ISBN: 978-3-95450-231-8 ISSN: 2673-5490 doi: 10.18429/JACoW-IPAC2023-TUXD1

ARBITRARY BUNCH SHAPING VIA WAKE POTENTIAL TAILORING

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Abstract

A method is described whereby any desired longitudinal electron bunch profile may be generated in a storage ring by tailoring the wake potential. The required wake function is found by implicitly solving the Haïssinski equation through the usage of a regularization parameter. For two coveted longitudinal profiles—a lengthened profile and a triangular profile—the required solutions are obtained and verified through particle simulations in longitudinal phase space, as well as through full particle tracking simulations. Auxiliary variables such as energy spreads/chirps and transverse phase-space distributions are found to be unaffected by the additional potentials. A possible implementation means is discussed in the context of using multiple harmonic cavities.

INTRODUCTION

In applications of relativistic electron bunches, various longitudinal bunch profiles are desired for various purposes. In particular, bunch lengthening and resultant particle density reduction are desired for longer Touschek lifetimes in electron storage rings. Active or passive high harmonic cavities are constantly being developed and employed in order to achieve this goal [1, 2]. Another much-desired bunch profile is the triangular shape for use as the drive beam in beam-driven accelerators. This profile allows for a transformer ratio that is much larger than two, which is the theoretical limit for Gaussian drive and witness bunch profiles. Methods such as emittance exchange [3] are being developed in linac contexts, but there are unfortunate drawbacks such as relatively poor distributions in transverse phase space, large energy chirps or spreads, and beam intensity reduction due to masking.

Here we describe a method in which arbitrary bunch shapes may in principle be obtained in a storage ring through wake potential tailoring. In the process we show how the self-consistency equation for a steady-state distribution in longitudinal phase space in a storage ring—the Haïssinski equation—may be solved for the longitudinal wake function for any given bunch shape. We also briefly touch upon the possibility of retrieving the machine impedance from a measured bunch distribution.

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INVERSE HASSINSKI PROBLEM

Problem Formulation and Solution

We begin with the equations of motion of an electron in longitudinal phase space (z, δ) in a storage ring subject to RF and wake potentials in normalized quantities:

$$\frac{dz}{ds} = -\eta\delta, \tag{1}$$

$$\frac{d\delta}{ds} = V_0 \sin\omega_{rf} z - \int_{-\infty}^{\infty} dz' \rho(z') W(z - z'), \tag{2}$$

where s is the abscissa along the accelerator circumference, z the spatial deviation from the synchronous particle, $\delta = \Delta E/E_0$ the relative energy deviation, η the slippage factor, ω_{rf} the RF frequency, V_0 the RF amplitude, $\rho(z)$ the line number density, and $W(z)$ the longitudinal wake function. We assumed zero energy loss per turn for simplicity. Placing Eqs. 1 and 2 in Hamiltonian context, the normalized Hamiltonian is

$$H = \frac{1}{2}\eta\delta^2 + \frac{2V_0}{\omega_{rf}} sin^2 \frac{\omega_{rf} z}{2} - \int_0^z dz'' \int_{-\infty}^{\infty} dz' \rho(z') W(z'' - z') \tag{3}$$

If the distribution function $\psi(z, \delta)$ is a function of H, i.e., $\psi = \psi(H)$, then it is a stationary solution of the Vlasov equation. Choosing $\psi \sim \exp\left(-\frac{H}{\eta\sigma_\delta}\right)$ where σ_δ is the energy spread of the beam, writing $\rho(z) = \int_{-\infty}^{\infty} \psi d\delta$, and differentiating both sides yield the Haïssinski equation in differential form,

$$\eta\sigma_\delta^2 \frac{d\, ln\, \rho(z)}{dz} + V_0\, sin\, \omega_{rf} z = \int_{-\infty}^{\infty} dz' \rho(z') W(z'' - z'). \tag{4}$$

Equation 4 is usually solved for ρ given W. Here, we try to do the opposite and solve for W given a particular desired or measured ρ. The proposed method is as follows. First, we defined the left-hand side as $L(\rho(z))$, which is entirely a function of $\rho(z)$. Then, we Fourier transform both sides and use the fact that convolution in physical space is multiplication in Fourier space. Then we have end up with an expression for the longitudinal impedance:

$$Z_\parallel(\omega) = \frac{\tilde{L}(\tilde{\rho}(\omega))}{\tilde{\rho}(\omega)}, \tag{5}$$

where $\tilde{\rho}(\omega)$ is the Fourier transform of $\rho(z)$, and $\tilde{L}(\omega)$ is the Fourier transform of $L(z)$.

This is seemingly straightforward, but there are two problems with this expression. First, there is no guarantee that Eq. (5) leads to a physical, causal wake function with $W(z > 0) = 0$. Mathematically, this means that the impedance should satisfy the Kramers-Kronig relations

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$$\Re Z_\parallel(\omega) = \frac{1}{\pi} P.V. \int_{-\infty}^{\infty} d\omega' \frac{\Im Z_\parallel(\omega')}{\omega' - \omega}, \qquad (6)$$

$$\Im Z_\parallel(\omega) = -\frac{1}{\pi} P.V. \int_{-\infty}^{\infty} d\omega' \frac{\Re Z_\parallel(\omega')}{\omega' - \omega}. \qquad (7)$$

Because $Z_\parallel$ in Eq. 5 is entirely a function of $\tilde{\rho}(\omega)$, Eqs. (6) and (7) are restrictions on the physically possible bunch shapes. The solution to finding physical bunch shapes is still under active investigation. Thus, in this work, we will leave out the problem of bunch shape physicality.

The second problem with Eq. 5 is that high-frequency noises are amplified. For instance, if we assume that the bunch distribution is Gaussian, $\tilde{\rho}(\omega) \sim \exp(-\omega^2)$ and so $Z_\parallel \sim \tilde{\rho}^{-1} \sim \exp\omega^2$. Therefore, high frequency noises get amplified. The solution to this problem is motivated from the Wiener filter [4] and is proposed as follows:

$$Z_\parallel = \frac{\tilde{L}(\tilde{\rho}(\omega))\tilde{\rho}^*(\omega)}{|\tilde{\rho}(\omega)|^2 + \epsilon_r}, \qquad (8)$$

where ϵ_r is a regularization parameter that suppresses high-frequency elements.

Verification via Simulation

In order to verify the validity of Eq. 8, a simple macro-particle tracking code was developed that solves Eqs. 1 and 2 with more realistic effects such as energy loss per turn, radiation damping, and quantum excitation. Machine parameters for a 4th-generation storage ring being designed in South Korea (Korea-4GSR) were used.

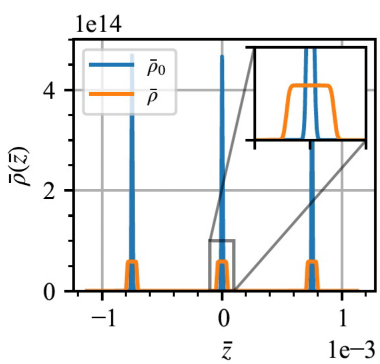

Figure 1: Profile of a Gaussian bunch train in a storage ring without for a zero wake function (blue). The desired lengthened bunch distribution (orange). The bars represent normalized quantities.

Figure 1 shows the desired lengthened bunch train distribution (orange), compared to the original Gaussian bunch train (blue). To obtain the impedance, the orange profile is Fourier transformed to obtain $\tilde{\rho}(\omega)$, and then Eq. 8 is used to obtain the require impedance. Figure 2 shows the obtained impedance. The peaks are harmonics of the RF frequency. It is emphasized here that the impedance is unphysical, i.e., that it leads to an acausal wake function, but in this work the mathematical validity of Eq. 8 itself is being verified.

The obtained impedance was used in the macro-particle tracking simulation, and the results at 29,900 turns are shown in Figure 3. It can be seen that a lengthened bunch distribution is indeed obtained (Fig. 3c), while the energy distribution is maintained (Fig. 3b). The total potential develops a plateau on which the beam sits on. Thus, the mathematical validity of Eq. 8 is confirmed.

Similar verification was done for a triangular bunch train but are not shown here.

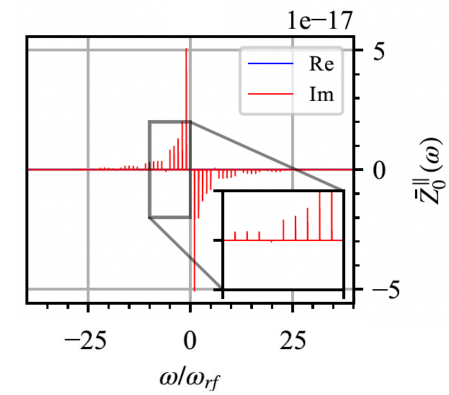

Figure 2: The impedance obtained from Eq. 8 using a lengthened bunch profile. The peaks are harmonics of the RF frequency.

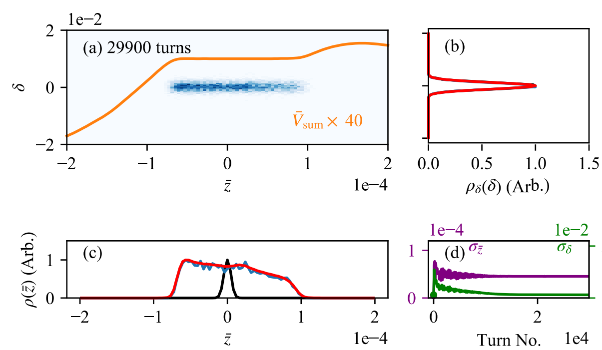

Figure 3: Results from the longitudinal particle-tracking simulation showing (a) $\psi(z, \delta)$ in arbitrary units (blue color) and the total potential experienced by the beam V_{sum} (orange; amplified by a factor of 40 for visibility). (b) The initial (black), final (blue), and 10,000-turn-averaged energy distributions. The lines are not discernible because the profiles are nearly the same. (c) The initial (black), final (blue), and 10,000-turn-averaged (red) longitudinal bunch distributions in arbitrary units. (d) Time evolution of σ_z (purple) and σ_δ (green).

IMPEDANCE RETRIEVAL

Another way Eq. 8 can be used is in retrieval of the machine impedance through bunch distribution measurements. In other words, if we know the stationary bunch profile $\rho(z)$, we can use Eq. 8 to retrieve the impedance that led to that bunch profile. While traditional methods to measure the machine impedance relies on measuring the impedances of different components and then adding them together, this method may provide a simpler alternative.

To check the validity of this method, the macro-particle simulation was conducted under a broad-band resonator impedance of the form

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$$Z_{\parallel}(\omega) = \frac{R_s}{1 + iQ\left(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r}\right)}, \tag{9}$$

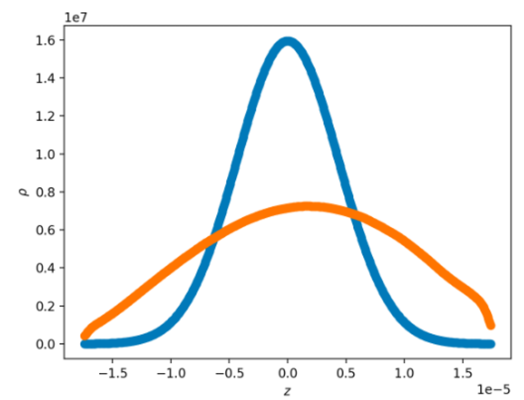

Figure 4: The equilibrium bunch profile without a wake field (blue) and the equilibrium distorted bunch profile under the impedance given by Eq. 9.

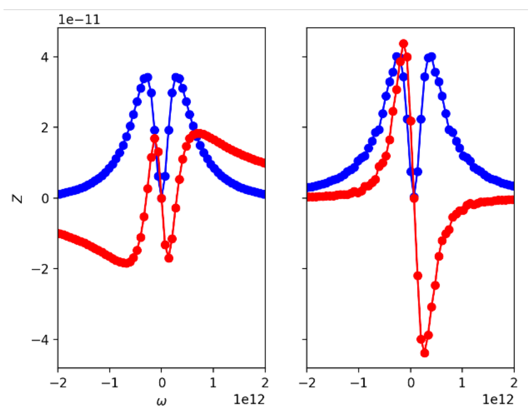

Figure 5: The left panel shows imposed impedance. The right panel shows the retrieved impedance by Fourier transforming the orange profile in Fig. 4 and inserting it into Eq. 8.

Fourier transforming the distorted bunch profile and inserting it into Eq. 8 yields the retrieved impedance, as shown in the right panel of Fig. 5. Comparing it to the imposed impedance shown in the left panel, the real part is accurately retrieved, but there is a mismatch of the imaginary part.

To see why there is a mismatch, an inverse Fourier transform is done on the retrieved impedance to obtain the retrieved wake function, as shown in Fig. 6. It can be seen that some discrepancies exist. In particular, the sharp drop at the origin in the imposed wake function is not retrieved, and this leads to a shift in the wake function. Nevertheless, the overall order and the shape is relatively well retrieved.

where R_s is the shunt impedance, Q the quality factor, and ω_r the resonant frequency. The parameters were chosen so that the equilibrium distribution changes to the orange line in Fig. 4.

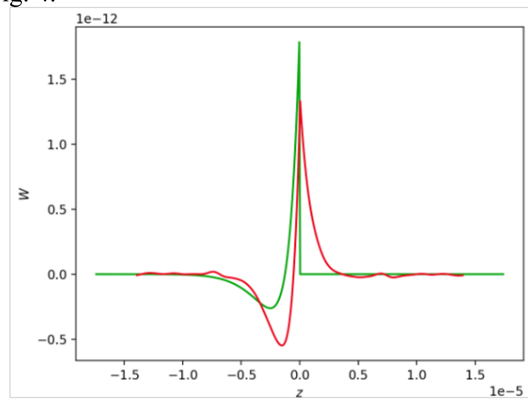

Figure 6: The imposed wake function (green) and the retrieved wake function (red)

CONCLUSION

A method to obtain the impedance given a desired or a measured bunch profile was presented. The Haissinski equation can be solved in an inverse manner, yielding an impedance that is entirely a function of the bunch shape. The method was verified through macro-particle simulations. Several outstanding problems were discussed and will be subject to future investigations.

ACKNOWLEDGEMENTS

This work was supported by an appointment to the JRG Program at the APCTP through the Science and Technology Promotion Fund and Lottery Fund of the Korean Government, and also by Korean local governments—Gyeongsangbuk-do province and Pohang city. Y.D.Y. was also supported by the POSCO Science Fellowship of POSCO TJ Park Foundation. We also thank Alex Chao and Jiazhen Tang for useful discussions.

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