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Observations of the Beam-Beam Interaction in Hadron Colliders.

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1 Introduction

Experimental data on hadron beam-beam interactions come from CERN ISR, CERN SPS and FERMILAB TEVATRON colliders. They all successfully collided hadron beams and produced luminosities that resulted in physics discoveries. The ISR pioneered the stochastic cooling techniques paving the way for proton-antiproton colliders, achieving a maximum luminosity of 1.3×10^{32} $cm^{-2}sec^{-1}$ colliding stochastically cooled coasting proton beams at energies up to 31 GeV. The SPS and the TEVATRON in their 1988-89 runs both reached 2×10^{30} $cm^{-2}sec^{-1}$ and delivered integrated luminosities of 8100 nb^{-1} and 9600 nb^{-1}, respectively, while colliding 6 proton bunches with 6 antiproton bunches.

The SPS was built with conventional magnets, limiting its beam energy to 315 GeV for stored beams. The TEVATRON, with approximately the same circumference as the SPS, was the first large accelerator built with superconducting magnets. This allowed beams to be accelerated to 900 GeV. In the very near future the TEVATRON beam energy will be raised to 1 TeV by reducing the temperature of the cooling system from 4.6°K to 4.2°K.

In bunched beam hadron colliders, the beam brightness (intensity/emittance) and therefore the luminosity is limited by head-on beam-beam interactions. However, the effect of the beam-beam interaction can be reduced by separating the closed orbits of protons and antiprotons. Unnecessary head-on interactions are eliminated, making it possible to increase the luminosity. In the SPS, beams were horizontally separated around 3/4 of the ring. Three interaction points were located in the remainder of the ring. In the TEVATRON beams will be separated in both planes (helical separation) so that the machine can be operated with more than 6 bunches per beam [1]. Beams

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will only be brought into collision at 2 locations, B0 and D0. The helical separation scheme will be operational for the first time during the 1992 TEVATRON collider run. Major components of the system have been tested (new low-beta lattice, electrostatic separators, feed-down circuits) during two study periods. These studies concentrated particularly on long-range beam-beam interactions.

This paper has three parts. In the first part the basic beam-beam theory will be reviewed. Theoretical issues relevant to e^+e^- colliders will not be mentioned. In the second part we summarize the operational experiences at FERMILAB and CERN. In the last part of the paper, experiments on long-range beam-beam interactions in the TEVATRON are reviewed.

2 Hadron Beam-Beam Theory

Resonances play an important role in periodic Hamiltonian systems such as hadron storage rings[2]. If the fractional tune of the betatron oscillation is close to a rational number the particle will experience correlated kicks from various nonlinear fields such as lattice imperfections, beam-beam interactions et cetera. Even if the perturbation is very weak the effect will be magnified, and the particle amplitude changes. In practice, in the absence of damping, the emittance grows.

The resonance condition cannot be sustained if the tune changes with amplitude. Therefore the curve showing the tune as a function of amplitude is very important in the understanding of resonances. The "detuning function" is uniquely determined by the source of the nonlinearity, and its strength.

A resonance is characterized by three attributes; the "resonance-width function", the strengths of the perturbations, and the phase advances between them. In practice, excited resonances cause emittance growth and beam lifetime effects. In theory, the amplitude growth mechanism (diffusion) is not well understood. The condition of "chaos" seems to be necessary but not sufficient for the diffusion of particles. There is no theory linking the beam lifetime to beam-beam resonances in hadron storage rings. The hadron beam-beam theory reviewed below is mostly concerned with the onset of chaos.

2.1 Beam-Beam Resonances

Consider a collider with a single beam-beam collision per turn. The betatron tune of a test particle depends on its amplitude, according to

$$Q(\alpha) = Q_0 + \xi D(\alpha) \tag{1}$$

Here Q_0 is the unperturbed tune, $\alpha \equiv a/\sigma$ is the normalized amplitude where a is the betatron amplitude and σ is the (transverse) rms size of the source bunch. The source bunch generates the beam-beam force and perturbs the test particle.

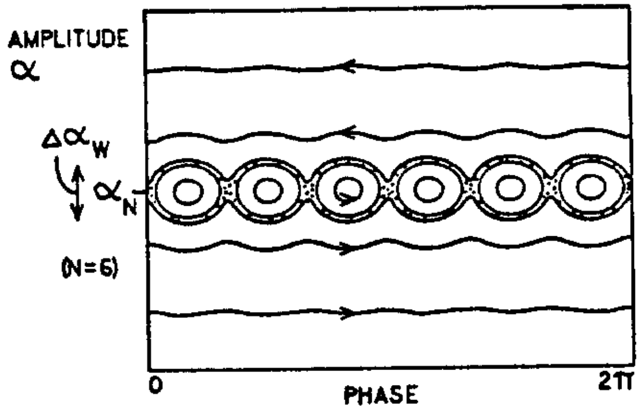

Figure 1: A sketch of phase-space flow near a beam-beam resonance. Note that the number of islands is equal to N, the order of the resonance.

$D(\alpha)$ is the "detuning function" and ξ is the "beam-beam parameter", defined as

$$\xi = \frac{N_p r_p \beta^*_{x,y}}{2\pi\gamma\sigma_{x,y}(\sigma_x + \sigma_y)} \tag{2}$$

Here $\beta^*_{x,y}$ are the betatron functions at the collision point, $\sigma_{x,y}$ are the rms beams sizes for the source bunch, r_p is the classical proton radius, N_p is the number of particles in the source bunch and γ is the relativistic factor. The beam-beam parameter ξ is equal to the tune shift experienced by a small amplitude particle. For round beams ($\sigma_x = \sigma_y$) the beam-beam parameter ξ can be written as

$$\xi = \frac{N_p r_p}{\pi(\varepsilon\beta\gamma)} \tag{3}$$

where ε is the source bunch emittance. Beams in hadron colliders are almost round, both due to design and due to residual coupling between the x- and the y-planes. Therefore the round beam expressions for detuning and resonance-width functions will be sufficient to describe the beam-beam dynamics. If colliding bunches have Gaussian transverse charge distributions, the detuning function for the beam-beam interaction of round beams has the exact analytic form [3]

$$D(\alpha) = 4\alpha^{-2}\left[1 - I_0(\alpha^2/4)\exp(-\alpha^2/4)\right] \tag{4}$$

Here I_0 is a modified Bessel function. A beam-beam resonance of order N is present if the tune is equal to a rational fraction n/N at some amplitude α_N. A schematic representation of the phase-space flow of a resonance is shown in Fig.1.

The beam-beam resonance islands, seen in the phase-space flow diagrams, have a half width

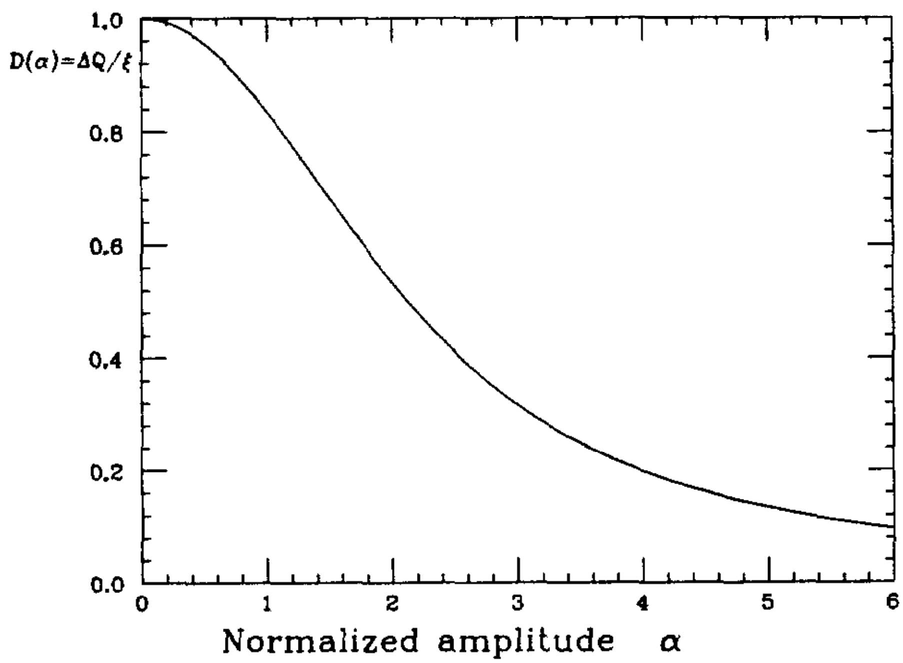

Figure 2: Beam-beam detuning function.

given by [3]

$$\Delta\alpha_w = 2\left[\frac{V_N(\alpha)}{\alpha D'(\alpha)}\right]^{1/2} \tag{5}$$

$D'(\alpha)$ is the derivative of $D(\alpha)$ with respect to α. For round beams the "resonance width function" $V_N(\alpha)$ is (even order N only)[4]

$$V_N(\alpha) = \int_0^\alpha \frac{8}{\alpha}\, I_{N/2}(\alpha^2/4)\exp(-\alpha^2/4)\, d\alpha \tag{6}$$

The detuning function is shown in Fig.2. Beam-beam resonance half-widths for resonances up to the 12^{th} order are shown in Fig.3.

One important observation from Fig.3 is that only large amplitude particles can excite the high order resonances. As will be discussed in the following sections, this theoretical result was used to explain the high background rates in CERN SPS collider operation when proton and antiproton bunches had unequal emittances.

2.2 Tune Modulation

Tune modulation caused by the synchrotron oscillations is practically unavoidable. If the chromaticity is not exactly zero, the oscillation in the particle energy is translated into an oscillation in tune. Another source of tune modulation is noise in the current supplied to the magnets. From

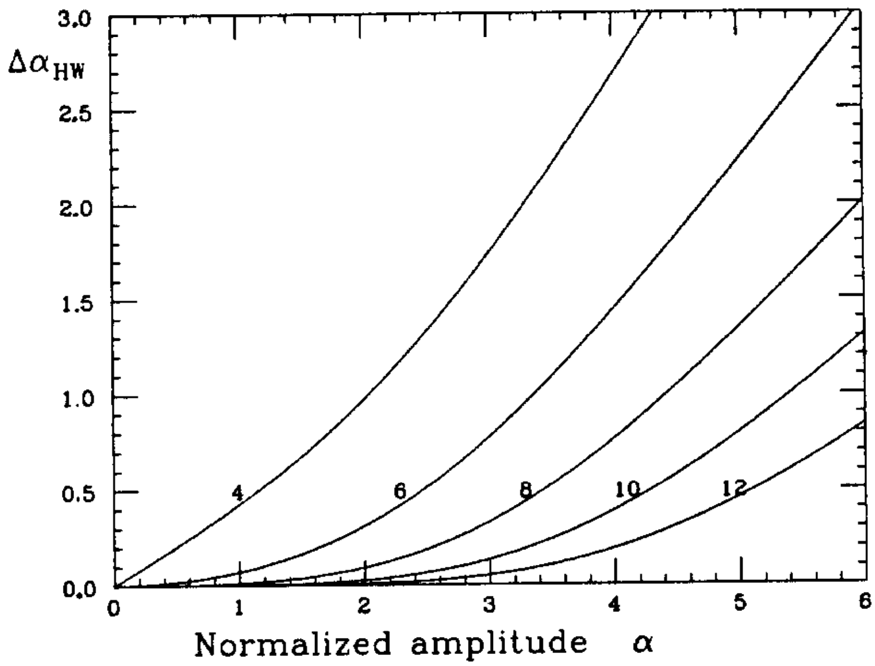

Figure 3: Beam-beam resonance island half-widths for resonances up to order 12.

experiment[5] and theory[6][7][8] it is known that tune modulation enhances the effect of resonances. The enhancement can be understood in terms of Chirikov's resonance overlap criterion [9].

It is supposed that, owing to an external modulating source, the perturbed betatron tune is given by

$$Q = Q_0 + q\ sin(2\pi Q_s t) \tag{7}$$

where Q_0 is the unperturbed betatron tune, q is the amplitude of the modulation (modulation depth), Q_s is the modulation tune, and t is the turn number. The resonance analysis is done at a particular point in the ring and "time" for the purposes of this analysis is quantized.

Tune modulation causes a family of synchrobetatron sideband resonances to appear, at time-averaged tunes of

$$Q(\alpha) = n/N + p\ Q_s/N \tag{8}$$

where p is an integer. This situation is depicted in Fig.4(a,b) where the sideband islands surround the fundamental islands.

The full width of the p^{th} sideband is given (if the sidebands do not overlap) by

$$\Delta\alpha_{wp} = 4\left[\frac{V_N(\alpha_p)J_p(Nq/Q_s)}{\alpha_p D'(\alpha_p)}\right]^{1/2} \tag{9}$$

Here J_p is the p^{th} integer order Bessel function, and α_p is the betatron amplitude corresponding to

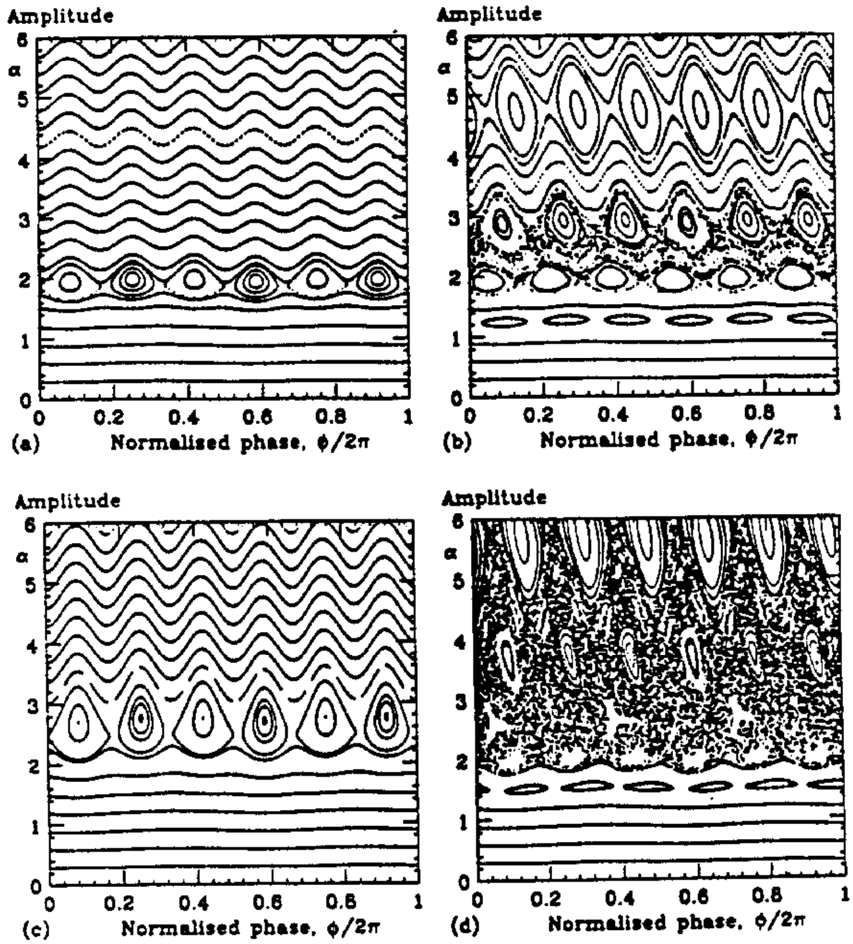

Figure 4: Simulated trajectories tracked for 2000 modulation periods, with $Q_s = 0.005$ and an unshifted tune of 0.331, near a sixth-order beam-beam resonance. The two left figures have no tune modulation, while the two right figures have modulation amplitude $q = 0.001$. The two top figures have a tune shift parameter of $\xi = 0.0042$, while the two bottom figures have a value $\xi = 0.0060$. Side bands $p = +1, 0, -1$, and -2, visible in (b) at increasing amplitudes, overlap and are submerged in a chaotic sea in (d).

this sideband. The magnitude of J_p is of the order of

$$J_p(Nq/Q_s) \approx (Q_s/\pi Nq)^{1/2} \tag{10}$$

if

$$\frac{n}{N} - q < Q(\alpha_p) \ < \ \frac{n}{N} + q \tag{11}$$

and very small if condition 11 is violated. The physical interpretation of this condition is as follows. Because of the tune modulation, the "instantaneous" tune varies between $Q(\alpha)-q$ and $Q(\alpha)+q$. For the resonance to have effect, this tune must cross n/N. So, if $Q(\alpha) < (n/N-q)$ or $Q(\alpha) > (n/N+q)$, the tune never reaches the resonance condition and the sidebands are suppressed. Sidebands are separated in amplitude from each other by

$$\Delta\alpha_s \equiv \frac{(Q_s/N)}{Q'(\alpha)} = \frac{Q_s}{N\xi D'(\alpha)} \tag{12}$$

As the beam-beam tune shift parameter ξ is increased, the sidebands remain constant in size while their separations decrease. When $\Delta\alpha_s \ < \ \Delta\alpha_{wp}$, the sidebands overlap and a chaotic layer is formed in phase-space flow as shown in Fig.4.d. In other words, there is overlap if

$$\xi \ > \xi_{\max} \equiv \ \frac{1}{4}(\pi q)^{1/4}(Q_s)^{3/4}\left(\frac{\alpha}{N^{3/2}V_N(\alpha)D'(\alpha)}\right)^{1/2} \tag{13}$$

This needs to be generalized to multiple collisions. The generalized $\Delta\alpha_s$ and $\Delta\alpha_{wp}$ are [8]

$$\Delta\alpha_s = Q_s/\left[N\cdot\sum_{i=1}^{m}\mid\vec{\xi}_{Ni}\mid\cdot D'(\alpha)\right] \tag{14}$$

$$\Delta\alpha_{wp} = 4\left(\left[\mid\sum_{i=1}^{m}\vec{\xi}_{Ni}\mid\cdot V_N(\alpha)J_p(Nq/Q_s)\right] / \left[\sum_{i=1}^{m}\mid\vec{\xi}_{Ni}\mid\alpha D'(\alpha)\right]\right)^{1/2} \tag{15}$$

Using the overlap condition $\Delta\alpha_s \ < \ \Delta\alpha_{wp}$ and Eq.(14), Eq.(15) we obtain

$$\left(\mid\sum_{i=1}^{m}\vec{\xi}_{Ni}\mid\cdot\sum_{i=1}^{m}\mid\vec{\xi}_{Ni}\mid\right)^{1/2} \ > \ \xi_{\max} = \frac{1}{4}(\pi q)^{1/4}(Q_s)^{3/4}\left[\frac{\alpha}{N^{3/2}V_N(\alpha)D'(\alpha)}\right]^{1/2} \tag{16}$$

where m is the number of head-on beam-beam interactions. The magnitude of the beam-beam resonance vector $\vec{\xi}_{Ni}$ is equal to the beam-beam parameter given in Eq.(2).

The calculation of $|\sum\vec{\xi}_{Ni}|$ requires the knowledge of phases at crossing points. Typically there is a several percent error in the lattice functions, and it is difficult to know the phase at the crossing points to sufficient accuracy. It is usual to simply take the root mean square average of the resonance vectors $\vec{\xi}_{Ni}$, namely, we approximate

$$|\sum_{i=1}^{m} \vec{\xi}_{Ni} | \approx (m)^{1/2}\xi \tag{17}$$

The other summation is easier since the phase information is not needed.

$$\sum_{i=1}^{m} | \vec{\xi}_{Ni} | = m\xi \tag{18}$$

Eq.(16) gives the "threshold equation". Given the order of the betatron resonance N, the particle amplitude α, the tune modulation frequency Q_s and depth q, the threshold equation tells whether the beam-beam parameter ξ is large enough to cause an overlap of sideband resonances. The threshold condition, Eq.(16), also defines the highest order betatron resonance that allows sideband overlap in the presence of tune modulation for a particular amplitude. From here on these will be called "critical resonances".

2.3 Beam-Beam Tune Shift and Spread

The tune shifts and spreads arising from beam-beam interactions can be calculated numerically or analytically. Tune shift from head-on interactions is well understood and given by [10], [11], [12].

$$\begin{aligned}
\Delta\nu_x &= \frac{1}{4\pi}\frac{N_p r_p}{\gamma}\frac{\beta_x}{\sigma_x\sigma_y}\int_0^1 \frac{dw}{\sqrt{\lambda_x^3\lambda_y}}[Z_0(\zeta_x) - Z_1(\zeta_x)]Z_0(\zeta_y) \qquad (19)\\
\lambda_x &\equiv 1 + (\frac{\sigma_x}{\sigma_y} - 1)w \\
\lambda_y &\equiv 1 + (\frac{\sigma_y}{\sigma_x} - 1)w \\
\zeta_x &\equiv \frac{\beta_x J_x}{2\sigma_x\sigma_y}\frac{w}{\lambda_x} \\
\zeta_y &\equiv \frac{\beta_y J_y}{2\sigma_x\sigma_y}\frac{w}{\lambda_y} \\
Z_n(\zeta) &\equiv e^{-\zeta}I_n(\zeta)
\end{aligned}$$

where γ is the relativistic factor, r_p is the classical proton radius, I_n are Modified Bessel functions. J_x and J_y are the so-called action variables ($J_x = \alpha_x^2/2,\ J_y = \alpha_y^2/2,$ here α_x and α_y are the normalized amplitudes for the x- and the y-planes, respectively). A similar expression can be written for $\Delta\nu_y$ by interchanging x and y subscripts.

Analytical expressions for tune shifts arising from a long-range beam-beam interaction are more complicated. Expressions calculated from the multipole expansion of the long-range beam-beam kick are given in Ref.[11]. Other theoretical issues related to hadron beam-beam interactions can be found in references[13].

3 Experience at CERN

3.1 ISR Operation

The ISR was a high current, high luminosity collider consisting of two interleaved rings [14] [15]. It collided unbunched beams. Protons were brought into collision in 1971. Later operations stored alpha particles, deuterons and antiprotons [16] [17]. Beam energy was variable between 26 Gev and 31 GeV for protons. High luminosity was possible when the beams were stochastically cooled. The maximum beam current was 60 Amps producing the maximum luminosity of 1.3×10^{32} cm^{-2}s^{-1} [18].

ISR beams crossed horizontally with an angle of 14.77° at 8 interaction regions around the ring. There was no tune shift in the horizontal plane. The vertical tune shift was of order 0.001 per crossing. The fractional tune (working point) was normally chosen to be between the 7^{th} and the 9^{th} order resonances where the beam-beam interaction did not affect the beam lifetimes [21].

Tune modulation, which plays an important role in bunched beam colliders, did not influence the beam behaviour in the ISR. In a debunched beam there is no mechanism for tune modulation other than external sources. Review articles on ISR can be found in Ref.[19] and Ref.[20].

3.2 Beam-Beam Interactions in the SPS

The CERN SPS was the first hadron collider that operated with bunched beams (first operation in 1981). At the end of the second run (1982) a peak luminosity of 5.3×10^{28} cm^{-2}s^{-1} was achieved. In subsequent runs the peak luminosity was increased, culminating in peak luminosities consistently above 2×10^{30} cm^{-2}s^{-1} during 1988-89 operation. Prior to the 1987 run 3 proton bunches collided head-on with 3 antiproton bunches. Horizontal separators were installed for the 1987-1988 runs, allowing 6 bunches per beam and 3 head-on interaction regions.

Early beam-beam experiments in the SPS (a single antiproton bunch colliding head-on with 3 proton bunches, no separators) showed that the antiproton intensity lifetime is very sensitive to the tune. In particular, the experiment demonstrated that the 7^{th} order resonance was excited by beam-beam interactions [18]. Under ideal head-on conditions only even-order resonances are excited. The excitation of an odd resonance can be explained as follows. The average tunes and therefore the closed orbits of the proton and antiproton beams were different during this experiment, causing a small displacement at the interaction regions. Beam-beam interactions of transversely displaced beams excite odd-order resonances. Small residual dispersions at the interaction regions also excite odd resonances.

The 7^{th} order resonance did not affect the proton intensity lifetime significantly in this experiment (Fig.5). Lattice nonlinearities affect proton and antiproton beams equally. If the proton intensity lifetime is longer than that of the antiproton, one suspects another source of nonlinearity

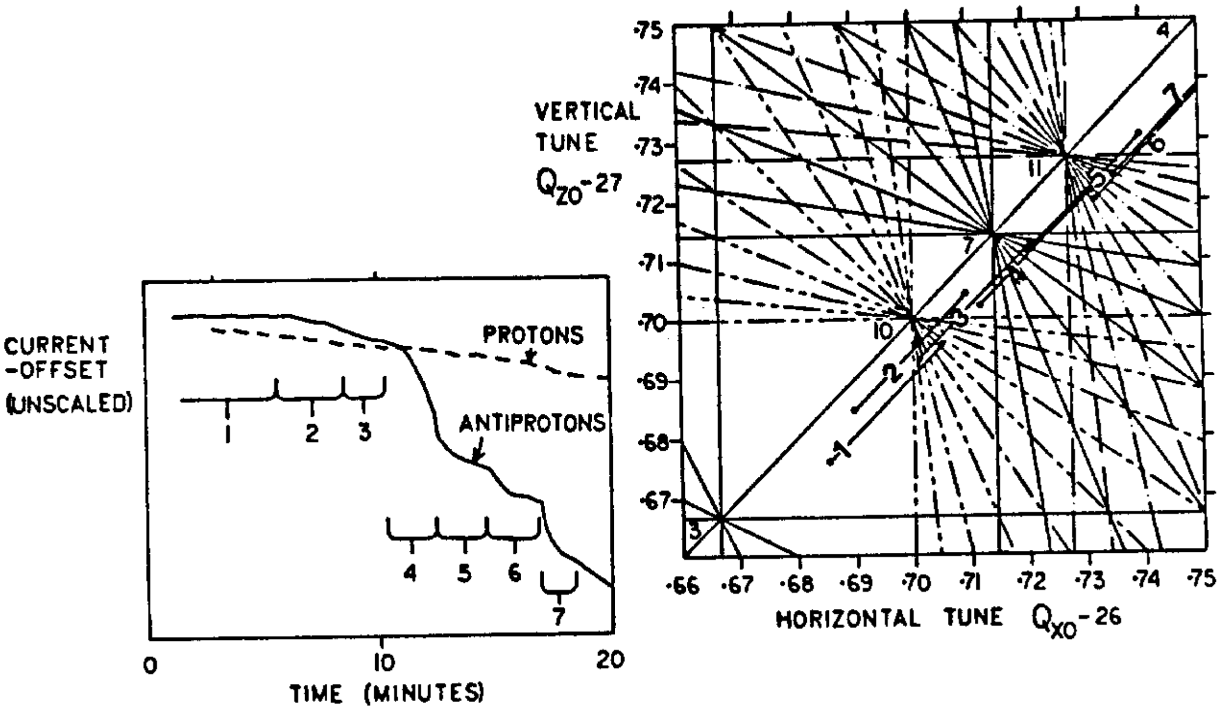

Figure 5: Measured proton and antiproton currents as a function of time in the early SPS beam-beam experiment.

- the beam-beam interaction. From this it was concluded that the SPS lattice nonlinearities did not significantly excite the 7^{th} order resonance. The early beam-beam experiments at the SPS also demonstrated that the 10^{th} order resonance was excited by the head-on beam-beam interactions. These results established the operating point (unshifted, fractional tunes) for the SPS to be near 0.68, a safe distance from 0.70 (10^{th} order resonance).

An important aspect of head-on beam-beam interactions is that the particles in the transverse tails sample a nonlinear force (see Fig.6) and thus become sensitive to resonances, while small amplitude particles sample linear beam-beam forces and therefore suffer only a tune shift. The significance of the beam-beam nonlinearity was demonstrated in SPS experiments studying the weak-strong case. Normally, the antiproton (weak beam) emittance is lower than the proton (strong beam) emittance. It was shown that when the antiproton emittance was larger than the proton emittance (due to malfunction or deliberate intervention) the large amplitude antiprotons diffused out faster. In one study [18], 3 antiproton bunches with successively larger emittances were injected into the SPS, and collided with 3 proton bunches of lower emittance. The antiproton bunch with the largest emittance decayed faster than the other bunches initially, and its lifetime approached that of the other bunches after 5 hours. This phenomenon is sometimes refered to as "self-scraping",

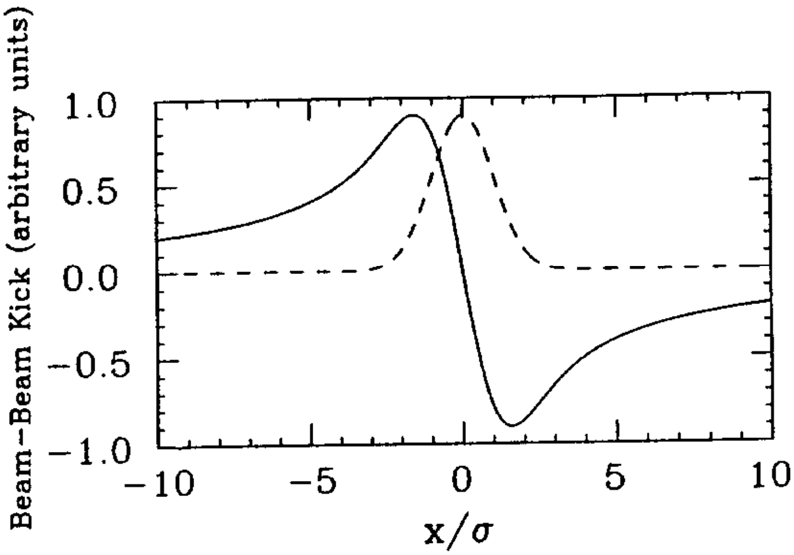

Figure 6: The kick experienced by an oppositely charged probe particle as it passes by a round gaussian source bunch. The source bunch distribution is indicated with the dashed curve. The symbol σ is the rms size of the source bunch while x is the distance between the center of the source bunch and the probe particle.

reflecting the observed effect that the beam emittance can actually decrease due to loss of particles from the transverse tails. This finding was later reproduced in the TEVATRON.

A variant of the self-scraping phenomenon is observed in proton beams when the antiproton bunch intensity is close to the proton bunch intensity (strong-strong case). In early SPS collider runs, the antiproton bunch intensity was typically 1.9×10^{10} particles per bunch while the proton bunch intensity was 15×10^{10} particles per bunch, clearly a weak-strong situation where the proton (strong) beam is not perturbed by the antiproton (weak) beam. In the 1988-89 SPS collider run the intensities were 5×10^{10} and 11×10^{10} for antiproton and proton bunches, respectively. This meant that the protons were significantly perturbed by the antiproton bunches, resembling the strong-strong case in e^+e^- colliders. In addition to a measurable beam-beam proton tune shift (0.0049 per crossing in the horizontal plane) there was also a decreased intensity lifetime for protons and very large background rates in the physics detectors[22].

This effect was caused by the difference in proton and antiproton beam emittances. What works in the weak-strong case does not work in the strong-strong case. In the weak-strong case the antiproton beam size had to be kept smaller than the proton beam size in order prevent antiproton losses from the tails. The tail antiprotons (large amplitude particles) experience nonlinear forces from head-on beam-beam interactions. In the strong-strong case, if the proton emittance is larger than the antiproton emittance, the moderate amplitude protons as well as the tail protons feel

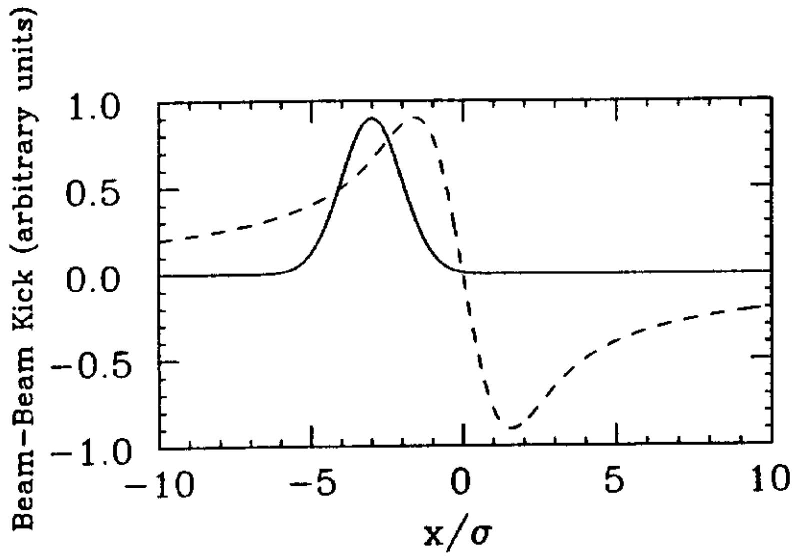

Figure 7: Long-range beam-beam interaction. The solid curve is the probe bunch positioned to ilustrate a 3σ separation between the centers of source and probe bunches. σ is still the rms size of the source bunch. Note that 1σ-amplitude particles in the probe bunch are sampling a nonlinear force.

a nonlinear beam-beam force. This causes the diffusion of protons. Background rates decreased greatly when the proton emittance was comparable to the antiproton emittance.

Since the operating point of the SPS was chosen to avoid resonances lower than the 10^{th} order, the proton diffusion in the case of unequal emittances must have been caused by higher order resonances. Theory states that the resonance-width of high-order resonances increases with amplitude (see Fig.3). The particles in the larger emittance beam have larger amplitudes, and therefore become more sensitive to higher order resonances. It was determined that resonances of order 13 and 16 were affecting the large amplitude protons[23]. This phenomenon was further studied in a series of experiments [24] which concluded that the linear beam-beam tune shift parameter is not sufficient to assess the strength of beam-beam effects in hadron colliders. Their conclusion was that the ratio of proton and antiproton emittances should be used in the parameterization.

More insight can be gained about relevant parameters from the threshold equation (Eq.16) which stresses the importance of vector addition. The magnitude of a resonance vector is equal to the beam-beam tune shift parameter for that interaction point. Simply adding the magnitudes of the vectors, i.e. using the total tune shift in the parametrization is not sufficient to describe the onset of chaos. The vector addition requires the knowledge of phase advances between the beam-beam kicks. Therefore the phase advances between the kicks should also be used in the parameterization. The threshold equation was used to study the combined effect of tune modulation

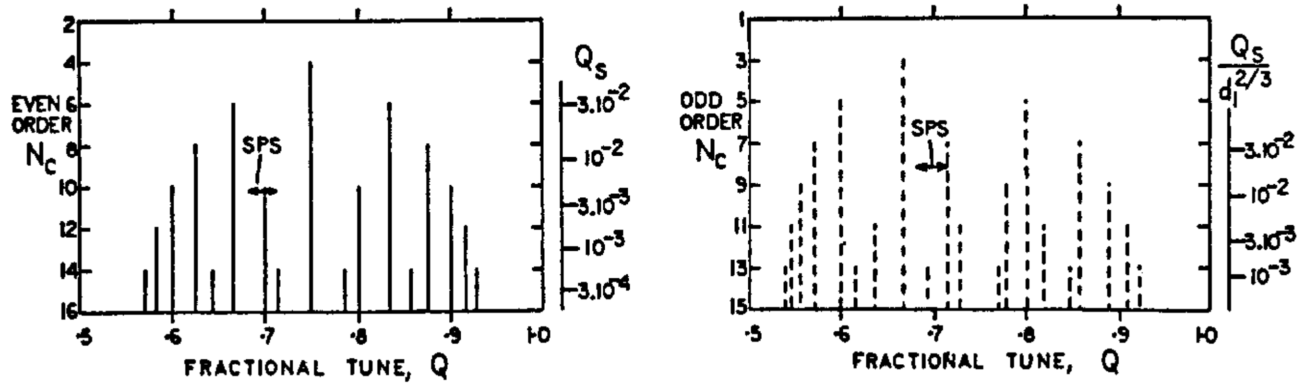

Figure 8: Critical resonances in the SPS collider operation. Even-order resonances are shown by solid lines and odd-order resonances by dashed ones. Lower order resonances are represented by taller nails. The tune spread is shown by the horizontal error bar. Its vertical position indicates the "critical resonance".

and beam-beam interactions in the SPS in Ref.[8]. The results of this study regarding critical resonances are summarized in Fig.8.

When the closed orbits of protons and antiprotons are separated the situation is completely different. Small amplitude particles as well as large amplitude particles can sample the nonlinear part of the beam-beam force. Fig.7 illustrates this situation. SPS data on long-range beam-beam interactions come from the 1988 and 1989 runs [22] [23] and experiments that investigated the effects of separation [25]. In one experiment a single proton bunch collided with two antiproton bunches. There were two head-on and two long-range interactions. The antiproton beam was scraped to achieve a small emittance in order to cause maximum effect on the proton beam. High background rates from large amplitude protons were the biggest concern in the SPS. This is why the proton beam was used as the probe.

In the SPS experiment, the proton emittance was larger than the antiproton emittance by a factor of 3, and intensities were 1.4×10^{10} and 7.5×10^{10} particles per bunch for antiproton and proton beams, respectively. At the long-range interaction points the separations were 5.9σ and 6.7σ when the separators were powered to full strength (100% separation). Tune scans were performed at 100% and 50% separations. During the tune scan with 50% separation, resonances of order 13 and 16 affected the background rates. Presence of the 13^{th} proved that long-range beam-beam interactions can indeed excite the odd-order resonances. During the tune scan with 100% separation, proton intensity lifetime decreased from an initial value of 100 hours to 80 hours on the 16^{th} and to 60 hours on the 13^{th}. This demonstrated that the 13^{th} order resonance can influence the tail particles even when beams are separated by 6σ.

4 Experience at FERMILAB

The first physics run of the TEVATRON collider was in 1987. During this run the luminosity lifetime was unexpectedly low (8 hours), due to transverse emittance growth rates of 8 πmm-mr/hr (95% definition). The causes of this emittance growth were found and fixed. In the 1988-89 TEVATRON collider run the luminosity lifetime was 15 hours in the beginning of a typical 30 hours store, increasing to 40 hours at the end of the store. During the 1988-89 run a peak luminosity of 2×10^{30} cm^{-2} sec^{-1} was reached and a total integrated luminosity of 9600 nb^{-1} was delivered to the CDF detector [26] [27] [28].

4.1 TEVATRON 1988-89 Collider Run

The 1988-89 TEVATRON Collider Run involved only head-on beam-beam interactions. Six antiproton bunches collided with six proton bunches, at 12 crossing points symmetrically distributed around the ring. Typical intensities were 7×10^{10} and 2.5×10^{10} particles per bunch, for protons and antiprotons, respectively. The normalized transverse proton emittance was typically 25 π mm-mr in both planes and the antiproton transverse emittance was typically 18 π mm-mr. The proton emittance was increased by artificial means to place the antiproton beam in the linear region of the beam-beam force. When the emittances were approximately the same the antiproton lifetime was shorter than the proton lifetime - antiprotons sampling the nonlinear part of the beam-beam force were influenced by resonances. This confirmed the self-scraping phenomenon observed in the SPS. By blowing up the proton emittance in a controlled manner the antiproton lifetime was improved, and a higher integrated luminosity was achieved. The unshifted horizontal and vertical tunes were near 19.41.

It is worth pointing out the difference between the operating points of the SPS and the TEVATRON colliders. The SPS operated between the 3^{rd} and the 10^{th} while the TEVATRON operated between 5^{th} and 7^{th} order resonances. The SPS working space covers the 13^{th} and the 16^{th} while TEVATRON's covers the 12^{th} and the 17^{th} order resonances. Fig.9 shows the tune spreads at various phases of the collider operations in the SPS and the TEVATRON.

The procedure of artificial enlargement of proton emittance caused proton loss from tails in the SPS while it was beneficial in the TEVATRON. The difference can be explained by the following facts.

In the SPS protons could not avoid the 13^{th} and the 16^{th} which were observed to be important for the large amplitude particles sampling the nonlinear beam-beam forces. In the TEVATRON, the proton tune spread was such that the large amplitude protons did not touch the 12^{th}. They

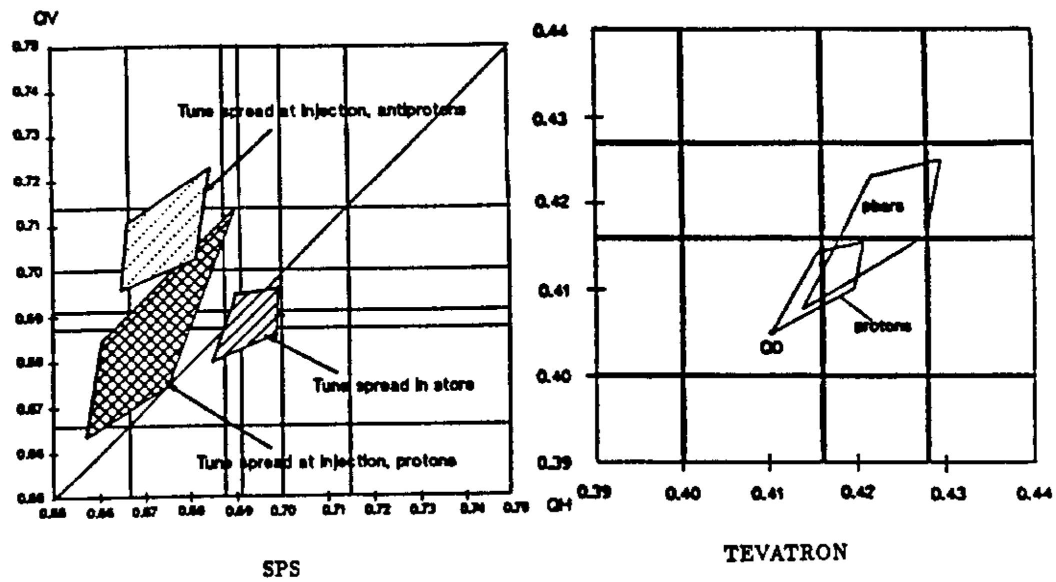

Figure 9: Tune spreads [23].

touched the 17^{th} order resonance - the higher the resonance order the lower the impact on beam lifetime.

The TEVATRON under storage conditions exhibits less tune modulation than SPS. In the TEVATRON a single low voltage power supply provides the current to all superconducting magnets, except the low-beta quadrupoles. Also, the effect of tune modulation due to synchrotron oscillations is smaller since the synchrotron tune of TEVATRON at 900 GeV is 0.0008, much smaller than the SPS value of 0.004. The combined effect of tune modulation and beam-beam interaction in the TEVATRON was studied in Ref.[29].

In the TEVATRON the tunes of protons and antiprotons were varied between the 5^{th} and the 7^{th} order resonances to find the optimum working point. During this run there was no mechanism to control proton and antiproton tunes independently. From Fig.9 it can be seen that when the tunes were moved up antiprotons touched the 7^{th} order resonance, and when the tunes were moved down protons touched the 5^{th} order resonance. These resonances affected the intensity lifetimes. The proton lifetime decreased while the antiproton lifetime was unaffected when the tunes were lowered towards 0.4. The opposite effect was observed when the tunes were moved up towards 0.4286.

The excitation of the 7^{th} order resonance is dominated by the beam-beam interaction. The contribution from the TEVATRON lattice is negligible[30]. On the other hand the 5^{th} order resonance is significantly excited by the TEVATRON lattice.

In both SPS and TEVATRON a beam-beam tune spread of 0.025 proved to be the limiting value. Operational experience dictates that the tune spread must be kept smaller than the tune space

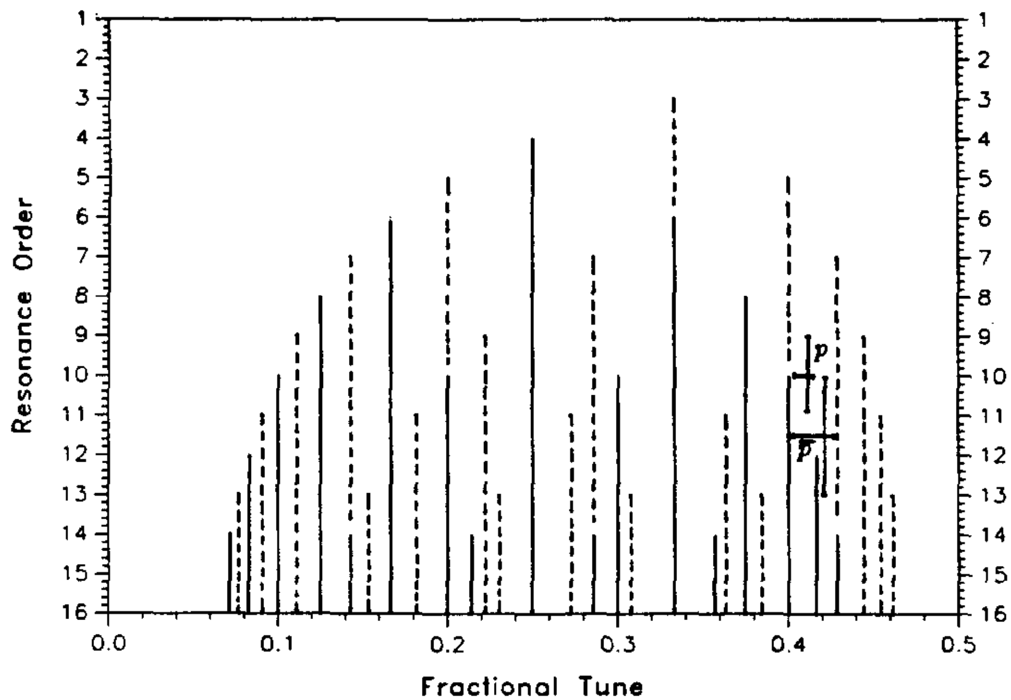

Figure 10: Critical resonances during the TEVATRON 1988-89 collider operation. The vertical error bar shows the range of critical resonances for particles in the range $\alpha = 2$ to $\alpha = 3$.

between the limiting resonances. For stored beams the limiting resonances for the TEVATRON were 2/5 and 3/7, a tune space of 0.0286. In the SPS the limiting resonances were 2/3 and 7/10 , a tune space of 0.0333. However, the width of the 3^{rd} order resonance is large in the SPS, since it is a low-order resonance, driven to first order in sextupole strength. Therefore the SPS could not use the whole 0.0333 tune space. Similarly the widths of 2/5 and 3/7 resonances in the TEVATRON force the total tune spread to be less than 0.025.

4.2 Long-Range Beam-Beam Interactions in the TEVATRON

In the following sections we will review the TEVATRON experiments that concentrated on the long-range beam-beam interactions. Details of these experiments can be found in Ref.[31]. All experiments were conducted at 150 GeV using the fixed target lattice configuration. Low-beta quadrupoles were not turned on. Two modules of electrostatic separators were available, at B17 and C48, providing 85μrad horizontal and vertical kicks, respectively. Since the phase advance between the B17 and C48 locations is approximately an integer multiple of 90° the orbits were helical, providing separation everywhere. The relative strength of these separators was not changed during the experiments. The helix amplitude and therefore the beam separation was changed by varying both separator voltages together.

When beams are separated in the TEVATRON they go through the chromaticity sextupoles

% Helix	Separation	Proton Lifetime [hrs]	Antiproton Lifetime [hrs]
0	0.0 σ	32.46 $\pm$ 0.50	-
20	0.6 σ	39.78 $\pm$ 1.09	7.69 $\pm$ 1.64
40	1.2 σ	48.43 $\pm$ 2.05	3.19 $\pm$ 0.22
60	1.8 σ	49.61 $\pm$ 5.50	1.79 $\pm$ 0.05
80	2.4 σ	-	6.92 $\pm$ 0.71
100	3.0 σ	48.96 $\pm$ 1.87	15.32 $\pm$ 3.08

Table 1: Lifetimes during the June 1989, 3x1 experiment.

off-axis and experience quadrupole fields (feed-down effect). Also, the superconducting TEVATRON magnets have a large sextupole moment (b_2) which dominates the feed-down effect [32]. To control the proton and antiproton tunes independently, "feed-down" sextupole circuits were instrumented in the TEVATRON. These "feed-down" sextupoles have no effect when beams circulate on the central orbit.

4.2.1 June 1989, TEVATRON 3x1 Experiment

A single antiproton bunch collided with 3 proton bunches at 150 GeV. Initial proton intensity was 7×10^{10} particles per bunch. Initial emittances were 20 and 10 πmm-mr for protons and antiprotons, respectively. The average value of β at 6 crossing points was approximately 31 meters and 91 meters in the horizontal and vertical planes, respectively. After injection separators were powered to 100% Helix, which corresponded to 3σ average separation. Beam lifetimes, emittance growth rates and tunes were measured at different separations. Table 1 reproduces the intensity lifetime data [31].

At 60% Helix (1.8σ average separation) the antiproton lifetime was significantly decreased. The beam-beam interaction is most nonlinear when the separation is near 1.6σ, therefore one expects a lifetime effect at 1.8σ separation if the tune is on one of the beam-beam resonances. The antiproton tune at 60% Helix was measured to be 0.412, close to the 5/12 resonance. However, the tunes were not adjusted by the feed-down sextupoles during this experiment. This meant that the beam-beam tune shift, and hence the instantaneous tunes were changing according to the changes in beam intensity and emittance. For instance, the proton tune was 0.42, very different from the antiproton tune. Therefore, 3x1 data were inconclusive about the excitation of the 12^{th} by long-range beam-beam interactions.

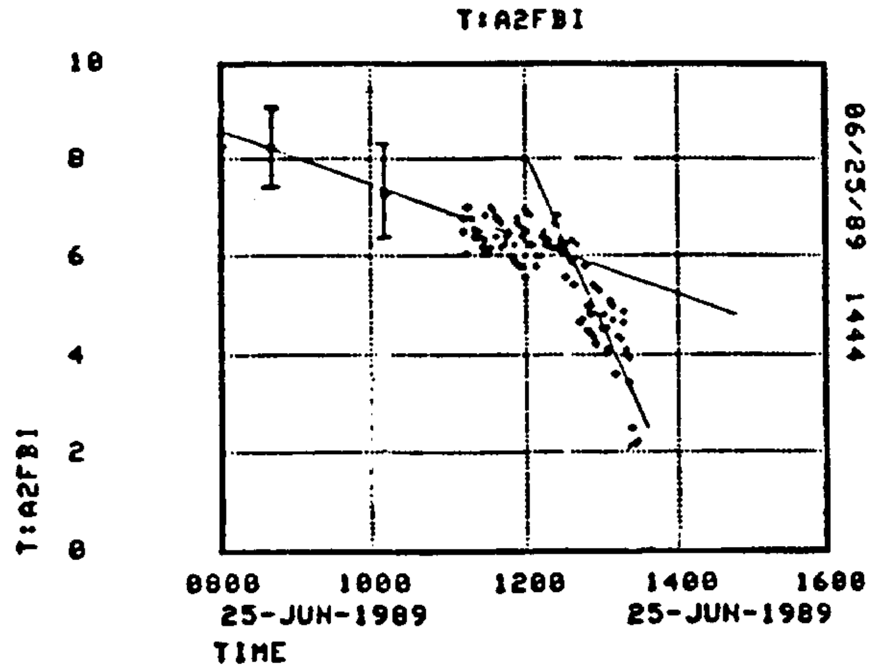

Figure 11: Antiproton intensity (particles per bunch, 10^9 scale) during the June 1989, 6x1 experiment.

4.2.2 June 1989, TEVATRON 6x1 Experiment

The 6x1 configuration was studied to answer questions regarding operational conditions that will exist in the 1992 Collider Run. Six proton bunches collided with a single antiproton bunch. Instead of a separation scan a tune scan was conducted. In other words the beam separation was held fixed at 100% (3σ average separation) and the tunes were adjusted using the feed-down sextupoles. Antiproton bunch intensity data from this experiment are presented in Fig.11.

The unperturbed bunch intensity lifetime is 13 hours in the TEVATRON at 150 GeV. This lifetime is caused by particles falling out of the rf buckets. With horizontal and vertical tunes sitting on the 12^{th} order resonance and with 3σ average separation the antiproton lifetime was measured to be 13 hours. No lifetime effect due to the excitation of the 12^{th} order resonance by long-range beam-beam interactions was observed.

This experiment suggests that the long-range beam-beam interactions will be benign at 3σ separation with the nominal 1992 beam parameters.

4.3 TEVATRON April 1990 Studies

The goal of this study period was to study long-range beam-beam interactions under the conditions that will exist during the 36x36 operation. Ideally, 36 proton bunches would collide with 36 antiproton bunches. In reality there will be 34 bunches per beam, as dictated by the abort kicker timing requirements. Proton and antiproton bunch intensities will be 50×10^{10} and 3×10^{10} particles per bunch, emittances will be 30 and 22 πmm-mr for protons and antiprotons, respectively. During April 1990 studies, 34 proton bunches collided with a single antiproton bunch. At 100% Helix, the average separation was 4.5σ. The helix was collapsed from 100% to 0% in 20% steps. At each step, total beam current, proton and antiproton bunch intensities, emittances, beam sizes and tunes were measured.

4.3.1 34x1 Tune Shift Measurements

The tune shift data are presented in Table 2. The relevant beam parameters are summarized in Table 3. The measurement of antiproton tunes was difficult, due to low bunch intensities. In order to increase the signal to noise ratio the antiproton bunch was excited coherently in the horizontal plane, using the TEVATRON Superdamper System, and tunes were read from the Schottky plate signals using a spectrum analyzer. Tunes were cross-checked with calculation by using the known currents from the feed-down circuits. Since the feed-down circuits have no effect at 0% Helix, the antiproton tunes were outside the working space (i.e. greater than 19.4286)

Vertical excitation of the antiproton bunch was not possible due to problems associated with the vertical superdamper kicker. Proton and antiproton tunes were differentiated by turning on the feed-down circuit and watching the tune lines for protons and antiprotons move in opposite directions. The beam-beam tune shifts were obtained by subtracting bare tunes (tunes measured when there is no beam-beam interaction while the feed-down circuit is on) from actual tunes.

Tune shifts were also simulated using the beam-beam code HOBBI [33]. The results are compared to data in Fig.13.

4.3.2 34x1 Lifetime and Emittance Measurements

The difference between proton and antiproton lifetimes was carefully observed during this experiment. Proton and antiproton tunes were kept on top of each other using the feed-down circuits. Therefore, any difference in lifetimes was due to long-range beam-beam interactions. Lifetime effects showed up at 60% and 40% Helix. Fig.14 and Fig.15 show the beam intensities as a function of time.

The agreement between simulation of tuneshift and data is very good with separated orbits. The discrepancy in the case of 0% Helix (68 head-on collisions) is striking. It should be mentioned that the spectrum analyzer measurement of the antiproton tune at 0% Helix could not be

% Helix	separation	Horz.Tune Shift (antiproton)
100	4.5σ	0.0025
80	3.6σ	-
60	2.7σ	0.0045
40	1.8σ	0.0131
20	0.9σ	0.0213
0	0.0σ	0.0425

Table 2: Antiproton tune shift data from April 1990, 34x1 experiment.

% Helix	ε_{p_x} [πmm-mr]	ε_{p_y} [πmm-mr]	$\varepsilon_{\bar{p}_x}$ [πmm-mr]	$\varepsilon_{\bar{p}_y}$ [πmm-mr]	protons/bunch [$\times 10^{10}$]
100	9.7	13.0	6.5	15.0	3.52
80	10.0	13.0	7.5	16.5	2.64
60	10.0	13.0	8.0	17.0	2.50
40	11.3	15.2	8.5	17.5	2.35
20	11.3	17.6	9.0	18.0	2.20
0	11.3	17.6	9.5	18.5	2.05

Table 3: Beam parameters used in the 34x1 simulation. Emittance values are given according to a 95% definition.

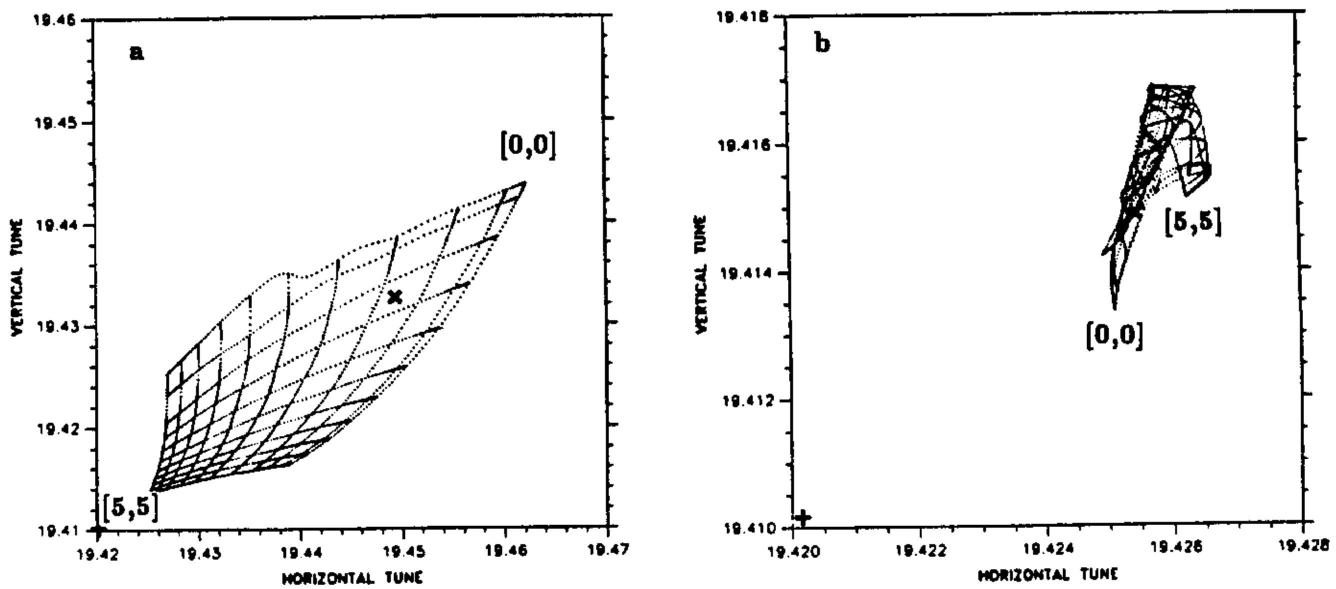

Figure 12: Tune shift footprint at a) 0% Helix and b) 60% Helix as calculated by HOBBI for the April 1990, 34x1 experiment. The footprint is generated for a mesh of amplitudes. The [0,0] and [5,5] refer to normalized amplitudes representing "small" and "large" amplitudes, respectively.

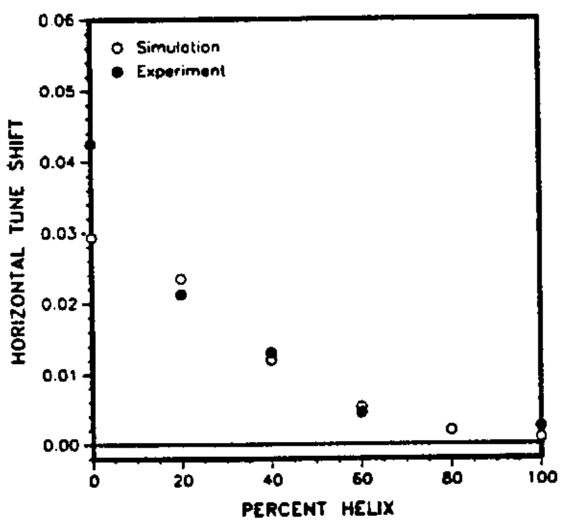

Figure 13: Comparison of tune shifts.

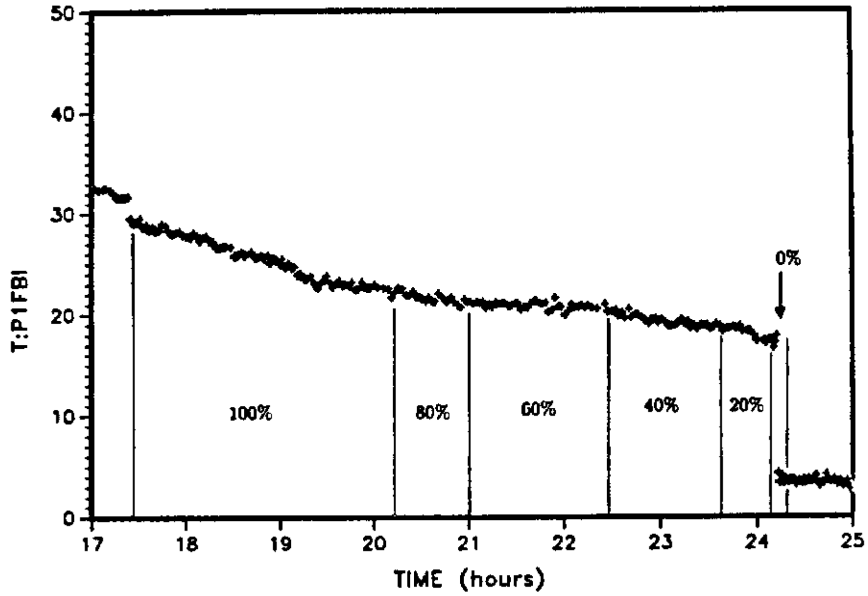

Figure 14: Proton intensity during the April 1990, 34x1 experiment. P1FBI is the number of protons in the P1 bunch (10^9 scale).

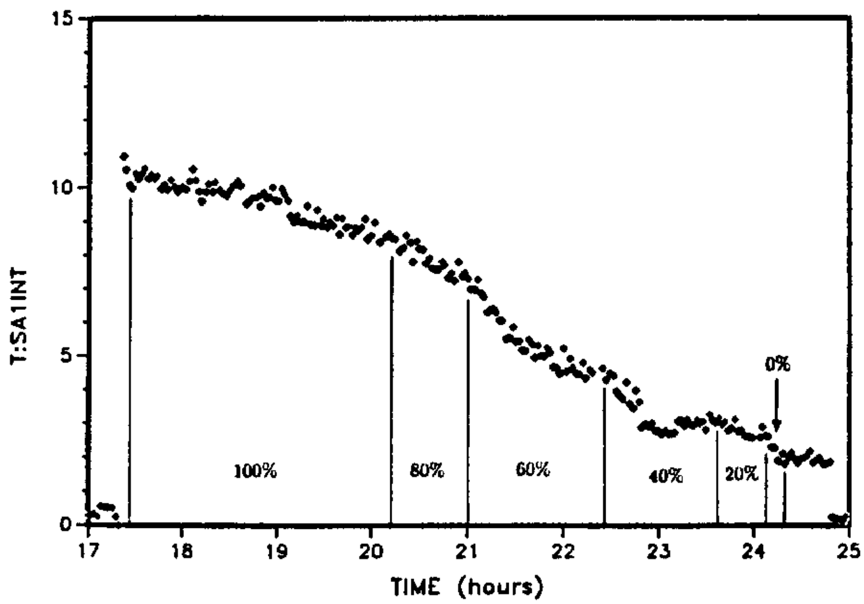

Figure 15: Antiproton intensity during the April 1990, 34x1 experiment. SA1INT is the number of antiprotons in the A1 bunch (10^9 scale).

confirmed using the feed-down currents. A plausible but unconfirmed explanation is that the tune corresponding to the π-mode of the coherent beam-beam oscillations was observed. Recall that the antiproton bunch was excited coherently during the experiment. Coherent beam-beam oscillations in the TEVATRON were observed[34] during the 1988-89 collider run.

The lifetime measurements showed clearly that at 60% Helix (2.7σ average separation) the antiproton lifetime decreased drastically. One does not expect a drastic lifetime effect at 2.7σ because in the simple beam-beam kick curve (Fig.6) this region is almost linear. One should also remember however that Fig.6 refers to a single interaction and that the separation is average. When the actual separation is taken into account for each interaction point the tuneshift-footprint shown in Fig.12.b results. This diagram illustrates a flip in the tuneshifts for small and large amplitude particles. When all beam crossings involve head-on interactions (0% Helix, Fig.12.a), small amplitude particles experience larger tune shifts. This relationship persists at 20% Helix and 40% Helix, but at 60% Helix the trend is reversed, and larger amplitude particles experience larger tune shifts. How is this related to the beam lifetime? A plausible answer lies in connecting beam lifetimes and resonance widths. Beam-beam resonance widths are inversely proportional to the square root of the slope of the detuning curve (Eq.(5)). The detuning curve resulting from Fig.12.b would be very distorted, facilitating large resonance-widths and, plausibly, short lifetimes.

Fig.15 shows that at 40% Helix (1.8σ average separation) there was also a lifetime effect. This effect can be attributed to the average separation being near 1.6σ, where the beam-beam kick is most nonlinear.

The 34x1 experiment has shown that the 36x36 scenario is viable provided the average separation is kept above 5σ separation. The proton bunch intensity will be much higher in the Collider Run with the Main Injector. This condition was studied in simulations which conclude that there is a beam-beam upper limit for proton bunch intensity, 40×10^{10} particles per bunch, coupled with a lower bound on proton emittance, 32 πmm-mr [35].

5 Conclusions

The beam-beam issues in three hadron colliders were reviewed. The beam-beam interaction was not a major problem in the ISR since it collided unbunched beams. The performance of bunched beam colliders, however, were seriously limited by the beam-beam interaction. Observations at the CERN SPS showed that the beam-beam interaction excites resonances up to order 16. The SPS experiments demonstrated the "self-scraping" phenomenon which was reproduced in the TEVATRON later. Other beam-beam experiments in the SPS showed the detrimental effects of having different emittances for protons and antiprotons, and the excitation of the 13^{th} order resonance in the case of separated beams.

Experiments on long-range beam-beam interactions in the TEVATRON have shown that there are two different processes that lead to beam loss. The first one occurs when the average separation is between 1.5-2σ. Particles experience very nonlinear fields and the resonances are excited. The second one occurs when the tune shift footprint exhibits a flip. Larger amplitude particles experience larger tune shifts. The distorted shape of the detuning curve enlarges the resonance-width.

Special attention was paid to the 12^{th} order resonance in the TEVATRON experiments. At 150 GeV, with proton and antiproton tunes sitting on the 12^{th} and with 3σ average separation, no lifetime effect was observed in the 6x1 experiment.

The long-range beam-beam experiments in the TEVATRON also demonstrated that in the 6x6 mode, with nominal intensities and emittances given for the 1992 collider run, even 3σ average separation can be tolerated. In the 36x36 mode, with the nominal beam parameters that can be provided by the Fermilab Main Injector, the average separation must be kept above 5σ.

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