3 \gamma_2 \gamma_3 \gamma_5 \\
& + 192\gamma_1^2 \gamma_2^2 \gamma_3 \gamma_5 - 16\gamma_1^4 \gamma_4 \gamma_5 \\
& + 768\lambda^2 \gamma_0^2 \gamma_1^2 \gamma_5^2 + 192\lambda \gamma_0 \gamma_1^3 \gamma_5^2 \\
& + 4\gamma_1^4 \gamma_5^2
\end{aligned}$$

$$\begin{aligned}
& + 768\lambda\gamma_0\gamma_1^2\gamma_2\gamma_5^2 + 96\gamma_1^3\gamma_2\gamma_5^2 \\
& + 192\gamma_1^2\gamma_2^2\gamma_5^2 - 256\lambda^2\gamma_0^2\gamma_1^4\gamma_6 - 64\lambda\gamma_0\gamma_1^5\gamma_6 \\
& + 4\gamma_1^6\gamma_6 - 256\lambda\gamma_0\gamma_1^4\gamma_2\gamma_6 \\
& + 32\gamma_1^5\gamma_2\gamma_6 + 64\gamma_1^4\gamma_2^2\gamma_6 \\
& - 16\gamma_1^4\gamma_3\gamma_6 - 64\gamma_1^4\gamma_4\gamma_6 \\
& - 64\gamma_1^4\gamma_5\gamma_6 - 128\gamma_1^4\gamma_6^2.
\end{aligned} \quad (99)$$

The parameters $\gamma_{0,1,2,\dots,6}$ in these expressions represent the ambiguity in the parametrization of quantum metric (5). Let us note that these bulky formulas, anyway, admit a completely invariant result when used in the potential (73), providing the very compact Eq. (74).

Appendix B

Here we derive the equation DM(35). All equations like DM(35) are numbered according to Ref. [11]. For comparison reasons, the notations in this section also follows [11]. The principal difference with the main text conventions is the opposite sign for the metric signature.

The starting expression is

$$\Delta S_{mM} = -\frac{1}{32\pi^2} \int d^4y \sqrt{-g} m_{\mu\nu\rho\sigma} \ln(-\square) M^{\rho\sigma\mu\nu}, \quad (100)$$

where DM(15) gives

$$\begin{aligned}
M^{\rho\sigma\mu\nu} &= \frac{M\kappa^2}{8} \int d\tau' \delta^4(y - x(\tau')) (2\dot{x}^\rho \dot{x}^\sigma \dot{x}^\mu \dot{x}^\nu + \dot{x}^\rho \dot{x}^\sigma g^{\mu\nu}), \\
m_{\mu\nu\rho\sigma} &= \frac{m\kappa^2}{8} \int d\tau \delta^4(y - z(\tau)) (2\dot{z}_\rho \dot{z}_\sigma \dot{z}_\mu \dot{z}_\nu + \dot{z}_\mu \dot{z}_\nu g_{\rho\sigma}).
\end{aligned} \quad (101)$$

Hereafter, $\dot{x}_\mu = \frac{dx_\mu}{d\tau}$ and $\dot{z}_\mu = \frac{dz_\mu}{d\tau}$. Before substituting (101) into (100), we take into account that the massive source M is static and situated at the origin, such that $x^0 = \tau$ and $x^k \equiv 0$. Furthermore,

$$g^{\mu\nu} \dot{z}_\mu \dot{z}_\nu = -1. \quad (102)$$

Then, the contractions in (100) give

$$\begin{aligned}
\Delta S_{mM} &= -\frac{mM\kappa^4}{64 \cdot 32\pi^2} \int dy^0 d^3\mathbf{y} \sqrt{-g(y)} \\
&\times \int d\tau \delta^4(y - z(\tau)) \\
&\times (2\dot{z}_\rho \dot{z}_\sigma \dot{z}_\mu \dot{z}_\nu + \dot{z}_\rho \dot{z}_\sigma g_{\mu\nu}) \\
&\times \ln(-\square) \int d\tau' \delta(y^0 - \tau') \delta^3(\mathbf{y}) \\
&\times (2\delta_0^\rho \delta_0^\sigma \delta_0^\mu \delta_0^\nu + \delta_0^\rho \delta_0^\sigma g^{\mu\nu})
\end{aligned}$$

$$\begin{aligned}
&= -\frac{mM\kappa^4}{64 \cdot 32\pi^2} \int dy^0 d^3\mathbf{y} \sqrt{-g(y)} \\
&\times \int d\tau \delta(y^0 - z^0(\tau)) \delta^3(\mathbf{y} - \mathbf{z}(\tau)) \\
&\times (g_{00} g^{\rho\sigma} \dot{z}_\rho \dot{z}_\sigma + 2\dot{z}_0 \dot{z}_0 g^{\rho\sigma} \dot{z}_\rho \dot{z}_\sigma + \\
&\times 2g_{00} \dot{z}_0 \dot{z}_0 + 4\dot{z}_0 \dot{z}_0 \dot{z}_0 \dot{z}_0) \\
&\times \ln(-\square) \int d\tau' \delta(y^0 - \tau') \delta^3(\mathbf{y}).
\end{aligned} \quad (103)$$

Taking into account (102), the formula (103) boils down to

$$\begin{aligned}
\Delta S_{mM} &= -\frac{mM\kappa^4}{64 \cdot 32\pi^2} \int dy^0 d^3\mathbf{y} \sqrt{-g(y)} \\
&\times \int d\tau \delta(y^0 - z^0) \delta^3(\mathbf{y} - \mathbf{z}(\tau)) \\
&\times (-g_{00} - 2\dot{z}_0 \dot{z}_0 + 2g_{00} \dot{z}_0 \dot{z}_0 + 4\dot{z}_0 \dot{z}_0 \dot{z}_0 \dot{z}_0) \\
&\times \ln(-\square) \delta^3(\mathbf{y}).
\end{aligned} \quad (104)$$

Taking the integral over y^0 and changing the order of integration, we get $y^0 \rightarrow \tau$. Similarly, taking $d^3\mathbf{y}$ results in $\mathbf{y} \rightarrow \mathbf{z}(\tau)$. Assuming non relativistic z^μ , i.e., $z^0 = \tau$ and $\dot{z}_k = 0$, we get

$$\begin{aligned}
\Delta S_{mM} &= -\frac{mM\kappa^4}{64 \cdot 32\pi^2} \int d\tau \\
&\times (-g_{00} - 2\dot{z}_0 \dot{z}_0 - 2\dot{z}_0 \dot{z}_0 + 4\dot{z}_0 \dot{z}_0 \dot{z}_0 \dot{z}_0) \\
&\times \ln(-\square) \delta^3(\mathbf{z}(\tau)) \\
&= -\frac{mM\kappa^4}{512\pi^2} \int d\tau \frac{1}{4} \ln(-\square) \delta^3(\mathbf{z}(\tau)),
\end{aligned} \quad (105)$$

that is four time more than DM(35). Correcting the missing factor of 4, the coefficient in DM(43) is $-1/6$ instead of $7/12$. This gives a Newtonian potential in the form (74).

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