# 36 The quark NNI textures rising from $SU(5) \times Z_4$ symmetry

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**Abstract** In this work we explored the consequences of the $SU(5) \times Z_4$ symmetry, where the quark mass matrices are in the Nearest-Neighbor-Interaction, on the leptonic sector. The model is based on the minimal $SU(5)$ Grand Unification (GUT) model with three right-handed neutrinos and two Higgs quintets. Due to the $SU(5)$ symmetry, the charged lepton mass matrix gets the same NNI form as the quark sector. However, in the context of the type-I seesaw mechanism, the effective neutrino mass matrix can have six different textures, of which only two are compatible with the leptonic experimental data.

## 36.1 Introduction

One of the open questions in particle physics is the explanation of the observed pattern of fermion masses and their mixings. One way to study this puzzle is for example by playing with texture zeroes; one example is the Nearest-Neighbour-Interaction (NNI) [1]. It has zeroes on the (1,1), (1,3), (2,2) and (3,1) elements and together with the hermiticity condition leads to the well known Fritzsch form [2–4].

It was shown in Ref. [5] that it is possible to obtain the quark mass matrices in the NNI form, in the context of the two-Higgs doublet model, through the implementation of a $Z_4$ flavour symmetry.

The goal of this work is to extend the idea developed in Ref. [5] to $SU(5)$ and study the consequences of such an implementation on the leptonic sector. This work is organised as follows: in section 36.2 we present the model and discuss how to obtain the quark mass matrices in the NNI form; in section 36.3 we explore the leptonic sector concerning the viability of the mass matrices, then we conclude.

## 36.2 The model

The model is based on the minimal $SU(5)$ [6] with three generations of 10 and $5^*$ fermionic representations as $10_i = (Q, u^c, e^c)_i$ and $5^*_i = (L, d^c)_i$. In order to generate neutrino masses we have introduced three right-handed neutrinos, $\nu^c_{1,2,3}$, singlets of $SU(5)$ that acquire mass via type-I seesaw mechanism [7–10].

The Higgs sector is composed by one 24 dimensional representation, $\Sigma$, and two quintets $H_1$ and $H_2$. The adjoint Higgs representation $\Sigma$ is introduced to break the $SU(5)$ down to the standard model (SM) gauge group ($SU(3)_c \times SU(2)_L \times U(1)_Y$) through the vacuum expectation value (VEV), $\langle\Sigma\rangle = \sigma \mathrm{diag}(2, 2, 2, -3, -3)$ where $\sigma = \frac{a}{2\lambda}\frac{1+\sqrt{1+4\xi(60\eta+7)}}{60\eta+7}$ (see Ref. [11]). The quintets break the SM gauge group down to $SU(3)_c \times U(1)_{em}$ when the neutral component of each doublet acquires a VEV $\mathrm{v}_1$, $\mathrm{v}_2$ such that $\mathrm{v}^2 \equiv |\mathrm{v}_1|^2 + |\mathrm{v}_2|^2 = (246.2\,\mathrm{GeV})^2$ and generate the fermion masses via the Yukawa interactions. Note that at low energy scale one falls into a two Higgs doublet model.

The most general Yukawa Lagrangian is given by,

$$\begin{aligned}-\mathcal{L}_{\mathrm{Y}} =& \frac{1}{4}\left(\Gamma_u^1\right)_{ij} 10_i 10_j H_1 + \frac{1}{4}\left(\Gamma_u^2\right)_{ij} 10_i 10_j H_2 \\ &+ \sqrt{2}\left(\Gamma_d^1\right)_{ij} 10_i 5_j^* H_1^* + \sqrt{2}\left(\Gamma_d^2\right)_{ij} 10_i 5_j^* H_2^* \\ &+ \left(\Gamma_D^1\right)_{ij} 5_i^* \nu_j^c H_1 + \left(\Gamma_D^2\right)_{ij} 5_i^* \nu_j^c H_2 + \frac{1}{2}(M_R)_{ij} \nu_i^c \nu_j^c + \mathrm{H.c.}\end{aligned} \tag{36.1}$$

where $\Gamma_u^{1,2}$ and $\Gamma_d^{1,2}$ are the up- and down-quark Yukawa matrices, $\Gamma_D^{1,2}$ and $M_R$ are the Dirac Yukawa and Majorana matrices for neutrinos. The up- and down-quark mass matrices are then given by $M_u = \mathrm{v}_1\,\Gamma_u^1 + \mathrm{v}_2\,\Gamma_u^2$ and $M_d = \mathrm{v}_1^*\,\Gamma_d^1 + \mathrm{v}_2^*\,\Gamma_d^2$.

The NNI form of the quark mass matrices is achieved through the introduction of a $Z_n$ discrete flavour symmetry. Under this $Z_n$ symmetry all fields except the adjoint Higgs field are charged. The two quintets $H_1$, $H_2$ carry charges $\phi_1$, $\phi_2$, the fermionic fields $10_i$, $5_i^*$ and $\nu_i^c$ carry charges $q_i$, $d_i$ and $n_i$.

In order to have mass matrices with the NNI form one should ensure that the zero entries in the mass matrices correspond to a non zero $Z_n$ charge of the trilinear terms and vice-versa. Following the method on Ref. [5] and choosing that the (3,3) entry of $M_u$ does not vanish we obtain $\phi_2 = -2q_3$, which leads to the following $Z_n$ charges,

$$\mathcal{Q}(10_i) = (3q_3 + \phi_1, -q_3 - \phi_1, q_3), \qquad Q(5_i^*) = (q_3 + 2\phi_1, -3q_3, -q_3 + \phi_1)\,. \tag{36.2}$$

The charge matrix of the up- and down-quark bilinears is then given by,

$$Q(10_i\,10_j) = \begin{pmatrix} 6q_3 + 2\phi_1 & 2q_3 & 4q_3 + \phi_1 \\ 2q_3 & -2\phi_1 - 2q_3 & -\phi_1 \\ 4q_3 + \phi_1 & -\phi_1 & 2q_3 \end{pmatrix}, \quad Q(10_i\,5_j^*) = \begin{pmatrix} 4q_3 + 3\phi_1 & \phi_1 & 2q_3 + 2\phi_1 \\ \phi_1 & -\phi_1 - 4q_3 & -2q_3 \\ 2q_3 + 2\phi_1 & -2q_3 & \phi_1 \end{pmatrix}, \tag{36.3}$$

from which we conclude that $\phi_1$ must be different from $\phi_2$ and that the minimal realization of $Z_n$ that makes the NNI structure possible is $Z_4$ as in Ref. [5].

The up- and down-quark mass matrices in terms of the Yukawa matrices are given as

$$M_u = \mathrm{v}_1 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & b_u \\ 0 & b_u & 0 \end{pmatrix} + \mathrm{v}_2 \begin{pmatrix} 0 & a_u & 0 \\ a_u & 0 & 0 \\ 0 & 0 & c_u \end{pmatrix}, \quad M_d = \mathrm{v}_1^* \begin{pmatrix} 0 & a_d & 0 \\ a'_d & 0 & 0 \\ 0 & 0 & c_d \end{pmatrix} + \mathrm{v}_2^* \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & b_d \\ 0 & b'_d & 0 \end{pmatrix}. \tag{36.4}$$

This being a GUT model some comments on proton decay and unification are in order.

**$\mathbf{M_e = M_d^T}$ Relation** As a consequence of the $SU(5)$ symmetry the charged-lepton mass matrix is equal to the down-type quark mass matrix transposed which is not compatible with the down-type quark and charged-lepton masses hierarchies observed at low energy scale. One possibility to correct this relation is to introduce non-renormalisable higher dimension operators [12, 13] due to physics at $\Lambda'$ scale above the GUT scale. For instance, dimension 5 operators contribute as

$$\sum_{n=1,2} \frac{\sqrt{2}}{\Lambda'} (\Delta_n)_{ij}\, H^*_{n\,a}\, 10^{ab}_i\, \Sigma^c_b\, 5^*_{jc}\,, \tag{36.5}$$

leading to the mass difference, $M_d - M_e^{\mathsf{T}} = 5\frac{\sigma}{\Lambda'}(\mathrm{v}_1^*\,\Delta_1 + \mathrm{v}_2^*\,\Delta_2)$ without destroying the NNI structure once $\Sigma$ is trivial under $Z_4$. Another alternative to correct the relation $M_e = M_d^T$ is to substitute the second Higgs quintet by a 45 dimensional Higgs representation [14]. In this case the mass difference will be given by $M_d - M_e^{\mathsf{T}} = 8\,\Gamma_d^2\,\mathrm{v}_{45}^*$, where $\mathrm{v}_{45}$ is the VEV of the 45. In any of those situations the up-quark mass matrix is no longer symmetric, which is the reason why we have considered arbitrary NNI mass matrices in section 36.3.

**Proton Decay** The proton decay can occur through the exchange of X and Y heavy gauge bosons or the exchange of the colour Higgs triplets, $T_1$ and $T_2$ contained in the quintets.

For the proton decay via the exchange of heavy gauge bosons the decay width can be estimated [15] as $\Gamma \approx \alpha_U^2 \frac{m_p^5}{M_V^4}$. Using the partial proton lifetime [16] $\tau(p \to \pi^0 e^+) > 8.2 \times 10^{33}$ years the mass of the heavy gauge bosons is estimated as $M_V > (4.0 - 5.1) \times 10^{15}$ GeV for a unified gauge coupling in the range $\alpha_U^{-1} \approx 25 - 40$.

Concerning the proton decay via the exchange of the colour Higgs triplets, the dimension 6 operators contributions at tree-level are given by

$$\sum_{n=1,2} \frac{\left(\Gamma_u^n\right)_{ij}\left(\Gamma_d^n\right)_{kl}}{M_{T_n}^2}\left[\frac{1}{2}(Q_iQ_j)(Q_kL_l) + (u_i^c e_j^c)(u_k^c d_l^c)\right]\,, \tag{36.6}$$

that in fact vanish due to the Yukawa matrices form.

**Unification** We have found unification of the gauge couplings at two-loop level without considering the threshold effects and performing the splitting between the masses of the $\Sigma_3$ and $\Sigma_8$. In our computation, we have set the fields X, Y, $T_1$, $T_2$ at GUT scale, $\Lambda$, and $H_1$, $H_2$ around electroweak scale. We found a GUT scale around $\Lambda \approx (1.3 - 2.4) \times 10^{14}$ GeV and the masses of the $\Sigma_3$ and $\Sigma_8$ components of $\Sigma$ in the range $M_Z \le M_{\Sigma_3} \le 1.8 \times 10^4$ GeV and $5.4 \times 10^{11}$ GeV $\le M_{\Sigma_8} \le 1.3 \times 10^{14}$ GeV. Unfortunately, the unification scale found is smaller than what we expect from the computation of the proton decay through the exchange of the heavy X and Y gauge bosons and the mass splitting between $M_{\Sigma_3}$ and $M_{\Sigma_8}$ is unnaturally large. This discrepancy can be avoid by the introduction of a 24 fermionic representation [17]. In such case the neutrino masses will get contributions also from type-III seesaw mechanism in addition to the usual type-I.

| parameters | NH | IH |
|---|---|---|
| $\Delta m^2_{21}\ \left(\times 10^{-5}\,\mathrm{eV}^2\right)$ | $7.62 \pm 0.19$ | |
| $\left\|\Delta m^2_{31}\right\|\ \left(\times 10^{-3}\,\mathrm{eV}^2\right)$ | $2.53^{+0.08}_{-0.10}$ | $2.40^{+0.10}_{-0.07}$ |
| $\sin^2\theta_{12}$ | $0.320^{+0.015}_{-0.017}$ | |
| $\sin^2\theta_{23}$ | $0.49^{+0.08}_{-0.05}$ | $0.53^{+0.05}_{-0.07}$ |
| $\sin^2\theta_{13}$ | $0.026^{+0.003}_{-0.004}$ | $0.027^{+0.003}_{-0.004}$ |

$m_e(M_Z) = 0.486661305 \pm 0.000000056$ MeV ,

$m_\mu(M_Z) = 102.728989 \pm 0.000013$ MeV ,

$m_\tau(M_Z) = 1746.28 \pm 0.16$ MeV ,

Table 36.1: The three-flavour oscillation parameters with 1 $\sigma$ errors, from Ref. [18], for normal hierarchy (NH) and inverted hierarchy (IH) (on the left) and the charged-lepton mass at $M_Z$ scale (on the right) [11].

## 36.3 Mass matrices

In Ref. [5] we have shown that the quark mass matrices in the NNI form accommodate all observed up- and down-quark masses and the CKM mixing matrix. As a consequence of $SU(5)$ symmetry and since the NNI form has zeroes in symmetric positions, the charged lepton mass matrix, $M_e$, has also NNI form.

Both quark and charged-lepton mass matrices can be written as,

$$M_x = \begin{pmatrix} 0 & A_x(1-\epsilon^x_a) & 0 \\ A_x(1+\epsilon^x_a) & 0 & B_x(1-\epsilon^x_b) \\ 0 & B_x(1+\epsilon^x_b) & C_x \end{pmatrix} , \tag{36.7}$$

where $x = u,\ d,\ e$ and $\epsilon$ measures the deviation from the Hermiticity; a global measurement of the asymmetry in the quark, $\varepsilon_q$, and leptonic, $\varepsilon_\ell$, sectors is given by

$$\varepsilon_q \equiv \frac{1}{2}\sqrt{\epsilon^{u\,2}_a + \epsilon^{u\,2}_b + \epsilon^{d\,2}_a + \epsilon^{d\,2}_b} \quad \text{and} \quad \varepsilon_\ell \equiv \sqrt{\frac{\epsilon^{e\,2}_a + \epsilon^{e\,2}_b}{2}} . \tag{36.8}$$

For $\varepsilon_q = \varepsilon_e = 0$ one recovers the Fritzsch form [2–4].

The fact that the $Z_4$ neutrino charges are free parameters obliges us to scan all charge combinations and select the viable textures by confronting them with neutrino experimental data. The effective neutrino mass matrix is given by the type-I seesaw formula [7] $m_\nu = -m_D\,M_R^{-1}\,m_D^{\mathsf{T}}$ to an excellent approximation ($m_D \ll M_R$). Performing the scan of all $Z_4$ charges for $\phi_1$, $q_3$ and neutrinos one is able to determine the shape of the effective neutrino mass matrix. After its analysis one concludes that among the six different possibilities (see Ref. [11]) only two textures are viable: II and $\mathrm{II}_{(12)}$ where $\mathrm{II} = P_{12}^{\mathsf{T}}\mathrm{II}_{(12)}\,P_{12}$.

$$\mathrm{II} = \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix} , \qquad \mathrm{II}_{(12)} = \begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & * \end{pmatrix} . \tag{36.9}$$

In order to confront the predictions from $M_e$ and $m_\nu$ with the neutrino oscillation data at $M_Z$ energy scale one needs to diagonalize both $M_e$ and $m_\nu$ and compute the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [19–21]. The leptonic mixing matrix is given by $U_{PMNS} = U_\ell^\top P_{12}\, U_\nu$ where $U_\ell$ and $U_\nu$ are the diagonalizing matrices of charged-leptons and neutrinos respectively.

In our numerics we have varied all charged-lepton masses and neutrino mass differences within their allowed range (see Table 36.1), scanned the mass of the lightest neutrino for different magnitudes below 2 eV and computed the other two masses through $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$ the mass squared difference, using the actual neutrino oscillation data [18]; the free parameters of $M_e$ and $m_\nu$ were also properly taken into account (see Ref. [11]).

We have considered as additional constraints the effective Majorana mass [22–24] $m_{ee} \equiv \sum_{i=1}^3 m_i U_{1i}^{*2}$; the constraint from Tritium $\beta$ decay [16] $m_{\nu_e}^2 \equiv \sum_{i=1}^3 m_i^2 |U_{1i}|^2 < (2.3\,\mathrm{eV})^2$ at 95% C.L. and constraints on the sum of light neutrino masses from cosmological and astrophysical data [25] $\mathcal{T} \equiv \sum_{i=1}^3 m_i < 0.68\,\mathrm{eV}$ at 95% C.L..

We got that texture II is compatible just with normal hierarchy (NH) while texture $\mathrm{II}_{(12)}$ is compatible just with inverted hierarchy (IH). For texture II and normal hierarchy we found that

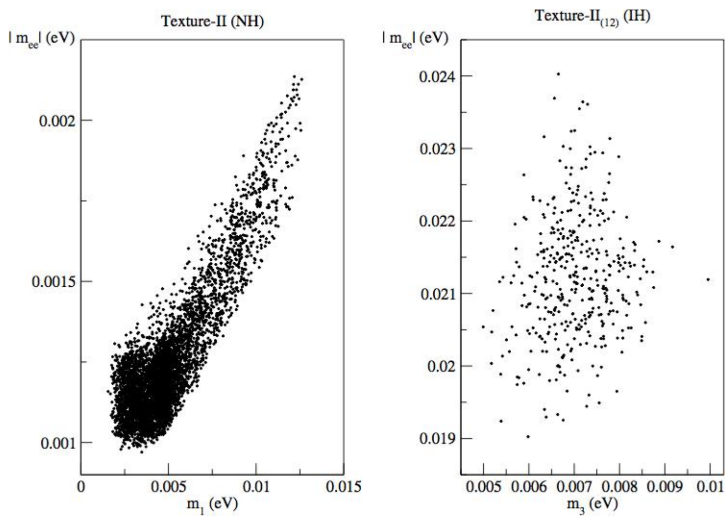


Figure 36.1: Plot of the effective majorana mass, $|m_{ee}|$, as a function of the lightest neutrino mass $m_1$ for the Textures-II (NH) (left) and $m_3$ for Texture-$\mathrm{II}_{(12)}$ (IH) (right).

the lightest neutrino mass varies in the range $m_1 = [0.0015, 0.013]$ eV while the global deviation is $\varepsilon_\ell > 0.005$; the effective Majorana mass found was $0.00097\,\mathrm{eV} < |m_{ee}| < 0.0021\,\mathrm{eV}$.

Concerning texture $\mathrm{II}_{(12)}$ where inverted hierarchy applies we found the lightest neutrino mass to be in the range $m_3 = [0.005, 0.010]$ eV, the global deviation is $\varepsilon_\ell > 0.003$ and the $|m_{ee}|$ parameter is given by $0.015\,\mathrm{eV} < |m_{ee}| < 0.021\,\mathrm{eV}$.

## 36.4 Conclusion

In this work we showed that it is possible to implement a $Z_4$ flavour symmetry, in the context of $SU(5)$ with minimal fermionic content plus three right-handed neutrinos and two Higgs quintets, that leads to quark mass matrices in the NNI form. We have studied the implications of this $SU(5) \times Z_4$ symmetry on the leptonic sector and found that, among the six possible textures for the effective neutrino mass matrix, only two are phenomenologically viable and it is possible to distinguish them by the light neutrino mass spectrum hierarchy.


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# Bibliography


[1] G. C. Branco, L. Lavoura, and F. Mota, Phys. Rev. D **39**, 3443 (1989).

[2] H. Fritzsch, Phys. Lett. B **73**, 317 (1978).

[3] L.-F. Li, Phys. Lett. B **84**, 461 (1979).

[4] H. Fritzsch, Nucl. Phys. B **155**, 189 (1979).

[5] G. Branco, D. Emmanuel-Costa, and C. Simões, Phys.Lett. B **690**, 62 (2010), arXiv:arXiv:1001.5065 [hep-ph] .

[6] H. Georgi and S. Glashow, Phys.Rev.Lett. **32**, 438 (1974).

[7] P. Minkowski, Phys.Lett. B **67**, 421 (1977).

[8] T. Yanagida, In Proc. of the Workshop on Unified Theory and Baryon Number in the Universe, KEK, March 1979.

[9] M. Gell-Mann, P. Ramond, and R. Slansky, To be published in Supergravity, P. van Nieuwenhuizen & D.Z. Freedman (eds.), North Holland Publ. Co., 1979.

[10] R. N. Mohapatra and G. Senjanovic, Phys.Rev.Lett. **44**, 912 (1980).

[11] D. Emmanuel-Costa and C. Simoes, Phys.Rev. **D85**, 016003 (2012), arXiv:1102.3729 [hep-ph] .

[12] B. Bajc, P. Fileviez Pérez, and G. Senjanovic, Phys. Rev. D **66**, 075005 (2002), arXiv:hep-ph/0204311 .

[13] D. Emmanuel-Costa and S. Wiesenfeldt, Nucl. Phys. B **661**, 62 (2003), arXiv:hep-ph/0302272 .

[14] P. Fileviez Pérez, Phys. Lett. B **654**, 189 (2007), arXiv:hep-ph/0702287 .

[15] P. Langacker, Phys. Rept. **72**, 185 (1981).

[16] K. Nakamura *et al.* (Particle Data Group), J. Phys. G **37**, 075021 (2010).

[17] B. Bajc and G. Senjanovic, JHEP **08**, 014 (2007), arXiv:hep-ph/0612029 .

[18] T. Schwetz, M. Tortola, and J. W. F. Valle, New J. Phys. **13**, 109401 (2011), arXiv:1108.1376 [hep-ph] .

[19] B. Pontecorvo, Sov. Phys. JETP **6**, 429 (1957).

[20] B. Pontecorvo, Sov. Phys. JETP **7**, 172 (1958).

[21] Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).

[22] S. Pascoli, S. T. Petcov, and L. Wolfenstein, Phys. Lett. B **524**, 319 (2002), arXiv:hep-ph/0110287 .

[23] S. Pascoli and S. T. Petcov, Phys. Lett. B **544**, 239 (2002), arXiv:hep-ph/0205022 .

[24] S. Pascoli and S. T. Petcov, Phys. Lett. B **580**, 280 (2004), arXiv:hep-ph/0310003 .

[25] D. N. Spergel *et al.* (WMAP), Astrophys. J. Suppl. **170**, 377 (2007), arXiv:astro-ph/0603449 .