Siam Physics Congress 2022: Carbon Neutrality (SPC 2022) IOP Publishing
Journal of Physics: Conference Series **2431** (2023) 012090 doi:10.1088/1742-6596/2431/1/012090

Splitting of the atomic hydrogen 21-cm line induced by gravitational waves from a supermassive black hole binary

N Wanwieng[1], N Chattrapiban[1] and A Watcharangkool[2]

[1] Department of Physics and Materials Science, Faculty of Science, Chiang Mai University, Chiang Mai 50200, Thailand

[2] National Astronomical Research Institute of Thailand, Chiang Mai 50200, Thailand

E-mail: apimook@narit.or.th

Abstract. We investigate the characteristic of the hyperfine splitting of a ground-state hydrogen atom subjected to a field of a gravitational wave. In our consideration, hydrogen atoms are in the source's local wave zone. The interaction between the atoms and the gravitational wave induces the shifts of some energy levels in the hyperfine structure, computed by the first-order perturbation theory. The splitting of energy levels leads to more than one hyperfine transition. Our analysis suggests that a gravitational wave emitted by supermassive black hole binaries could be strong enough to induce a significant effect on the hyperfine splitting. This radio-wave spectrum provides the gravitational-wave signature and thus offers a way of identifying the merger of supermassive black hole binaries.

1. Introduction

To date, at least ninety gravitational-wave (GW) events have been detected by a network of ground-based GW detectors. All GW events so far come from merging binaries consisting of stellar-mass black holes and neutron stars [1]. However, there are other interesting GW sources, e.g. merger of supermassive black hole binaries, that the current ground-based detectors cannot detect due to limitations of sensitivity to ambient noise on Earth. Supermassive black hole (SMBH) binaries are the last stages of galactic mergers in which the two SMBHs form a Keplerian orbit [2]. A future space-based detector such as the laser interferometer space antenna (LISA) could detect GW produced by these sources. Unlike the mergers of stellar-mass black holes, the mergers of SMBHs resulting from the collision of two galaxies are expected to take place in rich gaseous environments [3]. They emit not only intense GWs but also EM waves, the latter originating from the cloud of interstellar gas of the two colliding galaxies. This feature significantly enhances the possibility of detecting the SMBH binaries from their EM counterpart. The observational techniques used to search for SMBH binaries so far have mostly relied on direct imaging, photometry, and spectroscopic measurements [4]. However, there are uncertainties related to the uniqueness of observational signature when two SMBHs nearly merge.

To seek the GW signature in the EM spectrum, in this paper, we investigate the effects of GW on the hyperfine structure of the ground-state hydrogen atom which is the source of the well-known 21-cm line of hydrogen. The hyperfine structure is due to the interaction of the magnetic moments associated with the spin of the proton and electron, which gives a slightly different magnetic energy for each spin state. As a result, the ground state has its energy split into two closely separated levels by 5.9×10^6 eV, corresponding to a wavelength of 21 cm or a frequency of 1420 MHz. This 21-cm line is one of the most ubiquitous and important observations in astrophysics [5].

2. The Hamiltonian for the hyperfine interaction with a GW

We consider a hydrogen atom floating freely in the local wave zone of a GW's source (in the source's asymptotic rest frame). Since a typical wavelength and period of GWs generated by astrophysical sources are much larger than the atomic size and the atomic lifetime of excited states. So, it is reasonable to assume that the GW field, $h_{\rm GW}$, at the location of the atom is spatially homogeneous and it is instantaneously constant in time (the quasi-static approximation). The origin of our reference frame is chosen to be at the position of the proton. Without loss of generality, we take the z-axis along the direction that the GW propagates through the atom. For simplicity, we take into account only the effect of a plus-polarized GW, described by $h_{\rm GW}$, so that the spacetime metric in the TT-gauge [6] takes the form

$$ds^2 = -dt^2 + (1 + h_{\rm GW})dx^2 + (1 - h_{\rm GW})dy^2 + dz^2. \tag{1}$$

When a GW passes through a hydrogen atom, it produces a tidal force that stretches and squeezes space between electron and proton so that the proper distance between the electron and proton changes as

$$r \quad \to \quad r\left[1 + \frac{h_{\rm GW}}{2}\left(\frac{x^2 - y^2}{r^2}\right)\right] = r\left[1 + \frac{h_{\rm GW}}{2}\sin^2\theta\cos 2\phi\right]. \tag{2}$$

In the last equality, the expression is written in terms of the usual spherical coordinate. Consequently, in the presence of GW, the hyperfine interaction Hamiltonian composes of two terms as

$$H_{\rm hf} = H^0_{\rm hf} + H'_{\rm GW}, \tag{3}$$

where

$$H^0_{\rm hf} = \frac{\mu_0 g_p e^2}{8\pi m_p m_e}\frac{3(\mathbf{S}_p \cdot \hat{r})(\mathbf{S}_e \cdot \hat{r}) - \mathbf{S}_p \cdot \mathbf{S}_e}{r^3} + \frac{\mu_0 g_p e^2}{3 m_p m_e}\mathbf{S}_p \cdot \mathbf{S}_e \delta^3(\mathbf{r}) \tag{4}$$

is the ordinary hyperfine Hamiltonian when there is no GW (see [7]), and

$$H'_{\rm GW} = -\frac{3h_{\rm GW}}{2}\frac{\mu_0 g_p e^2}{8\pi m_p m_e}\sin^2\theta\cos 2\phi\left[\frac{3(\mathbf{S}_p \cdot \hat{r})(\mathbf{S}_e \cdot \hat{r}) - \mathbf{S}_p \cdot \mathbf{S}_e}{r^3}\right] \tag{5}$$

represents the coupling of the atom with a GW, accurate up to first order of $h_{\rm GW}$. g_p is the proton g-factor. m_p and m_e are proton and electron mass. In general, $h_{\rm GW} \ll 1$, therefore, $H'_{\rm GW}$ is treated as a perturbation to $H^0_{\rm hf}$.

3. The energy levels

The energy of the $1s$-state hydrogen atom can be computed via the first-order perturbation theory. The unperturbed energies — the eigenenergies of $H^0_{\rm hf}$, is well known in quantum mechanics textbooks:

$$E^0_{\rm hf} = \begin{cases} +\beta/4 & \text{(triplet)} \\ -3\beta/4 & \text{(singlet)} \end{cases} \quad \text{where } \beta \equiv \frac{4 g_p \hbar^4}{3 m_p m_e^2 c^2 a^4} = \frac{4}{3} g_p \left(\frac{m_e}{m_p}\right) m_e c^2 \alpha^4 \tag{6}$$

(α is the fine structure constant). The energy correction is obtained by solving the eigenvalue problem of the 4×4 matrix

$$\langle i| H'_{\mathrm{GW}} |j\rangle = -\kappa\gamma h_{\mathrm{GW}} I_{ij}, \quad (i, j = 1,\, 2,\, 3,\, 4), \tag{7}$$

where we have defined

$$\gamma \equiv \frac{\mu_0 g_p e^2 \hbar^2}{20\pi a^3 m_p m_e} = \frac{g_p}{5}\left(\frac{m_e}{m_p}\right) m_e c^2 \alpha^4,\ \kappa \equiv \int_{r_0}^{\infty} \frac{1}{r^3} e^{-2r/a} r^2 dr. \tag{8}$$

Here r_0 is the cut-off radius with value around the radius of proton, and

$$I_{ij} \equiv \langle i| S_{px}S_{ex} - S_{py}S_{ey} |j\rangle\,. \tag{9}$$

$S_{px}, S_{py}, S_{ex}, S_{ey}$ are the spin operators along x, y axes acting on the proton and the electron states. The base states for two spin-1/2 particles to be used in the calculation are as follows

$$\begin{aligned}
|1\rangle &= |0\ 0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_p|\downarrow\rangle_e - |\downarrow\rangle_p|\uparrow\rangle_e\right)\\
|2\rangle &= |1\ -1\rangle = |\downarrow\rangle_p|\downarrow\rangle_e\\
|3\rangle &= |1\ 0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_p|\downarrow\rangle_e + \frac{1}{\sqrt{2}}|\downarrow\rangle_p|\uparrow\rangle_e\right)\\
|4\rangle &= |1\ 1\rangle = |\uparrow\rangle_p|\uparrow\rangle_e.
\end{aligned}$$

For ease of notation, $|\uparrow\rangle \equiv |s = 1/2, m_s = +1/2\rangle$ and $|\downarrow\rangle \equiv |s = 1/2, m_s = -1/2\rangle$. It is found that the non-vanishing elements are $I_{24} = I_{42} = \hbar^2/2$. Therefore, the perturbation matrix reads

$$-h_{\mathrm{GW}}\frac{\kappa\gamma\hbar^2}{2}\begin{pmatrix} 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 \end{pmatrix}. \tag{10}$$

Diagonalize this matrix yields the eigenvalues $\Delta E^1_{\mathrm{GW}} = 0,\ \pm\kappa\gamma h_{\mathrm{GW}}$. The total energies and eigenstates of the splitting levels are shown in table 1. The energy-level diagram is drawn in figure 1. The differences in energy between the state $|I\rangle$ and the others are $\beta, \beta \pm \kappa\gamma h_{\mathrm{GW}}$ correspond to the transition wavelengths

$$\lambda_0 = \frac{hc}{(4/3)\, g_p\, (m_e/m_p)\, m_e c^2 \alpha^4}, \quad \lambda_\pm \cong \lambda_0 \left[1 \pm (3\kappa/20) h_{\mathrm{GW}}\right], \tag{11}$$

where λ_0 is the wavelength of the ordinary hyperfine transition whereas $\lambda_\pm$ are the transition wavelength caused by a GW.

4. Discussion

In the presence of GW, the spin-singlet state of the atom is left unchanged, whereas the triplet states are split into three distinct energy levels separated equally from the level when there is no GW. The transition wavelength of the splitting lines are proportional to the GW field which is determined by the source's parameters (see [8] for details). The order-of-magnitude of the GW strength produced by a compact binary system orbiting at separation distance d apart can be estimated from the quadrupole formula

$$h_{\mathrm{GW}} \sim \frac{G^2 M^2}{2c^4 r d} = 10^{-13}\left(\frac{M}{M_\odot}\right)\left(\frac{r_S}{d}\right)\left(\frac{1\ \mathrm{ly}}{r}\right)\,, \tag{12}$$

Table 1. Total energies and eigenstates of the splitting levels.

Total energy	Eigenstate
$E_I = -(3/4)\beta$	$\lvert I\rangle = \lvert 1\rangle$
$E_{II} = \beta/4 + \kappa\gamma h_{\rm GW}$	$\lvert II\rangle = \frac{1}{\sqrt{2}}(-\lvert 2\rangle + \lvert 4\rangle)$
$E_{III} = \beta/4$	$\lvert III\rangle = \lvert 3\rangle$
$E_{IV} = \beta/4 - \kappa\gamma h_{\rm GW}$	$\lvert IV\rangle = \frac{1}{\sqrt{2}}(\lvert 2\rangle + \lvert 4\rangle)$

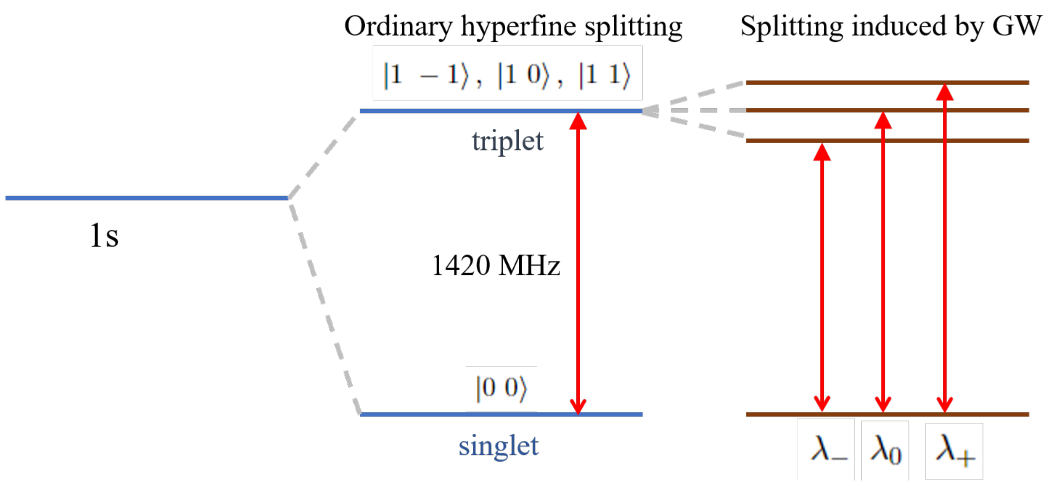

Figure 1. The energy-level diagram of the hyperfine splitting of ground-state hydrogen atom under gravitational wave.

where $r_S = 2GM/c^2$ is the Schwarzchild radius associated with the source's total mass M and r is the distance from the source to an observational point. In the perturbation regime, we expect $h_{\rm GW} \sim 10^{-2} - 10^{-4}$. The sources that could generate such strength should be supermassive black hole binaries. For example, the GW at distance ~ 1 ly from the center of SMBH binary with $M \sim 10^{10} M_\odot$, $d \sim r_S$ will give the strength $\sim 10^{-3}$. For this strength, the maximum and minimum wavelength in the hyperfine transition (Eq. (11)) are 21.1253 cm and 21.0645 cm, respectively, which could be measured by current instruments and techniques in radio astronomy. In fact, the pulsar timing Arrays (PTAs) is targeting the GW from SMBH binaries with masses $\sim 10^8 - 10^{10} M_\odot$ at separation distance $\sim 10^2 - 10^3 r_S$ [9]. Thus, the combined observation of PTAs and high-resolving radio spectroscopy could verify our prediction of the splitting 21-cm lines induced by the GW.

5. Conclusion

We investigated the effect of a GW in the hyperfine structure of the ground-state hydrogen atom. The interaction between the atom and the GW causes a splitting in the energy level of the spin-triplet states into three distinct levels. The energy shifts are proportional to the GW field, analogous to the Zeeman effect. When the atom is in a GW, there are three lines of the hyperfine transition instead of a single line. These radio-wave spectral lines provide the signature for identifying supermassive black hole binaries which produce strong enough GWs.

Acknowledgement
This research was supported by the Program Management Unit for Human Resources and Institutional Development, Research and Innovation [grant number B05F640051].

References

[1] Abbott R *et al* 2021 *Preprint arXiv:2111.03606*
[2] Enoki M, Inoue K T, Nagashima M and Sugiyama N 2004 *Astrophys. J.* **615** 19
[3] Barnes J E and Hernquist L E 1991 *Astrophys. J.* **370** L65–8
[4] Rosa D *et al* 2019 *New Astron. Rev.* **86** 101525
[5] Pritchard J R and Loeb A 2012 *Rep. Prog. Phys.* **75** 086091
[6] Hartle J B 2021 *Gravity: An Introduction to Einstein's General Relativity* (Cambridge: Cambridge University Press)
[7] Griffiths D J and Schroeter D F 2018 *Introduction to Quantum Mechanics* (Cambridge: Cambridge University Press) p 398
[8] Thorne K S 1995 *Preprint gr-qc/9506086*
[9] Foster III R S 1990 *Constructing a Pulsar Timing Array* (Berkeley: University of California)