The Relative Entropy of One Graviton Creation in Early Universe

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Abstract. The detection of nonclassicality of primordial gravitational waves (PGWs) is one way to prove the quantum theory of gravity. The nonclassical feature can be seen as a non-gaussianity, that could happen if there is a graviton creation in a squeezed state of Bunch-Davies vacuum. This condition would also produce an entanglement state. In this paper, we study that non-gaussianity by using relative entropy as a measurement. We find that, when the value of α (the entanglement constant) is near to one, the non-gaussianity would get its highest value. For the presence of matter field, the non-gaussianity still has the same characteristic for different values of α. But, in this case, the relative entropy value is constant for every value of the squeezed parameter.

1. Introduction

The successful unification theory of electromagnetic, strong, and weak interactions in the quantum framework led the scientists to try to formulate the theory of gravity in the same frame (quantum gravity). But experimentally as well as theoretically, the development of the quantum theory of gravity still has no significant result. This condition raises the question, "Do quantum gravity is really exist?". One way to answer this question is by proving the existence of the graviton, which is a hypothetical particle that is predicted by the quantum theory of gravity. Many ways have been proposed to prove the existence of the graviton. The latest is by detecting the non-classical properties of the gravitational waves [1,2,3]. The success of detecting gravitational waves in 2015 [4] became the main purpose to propose this method.

In this research, the gravitational waves that are studied, are sourced from the beginning universe. Based on the predictions from the cosmological inflation theory, the initial state of the universe which generated gravitational waves is the quantum vacuum state of the graviton (commonly referred to as the Bunch-Davies vacuum). This quantum state can be obtained by quantizing the second-order equations of motion in linearized flat spacetime. This equation of motion is similar to the Mukhanov- Sasaki [5,6,7], the only difference is the gravitational field has two degrees of freedom (due to the polarization of gravitational waves). So that, the quantum vacuum state would also be expressed in a form that containing two degrees of freedom. This condition becomes interesting because the annihilation and creation operators of the system are in a superposition of each operator in each degree of freedom, which means that if the creation operator works on a vacuum that will generate an entanglement state.

The development of quantum information theory provides several parameter options (measurement) that can be used to study the non-classical feature of gravitational waves. One of them is non-gaussianity. There are a lot of quantum states that can produce a non-gaussian distribution, like Fock states, Schrodinger cat states, photon added squeezed states, and many more [8]. In the cosmology context, as a result of the inflation at the beginning of the universe, the quantum vacuum state would evolve like squeezing processes in quantum optics [9]. Thus, if the graviton is created in that squeezed state, it will produce a state that is similar to the photon added squeezed state with entanglement. In this paper, that state would be studying by using relative entropy [10] as measurement.

This paper is divided into six-part. The first part is the introduction. In second, would be explained the measurement of non-gaussianity that is used (relative entropy). In third, will be a review of the Bunch-Davies vacuum in more detail. In fourth and fifth will be the calculation of the relative entropy for one graviton creation in the squeezed vacuum state with and without the presence of matter fields. The last one is the conclusion

2. Measurement of Non-Gaussianity

In recent years, the non-gaussianity of a state had a significant role in continuous-variable systems in quantum information science. A lot of measurements are developed to quantify non-gaussianity for that system. One of those is relative entropy. Before we explain more about measuring non-gaussianity using relative entropy, will be reviewed a little bit about quantum in continuous-variable systems.

Continuous variables in quantum system is a theory that the observable are in the continuous interval. For example free scalar real quantum field, the hamiltonian of the sistem will be a collection of harmonic oscillator and alaways can be represent in term of creation ($\hat{a}_k$) and anihilation ($\hat{a}_k^\dagger$) operator, satisfying the commutation relation $[\hat{a}_k, \hat{a}_k^\dagger]$. For each mode, one can defined quadrature operator

$$q_k = \frac{1}{\sqrt{2}}(\hat{a}_k + \hat{a}_k^\dagger) \tag{1}$$

$$p_k = \frac{1}{i\sqrt{2}}(\hat{a}_k - \hat{a}_k^\dagger). \tag{2}$$

These operator are obsevables with continuous eigenvalues that can be used to describe the entire quantum field system. These operator also satisfy the commutation $[q_i, p_j] = i\delta_{ij}$. In more general form that include all the modes, the commutation can be writen as

$$[R_i, R_j] = i\Omega_{ij} \tag{3}$$

with R_i is element of real vector $\mathbf{R} = (q_1, p_1, ..., q_n, p_n)^T$ and Ω_{ij} is element of sympletic matrix $\mathbf{\Omega} = i\bigoplus_{k=1}^{n}\sigma_2$, where σ_2 is a y-pauli matrix.

This paper will be used the relative entropy to quantify the non-gaussianity of the system. The basic idea of relative entropy is to measure the distinction between two quantities (entropy). The first one, the quantity from the state of the non-gaussian system (ρ_{NG}), and another one, the state of the gaussian reference (ρ_G). The gaussian reference is chosen to has the same vector mean value ($X[\rho_{NG}] = X[\rho_G]$) and the covariance matrix ($\sigma[\rho_{NG}] = \sigma[\rho_G]$). Where

$$X_j = \langle R_j \rangle \tag{4}$$

$$\sigma_{ij} = \frac{1}{2}\langle\{R_i, R_j\}\rangle - \langle R_i\rangle\langle R_j\rangle \tag{5}$$

Journal of Physics: Conference Series **2243** (2022) 012098 doi:10.1088/1742-6596/2243/1/012098

Mathematically the relative entropy is

$$\begin{aligned} S(\rho_{NG}||\rho_G) &= Tr\Big[\rho_{NG}(\log\rho_{NG} - \log\rho_G)\Big] \\ &= Tr\big[\rho_{NG}\log\rho_{NG}\big] - Tr\big[\rho_{NG}\log\rho_G\big] \end{aligned}$$

because $Tr\big[\rho_{NG}\log\rho_G\big] = Tr\big[\rho_G\log\rho_G\big]$ then

$$\begin{aligned} S(\rho_{NG}||\rho_G) &= Tr\big[\rho_{NG}\log\rho_{NG}\big] - Tr\big[\rho_G\log\rho_G\big] \\ &= S(\rho_G) - S(\rho_{NG}) \end{aligned} \tag{6}$$

with $S(\rho) = -Tr[\rho\log\rho]$. If the relative entropy of the system is equal to zero then the system is said to be a gaussian system. Otherwise if the relative entropy of the system is more than zero, that will represent the quantity of the non-gaussianity. For single mode system the most general gaussian reference is

$$\rho_G = D(\lambda)S(\alpha)v(n)S^\dagger(\alpha)D^\dagger(\lambda) \tag{7}$$

with $v(n)$ is a thermal state

$$v(n) = \Sigma_{n=0}^{\infty}\frac{\langle\hat{n}\rangle}{(1+\langle\hat{n}\rangle)^{n+1}}\,|n\rangle\,\langle n| \tag{8}$$

$D(\lambda)$ is a displacement operator and $S(\alpha)$ is a single mode squeezed operator, with $\alpha = re^{\varphi}, \lambda \in \mathbb{C}$. In this gaussian reference the element of covariance matrix are

$$\sigma_{11} = (\langle\hat{n}\rangle + \frac{1}{2})[\cosh(2r) - \sinh(2r)\cos(\varphi)] \tag{9}$$

$$\sigma_{22} = (\langle\hat{n}\rangle + \frac{1}{2})[\cosh(2r) + \sinh(2r)\cos(\varphi)] \tag{10}$$

$$\sigma_{12} = \sigma_{21} = (\langle\hat{n}\rangle + \frac{1}{2})\sinh(2r)\sin(\varphi). \tag{11}$$

Using this state, the Von-Neumann entropy [11] would only depend on the average number of the particles

$$S(\rho_G) = (\langle\hat{n}\rangle + 1)\log(\langle\hat{n}\rangle + 1) - \langle\hat{n}\rangle\log\langle\hat{n}\rangle \tag{12}$$

3. Bunch Davies Vacuum of Primordial Gravitational Wave

In this section, will be reviewing the quantum state of primordial gravitational wave [1,2,12]. The gravitational wave could happen if there is a small gravitational field perturbation on the flat space metric. From the cosmological point of view, the flat space can be represented as the FLRW flat metric. If the small gravitational perturbation field is written as $h_{\mu\nu}$, then the metric becomes

$$ds^2 = a(\eta)^2\Big[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j\Big] \tag{13}$$

with the metric element

$$\begin{aligned} g_{\mu\nu} &= a(\eta)^2\tilde{g}_{\mu\nu} \\ &= a(\eta)^2\Big(\eta_{\mu\nu} + h_{\mu\nu}\Big) \end{aligned}$$

and i, j (alphabet) indices start from 1 to 3, μ, ν (grace) indices start from 0 to 3. The tensor perturbation $h_{\mu\nu}$ has to be in transverse traceless gauge, which mean $h = 0$ (traceless), $h_{0\nu} = 0$ and $\partial_\mu h^{\mu\nu} = 0$ (transverse). The quantum state of this system can be found from the second-order of the action, which can be expanded to Einstein-Hilbert action and the matter action ($S = S_{EH} + S_{matter}$). Each of those actions in the second-order are

$$(S_{EH})^{(2)} = \int d^3x \left[\frac{1}{8} a^2 (h_{ij})'(h^{ij})' + \frac{1}{8} a^2 \partial_i h_{ij} \partial^i h^{jk} - \frac{1}{4} a^2 \big(\mathcal{H} + 2\mathcal{H}'\big) h_{ij} h^{ij} \right] \tag{14}$$

and

$$(S_{matter})^{(2)} = \int d^3x \frac{1}{4} a^2 \big(\mathcal{H}^2 + 2\mathcal{H}'\big) h_{ij} h^{ij} \tag{15}$$

with the total action

$$S^{(2)} = \frac{1}{8} \int d^3x \;\; a^2 \bigg((h_{ij})'(h^{ij})' + \partial_i h_{ij} \partial^i h^{jk} \bigg) \tag{16}$$

This action can be expressed in the fourier modes by using the fourier transfom

$$a(\eta) h_{ij}(\eta, x^i) = \frac{1}{\sqrt{2}} \sum_A \int \frac{d^3k}{(2\pi)^{3/2}} h_k^A(\eta) P_{ij}^A(k) e^{ikx} \tag{17}$$

In this transformation, we introduced the polarization tensor $P_{ij}^A(k)$ with the normalization $(P_{ij}^A(k))^* P_{ij}^B(k) = 2\delta^A B$. The index A in the polarization tensor represents the polarization modes of the gravitational wave. If the polarization is a linear polarization, then the index A could be in mode $(+)$ or $(\times)$. The total action in the Fourier mode is

$$S^{(2)} = \frac{1}{16} \int \;\; d\eta \; d^3k \bigg((h_k^A)'^2 + \bigg(k^2 - \frac{a''}{a}\bigg)(h_k^A)^2 \bigg) \tag{18}$$

So that, the equation of motion from this action

$$(h_k^A)'' + \bigg(k^2 - \frac{a''}{a}\bigg) h_k^A = 0 \tag{19}$$

This equation of motion is very similar to the Mukhanov-Sasaki equation in inflation perturbation cosmology [5]. The only difference is the gravitational field (h_k) in equation (7) has two degrees of freedom, rather than Mukhanov-Sasaki that has only one degree of freedom. That two degrees of freedom come from the polarization of the gravitational waves. Like the Mukhanov-Sasaki equation, the quantum state of this system can be found by defining the gravitational perturbation field h_k^A as an operator

$$\hat{h}_k^A(\eta) = \hat{a}_k^A h_k(\eta) + \hat{a}_k^{A\dagger} \big(h_k(\eta)\big)^* \tag{20}$$

with $\hat{a}_k^A$ is a non hermitian operator and h_k is the solution of equation motion (7)

$$h_k(\eta) = \frac{1}{\sqrt{2}k} \bigg(1 - \frac{i}{k\eta}\bigg) e^{-ik\eta} \tag{21}$$

The operator $(\hat{h}_k^A)$ should fullfil the equal time commutation relation $\big([\hat{h}_k^A, \hat{\Pi}_p^B] = i\delta^{AB}\delta^3(k-p)\big)$. With $\hat{\Pi}_k^A$ is the momentum conjugate of the field $\hat{h}_k^A$. The non hermitioan operator $\hat{a}_k^A$ will also fullfil the equal time commutation relation $\left([\hat{a}_k^A, \hat{a}_p^B] = \delta^{AB}\delta^3(k-p)\right)$. In this chase the non hermitian operator becomes creation and annihilation operator, so the quantum state of this system can be definned as

$$\hat{a}_k^A |0\rangle^A = 0 \qquad |m\rangle^A = \frac{1}{\sqrt{m!}}(\hat{a}_k^{A\dagger})^m |0\rangle^A \tag{22}$$

Which this quantum state usually called as Bunch-Davies vacuum state of particle graviton.

This quantum state could be represented in a more general form as a bipartite system that depends on the polarization mode of the gravitational waves [13]. In this form, the annihilation and creation operator is in the superposition of each polarization mode. If the gravitational wave polarization is linear, so the annihilation and creation operators are

$$\hat{a}_k^{(+,\times)\dagger} = \alpha\hat{a}_k^{(+)\dagger} + \beta\hat{a}_k^{(\times)\dagger} \tag{23}$$
$$\hat{a}^{(+,\times)} = \alpha\hat{a}_k^{(+)} + \beta^*\hat{a}_k^{(\times)} \tag{24}$$

with α and β are the ratio of each mode. So the quantum state now become $\left(|n,m\rangle = |n\rangle^{(+)} \otimes |m\rangle^{(\times)}\right)$. Where n is the total graviton particle in mode $(+)$ and m is in mode $(\times)$. In this form of the state, the annihilation and creation operator for each plarization mode will behave as

$$\hat{a}_k^{(+)} |n,m\rangle = \sqrt{n}\,|n-1,m\rangle\,, \qquad \hat{a}_k^{(+)\dagger} |n,m\rangle = \sqrt{n+1}\,|n+1,m\rangle \tag{25}$$

$$\hat{a}_k^{(\times)} |n,m\rangle = \sqrt{m}\,|n,m-1\rangle\,, \qquad \hat{a}_k^{(\times)\dagger} |n,m\rangle = \sqrt{m+1}\,|n,m+1\rangle \tag{26}$$

and for the general annihilation and creation equation (13,14) in the vacuum state

$$\left(\hat{a}_k^{(+,\times)}\right)^v |0,0\rangle = 0 \tag{27}$$

$$\left(\hat{a}_k^{(+,\times)\dagger}\right)^v |0,0\rangle = \sum_{n=0}^{v} \alpha^n \beta^{v-n} \sqrt{\binom{v}{n}}\,|n, v-n\rangle \tag{28}$$

In equation (16), one can see that creating graviton in a vacuum state would lead the state into an entanglement. As the simplest one ($m = 1$), the entanglement that produced is a Bell's state[14].

4. The Relative Entropy

One theory that could answer the problem that arises from the Cosmic Microwave Background (CMB) data is the inflation theory [15]. In this theory, the universe should have experienced an exponential expansion of space in the early condition after the singularity. This inflation process should also take effect on the Bunch-Davies vacuum. The effect is very similar to the vacuum state of the harmonic oscillator when the width of the potential energy is open up very abruptly [16]. The quantum state then is not in a vacuum state again. The state has now become a state called squeezed state.

The squeezed state is a quantum phenomenon commonly encountered in quantum optics [9]. A state is said to be a squeezed state when the variance from one of two non-commuting observables is lower than the minimum of its uncertainty . Mathematically a squeezed state can be obtained by operating a state to a squeezed operator. For a single-mode system, the squeezed operator is

$$\hat{S}^A(\zeta) = \exp\left[\frac{1}{2}(\zeta^*\hat{a}_k^{A2} + \zeta\hat{a}_k^{A\dagger 2})\right] \tag{29}$$

with $\zeta = re^{i\theta}$, r is known as squeezed parameter with $0 \leq r < \infty$ and $0 \leq \theta \leq 2\pi$. By using operator (29), the graviton creation of squeezed Bunch-Davies vacuum state can be written as

$$|\Psi\rangle = \frac{1}{\cosh r}\Big(\alpha\ \hat{a}_k^{(+)\dagger}\hat{S}^{(+)}(\zeta)\,|0\rangle\,\hat{S}^{(\times)}(\zeta)\,|0\rangle + \beta\ \hat{S}^{(+)}(\zeta)\,|0\rangle\,\hat{a}_k^{(\times)\dagger}\hat{S}^{(\times)}(\zeta)\,|0\rangle\Big) \tag{30}$$

Where $(1/\cosh r)$ is a normalization constant. Because we want to study the role of the entanglement to the non-gaussianity. The state would be represented by a density matrix in one of the Hilbert spaces. To do so, the matrix density from one of the Hilbert spaces equation (30) would be traced out. Chose the Hilbert space is $\mathcal{H}_{(+)}$ then

$$\begin{aligned}\rho_{(+)} &= Tr_{(\times)}\big(\,|\Psi\rangle\langle\Psi|\,\big)\\ &= \frac{1}{\cosh^2 r}\Big(\alpha^2\ \hat{a}_k^{(+)\dagger}\hat{S}^{(+)}\,|0\rangle\langle 0|\,\hat{S}^{(+)\dagger}\hat{a}_k + \beta^2\ \hat{S}^{(+)}\,|0\rangle\langle 0|\,\hat{S}^{(+)\dagger}\Big)\end{aligned} \tag{31}$$

To measure the relative entropy of this system the gaussian reference is needed. As explained in section 2, the gaussian references should have the same vector mean value and covariance matrix. To make the calculation simpler, the quadrature that is use is quadrature equations (1) and (2). Then, by using density matrix equation (31), one can calculated the element of covariance matrix (σ_{ij}) by only $\frac{1}{2}\langle\{R_i, R_j\}\rangle$, because the vector mean value is equal to zero. Therefore, all the elements of the covariance matrix are

$$\sigma_{11} = (\alpha^2 + \frac{1}{2})[\cosh(2r) - \sinh(2r)\cos(\varphi)] \tag{32}$$

$$\sigma_{22} = (\alpha^2 + \frac{1}{2})[\cosh(2r) + \sinh(2r)\cos(\varphi)] \tag{33}$$

$$\sigma_{12} = \sigma_{21} = (\alpha^2 + \frac{1}{2})\sinh(2r)\sin(\varphi). \tag{34}$$

In this covariance matrix, constant (α) would have a role as the average number of graviton $(\langle n\rangle)$. Therefore the Von Neumann entropy for the gaussian refrences become

$$S(\rho_G) = (\alpha + 1)\log(\alpha + 1) - \alpha\log\alpha \tag{35}$$

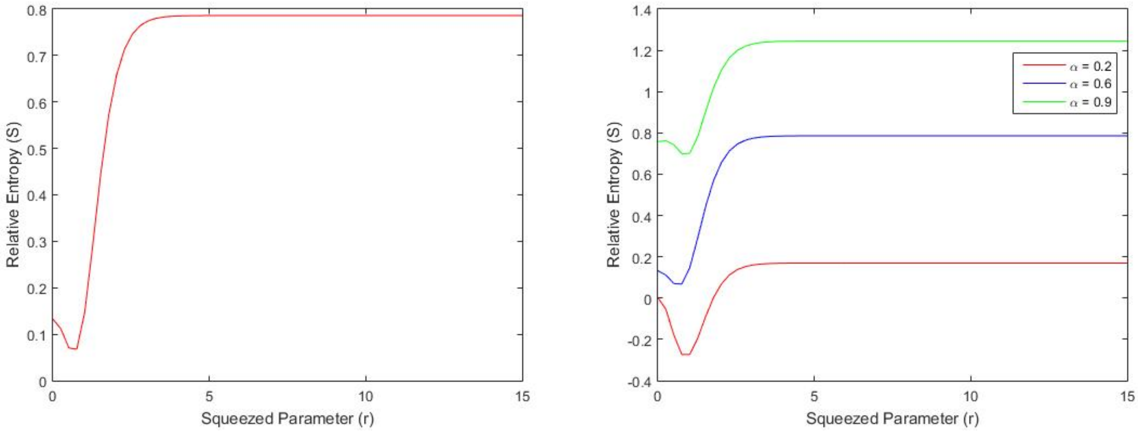
The Von Neumann entropy of density matrix equation (31) can be calculated by expanding it as

$$\begin{aligned}\rho_+ = &\frac{\alpha^2}{\cosh^3 r}\Big(\,|1\rangle\langle 1| + \sqrt{\frac{3}{2}}\tanh r\Big(\,|1\rangle\langle 3| + |3\rangle\langle 1|\,\Big) + \frac{3}{2}\tanh^2 r\Big(\,|3\rangle\langle 3|\,\Big) + ...\Big)\\ &+\frac{(1-\alpha^2)}{\cosh^3 r}\Big(\,|0\rangle\langle 0| + \frac{\tanh r}{\sqrt{2}}\Big(\,|0\rangle\langle 2| + |2\rangle\langle 0|\,\Big) + \frac{1}{2}\tanh^2 r\Big(\,|2\rangle\langle 2|\,\Big) + ...\Big)\end{aligned} \tag{36}$$

Evaluate the eigenvalue (A_n) of this matrix. Then the Von Neumann entropy is calculated by

$$S(\rho_{NG}) = S(\rho_+) = \sum_n -A_n \log A_n \tag{37}$$

Using the Von Neumann equation (35) and (37), the relative entropy is founded, plotting the relative entropy to the squeezed parameter (r) (figure 1.). One sees at the beginning, the relative entropy is come down a little bit. After that, the relative entropy will drastically increase until hitting the highest point and become constant. In figure 1. the relative entropy is also plotted with different α. We got, as the closer the value of α to one, the relative entropy will getting bigger.



(a) (b)

Figure 1: In figure (a), the relative entrophy is ploted to squeezed parameter with $\alpha = 0.2$. In figure (b), the relative entropy is ploted with three different α (green line $\alpha = 0.9$, blue line $\alpha = 0.6$ and red line $\alpha = 0.2$).

5. The Relative Entropy with Matter Field Presence

In the previous section, we calculate the relative entropy of one graviton creation in a squeezed bunch davies vacuum. In this section, we will calculate the relative entropy also, but we consider there is a linear interaction between the metric and matter field. To do so let's find out the hamiltonian interaction of the system.

The hamiltonian interaction of the system can be found from the first-order of the matter action ($S^{(1)}_{matter}$). Which can be written as

$$\begin{aligned} S^{(1)}_{matter} &= -\int d^4x\, \frac{a^2}{2}\, h^A_{\mu\nu}\Big(\partial^\mu\phi\partial^\nu\phi + \eta^{\mu\nu}\mathcal{L}_{matter}\Big) \\ &= -\int d^4x\, \frac{a^2}{2}\, h^A_{\mu\nu}\, T^{\mu\nu}. \end{aligned} \tag{38}$$

So the hamiltonian

$$\mathcal{H}_{int} = \frac{1}{2}a^2\hat{h}^A_{ij}T^{ij}. \tag{39}$$

Using this hamiltonian, a unitary operator could be constucted

$$\begin{aligned} \hat{D}^A(\lambda^A) &= \exp\Big[-i\int d\eta\, \mathcal{H}_{int}\Big] \\ &= \prod_A \exp\Big[\lambda^A\hat{a}^{A\dagger}_k - \lambda^{A*}\hat{a}^A_k\Big] \end{aligned} \tag{40}$$

With the coefficients λ^A are

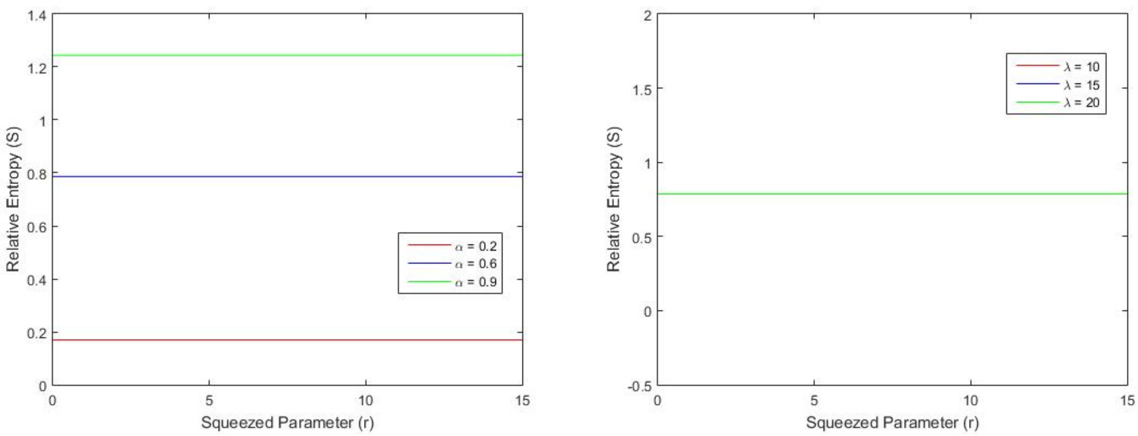
$$\lambda^A = \frac{i}{2\sqrt{2}}\int d\eta\, d^3k\, \Big(a^2\, P^A_{ij}(k)\, T^{ij}\, h^*_k\Big) \tag{41}$$

This unitary operator usually called as displacement operator. If this opeator work on vacuum, then the state become a coherent state. Using this unitary operator, then the density matrix become

$$\rho_+ = \frac{1}{\cosh^2 r}\Big(\alpha^2\, \hat{D}^{(+)}\, \hat{a}^{(+)\dagger}_k \hat{S}^{(+)}\, |0\rangle\, \langle 0|\, \hat{S}^{(+)\dagger}\hat{a}_k\, \hat{D}^{(+)\dagger} + \beta^2\, \hat{D}^{(+)}\, \hat{S}^{(+)}\, |0\rangle\, \langle 0|\, \hat{S}^{(+)\dagger}\hat{D}^{(+\dagger)}\Big). \tag{42}$$

By this density matrix, the element of covariance matix would not only depend on $\frac{1}{2}\langle\{R_i, R_j\}\rangle$, because $\langle R_i\rangle\langle R_j\rangle$ are not zero. But, the element of covariance matrix are still the same with equation (32), (33), and (34). Therefore, the α constant, still has the same role as the average number of graviton. The Von Neumann entropy for gaussian references then, would be the same like equation (35). Expanding the density matrix of equation (42)

$$\begin{aligned}\rho_{(+)} \;=\; & \frac{(1-\alpha^2)\, e^{-|\lambda|^2}}{\cosh^3 r} \sum_{m,n,i,j} \tanh^{(n+m)}(r)\, \frac{\sqrt{(2n)!(2m)!}\;\; (\lambda)^i(\lambda^*)^j}{(2^n\, n!)(2^m\, m!)\;\; i!j!}\, (\hat{a}^\dagger+\lambda^*)^{2n} \left|i\right\rangle \left\langle j\right| (\hat{a}+\lambda)^{2m} \\ & + \frac{\alpha^2\, e^{-|\lambda|^2}}{\cosh^3 r} \sum_{m,n,i,j} \tanh^{(n+m)}(r)\, \frac{\sqrt{(2n)!(2m)!(2n+1)(2m+1)}\;\; (\lambda)^i(\lambda^*)^j}{(2^n\, n!)(2^m\, m!)\;\; i!j!} (\hat{a}^\dagger+\lambda^*)^{2n+1} \\ & \left|i\right\rangle \left\langle j\right| (\hat{a}+\lambda)^{2m+1}. \end{aligned} \tag{43}$$



(a) (b)

Figure 2: Figure (a) is a graph of the relative entropy (S) to the squeezed parameter (r) in matter field presence with three different choice of α (green line $\alpha = 0.9$, blue line $\alpha = 0.6$ and red line $\alpha = 0.2$) and $\lambda = 10$. For the figure (b) the relative entropy is plotted with $\alpha = 0.6$ and different value of λ (green line $\lambda = 10$, blue line $\lambda = 15$ and red line $\lambda = 20$).

The relative entropy of the system is calculated in the same way as before. To calculate it, the indices i, j, m, n are run from 0 to 1, and the density matrix equation (43) would become 4x4. The result is plotted in figure 2. We can see that the relative entropy for this case is constant to the squeezing parameter. Different from the previous case, the relative entropy is growing up before hitting the constant value. We also can see that with different value of α the relative entropy give a different value and has the same character as in the previous case. We also found that that the relative entropy would not change by the constant λ.

6. Conclusion

In this work, we studied the non-gaussianity of one graviton creation in a squeezed vacuum state, with and without matter field presence that arising an entanglement. To do so, we calculated the relative entropy of the system. For the case where the presence of matter field is not involved, we got that, the entanglement constant (α), would play a role as the average number of the graviton. When the value of (α) is near to one, the non-gaussianity would get its highest value.

Journal of Physics: Conference Series **2243** (2022) 012098 doi:10.1088/1742-6596/2243/1/012098

In the presence of the matter field, the (α) constant would still play a role as the average number of graviton and still has the same characteristic. But, in this case, the relative entropy value is constant for every value of the squeezed parameter. This can happen because the unitary operator of the hamiltonian interaction is a displacement operator.

Even though this mechanism generates a non-gaussianity, we could not say that the non-gaussianity that happened in the early universe has come from graviton creation and claiming it as the non-classical feature of gravitational waves. Besides we need the calculation of more than one graviton creation, in this research we still do not have a solution that makes that non-gaussianity be observed and distinguish it from the other mechanism that has possibility to provides non-gaussianity (e.g magnetogenesis in early universe [3,17]). So, in the next research, we suggest using another way of detecting the non-gaussianity that could bring a unique solution.

7. Acknowledgement
This research is supported by KK theoretical high energy physics ITB.

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