

Isvector and isoscalar pairing in the nuclear shell model

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Abstract. Shell-model calculations based on a schematic hamiltonian are reported for several $N = Z$ nuclei in the $2p1f$ shell to assess the relative importance of isoscalar and isovector pairing in the presence of nuclear deformation and a spin-orbit interaction.

1. Introduction

Proton-neutron (pn) pairing correlations in $N \approx Z$ nuclei are typically treated using the generalized Hartree Fock Bogoylubov (HFB) approximation, with the pn pairing field included in addition to the alike-nucleon nn and pp fields [1]. Unfortunately, this method is not able to appropriately describe the physics of these competing modes without full restoration of symmetries [2].

The nuclear shell model, in contrast, can treat all pairing modes on an equal footing, without violation of symmetries. Furthermore, it can treat them in the presence of nuclear deformation, as is often present in nuclei with $N \approx Z$. Thus, in this paper we report a study of the various modes of pairing in $N = Z$ nuclei, using the nuclear shell model. We focus on nuclei in the $2p1f$ shell and consider a schematic hamiltonian that not only includes the various pairing modes, but also includes nuclear deformation and single-particle-splitting effects.

2. Our model

The model we use involves an inert ^{40}Ca core and an equal number of valence neutrons and protons interacting via a schematic hamiltonian

$$H = \chi \left(Q \cdot Q + aP^\dagger \cdot P + bS^\dagger \cdot S + \alpha \sum_i \vec{l}_i \cdot \vec{s}_i \right). \quad (1)$$

The first term in (1) is a quadrupole-quadrupole interaction (Q is the mass quadrupole operator) which by itself produces deformation. The second and third terms involve

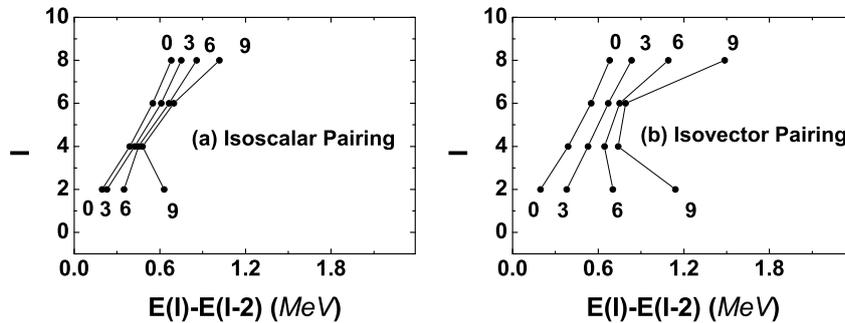


Figure 1. Spectra of the ground band of ^{44}Ti as a function of the strength of (a) isoscalar pairing interaction and (b) the isovector pairing interaction, with the optimal spin-orbit term present.

isoscalar and isovector pairing, respectively. The final term is the one-body part of a spin-orbit force. We carry out calculations as a function of the strength parameters a , b and α for a series of $N = Z$ nuclei, to assess how the various terms affect the rotational properties derived from the pure quadrupole-quadrupole force.

For an appropriate set of interaction parameters, the above hamiltonian in fact provides a fairly realistic description of the properties of nuclei in the lower part of the $2p1f$ shell. With the parameters $\chi = -0.05 \text{ MeV}$, $a = b = 12$ and $\alpha = 20$, we obtain a qualitative description of all nuclei up to ^{48}Cr , including the known backbend in ^{48}Cr . We refer to this as our *optimal Hamiltonian*. Note that in the optimal Hamiltonian, the strength of the isoscalar and isovector pairing interactions are the same.

3. Results and discussion

We first consider the nucleus ^{44}Ti , which has two active neutrons and two active protons. In the absence of a spin-orbit interaction, our results confirm the well-known result that isoscalar and isovector pairing produce precisely the same effect on the rotational properties of the system. More specifically, when we separately ramp up the strengths of the isoscalar pairing interaction and the isovector pairing interaction, with the optimal quadrupole strength but with $\alpha = 0$, we find exactly the same effect on the properties of the ground YRAST band from the two pairing interactions.

When we include the realistic spin-orbit interaction ($\alpha = 20$), however, this ceases to be the case. As can be seen from Figure 1, in the presence of the realistic spin-orbit splitting, isovector pairing dominates even for $N = Z$ nuclei. This conclusion, though already known from other studies, is borne out nicely in our model.

The next nucleus we consider is ^{46}V , an odd-odd nucleus with one additional neutron and one additional proton. We focus on the splitting between the lowest two states of the system, a $J^\pi = 0^+$, $T = 1$ state and a $J^\pi = 1^+$, $T = 0$ state. We consider equal isoscalar and isovector strengths ($a = b$), as in the optimal Hamiltonian, and study the effect of ramping up these two strengths together, first without and then with the spin-orbit term. When there is no spin-orbit term, $\alpha = 0$, these two states are degenerate in energy for any value of the common pairing strength $a = b$. In contrast,

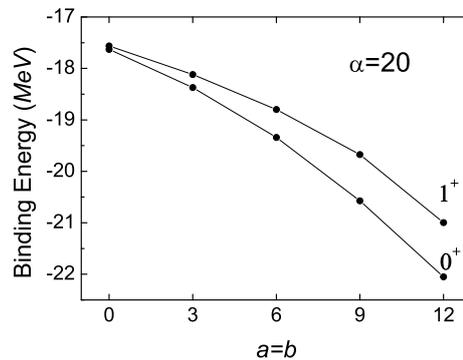


Figure 2. Calculated energies of the lowest $J^\pi = 0^+$ and $J^\pi = 1^+$ states of ^{46}V as a function of the equal strengths of isoscalar and isovector pairing, with the optimal spin-orbit term present.

in the presence of the optimal spin-orbit force ($\alpha = 20$), the degeneracy is removed. As seen in Figure 2, when the pairing strength is turned on, the 0^+ $T = 1$ state becomes the ground state, with its energy gain with respect to the 1^+ $T = 0$ state growing as the strength is increased. For the optimal pairing strength $a = b = 12$, the 0^+ state is at 1.05 MeV , fairly near the experimental value of 1.23 MeV . We conclude, therefore, that the fact that ^{46}V has a 0^+ ground state derives primarily from the spin-orbit interaction and its suppression of isoscalar pairing.

Lastly, we turn to ^{48}Cr , with four active neutrons and four active protons. Here we focus on a scenario with the optimal spin-orbit term ($\alpha = 20$) included. When we consider separately the effects of isoscalar pairing and isovector pairing on the YRAST band, we find that isoscalar pairing has essentially no effect whereas isovector pairing has a dramatic effect, producing a backbend at $J = 12$, as in experiment.

The backbend in ^{48}Cr was discussed earlier in the context of a realistic shell-model study [3], where it was shown to derive from isovector pairing. Our results support that conclusion. This can be seen in Figure 3, where we show the number of collective isoscalar and isovector pairs in the YRAST band for the optimal hamiltonian. For low angular momenta, the states are dominated by isovector pairs, with only few isoscalar pairs present. As we crank up the angular momenta, the number of isovector pairs gradually decreases, reaching a value comparable to the number of isoscalar pairs just prior to the backbend. In the backbend region, both the number of isoscalar and isovector pairs suddenly increase. After the backbend region, both then decrease to zero as rotational alignment is achieved.

We also studied the $K^\pi = 2^+$ YRARE band and found no evidence that it crosses the YRAST band in the vicinity of the backbend. We thus conclude that the backbend is not driven by band crossing. Indeed, the only unique feature we find in the region of the backbend is an anomalous growth in the number of collective pairs (both isovector and isoscalar), suggesting that perhaps this is driving the backbend. Further study of this result is called for.

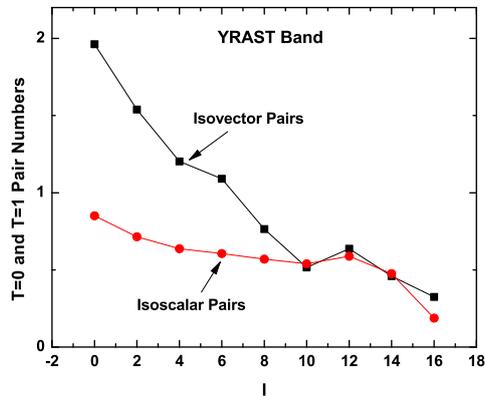


Figure 3. Calculated numbers of isovector pairs and isoscalar pairs in the ground band of ^{48}Cr for the optimal values of the hamiltonian parameters.

4. Closing remarks

We have reported a shell-model study of pn pairing in the $2p1f$ shell using a schematic hamiltonian that includes deformation, spin-orbit effects and both isoscalar and isovector pairing [4]. Working in a shell-model framework, we were able to assess the role of the various modes of pn pairing in the presence of nuclear deformation without violating symmetries.

Some of the key conclusions that emerged are: (i) the symmetry between isoscalar and isovector pairing disappears even at $N = Z$ in the presence of a spin-orbit force and isovector pairing dominates, (ii) that ^{46}V has a 0^+ ground state derives from the spin-orbit interaction and its relative effect on isoscalar and isovector pairing, (iii) isovector pairing dominates in ^{48}Cr and produces its backbend, and (iiii) isoscalar and isovector pairing exhibit an unusual behavior in the vicinity of the ^{48}Cr backbend.

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