The States in Deep Inelastic Scattering are Linked Chains of Color Dipoles

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Abstract

I consider the states in Deep Inelastic Scattering situations, as defined in the work of Marchesini and collaborators (the CMCF Model) and phrase the results in a language suitable for color dipole considerations. I show that the DIS states can be treated as the radiation from a set of linked *primary* dipoles, built up in "the initial state radiation" and decaying into "the final state radiation". The scenarium is valid both for energy momentum and space-time considerations and incorporates both the usual QCD parton model, the Boson-Gluon Fussion and the probe hard structure interactions. I end with describing some corrections to the well-known BFKL mechanism, resulting in an essential decrease of the power in the $x^{-\lambda}$-behaviour.

Résumé

Dans cet article, je considére les états en diffusion profondément inélastique, tels que définis dans le travail de Marchesini et collaborateurs (le modéle CMCF) et reformule les résultats dans un langage plus approprié aux considérations sur les dipoles de couleur.

1. Introduction and Remarks on the Color Dipole Cascade Model

There is one major difference between the calculation of the x-sections for the DIS and for the e^+e-annihilation events. The x-section for DIS events is not describable only by the lowest order perturbative terms but also contains the structure functions, ie the inclusive flux of the partons. For the total cross section we do not need to calculate all the radiation in the states. Instead the different states should after an inclusive sum over "the final state radiation" be amenable to a grouping into different (non-overlapping) classes.

It is the sum over the probability weights (ie the relevant Sudakov formfactors) of these classes which provides the cross section. The choice of the different classes is open but one may evidently make more or less intelligent partitionings, ie make choices containing less or more Sudakov suppression factors. Different states in a class contain a common "essential" set of partons emitted, usually called "the initial state radiation". Given this set one must calculate the "virtual corrections" leading to the Sudakov factors. They are in general possible to sum up to exponentials and can often be interpreted in probability language as "the probability to do nothing" inside a region where the available current will allow emission. A necessary consistency request is that it should be possible to obtain the final state radiation (which is summed up and therefore not included in the Sudakov weights) from the states chosen as the essential "initial" set.

In the first subsection I describe the states in e^+e^--annihilation in terms of color dipole radiation and introduce the triangular phase-space in the rapidity, y,

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and the logarithm of the squared transverse momentum, $\kappa = \log(k_\perp^2)$, natural in that context. I will also mention the approach in the Webber-Marchesini Model and the reason why the two models lead to equivalent results. After that I review the way Marchesini et al, [1], have defined the essential set of emissions in DIS. I will refer to their method as the CMCF Model and I will formulate their results in a language suitable for a treatment in terms of the triangular phase space used before for dipole emission.

After simplifications and modifications the results of the CMCF Model is the occurrence of a linked chain of dipoles. *In this way we may define the initial state radiation as the radiation necessary to obtain the precise dipole chain and the final state radiation is the result of the emissions from these dipoles.* I end by presenting the probabilities in this the Linked Dipole Chain (LDC) Model. Finally I consider some results for the structure functions and exhibit some large corrections to the BFKL-mechanism.

1.1. Gluon Emission in the Dipole Cascade Model and the Webber-Marchesini Cascades

In an e^+e^--annihilation (time-like) cascade there is an initial (color singlet) $q\bar{q}$-state formed. At high energies this state will emit gluons g by color dipole radiation with the approximate x-section:

$$dp \simeq \frac{\bar{\alpha}}{\pi}\frac{dk_\perp^2}{k_\perp^2}dy \tag{1}$$

Here $\bar{\alpha}$ is the "effective" coupling (containing suitable color factors) and the transverse momentum $k_\perp$ and rapidity y can be defined by Lorentz invariants

$$k_\perp^2 = \frac{s_{12}s_{23}}{s},\ y = \frac{1}{2}\log\left(\frac{s_{23}}{s_{12}}\right) \tag{2}$$

in terms of the two-parton mass squares, $s_{qg} \equiv s_{12}$, $s_{g\bar{q}} \equiv s_{23}$ and the total squared mass $s \equiv W^2$. For the phase space we have in the cms of the dipole

$$k_\perp \cosh(y) \leq \frac{W}{2} \quad \text{approx} \quad k_\pm \equiv k_\perp \exp(\pm y) < W (3)$$

corresponding to a triangular region in the $(y, \kappa \equiv \ln(k_\perp^2))$-plane, cf figure 1a.

Besides the color factor this is the same as in QED but there is a major change in the QCD final state. Emitting a photon in QED does not change the current. But the QCD current is changed because the emitted gluon is a color octet. There is, however, the simplification that instead of forming a complex charge system between the color 3, the $\bar{3}$ and the 8, the final state partons form *two independent dipoles*, [2]. Thus if

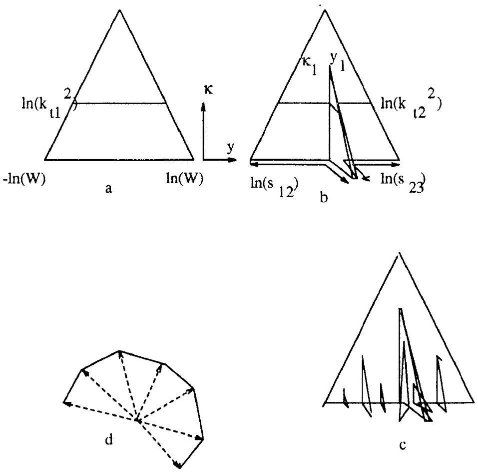

Figure 1. *The Dipole phase (triangular) space and multigluon emission according to the text*

we consider the emission of two gluons, indexed 1 and 2, the cross section is factorisable into

$$\begin{aligned} dp(q\bar{q} \to qg_1g_2\bar{q}) &= dp(q\bar{q} \to qg_1\bar{q}) \times \\ &\times(dp(qg_1 \to qg_2g_1) + dp(g_1\bar{q} \to g_1g_2\bar{q})) \end{aligned} \tag{4}$$

in case the transverse momenta are ordered ($k_{\perp 1} \geq k_{\perp 2}$). This factorisation property is better than a few percent all over the phase space, [3]. We note in particular that the two dipoles are not at rest with respect to each other or the original cms.

The Dipole Cascade Model (DCM) is then based upon the production of one dipole $\to$ two dipoles $\to$ three...etc. Every time a new gluon is emitted and the corresponding dipole partitioned. Actually the final state containing a set of dipoles has a strong similarity to the Lund String with a set of gluon excitations dragging out a set of straight string segments (corresponding to the dipoles), cf figure 1d.

It is useful to note that one gluon emission actually increase the rapidity region for further emissions. While the cms rapidity region, Δy, available for the first gluon emission (with transverse momentum $k_{\perp 1}$) is $\log(W^2/k_{\perp 1}^2)$ (the line shown in the triangle of figure 1a) the two "new" dipoles may emit a gluon (with transverse momentum $k_{\perp 2} < k_{\perp 1}$) inside the region

$$\begin{aligned} \lambda = (\Delta y)_{gen} &= \log(s_{12}/k_{\perp 2}^2) + \log(s_{23}/k_{\perp 2}^2) = \\ &= \log(W^2/k_{\perp 2}^2) + \log(k_{\perp 1}^2/k_{\perp 2}^2) \end{aligned} \tag{5}$$

Here we have used the lines after Eq (2). There is then by the emission be an increase in the allowed phase space, $\delta\lambda = \log(k_{\perp 1}^2/k_{\perp 2}^2)$. This can be "added" to the triangular phase space in figure 1a by extending a (double) fold as in figure 1b. Actually we are in this way adding a triangular region, made up of the two sides of the fold, to the original triangle. Each new emission will have the same effect on phase space, [4]. After several emissions we obtain a complex figure with many folds "sticking out" as in figure 1c (corresponding to the emission in figure 1d). The length of the baseline, which is what we called the generalised rapidity λ above in Eq (5), is irregular and behaves like a (multi)fractal with the so-called anomalous dimension of QCD basically equal to the fractal dimensions. The quantity λ is a useful measure of the multiplicity of the final state hadronic multiplicity, [4].

The Webber-Marchesini Independent Parton Cascade Model, [5] is a conceptually very different approach to multiple gluon emissions. The results are nevertheless very similar to those obtained in the Dipole Cascade Model. The reason for this similarity is that *the building of dipoles in the cascade is the same as the requirement of strong angular ordering in the emissions*, [6], in the Webber-Marchesini Model. They use one angular variable, θ, and one energy variable, E. Remembering the relation for the rapidity of a massless particle:

$$y = \frac{1}{2}\log\left(\frac{e+p_l}{e-p_l}\right) = -\log\tan(\theta/2)) \simeq -\log(\theta/2) \quad (6)$$

and the approximate relation $k_\perp \simeq E\theta$ we may map these variables onto the quantities $(y, k_\perp)$ and consider the emission of a Webber-Marchesini cascade in the triangular phase space. The strong angular ordering means that after emitting a "first" gluon along a parton line (emission angle θ) the next emission must have an emission angle smaller than θ. Therefore in this kind of cascade you may start with a first emission at some angle or rapidity and then afterwards go towards the q-end in rapidity and afterwards go towards the $\bar{q}$-end in rapidity (ie following along the q- and $\bar{q}$-lines, respectively).

In both cases the "new" emissions are evidently fulfilling the strong angular ordering requirement (ie diminishing angles along the parton line). In every "step" in rapidity one searches through the possible $k_\perp$- (or E-)values, ie one "goes from one side in rapidity, y, towards the other, all the time looking up and down in $k_\perp$". Every time one finds an emission a "new" parton line starts out and *this corresponds exactly to adding a folded triangle* in the Lund language. Then they are searched through in the same manner, producing subfold triangles etc. According to our description above one is in the Lund DCM "going down in $k_\perp$ all the time looking left and right inside the relevant rapidity region", also adding folds and resolving them for new emissions. The final result is the same (ie on an inclusive basis the events will coincide). It is only a question of different orderings in the cascades (and as a matter of fact there are different ways to take the recoils into account when the emitted gluons are "not" soft).

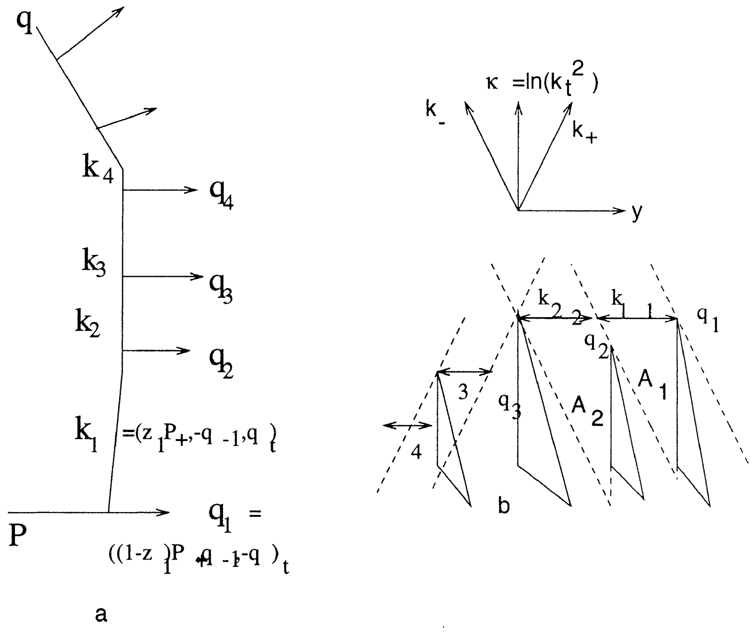

Figure 2. *A DIS fan-diagram and its description in the triangular phase space. The on-shell q-vectors are extended triangles from a single emission point and the (virtual) connectors k stretch betqween the $q_{+(j+1)}$ and $|q_{-j}|$. The areas A_j marked out correspond to the regions excluded according to the Sudakov suppression in Δ_{ne}*

2. The CMCF (Ciafaloni-Marchesini-Catani-Fiorini) Model

We start to consider the kinematics of the partonic emissions along any "fan-diagram", cf figure 2a. Independently of "the beginning" or "the end" there is a complete color connection along the diagram. In accordance with the LLA we will as in [1] neglect the process $g \to q\bar{q}$, ie the gluon splitting process (which has no z-pole) and consider the diagram as if it is just made up of gluon emissions. Then the fan diagram contains a connection from an "incoming parton" with a (mass-less) energy momentum vector P towards a "probe" with energy momentum q. There is a set of emissions "along" the color line under consideration and they are described by the energy momentum vectors q_j (which are always taken to be on-shell and massless). There is also a set of connectors described by the energy momentum vectors k_j, which are all space-like. At every "vertex" there is energy momentum conservation so that

$$k_j = P - \sum_{m=1}^{j} q_m \quad \text{ie} \quad k_j = k_{j-1} - q_j \qquad (7)$$

It is then in [1] noted that in case we use the variables z_m such that $k_{+j} = \prod^j (z_m) P_+$, some of the z_m are very small, ie stem from the z-pole dominance in the splitting function. In Ref [1] it is those emissions, ie when $q_{+j} = (1 - z_j)k_{+(j-1)} \simeq k_{+(j-1)}$ that are taken as "essential" and the virtual corrections are calculated to determine the cross section. We call the ensuing model the CMCF Model and we will next go over to the kinematical and dynamical results in some detail.

All emissions are in the CMCF Model ordered both in energy and in angle, ie in rapidity, which means that the strong angular ordering requirement is invoked. In case we actually follow the small-z contribution, eg at the vertex named j, it is evident that all the following radiation, ie the one "behind" the q_j, must have $q_{+m} \ll q_{+j}$, $j < m$ because there is only available the amount $k_{+j} \simeq z_j q_{+j} \ll q_{+j}$ for them. The emission situation is described for some different cases in figure 2b. We note that a given value of $k_{+j} = k_{\perp j}\exp(y_j) = z_j k_{+(j-1)}$ ($q_{+j} = q_{\perp j}\exp(y_{qj}) = (1 - z_j)k_{+(j-1)}$) corresponds to the points along a line with a distance $\log(1/z_j)$ ($(\log 1/(1 - z_1) \simeq 0$) to the "earlier" front $k_{+(j-1)}$.

The transverse momentum of the connectors k_j is in this logarithmical scenarium dominated by the "large" $\vec{q}_\perp$ emissions in "the neighborhood". All the emission steps can be partitioned into three different kinds: (1) $\vec{k}_{\perp j} \simeq -\vec{q}_{\perp j} \gg \vec{k}_{\perp(j-1)}$ (as an example cf the emission denoted 1 in figure 2b), (2) $\vec{k}_{\perp j} \simeq \vec{k}_{\perp(j-1)} \gg -\vec{q}_{\perp j}$ (cf the emission denoted 2 in figure 2b), (3) $-\vec{q}_{\perp j} \simeq \vec{k}_{\perp(j-1)} \gg \vec{k}_{\perp j}$ (cf the emission denoted 3 in figure 2b).

Each step in the emission chain is in the CMCF Model described in terms of the probability

$$\bar{\alpha}\frac{dz_j}{z_j}\frac{d^2 q_{\perp j}}{\pi \vec{q}^{\,2}_{\perp j}}\Delta_{ne}(z_j, k_{\perp j}, q_{\perp j})$$

$$\Delta_{ne}(z, k_\perp, q_\perp) = \exp(-\bar{\alpha}\log(1/z)\log\left(\frac{k_\perp^2}{z q_\perp^2}\right)) \quad (8)$$

The formula for the non-eikonal formfactor $\Delta_{ne}(z, k_{\perp j}, q_{\perp j})$ is of the Sudakov kind, ie the negative exponent (if Eq (9) below is fulfilled) of an integrated emission probability over the region which is excluded by the emission step j, for details cf Ref [7] and the the next subsection. From the expression for Δ_{ne} it seems necessary and it is also implicit in Ref [1] that only contributions with

$$k_{\perp j}^2 > z_j q_{\perp j}^2 \quad (9)$$

should be included in the essential emission set. Therefore a situation like the one, denoted 4 in figure 2b is excluded. It is essential both for dynamical reasons as well as for the interpretations we provide below that Eq (9) is fulfilled. We end by the remark that all the on-shell q-emission vectors are in this pictorial formalism described as extended folded triangles in the same way as in the DCM. The connector vectors k are, however not on the mass-shell. According to the way the CMCF Model choses the essential set we note that $k_{+j} \simeq q_{+(j+1)}$ and (besides the case 2. above) we also

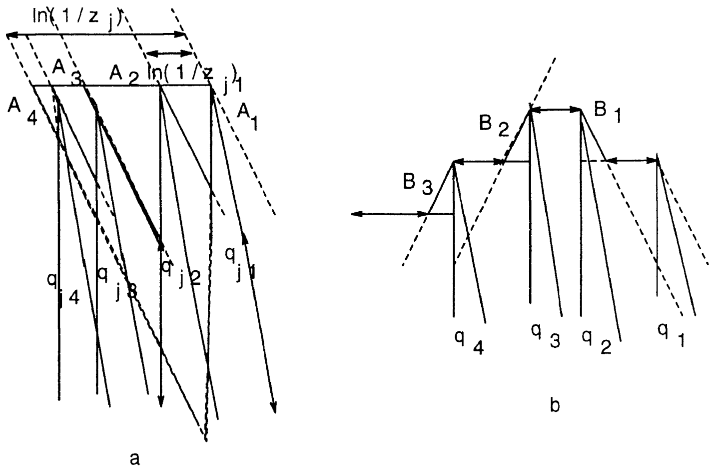

Figure 3. *The Sudakov suppressed regions in the single step* $\log(1/z_j)$ *and the regions corresponding to the "smaller" possible steps* z_{jk} *with* $\prod z_{jk} = z_j$ *are described in a. (note the property of the Sudakov regions that the "large"* $ln(1/z_j)$*-step area is "smaller" than the sum of the "small"* $\log(1/z_{jk}$*-step areas). The dipole regions spanned between neighbouring* q_j *emissions are shown with the regions* B_j *mentioned in the text in b.*

have that $k_{-j} \simeq -q_{-j}$. We may therefore conveniently represent the k vectors as lines on the height $\log(k_\perp^2)$ stretching between the neighbouring $q_\pm$ lines. The virtuality of the k vectors are in this way always given by the transverse momenta because we have $k_\perp^2 \gg |k_+ k_-|$.

2.1. Reformulation and Analysis, The Linked Dipole Chain Model

Some of the gluons, included in the chains described by the CMCF Model, have properties similar to those which we already have included in the final state interaction (and summed away from the Sudakov formfactor). Particularly this is so for the gluons included under case 2. above with transverse momenta smaller than their neighbouring $k_\perp$. We will now consider the sum over such "soft" gluon emissions.

By a variation in the $z_j \ll 1$ the value of $q_{+j} = (1 - z_j)k_{+(j-1)}$ is hardly affected but the value k_{+j} may change appreciably in the LLA. We consider all these soft q_j-emissions "close to the front $k_{+(j-1)}$" and then sum over their contributions to the cross section. In this way we produce a set of connectors with the same LLA value of $\kappa = \log(k_{\perp j}^2)$. Thus we may take "the step" z_j by emitting many soft q'_{jk} gluons with $\prod_k z'_{jk} = z_j$. The procedure is illustrated in figure 2b and figure 3a.

We show the relevant Sudakov regions (in figure 2a A_1 and A_2 and in figure 3a the A_j) corresponding in the CMCF Model to gluon emissions with rapidities $y(q) < y(q_j)$, ie smaller than the emitted gluon q_j ("angular ordering") but with q_+-values larger than the ensuing front $k_{+j} \simeq z_j q_{+j}$ (energy-momentum conservation). Those are not possible in the CMCF Model (note that gluons with $y(q) > y(q_j)$ are allowed and summed away under the assumption that they "did not change the front z-value appreciably"). In figure 3a we show a set of "soft" q'_{jk}-emissions. After summing over all the emissions of q'_{jk} we obtain the result that the procedure is equivalent to taking one step as in Eq (8) but *with the form factor* Δ_{ne} *equal to* 1. *This is the relevant weight for the case 2. and we obtain a primary dipole stretching between the (on-shell) gluon emissions* q_j *and* q_{j+1}. The radiation from this dipole, similar to the dipoles obtained in the $k_\perp$-ordered emissions in DCM, may provide all the gluons summed away above. Therefore the procedure is proven to be consistent.

We are then left with a chain of steps up and down in transverse momentum according to the types 1. and 3. It is then again possible to sum up a set of emissions that will compensate the Sudakov factor along the same lines as for the case 2. The result is a weight in each case equal to 1 as in Eq (8). The main question is, however, if it is possible to obtain the "summed-away" radiation as the final state radiation from the ensuing dipoles. The situation is illustrated in figure 3b and we note in particular the occurrence of the regions denoted B_j, which occur in the neighbourhood of an emission q with larger transverse momentum than the connector k.

It is a fact that if: (A) *we consider the primary dipoles to be stretched between the q-vectors, ie each dipole is made up of two neighbouring q-vectors,* (B) *we require that there should be no emission from such a dipole with transverse momentum larger than the relevant connector virtuality* $-k^2 \simeq k_\perp^2$ then we obtain radiation which will cover the region (including the parts called B_j) below the lines of the connectors stretched between the essential gluon set.

We note that the dipoles, defined from two neighbouring gluons, q_j and q_{j+1}, in the essential gluon set, are not pointlike in space-time. The two gluons are connected by the virtual connector k_j. The current associated with the two gluons therefore starts out with a space time size, corresponding to a transverse distance, b_j, defined by the virtuality $b_j \simeq 1/\sqrt{-k_j^2} \simeq 1/k_{\perp j}$. It is very difficult to obtain radiation with a wavelength smaller than this available antenna size. This result is also obtained in Ref [1] by a detailed analysis of the virtual corrections. This is the reason why the non-eikonal Sudakov in Eq (8) does not contain any reference to the regions above the virtuality lines $\log(k_{\perp j}^2)$. *The result* (and this is in our opinion the great achievement of the hard work done by Marchesini et al) *is that there is no emission density dp for* $q_{\perp j}^2 > -k_j^2$ *because the real and virtual contributions cancel in this region.*

To understand the cancellation we consider the radiation in the rest-frame of the two gluons q_j and q_{j+1}. In that frame there is then only gluon emission from the (q_j, q_{j+1}) dipole with $q_\perp < \sqrt{-k_j^2}$ with the transverse directions counted with respect to the dipole axis. In case we would boast back to the initial Lorentz frame specified by the probe and hadron directions (this will require both a longitudinal and a transverse boost, in general) then the radiation defined in the restframe will cover just the regions below the lines described above. The main point for the B_j-regions is that they occur because the two gluons defining the dipole in the probe-hadron frame will have different transverse momenta for the cases 1. and 3. (Note that the B_j-regions actually correspond to the cases of gluon emission "above" $k_\perp^2$ but "inside" the connection lines $k_\pm$ to the emitted q).

We note that by this procedure we are also making the states symmetric with respect to a treatment from the hadron and the probe side and we will in the next subsection further exhibit these properties with respect to the hadron and probe ends. In case we consider the same chain as before but this time analyse the results from the probe side we would consider the steps $l_j \equiv -k_j \rightarrow q_j + (-k_{j-1}) \equiv q_j + l_{j-1}$ and use the variables $-k_{-j} = z_-(-k_{-(j+1)})(\simeq q_{-j})$ for small z_- to define the essential set. It is in particular at this point that the requirement $k_{\perp j}^2 > z_{+j} q_{\perp j}^2$ for the essential set of the CMCF Model in Ref [1] becomes necessary.

In this picture we may, inside the LLA, define the transverse momentum for the emitted gluon as $q_{\perp j} = \max(k_{\perp j}, k_{\perp (j-1)})$. The two-dimensional description in the phase space triangle neglects the dependence on the azimuthal angles ϕ_j. An emitted gluon in the essential set is described by a single point, representing all values of $0 \leq \phi_j < 2\pi$ (but containing precise information of its $q_{\perp j}^2$ and its lightcone components $q_{\pm j}$ with $q_{+j} q_{-j} = q_{\perp j}^2$). This is not enough in order to define the relation between the $\vec{q}_\perp$ and $(\vec{k}_{\perp j}, \vec{k}_{\perp (j-1)})$. With this definition of $q_{\perp j}$ it is sufficient (although the connectors are not on the mass-shell) to provide either the values $(k_{\perp j}^2, k_{+j})$ or $(k_{\perp j}^2, k_{-j})$ and then use the connections in the dipoles to define the state. Therefore the probability for a step is given by

$$\bar{\alpha} \frac{dz_j}{z_j} \frac{d^2 k_{\perp j}}{\pi k_{\perp j}^2} \frac{k_{\perp j}^2}{\max(k_{\perp (j-1)}^2, k_{\perp j}^2)} \tag{10}$$

We have used $d^2 q_{\perp j} = d^2 k_{\perp j}$ and introduced $q_{\perp j}^2 \simeq \max(k_{\perp (j-1)}^2, k_{\perp j}^2)$ as in the discussion above. The last factor will make it difficult to "go down" in transverse momentum along the chain. The product over these factors integrated over all chains (as defined above with the requirement that $\prod_j z_j = x$) is then the structure function, ie the partonic flux. In this way we have defined the Linked Dipole Chain (LDC) Model by the requirements that the weights for all "steps" are defined by the Eq (10) and that the primary dipoles defined by two neighbouring vectors q_j will emit radiation in their restframes up to the $q_\perp$-order defined by the virtual connector k_j between them.

Actually the factors in Eq (10) imply that any maximum in transverse momentum along the chain will provide two contributions, ie there is one emission on one side (in order to "go up") and another on the other side (in order to "go down" again). Together they provide a combined factor $(q_\perp^{-2})^2 \simeq k^{-4}$, which is just the contribution valid for a Rutherford interaction. To be precise it is necessary to carry through all the calculations including the sums over the polarisations and the different color line contributions along the emission current. The result nevertheless stands, ie there is always at the "top" point of any fan diagram two emissions and they provide just well-known x-section for the Rutherford scatterings between two charges in a gauge field theory. In a "long" chain it is possible to have many maximum regions of this kind and we will come back to these situations.

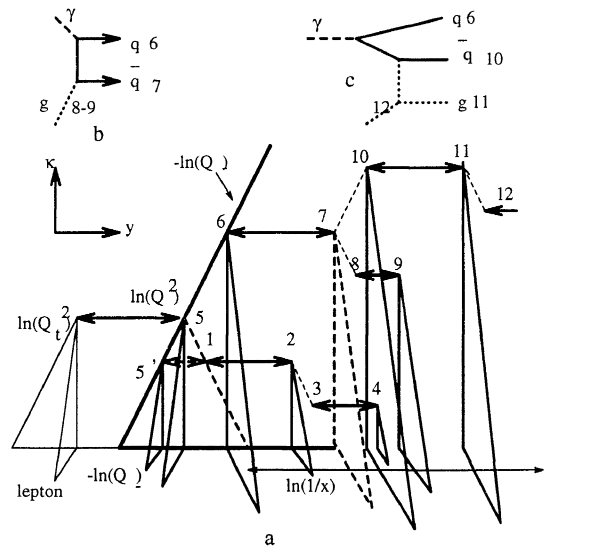

Figure 4. *A description of the "backend" interaction between a lepton probe scattering with $Q_\perp$ and Q_-, thereby producing the line Q_- as backward limit down to $\log(Q^2)$ where the line $xP_+ = Q^2/Q_-$ takes over. The notations are described in the text and the figs b and c correspond to Feynman diagram descriptions of Boson-Gluon Fussion and Rutherford scattering on the probe constituents*

3. The Probe End , Boson-Gluon Fussion, the Structure Function of the Probe and the General Structure Function of a Hadron

In the CMCF Model there is a tacit assumption that the partonic chain diagrams contains a largest transverse momentum, in general $\max_j(k_{\perp j}^2) \leq Q^2$. It is true that for large Q^2 the main contributions stem from these situations (this is the DGLAP mechanism). But for smaller values of Q^2 and for the investigation of states with large (Rutherford scattering) jets it is necessary to include all contributions even those containing some $k_{\perp j}^2 > Q^2$. This is straight-forward in the LDC Model. While the "hadron end" contains the splitting of the hadron into a color 3 and a color $\bar{3}$ (which due to the lack of a z-pole can be treated in the LLA as an entity, for more detailed studies cf Ref [7]) the probe end needs more refined analysis.

3.1. The Probe End and a Partitioning into Different Channels

For a description of the interactrion one may use different Lorentz frames, eg the cms frame between the field pulse and the hadron (the FPH frame) or the cms frame between the leptonic probe and the hadron (the LPH frame). We now show how to incorporate the description of the DIS events in both of the above-mentioned frames into the triangular phase space.

In a frame equivalent to the (LPH) frame, where the initial lepton has a (large) component along the negative lightcone, equal to (a positive) Q'_- it will send away a fraction Q_- with the bosonic field pulse and recoil with a transverse momentum $\vec{Q}_\perp$. This is described in figure 4 by means of the connector $\gamma^\star$ at the level $\log(Q^2)$ with $Q^2 \equiv \vec{Q}_\perp^2$ ending on the negative lightcone line Q_- with the lepton (after the scattering described by a triangle, although there is no gluonic emission in there) at the other end of the connector.

When the "incoming" propagator is described by the points marked 1 and 2 in figure 4 the event corresponds to an (upwards in transverse momentum) splitting of the gluon propagator denoted $3-4$ into a $\bar{q}$ (at 2) and a $q^\star$-propagator $1-2$ at the level $\log(k_\perp^2)$ with $k_\perp^2 \ll Q^2$ (on the logarithmic scale). The interaction is then the "usual" one in the QCD parton model, ie $\gamma^\star q^\star \to q$ with (the final) q "moved up" to the triangular fold marked at the point 5. The value of the Bjorken variable $x = -q^2/2pq$ is given by $xP_+ \equiv Q_+ = Q^2/Q_-$ and is shown by the hatched line. In a frame equivalent to the FPH frame we neglect the "backward" lepton triangle and the $\gamma^\star$ propagator and consider the probe to be given by $(-Q_+, Q_-)$ (with $-q^2 = Q^2 = Q_+Q_-$) interacting with the $q^\star$ propagator. This "extends" the propagator from the point 1 to the final (on-shell) q position as an emitted triangle at the point $5'$ (note that the transverse momentum for the q is in the FPH frame solely given by the "incoming" propagator $k_\perp$).

Next consider the case described by the points $6-7$ in figure 4. Then there is a quark propagator $q^\star$ at the level $k_\perp^2 \gg Q^2$. Therefore the gluon ($g^\star$) propagator $8-9$ (with transverse momentum $k_{\perp g}^2 \ll k_\perp^2$) is split up into the $\bar{q}$ (the triangle at the point 7) and the quark ($q^\star$) propagator $6-7$ at the level $\log(k_\perp^2)$ ending on the negative lightcone line Q_-. *This situation corresponds to a BGF event where the main transverse momentum, ie the "virtuality" of the state, is not the Q^2 of the field pulse but instead the $k_\perp^2$ splitting of the $g^\star \to \bar{q}q^\star$*, cf figure 4b. *According to the rules of perturbation theory one should always start at the largest virtuality and consider the emissions on either side as independently coherent.* For a BGF event we are no longer asking for a (virtual) parton line ending on the positive lightcone line $Q_+ = xP_+$. Instead we ask for the propagator to end on $Q_- = Q^2/Q_+$. In this way also the BGF events "which come from above the Q^2-level" may be incorporated into our description of the general events.

Finally we consider the case when the connected gluon (g^*) propagator is "above" the level $6-7$, ie the case described by the points $10-11$ in figure 4. Then the $q^\star$ propagator "dominates" the probe Q^2 but is itself dominated by the $g^\star$ at the level $10-11$. This situation, which may occur frequently for small boson virtualities, *corresponds to the evolvement of a structure function for the field pulse probe.* The state on the "backward" side of the largest virtuality, ie in this case the breakup state of the $\gamma^\star$ into q at 6 and $q^\star$ propagator $6-7$, is a coherent reaction. According to figure 4c the interaction corresponds to a *a quark-gluon Rutherford scattering (10–11) between a constituent $\bar{q}^\star$ from the $\gamma^\star \to q\bar{q}^\star$ and the hadron virtual state* ending on 12, (which then acts as a "probe" on the field pulse). The final state of the parton-parton scattering corresponds to the $\bar{q}$ triangle at 10 and the gluon triangle at 11.

3.2. Some Remarks on the Structure Functions

Space does not permit a detailed analysis of the results from the earlier sections. We obtain (just in Ref [1]) an equation for the structure functions, [7], containing contributions both from the DGLAP and the BFKL mechanisms. In the CMCF Model some of the "true" contributions to the anomalous dimensions are neglected with the argument that they are subleading, ie do not correspond to pole contributions in the Mellin transformation with respect to the Bjorken x-variable. Using the relation (valid for a smooth "non-integrated" structure function $F(\log(1/x), \log(k_\perp^2)))$

$$\theta(\vec{q}^2 - \min(k_\perp^2, k_\perp'^2))F(L', \kappa') \simeq F(L', \kappa') - \theta(\vec{k}_\perp^2 - \vec{q}^2)F(L', \kappa) \quad (11)$$

the authors are able to reproduce the BFKL result small x-values. In Ref [7] different scenarios are investigated. We find very large contributions from the sub-leading terms neglected in the CMCF Model, in particular if we use the "BFKL-approximation" in Eq (11) the power λ in $x^{-\lambda}$ decreases from around 0.5 to 0.33)

I would like to end by exhibiting a different result, partly because so many of my colleagues feel that the BFKL mechanism is stable. The main behaviour of the BFKL gluon structure function f is obtained from the iteration that the n:th step in the transverse momentum $\vec{k}'_\perp \to \vec{k}_\perp$ results from a particular kernel K:

$$f_n(\vec{k}_\perp^2) = \int d(\vec{k}'_\perp)^2 K(\vec{k}_\perp^2, \vec{k}_\perp'^2) f_{n-1}(\vec{k}_\perp'^2) \quad (12)$$

In case we know the eigenvalues of the kernel K (and the largest eigenvalue turns out to be $\lambda = 12\alpha\log(2)/\pi$ for a fixed coupling α) we may sum over all iterations to get the BFKL gluon structure function

$$f_{gBFKL} \propto \sum_n \frac{(\lambda \log(1/x))^n}{n!} \simeq \exp(\lambda \log(1/x) = x^{-\lambda} \quad (13)$$

using the integral (valid for positive y-values):

$$\int \prod_{j=1}^n dy_j \delta(\sum_{j=1}^n y_j - Y) = \frac{Y^{n-1}}{(n-1)!} \quad (14)$$

We note that that in every splitting in LLA one of the poles contributes to the results (we have conventionally followed the z-pole contributions). It is then necessary for consistency that it really does dominate, ie in this convention that $z < \exp(-a)$ for some real number a (which must be at least $a > \log(2)$ in order that $z < (1-z)$). (Another argument is that unless we restrict the z-variations we will break energy-momentum conservation along the emission line).

Introducing this simple restriction we obtain

$$\int \prod_{j=1}^n dy_j \delta(\sum_{j=1}^n y_j - Y) \to \int \prod_{j=1}^n dy_j \delta(\sum_{j=1}^n y_j - (Y - na)) = \frac{(Y-na)^{n-1}}{(n-1)!} \quad (15)$$

(keeping to the notations in Eq (14)). We have introduced the domain restriction in the second line and then the third line summed over all values of n will no longer provide the BFKL exponential. It is straightforward, using the Stirling approximation to the factorial to obtain the change to Eq (13) as a power in $1/x$ with $\lambda \to \rho$ with the relation $\log(\lambda/\rho) = a\rho$. Thus the power λ is diminished so that $\lambda \to \rho \simeq \lambda(1 - a\lambda)$. We conclude that the BFKL mechanism obtains a large part of its contributions from the possibility to emit gluons with moderate to large values of z (at the same time there are then many emitted gluons with $(1-z)$ small!). This decrease in the exponent is also found in a consistent evaluation of the structure function, cf Ref [7]. We note that the correction exhibited above is of the order α^2 (which is expected in the BFKL treatment). But it should also be noted that the correction is very large, ie $\lambda \simeq 0.5 \rightarrow 0.25$.

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