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Minimal Quantities and Measurable Variant of Gravity II. Strong Principle of Equivalence and Transition to High Energies

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Abstract

This paper continues a study of gravity within the scope of the **measurability** notion introduced by the author in his previous works. Based on the earlier results, it is shown that the **Strong Principle of Equivalence(SPE)** of General Relativity may be reformulated in terms of **measurable** quantities and is valid in this case at low energies far from the Planck's. Next, the possibility for generalization **SPE** of a **measurable** analog of gravity in the ultraviolet (Planck) energy region is analyzed.

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1 Introduction

This paper is a continuation of a study into a quantum theory and gravity in terms of the **measurability** notion, initiated in [1]–[9], with the aim to form the above-mentioned theories proceeding from the variations (increments) dependent on the existent energies.

These theories should not involve the infinitesimal variations $dt, dx_i, dp_i, dE, i = 1,...,3$ and, in general, any abstract small quantities $\delta t, \delta x_i, \delta E, \delta p_i, ...$
In work [10] in the general form it is demonstrated that all the basic ingredients of General Relativity (GR) have their measurable analogs, the way to derive every term in a measurable variant of the Einstein equations is presented. Passage of the **measurable** analog of GR to the ultraviolet (Planck) region is considered, showing that it is quite natural from the viewpoint of the methods and approaches developed in [10].
This paper directly follows from [10]. Here it is demonstrated that a **measurable** analog of the **Strong Principle of Equivalence(SPE)** is valid, i.e., **SPE** may be formulated *entirely* in terms of **measurable** quantities at low energies $E \ll E_p$.
Note, as in GR only low energy regions $E \ll E_p$ are considered, it is implied that **SPE** is valid in GR just in this energy region. The region of high energies $E \approx E_p$ belongs to **Quantum Gravity** that has not be formed by now.
Nevertheless, in terms of the **measurability** notion, we can perform an initial analysis of the possible generalization of **SPE** to the Planck (quantum-gravity) region. This is the principal object of the work.
The structure of this paper is as follows. Section 2 briefly outlines the necessary preliminary information from [1]–[9]. At the same time, for better understanding, some aspects are elucidated and supplemented. In particular, of importance are **Remark 2.3.**–**Remark 2.5.**. In Section 3, proceeding from the results of Section 4 in [10], it is indicated that the **Strong Principle of Equivalence (SPE)** may be reformulated in terms of the **measurability** notion at low energies $E \ll E_p$ and is valid in this case.
In Section 4, within the scope of the **space-time foam** notion, the possibility for generalization of **SPE** for a **measurable** analog of gravity in the ultraviolet (Planck) region is analyzed.

2 Necessary Preliminary Information

Let us briefly consider the earlier results [1]–[9] laying the basis for this study.
It is assumed that there is a minimal (universal) unit for measurement of the length ℓ corresponding to some maximal energy $E_\ell = \frac{\hbar c}{\ell}$ and a universal unit for measurement of time $\tau = \ell/c$. Without loss of generality, we can consider ℓ and τ at Plank's level, i.e. $\ell = \kappa l_p, \tau = \kappa t_p$, where the numerical constant κ is on the order of 1. Consequently, we have $E_\ell \propto E_p$ with the corresponding proportionality factor.
Then we consider a set of all nonzero momenta

$$\mathbf{P} = \{p_{x_i}\}, i = 1,..,3; |p_{x_i}| \neq 0. \qquad (1)$$

¿From this set we can isolate a set of the **Primarily Measurable** momenta characterized by the property

$$p_{x_i} \doteq p_{N_i} = \frac{\hbar}{N_i \ell}, \tag{2}$$

where N_i is an integer number and p_{x_i} is the momentum corresponding to the coordinate x_i.
From these formula it is not unreasonable to propose the following definition:

Definition 1. Primary Measurability
1.1. Any variation in Δx_i for the coordinates x_i and Δt of the time t is considered **primarily measurable** if

$$\Delta x_i = N_{\Delta x_i}\ell, \Delta t = N_{\Delta t}\tau, \tag{3}$$

where $N_{\Delta x_i} \neq 0$ and $N_{\Delta t} \neq 0$ are integer numbers.
1.2. Let us define any physical quantity as **primary or elementary measurable** when its value is consistent with point **1.1** of this Definition.

So, from **Definition 1.** it directly follows that all the momenta satisfying 2) are the **Primarily Measurable** momenta.
Then we consider formula (2) and **Definition 1.** with the addition of the momenta $p_{x_0} \doteq p_{N_0} = \frac{\hbar}{N_0 \ell}$, where N_0 is an integer number corresponding to the time coordinate ($N_{\Delta t}$ in formula (3)).
For convenience, we denote **Primarily Measurable Quantities** satisfying **Definition 1.** in the abbreviated form as**PMQ**.
It is clear that **PMQ** is inadequate for studies of the physical processes. To illustrate, the space-time quantities

$$\begin{gathered}\frac{\tau}{N_t} = p_{N_t c}\frac{\ell^2}{c\hbar} \\ \frac{\ell}{N_i} = p_{N_i}\frac{\ell^2}{\hbar}, 1 = 1, ..., 3,\end{gathered} \tag{4}$$

where $p_{N_i}, p_{N_t c}$ are **Primarily Measurable** momenta, up to the fundamental constants are coincident with $p_{N_i}, p_{N_t c}$ and they may be involved at any stage of the calculations but, evidently, they are not **PMQ** in the general case.
Thus, it is reasonable to use **Definition 2.**

Definition 2. Generalized Measurability
We define any physical quantity at all energy scales $E \leq E_\ell$ as **generalized measurable** or, for simplicity, **measurable** if any of its values may be obtained in terms of **PMQ** specified by points **1.1**,**1.2** of **Definition 1**.

Remark 2.1 What is the main difference between **Primarily Measurable Quantities (PMQ)** and **Generalized Measurable Quantities (GMQ)**? **PMQ** defines variables which may be obtained as a result of an immediate experiment. **GMQ** defines the variables which may be *calculated* based on **PMQ**, i.e. based on the data obtained in previous clause.

The main target of the author is to form a quantum theory and gravity only in terms of **measurable**quantities (or of **PMQ**).
Now we consider separately the two cases.

A) **Low Energies,** $E \ll E_\ell$.
In $\mathbf{P}$ we consider the domain $\mathbf{P}_{LE} \subset \mathbf{P}$ (LE is abbreviation of "Low Energies") defined by the conditions

$$\mathbf{P}_{LE} = \{p_{x_i}\}, i = 1,..,3; P_\ell \gg |p_{x_i}| \neq 0, \tag{5}$$

where $P_\ell = E_\ell/c$–maximal momentum.
In this case the formula of (2) takes the form

$$\begin{aligned} N_i &= \frac{\hbar}{p_{x_i}\ell}, or \\ p_{x_i} \doteq p_{N_i} &= \frac{\hbar}{N_i\ell} \\ |N_i| &\gg 1, \end{aligned} \tag{6}$$

where the last row of the formula (6) is given by the requirement (5).
As the energies $E \ll E_\ell$ are low, i.e. ($|N_i| \gg 1$), **primary measurable** momenta are sufficient to specify the whole domain of the momenta to a high accuracy $\mathbf{P}_{LE}$.
It is clear that

$$[N_i] \leq N_i \leq [N_i] + 1, \tag{7}$$

where $[\aleph]$ defines the integer part of $\aleph$. Then $|N_i|^{-1}$ falls within the interval with the finite points $|[N_i]|^{-1}$ and $|[N_i] + 1|^{-1}$ (which of the numbers is greater or smaller, depends on a sign of N_i). In any case we have $|N_i^{-1} - [N_i^{-1}]| \leq |([N_i] + 1)^{-1} - [N_i]^{-1}| = |([N_i] + 1)[N_i]|^{-1}$.
Thus, the difference between p_{N_i} and $p_{[N_i]}$ is negligibly small. Therefore, the **primary measurable** momenta p_{N_i}, ($|N_i| \gg 1$) are sufficient to specify the whole domain of the momenta to a high accuracy $\mathbf{P}_{LE}$.
This means that in the indicated domain a discrete set of **primary measurable** momenta $p_{N_i}, (i = 1, ..., 3)$ from formula (6) varies almost continuously, practically covering the whole domain.

That is why further $\mathbf{P}_{LE}$ is associated with the domain of **primary measurable** momenta, satisfying the conditions of the formula (5) (or (6)).
Of course, all the calculations of point A) also comply with the **primary measurable** momenta $p_{N_{tc}} \doteq p_{N_0}$ in formula (4). Because of this, in what follows we understand $\mathbf{P}_{LE}$ as a set of the **primary measurable** momenta $p_{x_\mu} = p_{N_\mu}, (\mu = 0, ..., 3)$ with $|N_\mu| \gg 1$.

Remark 2.2. It should be noted that, as all the experimentally involved energies E are low, they meet the condition $E \ll E_\ell$, specifically for LHC the maximal energies are $\approx 10TeV = 10^4 GeV$, that is by 15 orders of magnitude lower than the Planck energy $\approx 10^{19} GeV$. But since the energy E_ℓ is on the order of the Planck energy $E_\ell \propto E_p$, in this case all the numbers N_i for the corresponding momenta will meet the condition $min|N_i| \approx 10^{15}$,i.e., the formula of (6). So, all the experimentally involved momenta are considered to be **primary measurable** momenta,i.e. $\mathbf{P}_{LE}$ at low energies $E \ll E_\ell$.

In this way in the proposed paradigm at low energies $E \ll E_p$ any momentum with $p_{x_\mu}, \mu = 0, ...3$ takes the form $p_{x_\mu} = p_{N_\mu}$, where N_μ – integer with the property $|N_\mu| \gg 1$.
Further for the fixed point x_μ we use the notion $p_{x_\mu} = p_{N_{x_\mu}}$ or $p_{x_\mu} = p_{N_{\Delta x_\mu}}$.
Naturally, the small variation Δp_{x_μ} at the point $p_{x_\mu} = p_{N_{x_\mu}}$ of the momentum space $\mathbf{P}_{LE}$ is represented by the **primary measurable** momentum $p_{N'_{x_\mu}}$ with the property $|N'_{x_\mu}| \gg |N_{x_\mu}|$.
The problem is as follows: is any possibility that Δp_{x_μ} is infinitesimal? For the special point $p_{x_\mu} = p_{N_{x_\mu}}$ the answer is negative.
Indeed, the "nearest" points to $p_{N_{x_\mu}}$ are $p_{N_{x_\mu}-1}$ and $p_{N_{x_\mu}+1}$.
It is obvious that

$$\begin{aligned} |p_{N_{x_\mu}} - p_{N_{x_\mu}-1}| &= |p_{N_{x_\mu}(N_{x_\mu}-1)}|, \\ |p_{N_{x_\mu}} - p_{N_{x_\mu}+1}| &= |p_{N_{x_\mu}(N_{x_\mu}+1)}|. \end{aligned} \tag{8}$$

It is easily seen that the difference $|p_{N_{x_\mu}(N_{x_\mu}+1)}| - |p_{N_{x_\mu}(N_{x_\mu}-1)}|$ for $|N_{x_\mu}| \gg 1$ is infinitesimal, i.e., to within a high accuracy, we have $|p_{N_{x_\mu}(N_{x_\mu}+1)}| = |p_{N_{x_\mu}(N_{x_\mu}-1)}|$. And a small variation of $|\Delta p_{x_\mu}|$ at the point $p_{x_\mu} = p_{N_{x_\mu}}$ has a minimum that equals $|p_{N_{x_\mu}(N_{x_\mu}+1)}|$. Clearly, with an increase in $|N_{x_\mu}|$, we can obtain no matter how small $|p_{N_{x_\mu}(N_{x_\mu}+1)}|$.
So, in the proposed paradigm at low energies $E \ll E_p$ a set of the **primarily measurable** $\mathbf{P}_{LE}$ is discrete, and in every measurement of $\mu = 0, ..., 3$ there is the discrete subset $\mathbf{P}_{\mathbf{x}_\mu} \subset \mathbf{P}_{LE}$:

$$\mathbf{P}_{\mathbf{x}_\mu} \doteq \{..., p_{N_{x_\mu}-1}, p_{N_{x_\mu}}, p_{N_{x_\mu}+1}, ...\}. \tag{9}$$

In this case, as compared to the canonical quantum theory, in continuous space-time we have the following substitution:

$$dp_\mu \mapsto \mathbf{\Delta p_{N_{x_\mu}}} = \mathbf{p_{N_{x_\mu}}} - \mathbf{p_{N_{x_\mu}+1}} = \mathbf{p_{N_{x_\mu}(N_{x_\mu}+1)}};$$

$$\frac{\partial}{\partial p_\mu} \mapsto \frac{\mathbf{\Delta}}{\mathbf{\Delta p_\mu}}, \frac{\partial \mathbf{F}}{\partial \mathbf{p_\mu}} \mapsto \frac{\mathbf{\Delta F(p_{N_{x_\mu}})}}{\mathbf{\Delta p_\mu}} = \frac{\mathbf{F(p_{N_{x_\mu}}) - F(p_{N_{x_\mu}+1})}}{\mathbf{p_{N_{x_\mu}} - p_{N_{x_\mu}+1}}} = \frac{\mathbf{F(p_{N_{x_\mu}}) - F(p_{N_{x_\mu}+1})}}{\mathbf{p_{N_{x_\mu}(N_{x_\mu}+1)}}}. \quad (10)$$

It is clear that for sufficiently high integer values of $|N_{x_\mu}|$, formula 10) reproduces a continuous paradigm in the momentum space to any preassigned accuracy.
Similarly for sufficiently high integer values of $|N_t|$ and $|N_i \doteq N_{x_i}|$, the quantities $\tau/N_t, \ell/N_{x_i}$ from formula (4) may be arbitrary small.
Hence, for sufficiently high integer values of $|N_t|$ and $|N_i \doteq N_{x_i}|$, the quantities $\tau/N_t, \ell/N_{x_i}$ are nothing but a **measurable** analog of the small quantities $\delta x_i, \delta t$ and the infinitesimal quantities dx_i, dt, i.e. δx_μ, and dx_μ, $\mu = 0, ..., 3$.
As follows from formula (4), for sufficiently high integer values of $|N_{x_\mu}|, \mu = 0, ..., 3$, the **primarily measurable** momenta $\mathbf{P_{x_\mu}}$ (formula (9)) represent a**measurable** analog of small (and infinitesimal) space-time *increments* in the space-time variety $\mathcal{M} \subset \mathbf{R^4}$.
Because of this, for sufficiently high integer values of $|N_{x_\mu}|$, the space-time analog of formula (10) is as follows:

$$dx_\mu \mapsto \frac{\ell}{\mathbf{N_{x_\mu}}};$$

$$\frac{\partial}{\partial x_\mu} \mapsto \frac{\mathbf{\Delta}}{\mathbf{\Delta_{N_{x_\mu}}}}, \frac{\partial F}{\partial x_\mu} \mapsto \frac{\mathbf{\Delta F(x_\mu)}}{\mathbf{\Delta_{N_{x_\mu}}}} = \frac{\mathbf{F(x_\mu) - F(x_\mu + \ell/N_{x_\mu})}}{\ell/\mathbf{N_{x_\mu}}}. \quad (11)$$

Now we formulate the *principle of correspondence to a continuous theory.*

Principle of Correspondence to Continuous Theory (PCCT).

At low energies $E \ll E_p$ (or same $E \ll E_\ell$) the infinitesimal space-time quantities $dx_\mu; \mu = 0, ..., 3$ and also infinitesimal values of the momenta $dp_i, i = 1, 2, 3$ and of the energies dE form the basic instruments ("construction materials") for any theory in continuous space-time. Because of this, to construct the **measurable** variant of such a theory, we should find the adequate substitutes for these quantities.
It is obvious that in the first case the substitute is represented by the quantities ℓ/N_{x_μ}, where $|N_{x_\mu}|$ – no matter how large (but finite!) integer, whereas in the second case by $p_{N_{x_i}} = \frac{\hbar}{N_{x_i}\ell}; i = 1, 2, 3; \mathcal{E}_{N_{x_0}} = \frac{c\hbar}{N_{x_0}\ell}$, where N_{x_μ} – integer with the above properties $\mu = 0, ...3$.
In this way in the proposed approach all the **primary measurable** momenta $p_{N_{x_\mu}}, |N_{x_\mu}| \gg 1$ are small quantities at low energies $E \ll E_\ell$ and **primary**

measurable momenta $p_{N_{x_\mu}}$ with sufficiently large $|N_{x_\mu}| \gg 1$ being analogous to *infinitesimal* quantities of a continuous theory.
As, according to **Remark 2.2**, all the momenta at low energies $E \ll E_p$, to a high accuracy, may be considered to be the **primary measurable** momenta, from formula (4) we derive that at low energies the **primary measurable** momenta $p_{N_{x_\mu}}$ generate **measurable** small space-time *variations* and at sufficiently high $|N_{x_\mu}|$ – *infinitesimal variations.*

B)**High Energies,** $E \approx E_p$.
In this case formula (2) takes the form

$$N_i = \frac{\hbar}{p_{x_i}\ell}, or \qquad (12)$$
$$p_{x_i} \doteq p_{N_i} = \frac{\hbar}{N_i\ell}$$
$$|N_i| \approx 1.$$

where N_i is an integer number and p_{x_i} is the momentum corresponding to the coordinate x_i. The *discrete set* $p_{N_i} \doteq p_{N_{x_i}}$ is introduced as **primary measurable** momenta.
The main difference of the case B)**High Energies** from the case A) **Low Energies** is in the fact that at **High Energies** the **primary measurable** momenta are *inadequate* for theoretical studies at the energy scales $E \approx E_p$.
This is easily seen when we consider, e.g., the Generalized Uncertainty Principle (GUP) [11]–[20], that is an extension of Heisenberg's Uncertainty Principle (HUP) [22],[21], to (Planck) high energies

$$\Delta x \geq \frac{\hbar}{\Delta p} + \alpha' l_p^2 \frac{\triangle p}{\hbar} \qquad (13)$$

where α' is a constant on the order of 1.
Obviously, (13) leads to the minimal length ℓ on the order of the Planck length l_p

$$\Delta x_{min} = 2\sqrt{\alpha'} l_p \doteq \ell. \qquad (14)$$

In his earlier works [7],[9] the author, using simple calculations, has demonstrated that for the equality in (13) at high energies $E \approx E_p, (E \approx E_\ell)$ the **primary measurable** space quantity $\Delta x = N_{\Delta x}\ell$, where $N_{\Delta x} \approx 1$ is an integer number, results in the momentum $\Delta p(N_{\Delta x}, GUP)$:

$$\Delta p \doteq \Delta p(N_{\Delta x}, GUP) = \frac{\hbar}{1/2(N_{\Delta x} + \sqrt{N_{\Delta x}^2 - 1})\ell}. \qquad (15)$$

It is clear that for $N_{\Delta x} \approx 1$ the momentum $\Delta p(N_{\Delta x}, GUP)$ is not a **primary measurable** momentum.

On the contrary, at low energies $E \ll E_p, (E \ll E_\ell)$ the **primary measurable** space quantity $\Delta x = N_{\Delta x}\ell$, where $N_{\Delta x} \gg 1$ is an integer number, due to the validity of the limit

$$\lim_{N_{\Delta x}\to\infty} \sqrt{N_{\Delta x}^2 - 1} = N_{\Delta x}, \tag{16}$$

leads to the momentum $\Delta p(N_{\Delta x}, HUP)$:

$$\Delta p \doteq \Delta p(N_{\Delta x}, HUP) = \frac{\hbar}{1/2(N_{\Delta x} + \sqrt{N_{\Delta x}^2 - 1})\ell} \approx \frac{\hbar}{N_{\Delta x}\ell} = \frac{\hbar}{\Delta x}. \tag{17}$$

It is inferred that, for sufficiently high integer values of $N_{\Delta x}$ the momentum $\Delta p(N_{\Delta x}, HUP)$ within any high accuracy may be considered to be the **primary measurable** momentum.
This example illustrates that **primary measurable** momenta are insufficient for studies in the high-energy domain $E \approx E_p$ and we should use the **generalized measurable** momenta.

As noted above, the main target of the author is to construct a quantum theory at all energy scales $E \leq E_\ell$ in terms of **measurable** quantities.
In this theory the values of the physical quantity $\mathcal{G}$ may be represented by the numerical function $\mathcal{F}$ in the following way [8]:

$$\mathcal{G} = \mathcal{F}(N_i, N_t, \ell) = \mathcal{F}(N_i, N_t, G, \hbar, c, \kappa), \tag{18}$$

where N_i, N_t–integers for general form from the formula (2) and at high energies $E \approx E_\ell$ from the formula (12) and $G, \hbar, c$ are fundamental constants. The last equality in (18) is determined by the fact that $\ell = \kappa l_p$ and $l_p = \sqrt{G\hbar/c^3}$.
If $N_i \neq 0, N_t \neq 0$ (**nondegenerate** case), then it is clear that (18) can be rewritten as follows:

$$\mathcal{G} = \mathcal{F}(N_i, N_t, \ell) = \widetilde{\mathcal{F}}((N_i)^{-1}, (N_t)^{-1}, \ell) \tag{19}$$

Then at low energies $E \ll E_\ell$, i.e. at $|N_i| \gg 1, |N_t| \gg 1$, the function $\widetilde{\mathcal{F}}$ is a function of the variables changing practically continuously, though these variables cover a discrete set of values. Naturally, it is assumed that $\widetilde{\mathcal{F}}$ varies smoothly (i.e. practically continuously). As a result, we get a model, discrete in nature, capable to reproduce the well-known theory in continuous space-time to a high accuracy, as it has been stated above.
Obviously, at low energies $E \ll E_\ell$ the formula (19) is as follows:

$$\mathcal{G} = \mathcal{F}(N_i, N_t, \ell) = \widetilde{\mathcal{F}}((N_i)^{-1}, (N_t)^{-1}, \ell) = \\ = \widetilde{\mathcal{F}}_p(p_{N_i}, p_{N_{tc}}, \ell), \tag{20}$$

where $p_{N_i}, p_{N_{tc}}$ are **primary measurable** momenta.

Remark 2.3. What is the main point of this Section?
At low energies $E \ll E_p$ we replace the abstract small and infinitesimal quantities $\delta x_\mu, dx_\mu, \delta p_\mu, dp_\mu$ incomparable with each other, by the specific small quantities $\ell/N_{x_\mu}, p_{N_{x_\mu}}$, which may be made however small at sufficiently high $|N_{x_\chi}|$, still being ordered and comparable. It is very important that the quantities $\ell/N_{x_\mu}, p_{N_{x_\mu}}$ are directly associated with the existing energies; for $|N'_{x_\mu}| > |N_{x_\mu}|$ the momentum $p_{|N'_{x_\mu}|} < p_{|N_{x_\mu}|}$ and $p_{|N'_{x_\mu}|}$ corresponds to lower energy than $p_{|N_{x_\mu}|}$. The same is true for the space variations $\ell/N'_{x_\mu}, \ell/N_{x_\mu}$.

Remark 2.4.
At low energies $E \ll E_p$ we should emphasize the difference between the **primary measurable** *momenta $p_{N_{x_\mu}} \in \mathbf{P}_{LE}$ and the space-time quantities ℓ/N_{x_μ} corresponding to them in accordance with formula (4).*
The first, that is $p_{N_{x_\mu}}$, in accordance with **Remark 2.2.** *represent the whole set of the momenta $\mathbf{P}_{LE}$ at low energies $E \ll E_p$ in terms of* **measurable** *quantities, whereas the second, ℓ/N_{x_μ}, represent only the* **measurable** *small variations of space-time quantities.* Because of this, any point $p_{N_{x_\mu}} \in \mathbf{P}_{LE}$ is associated with the fixed **measurable** minimal variation $\Delta p_{N_{x_\mu}}$ from formula (10). At the same time, for a point with the space-time coordinates $\mathbf{x} \doteq \{x_\mu\}$ such **measurable** minimal variation is dependent on the number $|N_{x_\mu}|$ according to formula (11).

Remark 2.5.
Finally, according to **Definition 1.**, *in the relativistic case the* **primary measurable** *energy is of the form*

$$\mathcal{E} = \frac{\hbar c}{N_0 \ell}, N_0 \doteq N_{x_0}, \tag{21}$$

where N_0 is an integer number, and at low energies $E \ll E_p$ it is obvious that $N_0 \gg 1$.
Then at low energies $E \ll E_p$ from **Remark 2.2.** *it follows naturally that* **primary measurable** *energies, to a high accuracy, cover the whole low-energy spectrum. Then, considering that the formula*
$\mathcal{E}^2 = \boldsymbol{p}^2 c^2 + m^2 c^4$ low energies $E \ll E_p$ [23],[24] to a high accuracy is valid in terms of **measurable** *quantities and all components of the vector $\boldsymbol{p}$ are the* **primary measurable** *momenta, we can found the mass m in terms of the* **measurability** *notion* as follows:

$$m^2 = \frac{\hbar^2}{c^2}\left(\frac{1}{N_0^2 \ell^2} - \sum_{1 \leq i \leq 3} \frac{1}{N_i^2 \ell^2}\right). \tag{22}$$

3 Space-Time Metrics in Measurable Format and Strong Principle of Equivalence at Low Energy

The principal result of this section is based on Section 4 in [10] and we give all the required information from [10].
According to the above-mentioned results, the **measurable** variant of gravity should be formulated in terms of the small **measurable** space-time quantities $\ell/N_{\Delta x_\mu}$ or same **primary measurable** momenta $p_{N_{\Delta x_\mu}}$.
Let us consider the case of the random metric $g_{\mu\nu} = g_{\mu\nu}(x)$ [25],[26], where $x \in R^4$ is some point of the four-dimensional space R^4 defined in **measurable** terms. The phrase "some point of the four-dimensional space R^4 defined in **measurable** terms" means that all variations at the indicated point are determined in terms of **measurable** quantities (formula (18)–(20)). Specifically, as mentioned above, all small **measurable** variations, according to formula (4), take the from $\ell/N_{\Delta x_\mu} \propto p_{N_{\Delta x_\mu}}$, where $p_{N_{\Delta x_\mu}}$ are **primary measurable** momenta and $|N_{\Delta x_\mu}| \gg 1$.
Now, any such point $x \doteq \{x^\chi\} \in R^4$ and any set of integer numbers $\{N_{\Delta x^\chi}\}$ dependent on the point $\{x^\chi\}$ with the property $|N_{\Delta x^\chi}| \gg 1$ may be correlated to the bundle with the base R^4 as follows:

$$B_{N_{x^\chi}} \doteq \{x^\chi, \frac{\ell}{N_{\Delta x^\chi}}\} \mapsto \{x^\chi\}. \tag{23}$$

It is clear that $\lim\limits_{|N_{\Delta x^\chi}| \to \infty} B_{N_{\Delta x^\chi}} = R^4$.
Then as a *canonically measurable prototype* of the infinitesimal space-time interval square [25],[26]

$$ds^2(x) = g_{\mu\nu}(x) dx^\mu dx^\nu \tag{24}$$

we take the expression

$$\Delta s^2_{\{N_{\Delta x^\chi}\}}(x) \doteq g_{\mu\nu}(x, \{N_{\Delta x^\chi}\}) \frac{\ell^2}{N_{\Delta x^\mu} N_{\Delta x^\nu}}. \tag{25}$$

Here $g_{\mu\nu}(x, \{N_{\Delta x^\chi}\})$ – metric $g_{\mu\nu}(x)$ from formula (32) with the property that minimal **measurable** variation of metric $g_{\mu\nu}(x)$ in point x for coordinate χ has form

$$\Delta g_{\mu\nu}(x, \{N_{\Delta x^\chi}\})_\chi = g_{\mu\nu}(x + \ell/N_{\Delta x^\chi}, \{N_{\Delta x^\chi}\}) - g_{\mu\nu}(x, \{N_{\Delta x^\chi}\}), \tag{26}$$

Let us denote by $\Delta_\chi g_{\mu\nu}(x, \{N_{\Delta x^\chi}\})$ quantity

$$\Delta_\chi g_{\mu\nu}(x, N_{\Delta x^\chi}) = \frac{\Delta g_{\mu\nu}(x, N_{\Delta x^\chi})_\chi}{\ell/N_{\Delta x^\chi}}. \tag{27}$$

It is obvious that in the case under study the quantity $\Delta g_{\mu\nu}(x,\{N_{\Delta x^{\chi}}\})_{\chi}$ is a **measurable** analog for the infinitesimal increment $dg_{\mu\nu}(x)$ of the χ-th component $(dg_{\mu\nu}(x))_{\chi}$ in a continuous theory, whereas the quantity $\Delta_{\chi}g_{\mu\nu}(x,N_{\Delta x^{\chi}})$ is a **measurable** analog of the partial derivative $\partial_{\chi}g_{\mu\nu}(x)$.
In this manner we obtain the (23)-formula induced bundle over the metric manifold $g_{\mu\nu}(x)$:

$$B_{g,N_{\Delta x^{\chi}}} \doteq g_{\mu\nu}(x,\{N_{\Delta x^{\chi}}\}\} \mapsto g_{\mu\nu}(x). \tag{28}$$

Referring to formula (4), we can see that (25) may be written in terms of the **primary measurable** momenta $(p_{N_{\Delta x^{i}}}, p_{N_{\Delta x^{0}}}) \doteq p_{N_{\Delta x^{\chi}}}$ as follows:

$$\Delta s^2_{N_{\Delta x^{\chi}}}(x) = \frac{\ell^4}{\hbar^2} g_{\mu\nu}(x,\{N_{\Delta x^{\chi}}\}) p_{N_{\Delta x^{\mu}}} p_{N_{\Delta x^{\nu}}}. \tag{29}$$

Considering that $\ell \propto l_P$ (i.e., $\ell = \kappa l_P$), where $\kappa = const$ is on the order of 1, in the general case (29), to within the constant $\ell^4/\hbar^2$, we have

$$\Delta s^2_{N_{\Delta x^{\chi}}}(x) = g_{\mu\nu}(x,\{N_{\Delta x^{\chi}}\}) p_{N_{\Delta x^{\mu}}} p_{N_{\Delta x^{\nu}}}. \tag{30}$$

As follows from the previous formulae, the **measurable** variant of General Relativity should be defined in the bundle $B_{g,N_{\Delta x^{\chi}}}$

Remark 3.1
According to (25)–(27), a **measurable** analog of the metric $g_{\mu\nu}(x,\{N_{\Delta x^{\chi}}\})$ is differing from $g_{\mu\nu}(x)$ by the value of a "minimal" interval and by minimal variations of $g_{\mu\nu}(x,\{N_{\Delta x^{\chi}}\})$. However, the components $g_{\mu\nu}(x,\{N_{\Delta x^{\chi}}\})$ themselves are coincident with $g_{\mu\nu}(x)$.

For convenience, apart from formula (25), we use the equivalent formula

$$\Delta s^2_{\{N_{\Delta x_{\chi}}\}}(x) \doteq g^{\mu\nu}(x,\{N_{\Delta x_{\chi}}\}) \frac{\ell^2}{N_{\Delta x_{\mu}} N_{\Delta x_{\nu}}}, \tag{31}$$

that is a **measurable** analog of the formula

$$ds^2(x) = g^{\mu\nu}(x) dx_{\mu} dx_{\nu} \tag{32}$$

Since it has been demonstrated that the metric components in continuous and **measurable** cases are the same, they may be used to raise and to lower the indices in the **measurable** case as well. Specifically, instead of a set of the quantities $g_{\mu\nu}(x,\{N_{\Delta x^{\chi}}\}), N_{\Delta x^{\chi}}, \ell/N_{\Delta x^{\mu}}, p_{N_{\Delta x^{\mu}}}$, we can use the set $g^{\mu\nu}(x,\{N_{\Delta x_{\chi}}\}), N_{\Delta x_{\chi}}, \ell/N_{\Delta x_{\mu}}, p_{N_{\Delta x_{\mu}}}$.

Measurability and Strong Principle of Equivalence in Low Energies

We can easily show that because the energies are low ($E \ll E_p$ or same $|N_{\Delta x_\chi}| \gg 1$), the **Strong Principle of Equivalence (SPE)** ([27],p.69) is valid in terms of **measurable** quantities.
Indeed, let $\mathbf{x^0} \doteq (x^0_\mu), \mu = 0,..,3$ be some fixed point of the space-time variety $\mathcal{M} \subset \mathbf{R^4}$, with the metric $g^{\mu\nu}(x)$ i.e. $\mathbf{x^0} \in \mathcal{M}$.
According to **SPE**, in continuous space-time the point $\mathbf{x^0}$ has a sufficiently small neighborhood, where the metric $g^{\mu\nu}(x)$ is equivalent to the Minkowskian metric $\eta^{\mu\nu}(x); ||\eta^{\mu\nu}|| = Diag\,(-1,1,1,1)$.
We denote this neighborhood as $\mathbf{X^0}(\mathbf{g}^{\mu\nu})$.
Without loss of generality, we can calculate $\mathbf{X^0}(\mathbf{g}^{\mu\nu})$ for each of the coordinates $\mu = 0,..,3$ symmetric relative to $\mathbf{x^0}$, i.e., we have

$$\mathbf{X^0}(\mathbf{g}^{\mu\nu}) \doteq [(x^0_\mu - a_\mu < x_\mu < x^0_\mu + a_\mu) \doteq |x_\mu - x^0_\mu| < a_\mu, \\ \mu = 0,..,3; a_\mu > 0]. \tag{33}$$

Then we can easily find integer $N_{\Delta x^0_\mu}; |N_{\Delta x^0_\mu}| \gg 1$ sufficiently high in absolute value so that

$$|x_\mu - x^0_\mu| = \frac{\ell}{|N_{\Delta x^0_\mu}|} \ll a_\mu. \tag{34}$$

As noted above, for sufficiently high $|N_{\Delta x^0_\mu}|$, the metric $g^{\mu\nu}(x)$, to however high accuracy, is considered to be the **measurable** metric $g^{\mu\nu}(x, \{N_{\Delta x_\chi}\})$. As with an increase in $|N_{\Delta x^0_\mu}|$ the quantity $\ell/|N_{\Delta x^0_\mu}|$ is varying practically continuously, the metric $g^{\mu\nu}(x)$ to however high accuracy could be considered the **measurable** metric for

$$|x_\mu - x^0_\mu| \leq \frac{\ell}{|N_{\Delta x^0_\mu}|}. \tag{35}$$

Since the neighborhood of the point $\mathbf{x^0}$ assigned by the condition (35) is fully lying about the point specified by the condition (33), in this neighborhood the metric $g^{\mu\nu}(x)$ is equivalent to the Minkowskian metric $\eta^{\mu\nu}(x)$ in continuous space-time.
But, in turn, $\eta^{\mu\nu}(x)$ can be represented, to however high accuracy, for the integer number $N^{'}_{\Delta x_\chi}; |N^{'}_{\Delta x_\chi}| \gg 1$ sufficiently high in absolute value, in the form of **measurable** metrics $\eta^{\mu\nu}(x, \{N^{'}_{\Delta x_\chi}\})$.
So, within the concept of **measurability**, the **Strong Principle of Equivalence (SPE)** may be formulated as follows:

Definition 3.1. Measurable Variant of SPE at Low Energies.
For sufficiently small **measurable** *neighborhood of the point* $\mathbf{x^0}$*, (the term*

"**measurable** *neighborhood" means that all points of this neighborhood arise from* $\mathbf{x^0}$ *by means of* **measurable** *variations), the* **measurable** *metric* $g^{\mu\nu}(x, \{N_{\Delta x_\chi}\})$*, with the integer number* $N_{\Delta x_\chi}$ *sufficiently high in absolute value, is equivalent to the* **measurable** *Minkowskian metric* $\eta^{\mu\nu}(x, \{N^{'}_{\Delta x_\chi}\})$ *with the integer* $N^{'}_{\Delta x_\chi}$sufficiently high in absolute value. In other words, in a sufficiently small **measurable** neighborhood of the point $\mathbf{x^0}$ we can obtain, to however high accuracy, the equivalence of the two **measurable** metrics

$$g^{\mu\nu}(x, \{N_{\Delta x_\chi}\}) \equiv \eta^{\mu\nu}(x, \{N^{'}_{\Delta x_\chi}\}). \tag{36}$$

It is clear that, taking maximal absolute values from both sets $N_{\Delta x_\chi}$ and $N^{'}_{\Delta x_\chi}$, $|N^{*}_{\Delta x_\chi}| = Max\{|N_{\Delta x_\chi}|, |N^{'}_{\Delta x_\chi}|\}$, we can have for (36) the coincident sets $\{N_{\Delta x_\chi}\}$ and $\{N^{'}_{\Delta x_\chi}\}$:

$$g^{\mu\nu}(x, \{N^{*}_{\Delta x_\chi}\}) \equiv \eta^{\mu\nu}(x, \{N^{*}_{\Delta x_\chi}\}). \tag{37}$$

Remark 3.2
Again without loss of generality, we can takes as a *sufficiently small* **measurable** *neighborhood* of the point $\mathbf{x^0}$ the neighborhood $\mathbf{X^0}(\mathbf{g}^{\mu\nu})$ specified by formula (33).
It is clear that, as the energies under study are low ($E \ll E_p$), we have $a_\mu = N_{a_\mu}\ell$ and $N_{a_\mu} \gg 1$. Of course, the quantity $a_\mu = N_{a_\mu}\ell$ is not necessarily **primarily measurable**, i.e., the number N_{a_μ} is not necessarily integer. But we can always make it so, taking, instead of the number N_{a_μ}, its integer part $[N_{a_\mu}]$. Then the **primarily measurable** quantity $a_\mu = [N_{a_\mu}]\ell$ is also satisfying the condition specified in formula (33).

The condition "*sufficiently small* **measurable** *neighborhood*" indicates that the numbers N_{a_μ} should set the upper bound as follows:

$$1 \ll N_{a_\mu} \ll N_\mu(g^{\mu\nu}), \tag{38}$$

where the high positive number $N_\mu(g^{\mu\nu})$, (i.e. $N_\mu(g^{\mu\nu}) \gg 1$) is dependent on the metric $g^{\mu\nu}$.
For complete consideration of **SPE** at low energies $E \ll E_p$ in terms of **measurability** notion, we should study the coordinate transformations of a continuous theory in terms of **measurable** quantities.
Let us consider any coordinate transformation $x^\mu = x^\mu(\bar{x}^\nu)$ of the space–time coordinates in continuous space-time. Then we have

$$dx^\mu = \frac{\partial x^\mu}{\partial \bar{x}^\nu}\, d\bar{x}^\nu. \tag{39}$$

As mentioned at the Section 2 (formula (10)),in terms of **measurable** quantities we have the substitution

$$dx^{\mu} \mapsto \frac{\ell}{N_{\Delta x_{\mu}}}; d\bar{x}^{\nu} \mapsto \frac{\ell}{\bar{N}_{\Delta \bar{x}_{\nu}}}, \tag{40}$$

where $N_{\Delta x_{\mu}}, \bar{N}_{\Delta \bar{x}_{\nu}}$ – integers ($|N_{\Delta x_{\mu}}| \gg 1, |\bar{N}_{\Delta \bar{x}_{\nu}}| \gg 1$) sufficiently high in absolute value, and hence in the **measurable** case (39) is replaced by

$$\frac{\ell}{N_{\Delta x_{\mu}}} = \Delta_{\mu\nu}(x^{\mu}, \bar{x}^{\nu}, 1/N_{\Delta x_{\mu}}, 1/\bar{N}_{\Delta \bar{x}_{\nu}})\, \frac{\ell}{\bar{N}_{\Delta \bar{x}_{\nu}}}. \tag{41}$$

Equivalently, in terms of the **primary measurable** momenta we have

$$p_{N_{\Delta x_{\mu}}} = \Delta_{\mu\nu}(x^{\mu}, \bar{x}^{\nu}, 1/N_{\Delta x_{\mu}}, 1/\bar{N}_{\Delta \bar{x}_{\nu}})\, p_{\bar{N}_{\Delta \bar{x}_{\nu}}}, \tag{42}$$

where $\Delta_{\mu\nu}(x^{\mu}, \bar{x}^{\nu}, 1/N_{\Delta x_{\mu}}, 1/\bar{N}_{\Delta \bar{x}_{\nu}}) \doteq \Delta_{\mu\nu}(x^{\mu}, \bar{x}^{\nu}, p_{N_{\Delta x_{\mu}}}, p_{\bar{N}_{\Delta \bar{x}_{\nu}}})$ – corresponding matrix represented in terms of **measurable** quantities.
It is clear that, in accordance with formula (40), in passage to the limit we get

$$\lim_{|N_{\Delta x_{\mu}}| \to \infty} \frac{\ell}{N_{\Delta x_{\mu}}} = dx^{\mu} =$$
$$= \lim_{|\bar{N}_{\Delta \bar{x}_{\nu}}| \to \infty} \Delta_{\mu\nu}(x^{\mu}, \bar{x}^{\nu}, 1/N_{\Delta x_{\mu}}, 1/\bar{N}_{\Delta \bar{x}_{\nu}})\, \frac{\ell}{\bar{N}_{\Delta \bar{x}_{\nu}}} = \frac{\partial \bar{x}^{\mu}}{\partial x^{\nu}}\, dx^{\nu}. \tag{43}$$

Equivalently, passage to the limit (43) may be written in terms of the **primary measurable** momenta $p_{N_{\Delta x_{\mu}}}, p_{\bar{N}_{\Delta \bar{x}_{\nu}}}$ multiplied by the constant $\ell^2/\hbar$.
How we understand formulae (40)–(43)?
The initial (continuous) coordinate transformations $x^{\mu} = x^{\mu}\,(\bar{x}^{\nu})$ gives the matrix $\frac{\partial x^{\mu}}{\partial \bar{x}^{\nu}}$. Then, for the integers sufficiently high in absolute value $\bar{N}_{\Delta \bar{x}_{\nu}}, |\bar{N}_{\Delta \bar{x}_{\nu}}| \gg 1$, we can derive

$$\frac{\ell}{N_{\Delta x_{\mu}}} = \frac{\partial x^{\mu}}{\partial \bar{x}^{\nu}}\, \frac{\ell}{\bar{N}_{\Delta \bar{x}_{\nu}}}, \tag{44}$$

where $|N_{\Delta x_{\mu}}| \gg 1$ but the numbers for $N_{\Delta x_{\mu}}$ are not necessarily integer. Still, as noted above, the difference between $\ell/N_{\Delta x_{\mu}}$ and $\ell/[N_{\Delta x_{\mu}}]$ (and hence between $p_{N_{\Delta x_{\mu}}}$ and $p_{[N_{\Delta x_{\mu}}]}$) is negligible.
Then substitution of $[N_{\Delta x_{\mu}}]$ for $N_{\Delta x_{\mu}}$ in the left-hand side of (44) leads to replacement of the initial matrix $\frac{\partial x^{\mu}}{\partial \bar{x}^{\nu}}$ by the matrix $\Delta_{\mu\nu}(x^{\mu}, \bar{x}^{\nu}, 1/N_{\Delta x_{\mu}}, 1/\bar{N}_{\Delta \bar{x}_{\nu}})$ represented in terms of **measurable** quantities and, finally, to the formula (41). Clearly, for sufficiently high $|N_{\Delta x_{\mu}}|, |\bar{N}_{\Delta \bar{x}_{\nu}}|$, the matrix $\Delta_{\mu\nu}(x^{\mu}, \bar{x}^{\nu}, 1/N_{\Delta x_{\mu}}, 1/\bar{N}_{\Delta \bar{x}_{\nu}})$ may be selected no matter how close to $\frac{\partial x^{\mu}}{\partial \bar{x}^{\nu}}$.
Similarly, in the **measurable** format we can get the formula

$$d\bar{x}^{\mu} = \frac{\partial \bar{x}^{\mu}}{\partial x^{\nu}}\, dx^{\nu} \tag{45}$$

and correspondingly the matrix $\widetilde{\Delta}_{\mu\nu}(\bar{x}^{\mu}, x^{\nu}, 1/\bar{N}_{\Delta x_{\mu}}, 1/N_{\Delta x_{\nu}})$ with the property

$$\frac{\ell}{\bar{N}_{\Delta x_{\mu}}} = \widetilde{\Delta}_{\mu\nu}(\bar{x}^{\mu}, x^{\nu}, 1/\bar{N}_{\Delta x_{\mu}}, 1/N_{\Delta x_{\nu}})\,\frac{\ell}{N_{\Delta x_{\nu}}}, \tag{46}$$

Thus, any coordinate transformation may be represented, to however high accuracy, by the **measurable** transformation (i.e., written in terms of **measurable** quantities), where the principal components are the **measurable** quantities $\ell/N_{\Delta x_{\mu}}$ or the **primary measurable** momenta $p_{N_{\Delta x_{\mu}}}$.
From this it follows that all the components necessary for the formulation of a **measurable** variant of **SPE** at low energies $E \ll E_p$ are available – all of them are represented in terms of the **measurability** notion, making the above definition of a **measurable** variant of **SPE** at low energies $E \ll E_p$ correct.

4 Measurability in Gravity and Strong Principle of Equivalence at All Energy Scales

In this section, based on the results from [10], within the scope of the **spacetime foam** notion we perform an initial analysis of the possibility for generalization of **SPE** in a **measurable** analog of gravity to the ultraviolet (Planck) energy region.
As directly follows from the first part of Section 3, specifically from formulae (25)–(27), the principal components involved in gravitational equations of General Relativity have **measurable** analogs [10].
In particular, the Christoffel symbols [25],[26]

$$\Gamma^{\alpha}_{\mu\nu}(x) = \frac{1}{2}\, g^{\alpha\beta}(x)\, \Big(\partial_{\nu}\, g_{\beta\mu}(x) + \partial_{\mu}\, g_{\nu\beta}(x) - \partial_{\beta}\, g_{\mu\nu}(x)\Big) \tag{47}$$

have the **measurable** analog [10]

$$\Gamma^{\alpha}_{\mu\nu}(x, N_{x_{\chi}}) = \frac{1}{2}\, g^{\alpha\beta}(x, N_{x_{\chi}})\, (\Delta_{\nu} g_{\beta\mu}(x, N_{x_{\chi}}) + \Delta_{\mu} g_{\nu\beta}(x, N_{x_{\chi}}) - \\ - \Delta_{\beta} g_{\mu\nu}(x, N_{x_{\chi}})). \tag{48}$$

Similarly, for the *Riemann tensor* in a continuous theory we have [25],[26]:

$$R^{\mu}{}_{\nu\alpha\beta}(x) \equiv \partial_{\alpha}\Gamma^{\mu}_{\nu\beta}(x) - \partial_{\beta}\Gamma^{\mu}_{\nu\alpha}(x) + \Gamma^{\mu}_{\gamma\alpha}(x)\,\Gamma^{\gamma}_{\nu\beta}(x) - \Gamma^{\mu}_{\gamma\beta}(x)\,\Gamma^{\gamma}_{\nu\alpha}(x). \tag{49}$$

With the use of formula (48), we can get the corresponding **measurable** analog, i.e. the quantity $R^{\mu}{}_{\nu\alpha\beta}(x, N_{x_{\chi}})$ [10].
In a similar way we can obtain the **measurable** variant of *Ricci tensor*, $R_{\mu\nu}(x, N_{x_{\chi}}) \equiv R^{\alpha}{}_{\mu\alpha\nu}(x, N_{x_{\chi}})$, and the **measurable** variant of *Ricci scalar*:

$R(x,N_{x_\chi})\equiv R_{\mu\nu}(x,N_{x_\chi})\,g^{\mu\nu}(x,N_{x_\chi})$ [10].
So, for the *Einstein Equations* (EU) in a continuous theory [25],[26]

$$R_{\mu\nu}-\frac{1}{2}\,R\,g_{\mu\nu}-\frac{1}{2}\,\Lambda\,g_{\mu\nu}=8\,\pi\,G\,T_{\mu\nu} \tag{50}$$

we can derive their **measurable** analog, for short denoted as (EUM)*Einstein Equations Measurable* [10]:

$$R_{\mu\nu}(x,N_{x_\chi})-\frac{1}{2}\,R(x,N_{x_\chi})\,g^{\mu\nu}(x,N_{x_\chi})-\frac{1}{2}\,\Lambda(x,N_{x_\chi})\,g^{\mu\nu}(x,N_{x_\chi})= \\ =8\,\pi\,G\,T_{\mu\nu}(x,N_{x_\chi}), \tag{51}$$

where G – Newton's gravitational constant.
For correspondence with a continuous theory, the following passage to the limit must take place for all the points x:

$$\lim_{|N_{x_\chi}|\rightarrow\infty}\Lambda(x,N_{x_\chi})=\Lambda, \tag{52}$$

where the cosmological constant Λ is taken from formula(50).
Moreover, for high $|N_{x_\chi}|$, the quantity $\Lambda(x,N_{x_\chi})$ should be practically independent of the point x, and we have

$$\Lambda(x,N_{x_\chi})\approx\Lambda(x^{'},N^{'}_{x^{'}_\chi})\approx\Lambda, \tag{53}$$

where $x\neq x^{'}$ and $|N_{x_\chi}|\gg 1,|N^{'}_{x^{'}_\chi}|\gg 1$.
Actually, it is clear that formula (52) reflects the fact that (EUM) given by formula (51) represents **deformation** of the Einstein equations (EU) (50) in the sense of the Definition given in [28] with the deformation parameter N_{x_χ}, and we have

$$\lim_{|N_{x_\chi}|\rightarrow\infty}(EUM)=(EU). \tag{54}$$

We denote this deformation as $(EUM)[N_{x_\chi}]$. Since at low energies $E\ll E_P$ and to within the known constants we have $\ell/N_{x_\chi}=p_{N_{x_\chi}}$, the following deformations of (EU) are equivalent to

$$(EUM)[N_{x_\chi}]\equiv(EUM)[p_{N_{x_\chi}}]. \tag{55}$$

So, on passage from (EU) to the **measurable** deformation of $(EUM)[N_{x_\chi}]$ (or identically $(EUM)[p_{N_{x_\chi}}]$) we can find solutions for the gravitational equations on the metric bundle $B_{g,N_{x_\chi}}\doteq g_{\mu\nu}(x,\{N_{x_\chi}\})$ (formula (28)) given by formula (25) [10].

What are the advantages of this approach?

4.1. First, as $|N_{x_\chi}| \gg 1$, from the above formulae it follows that the metric $g_{\mu\nu}(x, \{N_{x_\chi}\})$ belonging to $B_{g,N_{x_\chi}}$ and representing a solution for $(EUM)[N_{x_\chi}]$, to a high accuracy, is a solution for the Einstein equations (EU) in a continuous theory. Besides, formula (54) shows that at sufficiently high $|N_{x_\chi}|$ this accuracy may be however high. In this way the **Principle of Correspondence to Continuous Theory (PCCT)** (Section 2) to a continuous theory takes place.

4.2. We replace the abstract infinitesimal quantities dx_μ, incomparable with each other, by the specific small quantities ℓ/N_{x_μ} which may be made however small at sufficiently high $|N_{x_\chi}|$, still being ordered and comparable. Because of this, we can compare small values of the squared intervals $\Delta s^2_{\{N_{x_\chi}\}}(x)$ from formula (25). Possibly, this will help to recover the *causality* property for all solutions in $(EUM)[N_{x_\chi}]$ without pathological solutions in the form of the Closed Time-like Curves (CTC), involved in some models of General Relativity [29]–[32].

4.3. Finally, this approach from the start is quantum in character due to the fact that the fundamental length ℓ is proportional to the Planck length $\ell \propto l_P$ and includes the whole three fundamental constants, the Planck constant $\hbar$ as well. Besides, it is naturally dependent on the energy scale: sets of the metrics $g_{\mu\nu}(x, \{N_{x_\chi}\})$ with the lowest value $|N_{x_\chi}|$ correspond to higher energies as they correspond to the momenta $\{p_{N_{x_\chi}}\}$ which are higher in absolute value. This is the case for all the energies E.
However, minimal measurable increments for the energies $E \approx E_P$ are not of the form ℓ/N_{x_μ} because the corresponding momenta $\{p_{N_{x_\chi}}\}$ are no longer **primary measurable**, as indicated by the results in Section 2.
So, in the proposed paradigm the problem of the ultraviolet generalization of the low-energy **measurable** gravity $(EUM)[N_{x_\chi}]$ (formula (51)) is actually reduced to the problem: what becomes with the **primary measurable** momenta $\{p_{N_{x_\chi}}\}, |N_{x_\chi}| \gg 1$ at high Planck's energies.
In a relatively simple case of GUP in Section 2 we have the answer. And, using the fact that $(EUM)[N_{x_\chi}] \equiv (EUM)[p_{N_{x_\chi}}]$ (55), based on the results of Section 2, we can construct a correct high-energy passage to the Planck energies $E \approx E_p$ [10]

$$(EUM)[p_{N_{x_\chi}}, |N_{x_\chi}| \gg 1] \mapsto (EUM)[p_{N_{x_\chi}}(GUP), |N_{x_\chi}| \approx 1], \tag{56}$$

where $p_{N_{x_\chi}}(GUP) = \Delta p(\Delta x_\chi, GUP)$ according to formula (15) of Section 2. In this specific case, we can construct the natural ultraviolet generalization $(EUM)[p_{N_{x_\chi}}, |N_{x_\chi}| \gg 1] \doteq (EUM)[p_{N_{x_\chi}}]$. The theoretical calculations $(EUM)[p_{N_{x_\chi}}(GUP), |N_{x_\chi}| \approx 1]$ derived at Planck's energies are obviously *dis-*

crete,**measurable**, and represent a high-energy deformation in the sense of the [28] **measurable** gravitational theory $(EUM)[p_{N_{x_\chi}}, |N_{x_\chi}| \gg 1]$.

Strong Principle of Equivalence in Measurable Variant at All Energy Scales

The Equivalence Principle (weak or strong) in its initial form has been formulated for a low-energy gravitational theory, i.e. for the energies $E \ll E_p$ in continuous space-time [27].There is nothing similar for the energies $E \approx E_p$.
However, in the proposed approach (or in the present paradigm) we go from continuous space-time to the **measurable** discrete space-time but in such a way that at low energies $E \ll E_p$ the introduced **measurable** discrete space-time is close to the continuous space-time, enabling the author to form a **measurable** analog of the **Strong Principle of Equivalence at Low Energies** in Section 3.
The basic parameters used to form the **measurability** notion for all the energy scales are the integer numbers $N_{x_\mu}, \mu = 0, ..., 3$ (or identically N_{x_μ}). At low energies $E \ll E_p$ these numbers satisfy the condition $|N_{x_\mu}| \gg 1$. As it has been demonstrated above, the corresponding **primarily measurable** momenta $p_{N_{\Delta x_\mu}}$ (and space-time variations $\ell/N_{\Delta x_\mu}$) are adequate to form a **measurable** variant of gravity at these energy scales.
At high energies $E \approx E_p$ (same $E \approx E_\ell$) (case B) from Section 2), due to the fact that for $|N_{x_\mu}| \approx 1$ a theory in terms of **measurable** quantities becomes *really discrete*, the **primarily measurable** momenta $p_{N_{\Delta x_\mu}}$, in line with formula (15), are *inadequate* for the correct examination of this case.
In the general case the transition from high $E \approx E_p$ to low energies for a **measurable** variant of gravity is given by reversal of the arrow from formula (56):

$$(EUM)[p_{N_{x_\chi}}, |N_{x_\chi}| \approx 1] \mapsto (EUM)[p_{N_{x_\chi}}, |N_{x_\chi}| \gg 1], \tag{57}$$

where $p_{N_{x_\chi}}$ for $|N_{x_\chi}| \approx 1$ the **generalized measurable** (or simply **measurable**) momenta are so that we have

$$p_{N_{x_\chi}}, (|N_{x_\chi}| \approx 1) \overset{|N_{x_\chi}|\approx 1 \rightarrow |N_{x_\chi}| \gg 1}{\Rightarrow} p_{N_{x_\chi}}, (|N_{x_\chi}| \gg 1). \tag{58}$$

The momenta in the right-hand part of formula (58),i.e. $p_{N_{x_\chi}}, (|N_{x_\chi}| \gg 1)$, are the **primary measurable** momenta at low energies $E \ll E_p$.
In Section 2 it is shown that the momenta $p_{N_{x_\chi}}(GUP), |N_{x_\chi}| \approx 1$ specified by formula (15) just satisfy the conditions of (57),(58). But it is obvious that in the general case at the energies $E \approx E_p$ the momenta $p_{N_{x_\chi}}, (|N_{x_\chi}| \approx 1$, meeting the conditions (57),(58), may be of a more complex form. For example, the form of GUP may be more complex than that considered in the survey work

[38]. In this case on passage to **quantum gravity** the formulas (56)–(58) are still valid.
In all the cases for a **measurable** variant of gravity the transition to the ultraviolet (i.e quantum) region may be realized by substitution of $\frac{\ell^2}{\hbar}p_{N_{\Delta x^\mu}}, |N_{\Delta x^\mu}| \approx 1$ in Section 3 for the quantities $\ell/N_{\Delta x^\mu} = \frac{\ell^2}{\hbar}p_{N_{\Delta x^\mu}}, |N_{\Delta x^\mu}| \gg 1$; by the corresponding corrections of formulae (25)–(31) from Section 3, of all the components necessary for derivation of gravitational equations in a **measurable** variant $\Gamma^{\alpha}_{\mu\nu}(x, N_{x_\chi}), R^{\mu}_{\nu\alpha\beta}(x, N_{x_\chi}), ...$, and of formulae (48),(50),... from this Section.
It is clear that, provided at high energies $E \approx E_p$ in the **measurable** case some analog of the **Strong Principle of Equivalence (SPE)**is involved, its formulation should be radically different from **(SPE)** in the **measurable** case at low energies $E \ll E_p$ considered in Section 3 for the two main reasons given below.

4.4A. As at high energies $E \approx E_p$ (and hence at $|N_{\Delta x^\mu}| \approx 1$) a **measurable** variant of gravity represents a discrete theory, where the notion of *locality* is senseless, we should involve the *minimal* **primarily measurable** spatial neighborhood and the *minimal* **generalized measurable** spatial variations $\frac{\ell^2}{\hbar}p_{N_{\Delta x^\mu}}, |N_{\Delta x^\mu}| \approx 1$ for the arbitrary point $\mathbf{x} \doteq \{x^\mu\}$ (with the naturally selected finite bounds of the numbers $N_{\Delta x^\mu}$).

4.4B. Besides, it is obvious that at high energies $E \approx E_p$ the space curvature becomes great and this space in any **measurable** neighborhood of the random point $\mathbf{x}$ is far from the flat space with the Minkowskian metric $\eta^{\mu\nu}(x)$.

As follows from remarks **4.4A.** and **4.4B.**, when for a **measurable** variant of gravity there is some form of an analog of the **Strong Principle of Equivalence (SPE)** at high energies $E \approx E_p$, its correct formulation should be completely coordinated with the transitions from high to low energies given in formulae (57), (58). In other words, on going from high to low energies, this high-energy analog of **SPE** should conform to **SPE** at low energies $E \ll E_p$ for a **measurable** variant of gravity considered in Section 3.
In accordance with the modern understanding of the problem, at high energies $E \approx E_p$ the space geometry, due to high Space-Time Quantum Fluctuations (STQF), represents the “space-time foam”(**stf**) [33],[34]. The notion of “space-time foam” was introduced by J. A. Wheeler about 60 years ago for the description and investigation of physics at Planck’s scales (Early Universe). Actually, because of high **quantum fluctuations** of the metric $g_{\mu\nu}$, the space has a quantity of geometries. Despite the fact that in the last time numerous works have been devoted to physics at Planck’s scales within the scope of this notion, by this time still their no clear understanding of **stf** as it is.

Still, some models based on *micro-black holes* are very interesting and fairly promising. In particular, the models studied in [35]–[37] and based on *micro-black holes*, i.e. black holes with a Schwarzschild radius of several Planck's units of length.
Without loss of generality, it may be considered that all the *micro-black holes* considered as "constituent parts" of **stf** are Schwarzschild's black holes.
It should be noted that the case of *micro-black holes* with the Schwarzschild metric in terms of **measurable** quantities has been already studied by the author in his papers [7], [9]. In these papers, within the scope of validity of the Generalized Uncertainty Principle (GUP) of Section 2, in terms of the **measurability** notion the gravitational equations at the event horizon surface of these holes have been derived and their basic thermodynamic characteristics (temperature, entropy) have been obtained.
It is obvious that these holes form a discrete finite set, provided their Schwarzschild radii r_{mbh} are considered **primarily measurable** quantities:

$$r_{mbh} = N_{r_{mbh}}\ell, N_{r_{mbh}} \approx 1, \tag{59}$$

where $N_{r_{mbh}}$ is an integer number.
Proceeding from all the above, a **measurable** variant of the **Strong Principle of Equivalence** at high energies $E \approx E_p$ for **stf** based on the geometry of Schwarzschild's *micro-black holes* may be formulated as follows.

In a sufficiently small **primarily measurable** *neighborhood of any spatial point* **x** *at the Planck scale the geometry of* **stf** *is equivalent to the geometry of some* micro-black hole *with the Schwarzschild metric and with the corresponding Schwarzschild radius r_{mbh} satisfying formula (59).*

As, in accordance with GUP of Section 2, we have

$$p(N_{\Delta x_i}, GUP) = \frac{\hbar}{1/2(N_{\Delta x_i} + \sqrt{N^2_{\Delta x_i} - 1})\ell}, i = 1, ..., 3, \tag{60}$$

on passage from high energies $E \approx E_p$ to low energies $E \ll E_p$, formula (58) is apparently valid and we can, to a high accuracy, obtain at low energies the **primarily measurable** momenta $p(N_{\Delta x_i}), |N_{\Delta x_i}| \gg 1$ and a **measurable** variant of the **Strong Principle of Equivalence at Low Energies** from Section 3.
In the process it is assumed that formula (57) is valid by default, i.e. passage from **stf** at high energies $E \approx E_p$ to low energies $E \ll E_p$ leads to the large-scale space-time structure and to Einstein Equations.
As noted in point **4.3.**, in a simple case of GUP considered in Section 2 passage to **quantum gravity** in a **measurable** variant of General Relativity is

represented by formula (56). However, GUP may be of a more complex as compared to the considered in the survey work [38]. In this case on passage to **quantum gravity** the formula (56) is still valid.

5 Conclusion

Thus, in this paper it has been demonstrated that the **Strong Principle of Equivalence (SPE)** may be correctly formulated in terms of **measurable** quantities, i.e. for a **measurable** analog of gravity (or same **measurable** variant of gravity) at low energies $E \ll E_p$. Besides, it has been shown that, within the scope of the specific models for **Space-Time Foam**, **SPE** may be also valid for a **measurable** variant of gravity and at the Planck scales, or at high energies $E \approx E_p$.
Since at the present time no direct or indirect experiments at the scales on the order of Planck's scales (i.e. at the energies associated with the quantum gravity scales) are known, all theoretical studies in this field are to some or other extent speculative. Nevertheless, considering that gravity should be formulated with the use of the same terms at all the energy scales, it must be governed by the particular unified principles the formulation of which varies depending on the available energies. Because of this, the results from Section 4 seem to be important. Of course, these results are tentative and may be corrected during further studies of gravity in terms of the **measurability** notion. But they give the main idea and define the trend towards the derivation of a **measurable** variant of gravity: framing of a correct gravitational theory at all the energy scales, with the use of a set of discrete parameters $p(N_{\Delta x_\mu})$ for all nonzero integer values of $N_{\Delta x_\mu}$, that is close to the General Relativity at low energies $E \ll E_p$ and is a new (discrete) theory at high energies $E \approx E_p$.
As noted in Section 4 (formula (54)) and in the earlier papers of the author, the above derivation of a **measurable** variant of gravity may be realized proceeding from the notion of the *deformation* of a physical theory introduced in [28]:

Deformation is understood as an extension of a particular theory by inclusion of one or several additional parameters in such a way that the initial theory appears in the limiting transition.

Denoting a **measurable** variant of gravity at low energies $E \ll E_p$ (that is yet incompletely derived) by $Grav[LE, meas]^{\ell}$, we obtain that the above-mentioned deformation is nothing else but the following mapping:

$$Grav[LE, meas]^{\ell} \stackrel{\ell \to 0}{\Rightarrow} GR, \qquad (61)$$

where the *deformation* parameters are **primarily measurable** momenta $p(N_{\Delta x_\mu}), |N_{\Delta x_\mu}| \gg 1$ (or the corresponding space-time variations $\ell/N_{\Delta x_\mu}$).
Then *Einstein Equations Measurable (EUM)* at low energies $E \ll E_p$ in Section 3 (formula (51)) is a low-energy deformation *deformation* of *Einstein Equations (EU)* in General Relativity (GR) as indicated by formula (54).
Considering that $Grav[LE, meas]^\ell$ and GR are very close but not identical, the author's hypothesis is as follows:

we can frame a **measurable** *variant of gravity* $Grav[LE, meas]^\ell$*, within the scope of which there is possibility for the effective solution of several problems at the joint of General Relativity and Quantum Theory: the above-mentioned Closed Time-like Curves (CTC) problem [29]–[32], black hole radiation problem, Hawking's Information Paradox [39] –[41], etc.*

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