

Additive versus Multiplicative Muon Conservation

INTRODUCTION

The question of the nature of muon conservation had been regarded as an unsolved curiosity for well over a decade, because it was not experimentally accessible. Recent progress in experimentation has finally made it possible to elucidate the answer.

The law of lepton conservation accounts for the absence of neutrinoless double-beta decay¹ but would allow muon to electron transitions such as

$$\begin{aligned}\mu^+ &\rightarrow e^+ \gamma \\ \mu^+ &\rightarrow e^+ e^- e^+ \\ \mu^- Z &\rightarrow e^- Z.\end{aligned}\tag{1}$$

The apparent absence² of these transitions led to the postulation of a new conserved quantum number, "muonness."³ The existence of a muon number was confirmed by the neutrino experiment of Danby *et al.*⁴ which demonstrated the distinct identity of neutrinos from pion decay (ν_μ) and neutrinos from nuclear beta decay (ν_e), by observing that

$$\nu_\mu Z \rightarrow Z \mu^-, \tag{2}$$

but

$$\nu_\mu Z \not\rightarrow Z e^-. \tag{3}$$

The possibility that muon conservation is a multiplicative law rather than an additive one was proposed by Feinberg and Weinberg⁵ and by Cabibbo and Gatto.⁶ Using the lepton numbers defined in Table I, one can require, in addition to the conservation of total lepton number

$$\Sigma L = \text{const}, \tag{4}$$

TABLE I
Lepton-number assignments

Particle	L	L_e	L_μ	P_μ
e^- , ν_e	1	1	0	1
μ^- , ν_μ	1	0	1	-1
e^+ , $\bar{\nu}_e$	-1	-1	0	1
μ^+ , $\bar{\nu}_\mu$	-1	0	-1	-1

either a new (charge-like) additive muon conservation law,

$$\Sigma L_\mu = \text{const}, \quad (5)$$

or a new (parity-like) multiplicative muon conservation law,

$$\Pi P_\mu = \text{const.} \quad (6)$$

There exist several formally equivalent ways of writing down either the additive or multiplicative law. A symmetric phrasing of the additive law is the separate conservation of electron and muon numbers ($\Sigma L_e = \text{const}$ and $\Sigma L_\mu = \text{const}$), which contains Eq. (5) and yields Eq. (4) since $L = L_e + L_\mu$. This version leads to a natural generalization for further lepton flavors, $\Sigma L_i = \text{const}$, $i = e, \mu, \tau, \dots$. The multiplicative law can be rewritten economically in terms of L_e and L_μ as $\Pi (-1)^{L_\mu} = \text{const}$ with $\Sigma (L_e + L_\mu) = \text{const}$. The symmetric formula $\Pi (-1)^{L_e} = \text{const}$ then follows as a result.⁷

The multiplicative and additive formulations are equally good *a priori*, since they both identically fulfil the requirement of prohibiting reactions (1) and reaction (3), but they do have different consequences. Of the two laws in Eqs. (5) and (6), the additive one is the more restrictive. The multiplicative law allows, but the additive law prohibits, muonium-antimuonium conversion⁸

$$\mu^+ e^- \leftrightarrow \mu^- e^+, \quad (7)$$

and muon decay with inverted subscripts on the neutrinos,

$$\mu^+ \rightarrow e^+ \nu_\mu \bar{\nu}_e, \quad (8)$$

while both laws allow muon decay with the conventional subscript assignment,

$$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu. \quad (9)$$

The observation of reactions (7) and (8), or their transposed equivalents

(such as $e^-e^- \rightarrow \mu^-\mu^-$ or $\bar{\nu}_\mu e^- \rightarrow \mu^-\bar{\nu}_e$) would signal a multiplicative law. We would then expect these two reactions to be weak interactions with a strength comparable to G_V^2 , the square of the weak coupling constant, $G_V \approx 1.17 \times 10^{-5}$ GeV $^{-2}$. For the charged-current reaction (8) the lifetime of the muon in fact puts a rigorous upper bound on the coupling. Since Eq. (8) is a partial decay mode of the muon, its branching ratio

$$R = \frac{\mu^+ \rightarrow e^+ \nu_\mu \bar{\nu}_e}{\mu^+ \rightarrow \text{all}} \quad (10)$$

is bounded by $0 \leq R \leq 1$. For the neutral current reaction (7) there is no corresponding rigorous bound. The new coupling could have a coupling strength greater than G_V^2 , although this would be unexpected.

Because of the symmetric appearance of decays (8) and (9), we might expect $R \approx 0.5$ in a multiplicative scheme. However, the selection rule cannot determine the dynamics; thus, a much smaller value of R has been predicted in a model theory by Derman.⁹

NEUTRAL CURRENTS: MUONIUM-ANTIMUONIUM EXPERIMENTS

For a standard V-A form of the interaction the probability of $M = \mu^+e^-$ converting to $\bar{M} = \mu^-e^+$ (Eq. 7) before decaying, *in vacuo*, is¹⁰

$$P(M \rightarrow \bar{M}) = 2.5 \times 10^{-5} (G_M/G_V)^2, \quad (11)$$

a respectable fraction for $G_M \sim G_V$. However, the conversion is strongly quenched in any finite-density target used to form the muonium, because the degeneracy between M and \bar{M} is broken by the external fields of collisions in a gas or the lattice in a crystal. The suppression factor is about $\sim 10^{-14}$ in a crystal and around 10^{-5} in a gas at one atmosphere.^{10,11}

It would certainly be desirable to look for $M \rightarrow \bar{M}$ conversion in a vacuum. Evidence for the observation of thermal muonium using thin gold foils in a vacuum has been reported by Barnett *et al.*¹² No formation of thermal muonium in a vacuum from thin foils was seen, however, in a similar experiment by Beer *et al.*¹³ Bolton *et al.*¹⁴ have presented evidence for production of fast muonium emerging into vacuum from a single foil.

Amato *et al.*¹⁵ have carried out a search for $M \rightarrow \bar{M}$ conversion in argon gas at the Nevis cyclotron. A μ^+ beam was stopped in a 1-atm Ar target, to form μ^+e^- . Upon conversion to μ^-e^+ the μ^- would be captured to form a muonic argon atom. The experiment therefore used the $2P \rightarrow 1S$ mu-mesic argon x ray to signal the conversion process. In this gas target the probability of conversion was

$$P(M \rightarrow \bar{M}) = 1.0 \times 10^{-10} (G_M/G_V)^2.$$

No antimuonium signal was seen in $5 \times 10^{+7} \mu^+$ stops on the target, corresponding to 4.2×10^6 muonium formations. The resulting lower limit on the coupling constant for $M \rightarrow \bar{M}$ (Eq. 7) is

$$G_M/G_V < 6800 \text{ (90\% confidence level).}$$

A search for the inverse process, $e^- e^- \rightarrow \mu^- \mu^-$, was carried out in colliding electron beams of 525 MeV/beam at the Princeton-Stanford electron-storage rings by Barber *et al.*¹⁶ Time-of-flight techniques were used to select beam-associated events, while lead absorbers and shower chambers distinguished muon and electron events. One $\mu^- \mu^-$ candidate passed the final selection criteria with an expected background of 3.7 ± 1.1 events. This result is consistent with no $M \rightarrow \bar{M}$ conversion and sets an upper limit

$$G_M/G_V < 610 \text{ (95\% confidence level).}$$

CHARGED CURRENTS: MUON-DECAY EXPERIMENTS

In order to differentiate muon-decay modes (8) and (9), it is necessary to observe the neutrinos from muon decay. This became possible with the high muon fluxes at the C.P. Anderson Meson Physics Facility (LAMPF). The charged current reactions

$$\nu_e Z \rightarrow e^- X, \quad (12)$$

and

$$\bar{\nu}_e Z \rightarrow e^+ X \quad (13)$$

distinguish ν_e from $\bar{\nu}_e$ because of the conservation of total lepton number (Eq. 4).

The first results on the multiplicative-law branching ratio R (Eq. 10) were obtained with the Gargamelle heavy-liquid bubble chamber in the CERN neutrino (antineutrino) beam, which is predominantly $\nu_\mu(\bar{\nu}_\mu)$ but has a small contamination of $\nu_e(\bar{\nu}_e)$ from muon and kaon decays. Using calculated ν_e and $\bar{\nu}_e$ fluxes from both sources, the observed events of reactions (12) and (13) were analyzed for the contribution of muon decay (8) and (9). Based on the analysis of 38 events Eichten *et al.*¹⁷ reported $R < 0.25$ (95% confidence level). A larger sample of 200 e^- and 60 e^+ events was examined by Blietschau *et al.*,¹⁸ who obtained the values of R shown in Table II. If we do a weighted average of those four values, we obtain $R = 0.13 \pm 0.15$.

TABLE II
Results for R from the Gargamelle experiment

ν exposure	$\bar{\nu}$ exposure
Excess e^+ : $R = 0.1 \pm 0.3$	Excess e^- : $R = 0.2 \pm 0.2$
Lack of e^- : $R = 0.00 \pm 0.6$	Lack of e^+ : $R = 0.3 \pm 0.6$

The neutrinos from a pure source of μ^+ decays at rest were observed in a neutrino experiment at LAMPF by a Yale/LASL/Saclay/SIN/Bern/NRC collaboration.¹⁹ The μ^+ originated from π^+ decay in the primary proton-beam stop where the π^- were captured before decaying into μ^- . A six-ton water Cerenkov counter²⁰ filled alternately with water and heavy water was used to observe $\bar{\nu}_e$ and ν_e by the elastic inverse beta-decay reactions



on the free protons in H_2O , and



on the deuterons in D_2O . The free proton in reaction (14) identified $\bar{\nu}_e$, since charge conservation prevents $\nu_e p$ inverse-beta decays. Neutrino events on oxygen were a negligible background because of Pauli-exclusion-principle effects; muon neutrinos were below the charged-current energy threshold.

Figures 1 and 2 show the beam-associated signal for D_2O and H_2O runs with 400 and 1000 Coulombs of protons, respectively. Low-energy-neutron

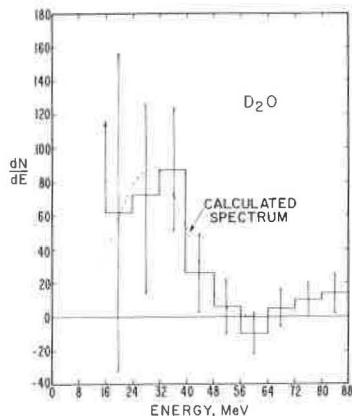


FIGURE 1. Yale/LASL-experiment neutrino spectrum: D_2O .

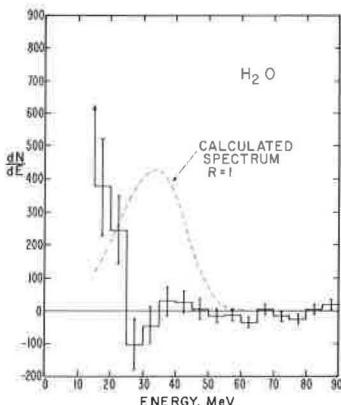


FIGURE 2. Yale/LASL-experiment neutrino spectrum: H_2O .

backgrounds are rejected by a 25-MeV energy cut. Figure 1 shows a 250-event neutrino signal in D_2O ; no corresponding signal is seen in the H_2O run (Figure 2). Comparison of the two spectra yields the value

$$R = 0.00 \pm 0.06$$

and clearly points to an additive law.

Neutrino-induced inverse muon decay provides an alternative means of searching for nonzero R . Chang²¹ obtained an upper limit of $G_M < 5G_V$ (corresponding to a nonphysical limit of $R < 25$) for the reaction $\nu_\mu Z \rightarrow \mu^+ e^- \bar{\nu}_e Z$ in a CERN neutrino beam. Jonker *et al.*²² have used the CHARM neutrino detector in the CERN wide-band neutrino beam to compare the rate of $\bar{\nu}_\mu e^- \rightarrow \mu^- \bar{\nu}_e$ (corresponding to Eq. 8) with that of $\nu_\mu e^- \rightarrow \mu^- \bar{\nu}_e$ (corresponding to Eq. 9) and reported $R < 0.09$ (90% confidence level). Table III summarizes the experimental limits on R in chronological order.

CONCLUSIONS

The neutral-current experiments have not yet yielded information about muonium-antimuonium conversion at the weak-interaction level.

TABLE III
Limits on multiplicative-law parameter R

Chang	1970	$R < 25$
Gargamelle	1973	$R < 0.25$, 95% confidence level
Gargamelle	1978	$R = 0.13 \pm 0.15$ ($R < 0.34$, 90% confidence level)
Yale/LASL	1980	$R = 0.00 \pm 0.06$ ($R < 0.10$, 90% confidence level)
CHARM	1980	$R < 0.09$, 90% confidence level

In the charged-current experiments we now have a clear picture. All the experiments agree that there is no evidence for a multiplicative law. The best limits, from the muon-decay neutrino experiment at LAMPF and from the inverse muon-decay experiment in the CERN neutrino beam, definitely exclude multiplicative law schemes with $R \sim 1/2$.

Unless the dynamics conspire to make a multiplicative law with very small R , we appear to live in a world with separately conserved additive lepton flavors.^{2,3}

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Acknowledgements

This work was supported in part by the US Department of Energy under contracts W-7405-ENG-48 and EY-76-C-3075

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