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TOWARDS SCREENING OF COLOR FIELD IN INSTANTON LIQUID

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The effect of external colour field impact on the instanton liquid is studied. In the course of this study the corresponding effective Lagrangians are derived for both regimes of weak and strong external field and in long wave-length approximation. The example of Euclidean colour point-like source is analyzed in detail and the feedback of field on the instanton liquid is estimated as a function of source intensity.

1 Introduction

The declarations of discovering new state(s) of matter in relativistic heavy ion collisions at RHIC which are actively wandering in the papers nowadays are sometimes based on the results of different nature. From one side it is the striking result of direct experimental measurements of a strong suppression (comparing to pp and pA collisions) of particle production at high transverse momentum well-known as a jet quenching. And although the jet reconstruction in these experiments is a nontrivial task the (and accompanying) result(s) is(are) interpreted as a degradation of hard parton (initiating a jet) energy induced by medium (new thermalized matter) produced in collision long before hadronizing in the QCD vacuum. On the other hand the convincing success of phenomenological analysis of the other measurable characteristics based on the perfect liquid hydrodynamics results in the question about the sort of quark-gluon plasma (QGP) (if produced) and intimately related one about the origin of the QCD vacuum Ref.[1]. These investigations devoted to exploring collisions of ultrarelativistic heavy ions and aimed at producing quark-gluon plasma under laboratory conditions pose the interesting problem of studying the effect of intense gluon fields on the QCD physical vacuum. It is assumed that such fields can be generated in the collision process within a relatively macroscopic region and that they can be described in the semiclassical approximation. Currently available models of radiative gluon fields rely on various premises, but they do not provide an unambiguous and sound prediction for the intensity of the field (see, for example, [2]). These difficulties could have been sidestepped if the detailed structure of the physical vacuum had been known. The corresponding threshold value could then have been extracted on the basis of knowledge of characteristic vacuum-fluctuation fields. Unfortunatelly, we know at the present time only rather general features of the physical vacuum, such as gluon condensates, preliminary data on virtualities [3], and data from some lattice simulations [4]. In this situation, it only remains to estimate relevant effects on the basis of some plausible models of the QCD vacuum. The instanton liquid model seems to be of great value in this respect. The present study is devoted to describing the effects of the screening of an external color field precisely within this model. In general, this formulation of the problem may seem somewhat unexpected from the point of view of the model, since the additional components are introduced in an instanton liquid in order to describe the confining component and to remove simultaneously the problem of large-size instantons [5, 6].

In the case of a week external field, we adopt a diametrically opposite approach in a sense, assuming that it will play as if subordinate role. Nevertheless, the conclusions at which we are arrive will perfectly correspond to the phenomenology of strong interactions. An instanton liquid will be considered within the simplest approximation – a the stochastic ensemble of instantons in the singular gauge. The generating functional is estimated on the basis of the variational principle proposed in Ref.[7]. Comparative simplicity of the superposition ansatz and variational procedure allows us to analyze the effects almost analytically, but, in principle, our analysis is applicable to any other saturating configuration. Further, we proceed to estimate the effect of a strong external field on the instanton liquid. We consider the simplest model problem of an Euclidean pointlike color source in order to get an idea of the characteristic scale of the phenomenon.

2 External weak field in an instanton liquid

As a major configuration saturating the generating functional

$$Z = \int D[\mathcal{A}]\; e^{-S(\mathcal{A})} \tag{1}$$

where $S(\mathcal{A})$ is a standard Yang-Mills action we take the approximate solution for the Yang-Mills equations in the form of the following superposition

$$\mathcal{A}^a_\mu(x) = B^a_\mu(x) + \sum_{i=1}^{N} A^a_\mu(x;\gamma_i), \tag{2}$$

here A^a_μ implies the field of (anti-)instantons in the singular gauge

$$A^a_\mu(x) = \frac{2}{g}\,\omega^{ab}\bar\eta_{b\mu\nu}a_\nu(y), \quad a_\nu(y) = \frac{\rho^2}{y^2+\rho^2}\frac{y_\nu}{y^2}, \quad y = x - z, \tag{3}$$

with the parameters $\gamma_i = (\rho_i, z_i, \omega_i)$ describing the i-th instanton of the ρ size centered at the pseudo-particle coordinate z, with the matrix of colour orientation ω, and g denotes the coupling constant of non-abelian field; for the anti-instanton the 't Hooft symbols should be changed according to $\bar\eta \to \eta$; and $B^a_\mu(x)$ is an external field. As was indicated in introduction, we are interested in quite a specific configuration generated in heavy-ion collisions rather than in an arbitrary external field. The localization of this field within a nuclear-size scale and its semiclassical character might be features peculiar to such configurations. An analogy with electrodynamics suggests that such fields can be approximately described by means of a multipole expansion. It is precisely this qualitative pattern that we will use here as a guideline.

The non-abelian strength tensor from external field and individual pseudo-particle is defined by

$$G^a_{\mu\nu}(\mathcal{A}) = \partial_\mu\mathcal{A}^a_\nu - \partial_\nu\mathcal{A}^a_\mu + g\; f^{abc}\mathcal{A}^b_\mu\mathcal{A}^c_\nu = G^a_{\mu\nu}(B) + G^a_{\mu\nu}(A) + G^a_{\mu\nu}(A,B) \tag{4}$$

with entirely anti-symmetric tensor f^{abc}, where the first two terms in the second relation correspond to standard strength tensors of non-abelian field. In particular,

$$G^a_{\mu\nu}(A) = -\frac{4}{g}\,\omega^{ak}\bar\eta_{k\alpha\beta}M_{\mu\alpha}M_{\nu\beta}\frac{\rho^2}{(y^2+\rho^2)^2}, \tag{5}$$

where $M_{\mu\nu} = \delta_{\mu\nu} - 2\hat y_\mu\hat y_\nu$, $\hat y_\mu = y_\mu/|y|$. The 'mixed' component of the instanton strength field looks like

$$G^a_{\mu\nu}(A,B) = g\,f^{abc}(B^b_\mu A^c_\nu - B^b_\nu A^c_\mu) = g\,f^{abc}\omega^{cd}\frac{2}{g}(B^b_\mu\bar\eta_{d\nu\alpha} - B^b_\nu\bar\eta_{d\mu\alpha})a_\alpha(y). \tag{6}$$

Calculating now G^2 we receive the partial contributions of external field and each separate pseudo-particle as

$$\begin{aligned} G^a_{\mu\nu}G^a_{\mu\nu} &= G^a_{\mu\nu}(B)G^a_{\mu\nu}(B) + G^a_{\mu\nu}(A)G^a_{\mu\nu}(A) + G^a_{\mu\nu}(A,B)G^a_{\mu\nu}(A,B) \\ &+2G^a_{\mu\nu}(B)G^a_{\mu\nu}(A) + 2G^a_{\mu\nu}(B)G^a_{\mu\nu}(A,B) + 2G^a_{\mu\nu}(A)G^a_{\mu\nu}(A,B). \end{aligned} \tag{7}$$

In order to keep the further steps as simple and transparent as possible we limit ourselves with the standard sum of partial contributions in the superposition ansatz action and hold the highest in IL density (precisely in packing fraction parameter $n\rho^4$) one particle contributions

$$S(B,\gamma) = \int dx\;\frac{G^a_{\mu\nu}\;G^a_{\mu\nu}}{4} \simeq \sum_i \int dx\;\frac{G^a_{\mu\nu}(i)\;G^a_{\mu\nu}(i)}{4}. \tag{8}$$

The crossing terms of different pseudo-particles (which are proportional to the IL density squared) are neglected here because of very small packing fraction parameter characteristic to IL, i.e. $n\rho^4 \sim 0.01$. Thus, the regularized generating functional for the IL model takes the following form (for denotations see Ref.[7])

$$Y = \int D[B]\;\frac{1}{N!}\int\prod_{i=1}^{N} d\gamma_i\;\; e^{-S(B,\gamma)}. \tag{9}$$

First we consider the case of weak external field. We assume that the characteristic parameters of the instanton liquid, such as average pseudoparticle size $\bar\rho$ and the IL density n, do not change, coinciding with their vacuum magnitudes. For the saturating configuration chosen here, these values of the pseudoparticle size is immaterial.

In order to avoid cumbersome expressions, we therefore assume that all pseudoparticles have the same size, $\bar{\rho}$. Those are fixed by some repulsive mechanism (see, however, the remark at the end of paper) for the particular choice of saturating configuration done above[1]. In calculating the generating functional (10), it therefore only remains to perform averaging over the pseudoparticle positions and color orientations.

In order to calculate the effective action, it is necessary to find the contribution of the fields of quantum fluctuation in the vicinity of the saturating configuration (1). By convention, this contribution can be written in terms of the running coupling constant as a function of the external field and characteristic pseudoparticle size, $g(\rho, B)$. With the aid of this quantity, one can correctly go over to the relevant scale. For the goals pursued in the present study, however, it is sufficient to use an approximate expression that is obtained upon the substitution $g(\bar{\rho}, B) \to g(\bar{\rho})$. Indeed, the fields at short distances (where according to our assumption, the external field is concentrated) are not singular by virtue of asymptotic freedeom. Dangerous singularities may arise at long distances, but an ensemble of pseudoparticles controls the situation there. Thus, we will describe the external field less accurately (but we do not aim at reaching a high accuracy here) but will not miss dangerous singular contributions. It turns out that even this extremely simple estimate of generating functional at the saddle point leads to the emergence of an infrared singularity, and we now proceed to describe it.

Making use the cluster decomposition we obtain the corresponding average of exponential as

$$\langle \exp(-S) \rangle_{\omega z} = \exp\left(\sum_k \frac{(-1)^k}{k!} \langle\langle S^k \rangle\rangle_{\omega z} \right), \qquad (10)$$

where $\langle S_1 \rangle = \langle\langle S_1 \rangle\rangle$, $\langle S_1 S_2 \rangle = \langle S_1 \rangle \langle S_2 \rangle + \langle\langle S_1 S_2 \rangle\rangle, \ldots$. The first cumulant is simply defined by the action averaged. Taking into account the direct form of field strength tensors (4) and (5) it is evident that the following terms will only be present in the partial contribution after averaging over colour orientation

$$\langle G^a_{\mu\nu} G^a_{\mu\nu} \rangle_\omega = G^a_{\mu\nu}(B) G^a_{\mu\nu}(B) + \langle G^a_{\mu\nu}(A) G^a_{\mu\nu}(A) \rangle_\omega + \langle G^a_{\mu\nu}(A,B) G^a_{\mu\nu}(A,B) \rangle_\omega + 2 \langle G^a_{\mu\nu}(A) G^a_{\mu\nu}(A,B) \rangle_\omega. \quad (11)$$

The colour averaging is performed by the help of equality

$$\langle \omega^{ak} \omega^{cd} \rangle = \frac{\delta^{ac} \delta^{kd}}{N_c^2 - 1}, \qquad (12)$$

implying N_c as the number of colours. Averaging over the pseudo-particle positions results in the following integral

$$\int \frac{dz}{V}\ a_\alpha(y)\ a_\gamma(y) = \delta_{\alpha\gamma}\ \frac{1}{V}\ \frac{\pi^2}{4}\ \rho^2, \qquad (13)$$

because the basic IL parameters, as we agreed, are unchanged. Handling the 'mixed' component average we have it in the form as reads (all the other terms disappear)

$$\langle G^a_{\mu\nu}(A,B)\ G^a_{\mu\nu}(A,B) \rangle_{\omega z} = \frac{18\ \pi^2\ \rho^2}{V} \frac{N_c}{N_c^2 - 1}\ B^b_\mu\ B^b_\mu, \qquad (14)$$

Finally, collecting all appropriate terms we find the effective action for the external field in IL as

$$\langle\langle S \rangle\rangle_{\omega z} = \int dx\ \left(\frac{G(B)\ G(B)}{4} + \frac{m^2}{2}\ B^2 \right) + N\ \beta, \qquad (15)$$

$$m^2 = 9\pi^2\ n\ \rho^2\ \frac{N_c}{N_c^2 - 1}, \qquad (16)$$

here N is the full number of particles in volume V with $n = N/V$ and a single pseudo-particle action $\beta = 8\pi^2/g^2$. The last term of Eq.(16) introduces the contribution of purely instanton component $\langle G(A) G(A) \rangle_{\omega z}$. The contribution of repulsive term which fixes the pseudo-particle size in IL is omitted in Eq.(16) so long as it is not a principal point in this context and adding it, leads to the insignificant correction to the last condensate term in Eq.(16). An amusing point is that the mass term of Eq.(17) has been well-known for rather long time and as a matter of fact fixing the pseudo-particle size in the variational procedure of Ref.[7] is provided just by this mechanism of mass generation. With the characteristic IL parameters ($N_c = 3$ and number of flavours $N_f = 2$) $n/\Lambda^4_{QCD} = 1.2$, $\bar{\rho}\Lambda_{QCD} = 0.27$, $\beta = 18$, see for example [32], the mass estimate is $m \sim 440$MeV for $\bar{\rho} \sim 1$GeV and Λ_{QCD} in the interval of 200 — 300 MeV. The screening properties of the repulsive interaction were highlighted in [7], and the value of $m \sim 350$MeV was presented there for the screening mass. The studies of Hütter [10], where the estimate $m \sim 480$MeV was obtained for the mass of the gluon in an instanton medium,

[1]In the literature three mechanisms for fixing ensemble of pseudoparticles are discussed: repulsive [7]; freezing of the coupling constant [8]; stabilization due to influence of confining vacuum component

is also worthly of note. One can see that all these estimates are rather close since, in all of the cases, the effect arises owing to the mixed term in the field strength (5). We also note that the compatibility conditions for the equations resulting from Eq.(16) is $\partial_\mu B_\mu = 0$ which is satisfied by the pseudo-particle field Eq.(9) as well. There may arise the question of why pseudoparticles oriented at random in color space lead to screening — which component plays the role of a distribution function. In the present case (in non-Abelian theory), this is the exponential function featuring the Yang–Mills action functional. A nontrivial contribution originating from mixed term in the field strength (5) is generated in it. In the Abelian case, there are no such contributions by virtue of the superposition principle.

Turning now to the next term of cluster decomposition to calculate the effective Lagrangian corrections we conclude immediately that in the second cumulant

$$\frac{1}{2}\left\langle\left\langle\int dx_x \frac{G\,G}{4}\int dx_2\frac{G_2\,G_2}{4}\right\rangle\right\rangle\ , \tag{17}$$

there are two nontrivial terms

$$\frac{1}{2}\left\langle\int dx_1\, 2\,\frac{G^a_{\mu\nu}(B)G^a_{\mu\nu}(A)}{4}\int dx_2\, 2\,\frac{G^b_{\alpha\beta}(B_2)G^b_{\alpha\beta}(A_2)}{4}\right\rangle, \tag{18}$$

$$\frac{1}{2}\left\langle\int dx_1\, 2\,\frac{G^a_{\mu\nu}(A)G^a_{\mu\nu}(A,B)}{4}\int dx_2\, 2\,\frac{G^b_{\alpha\beta}(A_2)G^b_{\alpha\beta}(A_2,B_2)}{4}\right\rangle, \tag{19}$$

here the index 2 underlines the fact that corresponding functions are dependent on x_2. The remaining terms originate from either the interference terms (and are cancelled by the contribution of the first cumulant squared) or lead to the contributions anharmonic in B which are not in our interest for this paper. It was analyzed for the first time in Ref.[11] that $G(B)G(A)$ in (6) generates the dipole interaction. However, this interaction does not manifest itself in the first term of cluster decomposition if the averaging over the colour orientation is performed. It comes into focus starting on the second order of decomposing. In particular Eq.(10) can be presented in the following form

$$\frac{1}{2}\left\langle\int dx_1\, 2\,\frac{G^a_{\mu\nu}(B)G^a_{\mu\nu}(A)}{4}\int dx_2\, 2\,\frac{G^b_{\alpha\beta}(B_2)G^b_{\alpha\beta}(A_2)}{4}\right\rangle_\omega =$$
$$= \frac{1}{2}\,\frac{1}{N_c^2-1}\int dx_1\,dx_2\,\frac{G^a_{\mu\nu}(B)G^b_{\alpha\beta}(B_2)}{4}\;G^b_{\mu\nu}(A)G^b_{\alpha\beta}(A_2), \tag{20}$$

if one exploits Eq.(4) and Eq.(12) keeping in mind that $G^b_{\mu\nu}(A)G^b_{\alpha\beta}(A_2)$ is colour independent because of the identity $\omega^{ab}\omega^{ac} = \delta^{bc}$. Eq.(20) should be also averaged over the pseudo-particle positions which results in the correlation function for the instantons in singular gauge developing the following form obtained in Ref.[1]

$$\int \frac{dz}{V}\,G^a_{\mu\nu}(A)G^a_{\alpha\beta}(A_2) = \frac{1}{V}\,\frac{16}{g^2}\,(\delta_{\mu\alpha}\delta_{\nu\beta}-\delta_{\mu\beta}\delta_{\nu\alpha}+\varepsilon_{\mu\nu\alpha\beta})\ I_s\left(\frac{\Delta}{\rho}\right), \tag{21}$$

where $\Delta = |x_1 - x_2|$, and for the anti-instanton the substitution $\varepsilon \to -\varepsilon$ should be done. The analytical form of function I_s is not our priority here, however, it is shown in Fig.1. If the numbers of instantons and anti-instantons are balanced then the term proportional to the tensor ε disappears.

Now collecting the terms together we find the contribution of Eq.(10) in the IL approach as

$$\frac{16}{g^2}\,\frac{1}{N_c^2-1}\,n\int dx_1dx_2\ I_s\left(\frac{\Delta}{\rho}\right)G^a_{\mu\nu}(B)G^a_{\mu\nu}(B_2). \tag{22}$$

Clearly, it leads to an abatement of initial action and it is more convenient for analyzing to present the non-local factor of dielectrical susceptibility type in the Fourier components Ref.[11]

$$\int dk\left(1-\frac{16}{g^2}\,\frac{1}{N_c^2-1}\,n\,\widetilde{I}_s(k\rho)\right)G^a_{\mu\nu}[B(k)]G^a_{\mu\nu}[B(-k)]. \tag{23}$$

Numerical estimate of $\widetilde{I}_s(k\rho)$ at the zero value of argument is $\widetilde{I}_s(0)\sim 6\rho^4$, and at $N_c = 3$, $N_f = 2$ the correction coefficient can be estimated as

$$\kappa = \frac{16}{g^2}\,\frac{1}{N_c^2-1}\,n\,\widetilde{I}_s(0)\sim 0.013. \tag{24}$$

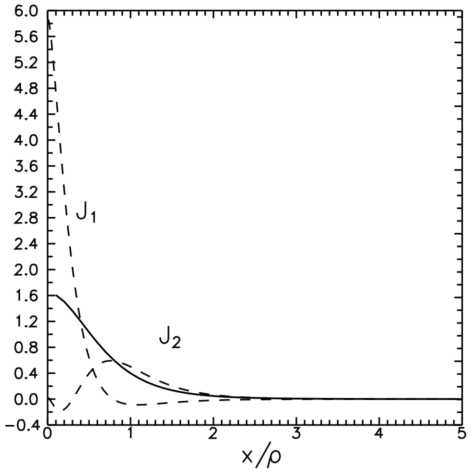

Figure 1. Correlation function I_s is given by solid line and the correlation functions J_1 and J_2 are given by the dashed lines.

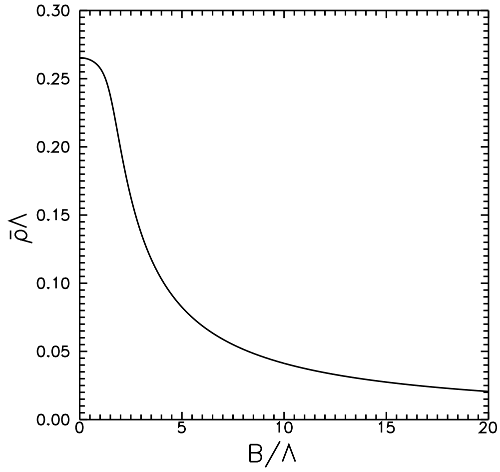

Figure 2. Pseudo-particle mean size as a function of applied external field.

Analyzing now the term Eq.(11) we present it as

$$\frac{1}{2}\left\langle\int dx_1\ 2\ \frac{G^a_{\mu\nu}(A)\ G^a_{\mu\nu}(A,B)}{4}\int dx_2\ 2\ \frac{G^b_{\alpha\beta}(A_2)\ G^b_{\alpha\beta}(A_2,B_2)}{4}\right\rangle_\omega =$$
$$=\frac{1}{2}\ \left\langle\int dx_1\ dx_2\ \omega^{ak}\ G^k_{\mu\nu}(A)\ f^{amn}\ \omega^{nl}(B^m_\mu\ \bar\eta_{l\nu\gamma}-B^m_\nu\ \bar\eta_{l\mu\gamma})\ a_\gamma\times\right.$$
$$\left.\times\omega^{bc}\ G^k_{\alpha\beta}(A_2)\ f^{bde}\ \omega^{ef}(B^d_{2\alpha}\ \bar\eta_{f\beta\delta}-B^d_{2\beta}\ \bar\eta_{f\alpha\delta})\ a_{2\delta}\right\rangle_\omega . \quad (25)$$

and imply the dependence of G on the colour matrix ω might be given by the common factor (without introducing new symbol for G). Formally, this term looks like the next one expanding in $1/N_c$, i.e. ($\sim\omega^4$). However, using the identity for colour matrices $f^{man}\omega^{ak}\omega^{nl}=\varepsilon^{klg}\omega^{mg}$, we have $\langle f^{man}\omega^{ak}\omega^{nl}f^{dbe}\omega^{bc}\times\omega^{ef}\rangle=\delta^{md}\left(\delta^{kc}\delta^{lf}-\delta^{kf}\delta^{lc}\right)/(N_c^2-1)$, and then Eq.(21) receives the following form

$$\frac{1}{2}\left\langle\int dx_1\,2\ \frac{G^a_{\mu\nu}(A)\ G^a_{\mu\nu}(A,B)}{4}\int dx_2\ 2\ \frac{G^b_{\alpha\beta}(A_2)\ G^b_{\alpha\beta}(A_2,B_2)}{4}\right\rangle_\omega =$$
$$=2\ \frac{1}{N_c^2-1}\ \int dx_1dx_2\ \left[G^k_{\mu\nu}(A)G^k_{\alpha\beta}(A_2)\bar\eta_{l\nu\gamma}\bar\eta_{l\beta\delta}-G^k_{\mu\nu}(A)G^l_{\alpha\beta}(A_2)\bar\eta_{k\beta\delta}\bar\eta_{l\nu\gamma}\right]a_\gamma a_{2\delta}B^m_\mu B^m_{2\alpha}, \quad (26)$$

The lower line here develops this form because of an asymmetric property of tensor G. Averaging over the pseudo-particle positions we may extract the correlation function in the following form

$$\int\frac{dz}{V}\ \left[\ G^k_{\mu\nu}(A)\ G^k_{\alpha\beta}(A_2)\ \bar\eta_{l\nu\gamma}\bar\eta_{l\beta\delta}-G^k_{\mu\nu}(A)\ G^l_{\alpha\beta}(A_2)\ \bar\eta_{k\beta\delta}\bar\eta_{l\nu\gamma}\right]\ a_\gamma a_{2\delta}=$$
$$=\frac{16}{g^2}\ \frac{1}{V}\ \left[J_1\left(\frac{\Delta}{\rho}\right)\ \delta_{\mu\alpha}+J_2\left(\frac{\Delta}{\rho}\right)\ \hat\Delta_\mu\hat\Delta_\alpha\right], \quad (27)$$

where $\hat\Delta=x_2-x_1/|x_2-x_1|$ is the unity vector.

The simple algebra allows us to calculate the functions

$$J_1=\int dy\frac{\rho^8(16t^3-8t+4pq+6(p^2+q^2)t-12t^2pq)}{3(y^2+\rho^2)^3(z^2+\rho^2)^3|y|\,|z|},$$
$$J_2=\int dy\frac{4\rho^8(4t^3+5t-4pq-6(p^2+q^2)t+12t^2pq)}{3(y^2+\rho^2)^3(z^2+\rho^2)^3|y|\,|z|}, \quad (28)$$

with $z=y+\Delta$, $t=\frac{(y\ z)}{|y||z|}$, $p=\frac{(y\ \Delta)}{|y||\Delta|}$, $q=\frac{(z\ \Delta)}{|z||\Delta|}$. Similarly to I_s we do not need their explicit forms here but one may estimate their behaviours looking at the dashed lines in Fig.1. Finally, the additional contribution to the mass term reads as

$$\frac{1}{N_c^2-1}\ \frac{32}{g^2}\ n\int dx_1dx_2\left[J_1\left(\frac{\Delta}{\rho}\right)\delta_{\mu\alpha}+J_2\left(\frac{\Delta}{\rho}\right)\hat\Delta_\mu\hat\Delta_\alpha\right]B^a_\mu B^a_{2\alpha}, \quad (29)$$

and in the Fourier components as

$$\int dk \left[\frac{m^2}{2} - \frac{32}{g^2}\,\frac{1}{N_c^2-1}\; n\left(\widetilde{J}_1(k\rho)\delta_{\mu\alpha} + \widetilde{J}_2(k\rho)\hat{k}_\mu\hat{k}_\alpha\right)\right] B^a_\mu(k)B^a_\alpha(-k). \tag{30}$$

Estimating numerically the nonlocal correction to the mass we find out that $\widetilde{J}_1(0) \sim -1.4\;\rho^2$ unlikely above result. Then the mass term and corresponding correction in Eq.(22) come about at zero momentum $9\pi^2\;N_c$ and $(-\frac{4\beta}{\pi^2}\;1.4)$, respectively. At the characteristic value $\beta \sim 18$ it means the quantitative correction smallness or, globally, the corrections initiated by the second term of cumulant expansion are negligible at the contemporary values of basic IL parameters. There is another contribution to the effective Lagrangian which comes from the interaction of sources generating the external field with (anti-)instanton superposition

$$S_{int} = \sum_{i=1}^{N} \int\; dx\; j^a_\mu(x) A^a_\mu(x;\gamma_i)\;.$$

Making use the cluster decomposition one expects the possibility to calculate corresponding small contributions (if the sources are treated in the quasiclassical approximation) which are given by the correlation functions of the form $\langle A^a_\mu(x;\gamma)A^b_\nu(y;\gamma)\rangle_\gamma$ Ref.[11].

To conclude this section, we will consider, for the effective Lagrangian in (16) a somewhat different interpretation following which one can obtain the infrared singularity mentioned at the beginning of this section, see also Ref.[13]. Let us suppose that the quasi-classical field B is described in the infra-red momentum region by the initial Yang-Mills action without the term breaking down gauge symmetry as before. In particular, we consider the field of point-like Euclidean source of intensity e with only one non-zero n-th component

$$B^a_\mu(x) = (\mathbf{0}, \delta^{an}\;\varphi),\;\; \varphi = \frac{e}{4\pi}\;\frac{1}{|\boldsymbol{x}|}\;.$$

Then B^2 integrated over the 4-dimensional space gives

$$\int dx \left(\frac{e}{4\pi\;|\boldsymbol{x}|}\right)^2 = \frac{e^2}{4\pi}\;X_4\;L\;,$$

where X_4, L are some formal upper limits of corresponding integrals. In this approach the contribution of the first cumulant Eq.(16) could be written down as

$$\langle\langle S\rangle\rangle_{\omega z} = E\;X_4, \quad E = \frac{e^2}{4\pi}\;\frac{1}{r_0} + \sigma\,L + \beta\,n\,L^3, \tag{31}$$

with $\sigma = \frac{9\pi}{8}\frac{N_c}{N_c^2-1}\;e^2\;n\overline{\rho^2}$. The first term in defining E comes from the Coulomb energy of point-like source and r_0 represents a formal particle radius. The last term is originated by the gluon condensate and the previous term looks like negligibly small correction to the condensate term. However, this contribution linearly increasing with L is proportional to e^2 and has different physical meaning as a term additional to the self-energy of source. In other words, it demonstrates an impossibility for the source with an open colour to be available in IL because the amplitude of such a state is very strongly suppressed (e^{-S}) comparing to the condensate contribution if the screening effects are not taken into account. For the dipole in 'isosinglet' (s) and 'isotriplet' (t) states (i.e. $N_c = 2$) we obtain

$$B^a_\mu(x) = (\mathbf{0}, \delta^{a3}\;\varphi),\;\; \varphi = \frac{e}{4\pi}\;\left(\frac{1}{|\boldsymbol{x}-\boldsymbol{z}_1|} \mp \frac{1}{|\boldsymbol{x}-\boldsymbol{z}_2|}\right)\;,$$

where $\boldsymbol{z}_1$, $\boldsymbol{z}_2$ are the dipole coordinates what leads to

$$\int dx\;B_s^2 = \frac{e^2}{4\pi}\;X_4\;l\;, \qquad \int dx\;B_t^2 = \frac{e^2}{4\pi}\;X_4\;(4\;L - l)\;,$$

with $l = |\boldsymbol{z}_1 - \boldsymbol{z}_2|$ to be the distance separating sources. We have another confirmation of suppression effect for the states with open colour in IL, i.e. the energy of 'isosinglet' dipole state increases with l enlarging and the corresponding coefficient is $\sigma \sim 0.6$ GeV/fm if we take $e \sim g$. In principle the same situation for arbitrary oriented in color space charges is valid, as comes from analysis of corresponding exact solution obtained in [14].

Thus, we are quite allowed to conclude the regime of weak external field in IL is described by effective Lagrangian Eq.(16) and basic IL parameters are within a well adapted interval. Moreover all the corrections originated by the second cumulant should be certainly neglected.

3 Long-wave approximation for a strong field

In the preceeding section, we have derived the effective action for a weak external field under the assumption that the parameters describing the state of the instanton liquid remain unchanged, but we did not formulate a corresponding criterion of weakness of the external field. In the case of a strong external field, the validity of the naive approximate solution to the Yang–Mills equations (1) is naturally questionable, since a substantial distortion of the pseudoparticle fields may be expected here. In order to estimate these effects, we have investigated in detail the behaviour of an (anti)instanton in the field of an Euclidean pointlike color source. The instanton-like configurations (9) having a variable size $\rho \to R(x,z)$ and a variable color orientation $\omega^{ab} \to \Omega^{ab}(x,z)$ were considered in [15]. The singular nature of the solution used in the instanton liquid model for the pseudoparticles makes it possible to apply the multipole expansions of deformation fields; that is,

$$\begin{aligned} R_{in}(x,z) &= \rho + c_\mu\ y_\mu + c_{\mu\nu}\ y_\mu\ y_\nu + \dots, \qquad |y| \le L, \\ R_{out}(x,z) &= \rho + d_\mu\ \frac{y_\mu}{y^2} + d_{\mu\nu}\ \frac{y_\mu}{y^2}\ \frac{y_\nu}{y^2} + \dots, \qquad |y| > L, \end{aligned} \tag{32}$$

(the same concerns instanton orientation in color space $\Omega(x,z)$). Here, L is a parameter that determines the radius of the sphere where the multipole expansion increasing with distance gives way to a decrising one, in accordance with requirement that the deformations be regular. The coefficients $c_\mu, c_{\mu,nu}, \dots$ and $d_\mu, d_{\mu,nu}, \dots$ are functions of the external field and are determined by solving the corresponding variational problem. It can easily be seen that, at nonzero coefficients, the opposite parts of a pseudoparticle may have different sizes and different color orientations. In view of this, we reffered to these configurations as crumpled instantons. Investigations revealed that, in the problem being considered, there appears a characteristic scale that is on the same order of magnitude as the pseudoparticle size and at which deformations become significant, but repulsion effects remain dominant. In a rough approximation, we can discard deformations completely since the instanton liquid density decreases fast at short distances from the source, as well see below. We also note that the deformation fields are of interest in themselves because they make it possible to describe excited states of instanton liquid [16].

We are going to modify slightly the variational procedure of Ref.[7] to implement possibility of the changing IL parameters. We retain here the same designations to demonstrate precisely where the changes are introduced and imply $S(B,\gamma)$ in Eq.(10) in the following form

$$S(B,\gamma) = -\sum \ln d(\rho_i) + \beta\ U_{int} + \sum U_{ext}(\gamma_i, B) + S(B). \tag{33}$$

The first term here describes one-instanton contributions with the following distribution function over the (anti-)instanton sizes

$$d(\rho) = C_{N_c} \Lambda^b_{QCD}\ \rho^{b-5} \widetilde{\beta}^{2N_c}, \tag{34}$$

where $b = \frac{11}{3}N_c - \frac{2}{3}N_f$, $\widetilde{\beta} = -b\ln(\Lambda_{QCD}\ \bar\rho)$, $\Lambda_{QCD} = \Lambda_{\overline{MS}} = 0.92\Lambda_{P.V.}$) with C_{N_c} dependent on renormalization scheme

$$C_{N_c} \approx \frac{4.66\ \exp(-1.68 N_c)}{\pi^2 (N_c-1)!(N_c-2)!}.$$

The second term of Eq.(33) is responsible for providing pseudo-particles with repulsive interaction which fixes their sizes. The characteristic single instanton action is defined on the scale of average pseudo-particle size $\beta = \beta(\bar\rho)$ where $\beta(\rho) = -\ln C_{N_c} - b\ln(\Lambda_{QCD}\ \rho)$.

The partial pseudo-particle contributions grouped in the third term and we take only

$$U_{ext}(\gamma_i, B) = \int dx\ \frac{G^a_{\mu\nu}(A_i,B)\ G^a_{\mu\nu}(A_i,B)}{4},$$

because the other contributions at the standard IL parameters are small as we have seen. At last, the fourth term represents simply the Yang-Mills action of the B field

$$S(B) = \int dx\ \frac{G^a_{\mu\nu}(B)\ G^a_{\mu\nu}(B)}{4}.$$

The well-known property of exponential makes it possible to estimate the generating functional of Eq.(10) with the approximating functional as

$$Y \ge Y_1\ \exp(-\langle S - S_1 \rangle), \tag{35}$$

where

$$Y_1 = \int D[B]\ \frac{1}{N!} \int \prod_{i=1}^{N} d\gamma_i\ \ e^{-S_1(B,\gamma) - S(B)}\ , \quad S_1(B,\gamma) = -\sum \ln \mu(\rho_i),$$

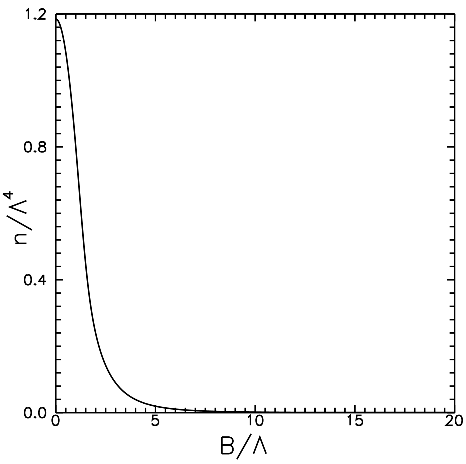

Figure 3. The IL density as a function of applied external field.

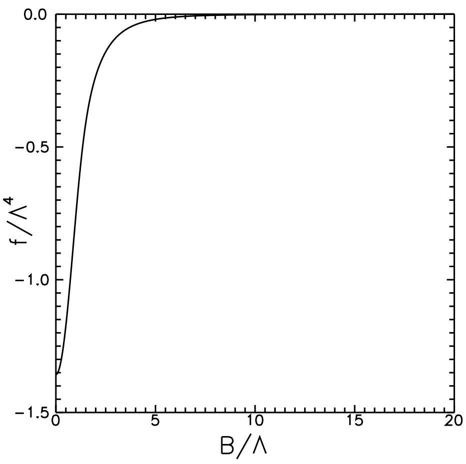

Figure 4. Free energy density as a function of external field B.

and $\mu(\rho)$ is an effective one-particle distribution function which may be derived with the variational procedure. In our particular situation a mean value of corresponding difference is given by

$$\begin{aligned}\langle S - S_1\rangle &= \frac{1}{Y_1 N!}\int\prod_{i=1}^{N} d\gamma_i[\beta\, U_{int} + U_{ext}(\gamma, B) - \sum \ln d(\rho_i) + \sum \ln \mu(\rho_i)]e^{\sum \ln\mu(\rho_i)} \\ &= \frac{N}{\mu_0}\int d\rho\, \mu(\rho) \ln\frac{\mu(\rho)}{d(\rho)} + \frac{\beta}{2}\frac{N^2}{V^2\mu_0^2}\int d\gamma_1 d\gamma_2\, U_{int}(\gamma_1,\gamma_2)\mu(\rho_1)\mu(\rho_2) + \int dx\,\frac{N}{V\,\mu_0}\int d\rho\,\mu(\rho)\rho^2\zeta\, B^2 \\ &= \int dx\; n\,\left(\frac{1}{\mu_0}\int d\rho\;\mu(\rho)\;\ln\frac{\mu(\rho)}{d(\rho)} + \frac{\beta\xi^2}{2}\; n\left(\overline{\rho^2}\right)^2 + \zeta\overline{\rho^2}\; B^2\right),\end{aligned} \tag{36}$$

with $\zeta = \frac{9}{2}\frac{\pi^2}{}\frac{N_c}{N_c^2-1}$, $\xi^2 = \frac{27}{4}\frac{N_c}{N_c^2-1}\pi^2$, $\mu_0 = \int d\rho\; \mu(\rho)$. Here we estimate the functional in the adiabatic (long wave-length) approximation. It means we consider the IL elements of some characteristic size (of the same order of magnitude as the mean distance between pseudo-particles) being equilibrated by the presence of some fixed field B. Then calculating the optimal configurations of pseudo-particles we found out the effective action in the mean field. Eq.(36) is given just in the form underlining that an integration is performed over liquid elements and the proper parameters describing their states could be dependent on the external field, i.e. could be the functions of coordinate x. Physical meaning of such a functional is quite transparent, it implies that each separate element of IL possesses a characteristic aptitude of screening external field assessed by U_{ext}.

Calculating the variation of $\langle S - S_1\rangle$ in $\mu(\rho)$ we have

$$\mu(\rho) = C\, d(\rho)\, e^{-(n\beta\xi^2\overline{\rho^2}+\zeta\, B^2)\rho^2},$$

where C is an arbitrary constant and we fix it demanding the coincidence of its value when the external field is absent with its vacuum average. Then

$$\mu(\rho) = C_{N_c}\widetilde{\beta}^{2N_c}\Lambda^b_{QCD}\rho^{b-5}\; e^{-(n\beta\xi^2\overline{\rho^2}+\zeta\;B^2)\rho^2}. \tag{37}$$

and making use the definition of an average as

$$\overline{\rho^2} = \frac{\int d\rho\;\rho^2\;\mu(\rho)}{\mu_0},$$

we obtain the practical relation between mean pseudo-particle size and the IL density

$$(n\;\beta\;\xi^2\;\overline{\rho^2} + \zeta\;B^2)\;\overline{\rho^2} \simeq \nu, \tag{38}$$

where $\nu = (b-4)/2$. Apparently, it results in a well-known form of pseudo-particle size distribution

$$\mu(\rho) = C_{N_c}\widetilde{\beta}^{2N_c}\Lambda^b_{QCD}\rho^{b-5}\; e^{-\nu\;\rho^2/\overline{\rho^2}}. \tag{39}$$

Now Eq.(38) allows us to formulate the criterion we are interested in. It looks like $\zeta B^2 \ll n\;\beta\;\xi^2\;\overline{\rho^2}$ and for the IL parameters mentioned above it is $B \ll 400$ MeV.

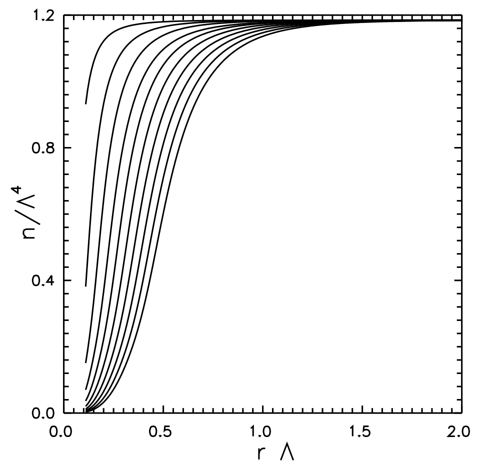

Figure 5. The IL density a a function of r for different values of intensity. Extreme right hand side line corresponds to $e/4\pi = 1$. Going to the left corresponds to changing $e/4\pi$ with a pace of 0.1 up to $e/4\pi = 0.1$ what corresponds to extreme left hand side line.

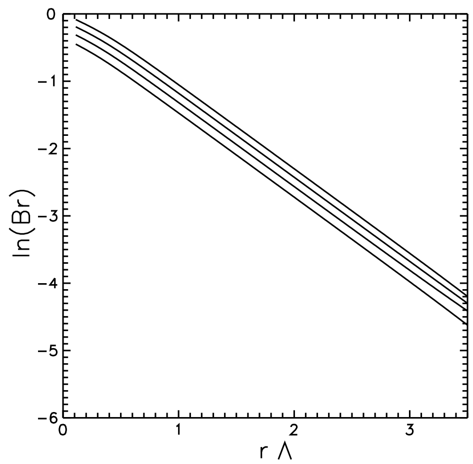

Figure 6. $ln(Br)$ as a function of r for four various solutions. The upper line corresponds to $e/4\pi = 1$. Going down the lines correspond to decreasing $e/4\pi$ with spacing 0.1.

Dealing with Eq.(36) and Eq.(39) the generating functional estimate Eq.(35) may be presented as

$$Y \geq \int D[B]\; e^{-S(B)}\; e^{-F}, \tag{40}$$

$$F = \int dx\; n\; \left\{ \ln \frac{n}{\Lambda^4_{QCD}} - 1 - \frac{\nu}{2} + \frac{\zeta\;\overline{\rho^2}\;B^2}{2} - \ln[\Gamma(\nu)\;C_{N_c}\;\widetilde{\beta}^{2N_c}] - \nu\;\ln\frac{\overline{\rho^2}}{\nu} \right\}.$$

Making use of the relation Eq.(38) it is not difficult to find the maximum of functional Eq.(40) in the IL parameters at the fixed B value as a solution of transcendental equation ($\frac{dF}{d\bar\rho} = 0$). As an information we give the simple expression of its derivative in n

$$F'_n = \ln\frac{n}{\Lambda^4_{QCD}} + \frac{1}{4}\;\frac{n^2\xi^4\beta\;b\;(\overline{\rho^2})^3}{2n\beta\;\xi^2\;\overline{\rho^2}\;\zeta\;B^2 - n\;\xi^2\;\frac{b}{2}\;\overline{\rho^2}} - \ln[\Gamma(\nu)\;C_{N_c}\;\widetilde{\beta}^{2N_c}] - 2N_c\;n\;\frac{\widetilde{\beta}'_n}{\widetilde{\beta}} - \nu\;\ln\frac{\overline{\rho^2}}{\nu}.$$

Fig.2 and Fig.3 demonstrate the solutions for $\bar\rho$ and n at $N_c = 3$ and $N_f = 2$ as the functions of field B. Fig.4 shows the plot of free energy density f/Λ^4_{QCD} where $F = \int dx\; f$ and convinces IL is steady as to an impact of external field. At strong external field the IL parameters are given by the following asymptotic formulae

$$\overline{\rho^2} \simeq \frac{\nu}{\zeta\;B^2}\;\left(1 - \frac{n\;\nu\;\beta\xi^2}{\zeta^2\;B^4}\right), \qquad n \simeq \frac{\Gamma(\nu)\;C_{N_c}\;\widetilde{\beta}^{2N_c}}{(\zeta\;B^2)^\nu}\;\left(1 + \frac{\Gamma(\nu)\;C_{N_c}\;\widetilde{\beta}^{2N_c}}{(\zeta\;B^2)^\nu}\;\frac{N_c\;b\;\nu\;\beta\;\xi^2}{\zeta^2\;B^4}\right).$$

This regime starts somewhere around $B\Lambda^{-1}_{QCD} \sim 10$ at all the plots given.

Thus, the effective action for the B field is given by the following nonlinear functional

$$S_{eff} = \int dx \left(\frac{G^a_{\mu\nu}(B)\;G^a_{\mu\nu}(B)}{4} + f[B] \right). \tag{41}$$

This functional makes possible to calculate the external field as a function of x and IL parameters $\bar\rho[B]$ and $n[B]$.

It is interesting to note that the variant of variational principle applied here makes it possible obtain self-consistent description of the instanton ensemble, with sligtly deviated parameters in comparison with singular instanton profile, see Ref. [17].

4 Charged sphere in an instanton liquid

To get any estimate of the IL feedback on the presence of external field could be very practical for instanton liquid model. If so let us try to extract such an estimate from very simple example. Now we will search the minimum of effective action resolving the following boundary value problem

$$\triangle_r B = \frac{df[B]}{dB}, \quad B|_{r=r_0} = p(e), \quad \nabla_r B\,|_{r=r_0} = -\frac{e}{4\pi}\frac{1}{r_0^2}.$$

The source intensity here is controlled by e, and parameter r_0 sets a radius of colour ball which we take as $\sim 0.1\bar{\rho}$ (albeit it is unessential) in order to avoid the difficulties in resolving the singular boundary value problem of Eq.(42). The solution could be accomplished numerically probing such values of potential $p(e)$ which provide with the solution going to zero magnitude at large values of r.

The IL density as a function of r is plotted in Fig.5 for ten various quantities of intensity. The extreme right hand side line corresponds to $e/4\pi = 1$ and the extreme left hand side corresponds to $e/4\pi = 0.1$. The same quantity of spacing corresponds to the lines running to the right with intensity increasing. As it was expected the solution has the Yukawa like behaviour which is well seen in Fig.6 where $\ln(Br)$ is plotted as a function of r for four various values of intensity with the pace of 0.1 and $e/4\pi = 1$ for the upper line. Fitting it with the linear function gives the estimate of screening radius which looks as follows

$$R_d \sim (1.24\ \Lambda_{QCD})^{-1}\ ,$$

Amazingly, this results remains practically unchanged for the whole interval of the intensities from $e/4\pi = 0.1$ to $e/4\pi = 1$ and implies that such a parameter characterizes (at least in this interval of values) the screening properties of IL itself. In a context of the model it looks like rather soft scale for the screening radius and might be taken as another confirmation of adiabatic approximation relevance for the Coulomb external field.

Eventually let us comment on how it is essential that we are dealing with singular (anti-)instanton ensemble as a saturating configuration. Apparently, the screening properties of effective Lagrangian for external field B could be provided by any stochastic configuration of small characteristic size. The assumption of superposition ansatz validity occurs crucial to have all the leading contributions coming from the 'mixed' (repulsive) component of $G(A, B)$ again. Another solution of the problem may appear, of course, in the quantum approach but this discussion is out of this paper scope. Studying the pseudo-particle behaviour while inside (anti-)instanton medium ($n \neq 0$) one could explore the interrelation of two mechanisms (the repulsive interaction and freezing the coupling constant out Ref.[8]) of fixing instanton size.

5 Conclusion

To summarize the foregoing, we will list the main results of our study. The effect of the screening of an external color field in an instanton liquid has been studied. For the case of a weak field and for the case of strong field in the long-wave approximation, we have derived the corresponding effective Lgrangians. It should be noted that, in the case of a strong field, there is a pronounced trend toward the restoration of gauge invariance. Thus, the Lagrangian in (41) demonstrates that it is possible to describe correctly the introduction of external sources at a qualitative level and to take effectively into account charge conservation [even within the simplest superposition form of an approximate solution to the Yang–Mills equations (1)], wherein precisely lies the physical meaning of gauge invariance. One encounters a similar situation in the case of superconductivity in Abelian theory (see, for example, [18]). We have derived a criterion that specifies the field strength above which the effect of the field on the instanton liquid may prove to be significant. For example of the model problem of an Euclidean charged color source, we have estimated the variations in the instanton liquid parameters versus the coupling constant. Also we have obtained an estimate for the Debye screening radius. We have indicated that the interplay of two mechanisms fixing the pseudoparticle size in an instanton liquid (repulsion and freezing of the coupling constant) is possible.

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