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Spin-encoded quantum computer near ultimate physical limits

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Abstract
Landauer's bound is applicable to irreversible quantum operations. In this study, we showcased that the Doppler temperature manifests the existence of Landauer's bound, which does not block a spin from (irreversibly) flipping with a tiny amount of energy via quantum tunneling. Verified by a spin–spin magnetic interaction experiment, this (energy) amount was determined to be only 1.25 times the theoretical value of Landauer's bound. Based on Heisenberg's principle, we defined information from a measuring perspective: one bit of information corresponds to the smallest error when quantifying the product of the measured energy uncertainty (ΔE) and the measured time duration (Δt). We then illustrate an optically manipulated, spin-encoded, near-Landauer-bound, near-Heisenberg-limit quantum computer that encompasses this new definition of information. This study may represent the last piece of the puzzle in understanding both quantum Landauer erasure and Heisenberg's quantum limit since a single spin is the smallest information carrier.

Keywords Quantum computer · Qubit · Spin · Landauer's bound · Heisenberg's quantum limit

1 Introduction

Quantum computing, described by unitary operations, is notably reversible. However, the projective initialization required to set up the system in an entangled state and the projective measurement needed to extract classical information from the computation are not reversible. Landauer's bound [29] restricts these irreversible operations, implying that the growth in the number of computations per joule of dissipated energy will plateau around 2050 [23, 24].

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Landauer's bound was introduced in 1961 when Landauer postulated that information has a physical nature. According to this, erasing a bit of classical information necessitates a minimum energy of $\Delta E = k_B T \ln 2$, where k_B represents the Boltzmann constant and T is the system's temperature. Importantly, Landauer's principle established the fundamental physical boundary for computations [29].

In March 2012, Bérut et al. experimentally confirmed Landauer's bound using a single silica glass bead (with a diameter of 2 μm) acting as a Brownian particle. The particle was confined in a double-well potential, and the observed mean dissipated heat was shown to saturate at the bound under long erasure cycles [5]. Subsequently, in June 2012, Alexei et al. presented the initial experimental validation of Landauer's principle in logically reversible operations. They measured energy dissipation significantly below Landauer's bound (at the sub-$k_B T$ level) for reversible operations, while irreversible operations exhibited energy dissipation exceeding Landauer's bound [1].

In 2014, Jun et al. confirmed the bound using a fluorescent particle (200 nm in size). They showcased this by utilizing small particles confined in traps and minimizing the work exerted to reach the Landauer limit during the erasure process [22].

In 2016, Hong et al. expanded Landauer's principle to encompass orientation-encoded information. They measured an energy dissipation of 4.2 zeptojoules in a single-domain nanomagnet, which consisted of over 10^4 spins. The energy dissipation was assessed using a laser probe while flipping a bit from the off to the on state [20].

In May 2018, a team led by Feng reported a demonstration of Landauer's principle in a fully quantum system involving a single atom. They utilized a trapped ultracold $^{40}Ca^+$ ion as an atom qubit, comprising its two internal states [42]. The erasure protocol involved the heat reservoir (the ion's own vibrational modes), and the associated work was measured [12, 42]. In June 2018, Gaudenzi et al. similarly extended Landauer's principle to the quantum domain, focusing on a collective $S_z = \pm 10\,(20\,\mu_B)$ giant spin at 1 K, employing a superconducting quantum interference device (SQUID) [13, 14].

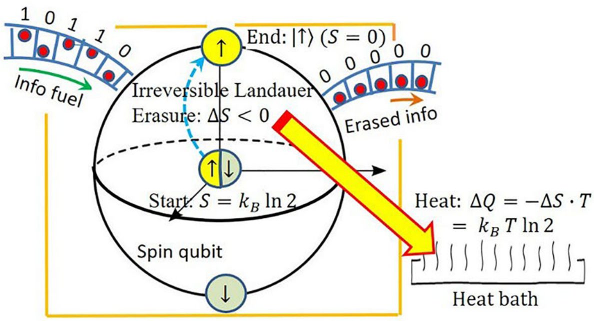
In March 2020, Saira et al. conducted measurements of Landauer's bound at 500 mK [37]. Following this, in June 2020, Çetiner et al. measured Landauer's bound in ion channels, which are smaller than fluorescence molecules but larger than spins [9].

In March 2021, Holtzman et al. demonstrated that the enforcement of Landauer's bound occurs through the contraction of the physical system's phase-space volume during bit erasure. They further proposed that precise knowledge of the system's energy allows for the implementation of an erasable bit with no thermodynamic cost in a Hamiltonian memory [19]. However, they emphasized the theoretical nature of their proposal, acknowledging that any uncertainty in energy (common in realistic situations due to limited knowledge of the system's energy) reinstates Landauer's bound [19].

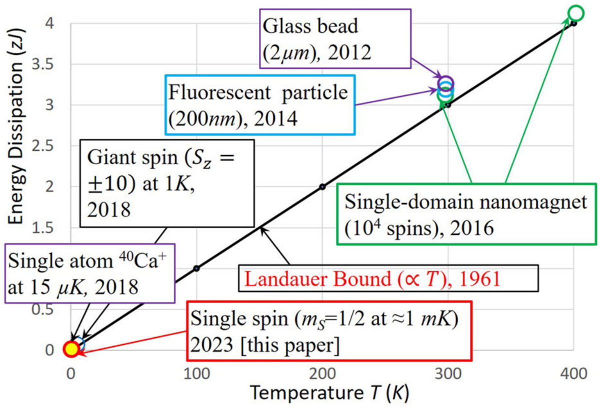
In April 2021, Chiribella et al. made a notable finding that even a logically reversible quantum operation, operating on a physical processor with different energy levels, necessitates energy. They provided quantified upper and lower bounds, which persist even in entirely reversible evolutions [11]. Interestingly, these bounds can be quantitatively compared with the classical Landauer bound, which manifests in irreversible evolutions [11].

In November 2021, Georgescu conducted a comprehensive review of 60 years of Landauer's principle. The review emphasized that Landauer's principle establishes a fundamental energy bound, not only for irreversible bit operations in classical systems (which is the traditional domain of Landauer's principle) but also for the reversible operations of quantum computation, highlighting the unifying aspect despite the distinction between these two types of operations [15].

In this study, we will explore Landauer's bound in a quantum computer using spin qubits, as illustrated in Fig. 1. Initially, during an erasure, the spin qubit is in a maximally mixed state [or (↑↓)] with the probability density matrix $\rho_{\text{qubit}} = (|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|)/2$ (i.e., the spin qubit is equally likely to be either up or down), corresponding to the center of the Bloch sphere, resulting in maximal entropy $S = k_B \ln 2$. As the qubit undergoes erasure, it converges to a pure quantum state (a point on the



(a) A quantum computer as an information "burning" engine;



(b) Various information carriers.

Fig. 1 **a** A quantum computer operates using a few spin qubits, whose states can be depicted by a point on the Bloch sphere, a representation of their possible orientations. **b** Numerous experimental validations of Landauer's bound have been conducted, involving diverse information carriers at their respective operating temperatures. This particular study, focusing on a single spin, may represent the last piece of the puzzle in understanding quantum Landauer erasure

Bloch sphere's surface), reducing its entropy to $S = 0$. This information erasure is an irreversible manipulation of the generated information, causing ΔS to be less than zero. In other words, the "Maxwell demon" [5] or the observer responsible for "creating" the information loses the ability to extract work from the system once the information is already "burnt." Remarkably, this energy constraint remains achievable even when complex quantum circuits are utilized, involving numerous individual unitary gates [19].

In this article, the maximally mixed state [or ($\uparrow\downarrow$)] of a single spin needs to be carefully distinguished from the superposition $[(|\uparrow\rangle \pm |\downarrow\rangle)/\sqrt{2}]$ of $|\uparrow\rangle$ and $|\downarrow\rangle$ in one spin, the four eigenstates ($|\uparrow\downarrow\rangle$, $|\downarrow\uparrow\rangle$, $|\uparrow\uparrow\rangle$, and $|\downarrow\downarrow\rangle$) of two spins, and the entanglement $[\psi\pm = (|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle)/\sqrt{2}$ and $\chi\pm = (|\uparrow\downarrow\rangle \pm i|\downarrow\uparrow\rangle)/\sqrt{2})]$ of two spins.

In the age of quantum computing, a natural inquiry arises regarding the potential approach toward this bound, given the fundamental disparity between quantum and classical bits [12]. Our aim in this article is to endeavor to address this very question.

2 Position-encoded information

In the scenario of one-dimensional Brownian motion, considering a position-encoded system [depicted in Fig. 2a as a solid particle serving as an information carrier confined in a chamber with an impenetrable (Landauer) wall], the system can be approximated to be in internal thermodynamic equilibrium for each specific value of the particle's coordinate x.

For a silica bead [5] or a fluorescent particle [22], the erasure (L) state is achieved from the random data state by transitioning through a free state (where the carrier can move freely between the two chambers) after removing the partition. This erasure, functioning as an isothermal contraction, generates $k_B T$ ln2 (Landauer's bound) by introducing a frictionless piston and moving it toward the L direction.

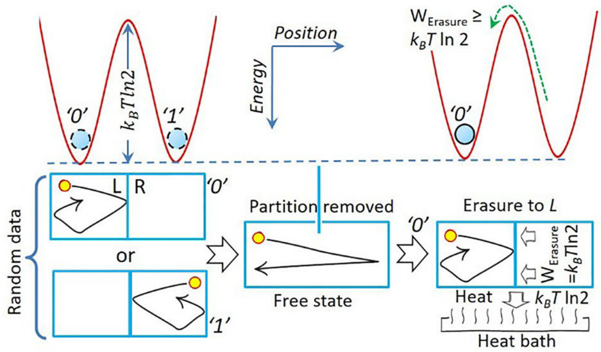
The statistical–mechanical expression for the free energy F is given by: $F = -k_B T \ln Z$, where Z represents the partition function [7]. The free energy for the subsystem in Fig. 2a is described by: $F(x) = -k_B T \ln Z(x)$, where $F(x)$ denotes the subsystem's free energy at position x and $Z(x)$ is computed by summing the microstates at x [7] [35].

In the case of a position-encoded (classical) information bit represented in Fig. 2a, the "random data" carried by the information carrier have an equal probability of being in either the L or R chamber, resulting in probabilities of $P(L) = P(R) = 1/2$. Following erasure, the carrier is unequivocally reset to a specific reference state, which in this instance is the L chamber: $P(L) = 1$; $P(R) = 0$.

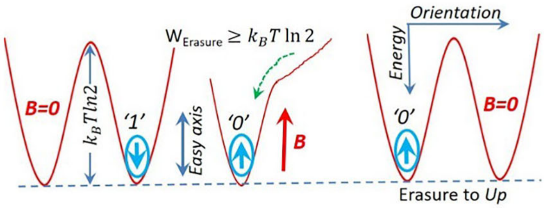
The work required to move the information carrier (with two possible positions) to the desired half (L) can be expressed as:

$$W \geq F(x) = k_B T \ln 2, \tag{1}$$

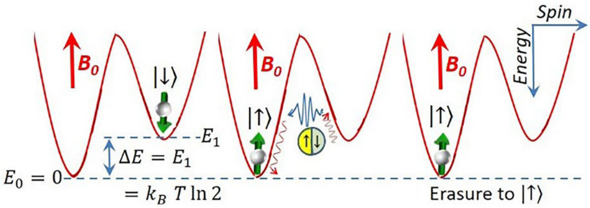
where $Z(x) = \frac{1}{2}$ since the information carrier has only two possible positions in the chamber.



(a) Erasure protocol of position-encoded information;



(b) Erasure protocol of orientation-encoded information;



(c) Erasure protocol of a spin qubit.

Fig. 2 Comparison of the three erasure protocols

Interestingly, the aforementioned operation could be conducted by a "Maxwell's demon" that expends energy to observe the position of the carrier and insert the partition. Remarkably, the energy consumed in this process equals the work exerted for (Landauer) erasure.

3 Orientation-encoded information

A single-domain nanomagnet, incorporating more than 10^4 spins [20] and considered large enough to be treated as classical [37], was employed to represent a piece of information by encoding its orientation (magnetization), as depicted in Fig. 2b. Thermal agitation causes the orientation (x) of a magnetic moment to fluctuate, allowing it to assume an arbitrary direction. The probability of finding x at thermal equilibrium, which is proportional to $Z(x)$, can be deduced from Eq. 1. Therefore, we have:

$$F(x) = -k_B T \ln Z(x) = k_B T \ln 2, \tag{2}$$

where $Z(x) = \frac{1}{2}$ since the direction of a magnetic moment is either "up" or "down" along the easy axis. The two potential orientations of a magnetic moment are comparable to the two possible positions of a Brownian particle within a position-encoded information system. Essentially, these two distinct information systems share the same underlying thermodynamics.

In Fig. 2b, the erasure (Up) state is attained from the random data state by applying a magnetic field B along the z-axis to overcome the Landauer barrier of $k_B T$ ln2 (this field also tilts the potential energy landscape).

As a paradigm in quantum computing, a single or giant spin can function as a carrier of quantum spin information, with its spin orientation serving as the encoding mechanism, illustrated in Fig. 2c. The spin angular momentum is quantized, resulting in only two possible z-components. Especially at extremely low temperatures, direct tunneling via the ground state becomes highly relevant and often dominates the spin relaxation process [13, 14]. As depicted in Fig. 2c, quantum spin tunneling through the (Landauer) barrier from $|\downarrow\rangle$ to $|\uparrow\rangle$ is coupled with excitation involving resonant phonons to reach the tunneling state, and de-excitation entails emitting phonons to return to the ground state [13, 14]. As also illustrated in Fig. 2c, two spins ($|\uparrow\rangle$ and $|\downarrow\rangle$), akin to free and oscillating waves, become entangled while tunneling.

As shown in Fig. 2c, an external (static) magnetic field B_0 eliminates the degeneracy between the two probe states ($|\uparrow\rangle$ and $|\downarrow\rangle$) and tilts the potential landscape to make it asymmetric such that the energy bias $\Delta E = k_B T \ln 2 \neq 0$. In the figure, the height of a wavefunction (in blue) represents the (lower-lying) quantum energy level, contrasting with the classical double-well potential landscape (in red). This energy bias ($\Delta E = k_B T \ln 2$), due to B_0, stops the previously erased spin qubit from tunneling back to its initial state, which ensures an irreversible Landauer erasure.

To compensate the decreased entropy ($\Delta S < 0$), the energy is "lost" by being dissipated as heat due to the existence of magnetic hysteresis [13, 14], although the damping of the magnetic reversal (with the aid of a tilted double-well energy landscape) is weak. This dissipative process is also evidenced by the fact that (phonon-mediated/assisted) quantum spin tunneling remains correlated with the 'surrounding world', inclusive of the environmental temperature T [13, 14]. Actually, this is the so-called red-sideband effect [27], in which a laser beam is used to couple the (spin) qubit and the micromotion of the ion (measured by the phonon number n) in such a way ($|\downarrow, n\rangle \leftrightarrow |\uparrow, n+1\rangle$). The flip of the spin will increment or decrement the phonon number n during the erasure process.

The energy of flipping a spin undergoing a magnetic field B is:

$$\Delta E_{\uparrow\rightarrow\downarrow} = \vec{\mu}_B \cdot \vec{B} = \vec{\mu}_B \cdot B\hat{z} = \gamma \hat{S}_z B = \gamma B \frac{\hbar}{2}(|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|) \tag{3}$$

where μ_B is the Bohr magneton, B is an external magnetic field, γ is the gyromagnetic ratio of an isolated electron, $\hat{S}_z = \frac{\hbar}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ is the quantum–mechanical operator associated with spin-$\frac{1}{2}$ observable in the z axis, and $\hbar$ is the reduced Planck constant.

At first glance, this energy bound associated with flipping a spin might appear disconnected from the environmental temperature T. However, as mentioned above, quantum spin tunneling is phonon-mediated/assisted and is thus correlated with the environmental temperature T [13, 14]. In essence, the spin relaxation time, τ_{rel}, approximately adheres to Arrhenius's law: $\tau_{rel} = \tau_0 \exp(\frac{U}{k_B T})$, where $\tau_0 = 10^{-8}$s, U is the activation energy determined by the tunneling channel and $\tau_{rel} \geq 100$s as T diminishes to 1 K [13, 14].

For a $S_z = \pm 10$ giant spin [13, 14] or a single spin as discussed in this article, the erasure state ($|\uparrow\rangle$) is reached from the mixed state () of the spin qubit, as shown in Fig. 2c. Quantum spin tunneling (represented by a blue wave) effectively surpasses the thermal energy barrier, which is significantly larger than Landauer's bound, presenting a "shortcut" for spin reversal. This stands in contrast to classical information manipulations where the erasure cost doesn't arise from "climbing a barrier" but rather from compressing phase space through dissipative dynamics.

4 Experiment of spin–spin magnetic interaction

In Fig. 3, notably weak magnetic interactions were measured between the two ground-state spin-1/2 ($1\mu_B$) valence electrons of two $^{88}Sr^+$ ions. The two ions were laser-cooled to their ground state and entangled across a separation ($d = 2.4$) µm. In this setup, if an ion moved toward the light source, it would absorb more photons, resulting in a reduction in the ion's speed. This effect effectively cooled the ion as temperature is a measure of random internal kinetic energy [27]. The experimental setup involved trapping the two ions in a linear radiofrequency (rf) Paul trap, with a radial trap frequency (Γ) set at $2\pi \times 2.5$MHz, and laser cooling the system to a temperature of ≈ 1 mK (even 30 µK was reached by adding resolved sideband cooling) [25]. The measurement made effective use of the quantum lock-in method [26] to distinctly isolate feeble signals from the background noise.

An external magnetic field $B_0 = 1.3 \times 10^{-3} T$ established the spin quantization axis and eliminated the degeneracy between the two probe states, creating a frequency of $f_0 = 13.16$MHz.

In this experiment, it was observed that the spin–spin magnetic interaction follows an inverse-cube law. This magnetic interaction has the capability to induce a change in their orientation. As the smallest magnetic unit (the Bohr magneton), a spin ($\overrightarrow{\mu}_B$) generates a magnetic field affecting another spin. When the two spins are aligned along the line connecting the two ions [27], the strength of the (equivalent) magnetic field at $d = 2.4$ µm is given by:

$$B_{spin-spin} = \frac{\mu_0}{4\pi}\frac{2\mu_B}{d^3} = 1.34 \times 10^{-13} T, \tag{4}$$

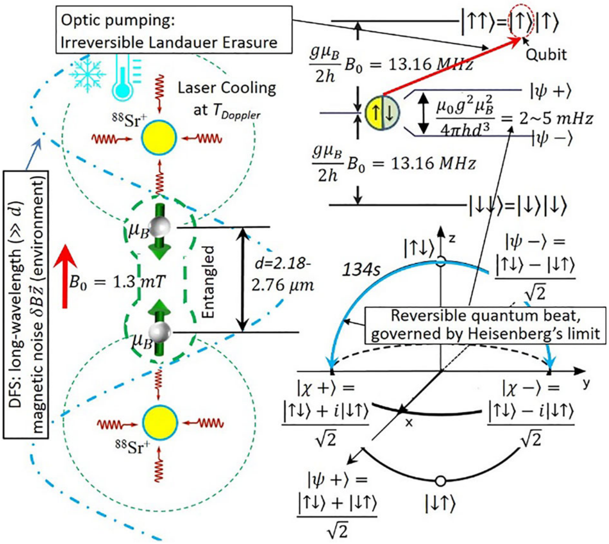


Fig. 3 A notably feeble magnetic interaction between the two ground-state spin-1/2 ($1\mu_B$) valence electrons of two $^{88}Sr+$ ions was observed in the experiment [27]. Rabi flopping between $|\uparrow\ \uparrow\rangle \leftrightarrow |\downarrow\ \downarrow\rangle$ (as illustrated in the Bloch sphere) was performed with angular frequency $4\xi = \mu_0\mu_B^2/\pi\hbar d^3$. The measured energy splitting (2×13.16MHz) between $|\uparrow\ \uparrow\rangle$ and $|\downarrow\ \downarrow\rangle$ in this experiment was utilized to validate the irreversible Landauer erasure (see main text and Fig. 5 for details). Another measured (interaction) energy ($2 \sim 5$mHz) between $|\psi+\rangle$ and $|\psi-\rangle$ in this experiment was utilized to validate the Heisenberg limit: $\Delta E_{\uparrow\downarrow\leftrightarrow\downarrow\uparrow}\Delta t = \mu_B B_{spin-spin}\Delta t = 9.2740 \times 10^{-24}\frac{J}{T} \times 1.34 \times 10^{-13}T \times 134\text{s} = 1.66 \times 10^{-34}$Js, which is the same as the Heisenberg quantum limit of $h/4 \approx 1.66 \times 10^{-34}$J · s (see main text for details). The redraw of the setup was generously provided by Shlomi Kotler from the Hebrew University of Jerusalem

where μ_0 is the vacuum permeability constant. This (equivalent) magnetic field $B_{spin-spin}$ is ten orders of magnitude smaller than the external magnetic field $B_0 = 1.3 \times 10^{-3}$T.

According to Fig. 2c and Eq. 3, we used B_0 to compute the energy of (irreversibly) flipping a spin as below:

$$\Delta E_{(\uparrow\downarrow)\rightarrow\uparrow} = \mu_B B_0 = 9.274 \times 10^{-24} J/_T \times 1.3 \times 10^{-3} T = 1.21 \times 10^{-26} J, \quad (5)$$

where ($\uparrow\downarrow$) denotes the maximally mixed state (◐) of the eigenstates $|\uparrow\rangle$ and $|\downarrow\rangle$ of the single spin. One spin in this spin–spin experiment is viewed as a spin qubit in our study, and another spin is viewed as a magnet to manipulate the (spin) qubit via the spin–spin magnetic interaction ($\boldsymbol{\xi} = \mu_0\mu_B^2/4\pi\hbar d^3$).

This energy can be converted to $\frac{\Delta E_{(\uparrow\downarrow)\rightarrow\uparrow}}{h} = \frac{1.21\times10^{-26}J}{6.63\times10^{-34}J\cdot s} = 18.3$MHz, which is reasonably close to half of the measured degeneracy (2×13.16MHz) between $|\uparrow\ \uparrow\rangle$ and $|\downarrow\ \downarrow\rangle$ in the spin–spin magnetic interaction experiment (Fig. 3) [27].

In practicality, a fault-tolerant quantum computer with imperfect quantum logic gates must endure extended computations without succumbing to inevitable errors and noise. Reliability and error probability become paramount concerns. We emphasize

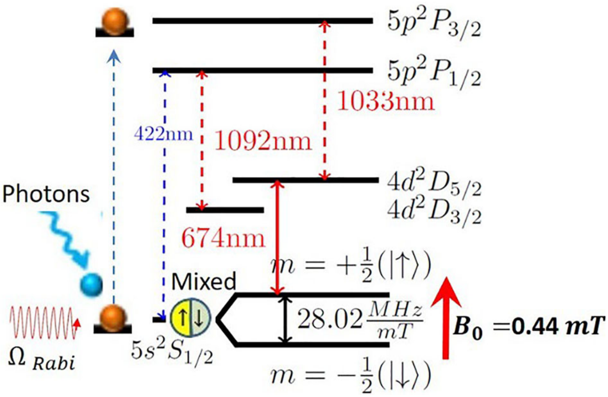


Fig. 4 Degeneracy between the probe states of a single $^{88}Sr^{+}$ ion was eliminated by applying a magnetic field $B_0 = 0.44\text{mT}$, resulting in a lifting frequency $f_0 = 12.34\text{MHz}$. The corresponding gyromagnetic ratio is $\gamma = \frac{12.34\text{MHz}}{0.44\text{mT}} = 28.02\frac{\text{MHz}}{\text{mT}}$. To induce spin rotations, an oscillating magnetic field resonant with this frequency was pulsed, with the field being perpendicular to the quantization axis set by the external magnetic field B_0. This led to a Rabi frequency of $\Omega_{\text{Rabi}} = 65.8\text{kHz}$. A resonant 422 nm laser pumped the two eigenstates ($|\uparrow\rangle$, $|\downarrow\rangle$) of the single spin to a maximally mixed quantum state ( ) [27]

that a single spin can be switched reliably, attaining a typical detection fidelity of 98%, even in the presence of magnetic noise that is six orders of magnitude greater than the applied magnetic field [27]. The spin evolution was effectively confined to a decoherence-free subspace (DFS) that remains resilient against collective magnetic field noise [27]. This resilience stems from both spins experiencing the same (time-dependent) magnetic noise from the environment, with a wavelength much larger than the separation d (Fig. 3).

As shown in Fig. 4, optical pumping, a widely used technique to elevate electrons from a lower energy level to a higher one within an atom or molecule, was employed in this experiment to pump electrons bound within the ions into a well-defined quantum state, either $|\uparrow\ \downarrow\rangle$ or $|\downarrow\ \uparrow\rangle$. The probe state is measured by shelving the $|\uparrow\rangle$ state to the appropriate metastable D sub-level with an on-resonant 674 nm laser, followed by state selective fluorescence with a resonant 422 nm laser. State preparation to $|\uparrow\rangle$ is done by optical pumping with a σ_+ circularly polarized on resonant 422-nm laser. The 1092-nm and 1033-nm lasers in the infrared range were utilized as repump lasers [27]. Optical pumping can reduce the entropy of an atom or ion [34], which is exactly what we need in our study.

The generation of entangled Bell states in the form of $|\boldsymbol{\psi}\pm\rangle = (|\uparrow\ \ \downarrow\rangle \pm |\downarrow\ \ \uparrow\rangle)/\sqrt{2}$ was accomplished using a Sörenson–Mölmer entangling gate [27]. Representing a pure quantum state via the Bloch vector, its location could be determined by measuring its projection on an equal superposition basis, for example, $|\boldsymbol{\chi}\pm\rangle = (|\uparrow\ \ \downarrow\rangle \pm \boldsymbol{i}|\downarrow\ \ \uparrow\rangle)/\sqrt{2}$ (i.e., y) when rotating it around x, through a parity observable. This collective rotation retained the relative orientation of the spins, leaving the spin–spin interaction unchanged [27]. The parity observable was utilized to measure coherence between $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$, yielding a value of + 1 if the spins were aligned and − 1 if they were anti-aligned.

In magneto-optical traps (MOTs), the actual temperature is $(10 \sim 30)T_{\text{Doppler}}$ [10]. The minimum Doppler temperature is:

$$T_{\text{Doppler}} = \frac{\hbar\Gamma}{2k_{\text{B}}} = \frac{1.05 \times 10^{-34}\text{J} \cdot \text{s} \times 2\pi \times 2.5 \times 10^{6}\text{s}^{-1}}{2 \times 1.38 \times 10^{-23}\text{J} \cdot \text{K}^{-1}} = 5.07 \times 10^{-5}\text{K}, \quad (6)$$

where Γ is broad natural linewidth (measured in radians per second), hence the calculated temperature is $T = (10 \sim 30) \times 5.07 \times 10^{-5}\text{K} = (0.51 \sim 1.52)\text{mK}$, which agrees reasonably with the actual temperature of $\approx 1\text{mK}$. Landauer's bound can be expressed by $k_{\text{B}}T\ln2 = k_{\text{B}}T\ln2 = 9.6 \times 10^{-27}J$ at 1 mK, which is 10^{-5} times Landauer's bound (3×10^{-21}J) at room temperature (300 K) as it is proportional to the temperature.

Significantly, based on Eq. 5, the input energy (1.21×10^{-26}J) required to (irreversibly) erase a spin qubit is only 1.25 times the theoretical value of Landauer's bound (9.6×10^{-27}J) at the corresponding temperature (1mK). This alignment serves as a strong validation, leveraging the data obtained from the spin–spin experiment dating back to 2014 [27]. Notably, Kotler expressed their excitement, stating "It's exciting to hear that our work is useful in new areas of research that we were not aware of when doing the experiment," upon reviewing the manuscript we shared with them.

Even though the spin–spin experiment [27] operates within a distinctly different context involving magnetic interactions governed by an inverse-cube law, it essentially encompasses a complete erasure protocol with $\Delta S < 0$ and provides the measurement of the work involved, as illustrated in Fig. 5.

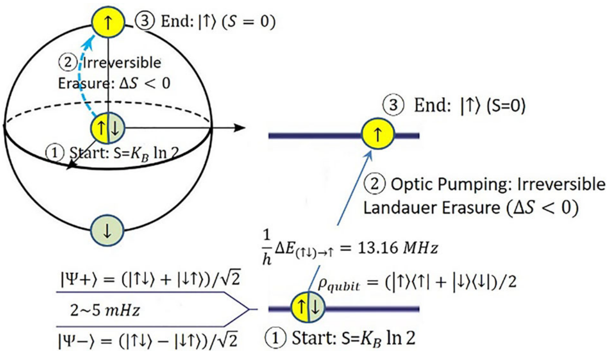


Fig. 5 Energy diagram of two $^{88}Sr^{+}$ ion. Even though the spin–spin experiment [27] operates in a distinct magnetic interaction context, it effectively encapsulates a complete erasure protocol including Step ①, ② and ③: ① Starting point, where the qubit is in an equal probability admixture (↑↓) of the eigenstates $|\uparrow\rangle$ and $|\downarrow\rangle$ with $\rho_{qubit} = (|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|)/2$ in the entangled state $|\psi +\rangle = \frac{|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle}{\sqrt{2}}$ or $|\psi -\rangle = \frac{|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle}{\sqrt{2}}$ (where entangled spins cannot be described as independent entities), indicating that the initial entropy of the system is maximal since we have no explicit information about the existing state of the (information) spin. ② Mediate step using optical pumping that can reduce the entropy of an atom or ion [34], where a laser beam pumps electrons bound within the ions into a well-defined quantum state, either $|\uparrow\uparrow\rangle$ or $|\downarrow\downarrow\rangle$, decreasing the entropy ($\Delta S < 0$). ③ End of the erasure, where the system is populated in $|\uparrow\rangle$

The complete erasure protocol entails all the crucial steps as defined in [42]: at the initial erasure step, a laser beam pumped the spin qubit (the spin on the left in Fig. 5) to a maximally mixed quantum state (): this setup ensured that the single spin was equally likely to be in either of the up/down eigenstates (corresponding to the center of the Bloch sphere in Fig. 5), resulting in maximal entropy S $= k_\mathrm{B}\ln 2$. Moving to the intermediate erasure step, the optically created qubit was subsequently erased by optic pumping. Finally, in the concluding step of erasure, the qubit ended up in a (ground) quantum state $|\uparrow\rangle$ (representing the north pole of the Bloch sphere in Fig. 1), resulting in zero entropy $S = 0$. Notably, the reduction in entropy ($\Delta S < 0$) at this stage ensures the irreversibility required in a Landauer erasure process.

To validate the comprehensiveness of the erasure protocol encompassing the initiation, erasure, and conclusion stages, a comparison between the spin–spin experiment [27] and the single-atom demonstration illustrating the completion of the erasure protocol supporting the quantum Landauer principle [42] can be made. In this study focusing on a single spin as the smallest information carrier, our primary objective is to ascertain and substantiate that Landauer's bound is upheld at the level of a single spin.

In the above irreversible information erasure with the population transfer from to $|\uparrow\rangle$, the work required to erase a spin qubit must offset the corresponding entropy drop $\Delta S < 0$. This entropy reduction is crucial, ensuring the irreversibility essential for the (irreversible) Landauer erasure according to the 2nd law of thermodynamics [12, 42].

In order to discern the primary influential factors in our specific problem, we reformulated the spin section of the Hamiltonian for the two-ion system as observed in the spin–spin experiment [27]:

$$H = \underbrace{0.5\hbar\left(\omega_{A,1}\sigma_{z,1} + \omega_{A,2}\sigma_{z,2}\right)}_{\text{Magnetic field } B_0=1.3mT\,(\text{MHz})} + \underbrace{2\hbar\zeta\sigma_{z,1}\sigma_{z,2}}_{\text{Spin-spin(mHz)}} \underbrace{-\hbar\zeta\left(\sigma_{x,1}\sigma_{x,2} + \sigma_{y,1}\sigma_{y,2}\right)}_{\text{Rabi flopping (kHz)}} \quad (7)$$

Here $\sigma_{j,i}$ is the $j \in \{x, y, z\}$ Pauli spin operator of the ith spin, within which $\sigma_{z,1}\sigma_{z,2}$ does not cause any spin flips and acts as a phase gate in quantum computing, whereas $\sigma_{x,1}\sigma_{x,2}$ and $\sigma_{y,1}\sigma_{y,2}$ lead to Rabi flopping of $|\uparrow\ \downarrow\rangle \leftrightarrow |\downarrow\ \uparrow\rangle$; $\omega_{A,i} = 2\mu_\mathrm{B}B_i/2\hbar$, where B_i is the external magnetic field. The spin–spin interaction strength is $\xi = \mu_0\mu_\mathrm{B}^2/4\pi\hbar d^3$.

The first term on the right-hand side of Eq. 7 describes the Zeeman shift of the spins' energy due to the external magnetic field $B_0 = 1.3$mT, which is equivalent to MHz in the spin Larmor frequency $\omega_{A,i}(i = 1, 2)$ [27] that characterizes the precession of a transverse magnetization about a static magnetic field. Kotler confirmed that "The energy splitting I had for the Sr^+ valence electron spins was a linear Zeeman shift: $\mu_\mathrm{B}B_\mathrm{z}$, which had a typical energy scale of Planck times 10MHz. The typical relaxation times in this case were extremely long. So to reset this spin one would need to use an auxiliary energy level and optical pumping. That is indeed an irreversible process by design."

The second term in Eq. (7) describes the spin–spin magnetic interaction, which is equivalent to 2–5 mHz [27]. Kotler confirmed that "For an experiment with a single electron, an external magnetic field can cause rotations to the spin. Those will be reversible, since the damping of the spin oscillations is very weak. For an experiment with two electrons, the effect of a homogenous magnetic field disappears. Indeed the oscillations were between spin up spin down (|*u*,*d* >) and spin down spin up (|*d*,*u* >). These two states are degenerate in the presence of a homogeneous magnetic field. Now the spin of one electron generates a magnetic field at the location of the other electron, causing them to rotate from up down to down up. This state too also has low dissipation and is essentially reversible."

The third term in Eq. (7) results in a collective spin flip, in which spin rotation is performed by pulsing a resonant oscillating magnetic field, resulting in a Rabi frequency in kHz [27]. The spin Larmor frequency in the first term is on the MHz order, whereas the Rabi frequency in the third term is on the kHz order, the former is dominant in terms of calculating the work of irreversibly erasing a spin qubit.

In our study here, it is the first term (MHz) and the second term (mHz) that are at the focuses. We will develop an analytical model to elucidate the experimental confirmations described above in the following section.

5 Spinor wavefunction of an isolated electron

The Schrödinger–Pauli equation for an isolated electron [the smallest magnet being an information carrier] is:

$$i\hbar\frac{\mathrm{d}|\Psi\rangle}{\mathrm{d}t}=\hat{H}|\Psi\rangle, \tag{8}$$

where the spinor wavefunction is $|\Psi(\mathrm{t})\rangle = C^{+}(t)|\uparrow\rangle + C^{-}(t)|\downarrow\rangle$, and the Hamiltonian is $\widehat{H} = -\gamma B\frac{\hbar}{2}(|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|)$ according to Eq. 3. Substitutions into Eq. 7 give:

$$\begin{aligned} i\hbar(\dot{C}^{+}|\uparrow\rangle+\dot{C}^{-}|\downarrow\rangle) &= -\gamma B\frac{\hbar}{2}(|\uparrow\rangle\langle\uparrow|-|\downarrow\rangle\langle\downarrow|)\big(C^{+}|\uparrow\rangle+C^{-}|\downarrow\rangle\big)\\ &= -\gamma B\frac{\hbar}{2}\big(C^{+}|\uparrow\rangle-C^{-}|\downarrow\rangle\big), \end{aligned} \tag{9}$$

$$\begin{bmatrix}\dot{C}^{+}\\ \dot{C}^{-}\end{bmatrix}=\frac{i}{2}\gamma B\begin{bmatrix}1&0\\0&-1\end{bmatrix}\begin{bmatrix}C^{+}\\ C^{-}\end{bmatrix}=\frac{i}{2}\gamma B\begin{bmatrix}C^{+}\\ -C^{-}\end{bmatrix} \tag{10}$$

The WKB (Wentzel–Kramers–Brillouin) approximation rewrites the (complex-valued) spinor wavefunction as:

$$|\Psi(\mathrm{t})=\begin{bmatrix}C^{+}(t)\\ C^{-}(t)\end{bmatrix}=\begin{bmatrix}C^{+}(0)e^{\Phi(\mathrm{t})}\\ C^{-}(0)e^{-\Phi(t)}\end{bmatrix} \tag{11}$$

The time evolution takes place in the presence of an external magnetic field B_0. To overcome the thermal perturbation (Landauer's bound), we should have $\Delta E_{\uparrow\to\downarrow} = \mu_B B \approx \mu_B B_0 \geq k_B T \ln 2$. Without losing generality, $\Delta E_{\uparrow\to\downarrow}(t)$ is assumed as a positive constant E during $-\Delta t/2 \leq t \leq \Delta t/2$. Then, we obtain:

$$\begin{aligned}\Phi\left(t=\frac{\Delta t}{2}\right) &= i\frac{1}{\hbar}\int_{-\infty}^{t}\left(-\gamma B\frac{\hbar}{2}\right)\mathrm{d}t\bigg|_{t=\Delta t/2} \\ &= i\frac{1}{\hbar}\int_{-\infty}^{t}\Delta E_{\uparrow\to\downarrow}(t)\mathrm{d}t\bigg|_{t=\Delta t/2} \\ &= i\frac{1}{\hbar}E\Delta t.\end{aligned} \tag{12}$$

Then, Eq. 11 simplifies to:

$$|\Psi(\mathrm{t}=\Delta t/2) = \begin{bmatrix} C^{+}(0)e^{\Phi(\mathrm{t})} \\ C^{-}(0)e^{-\Phi(t)} \end{bmatrix} = \begin{bmatrix} C^{+}(0)e^{i\frac{1}{\hbar}E\Delta t} \\ C^{-}(0)e^{-i\frac{1}{\hbar}E\Delta t} \end{bmatrix}. \tag{13}$$

Equation 13 illustrates that the single spin, acting akin to a free and oscillating wave, tunnels through the energy barrier, analogous to a particle surmounting a hill, emerging on the other side with a probability $|\Psi|^2$, effecting a reversal in the spin–spin magnetic interaction experiment [27]. A comparable quantum spin tunneling phenomenon was also observed in a collective $S_z = \pm 10(20\mu_B)$ giant spin [13, 14].

In Eq. 13, the wavefunction's definition is governed by $(E\Delta t)$. This implies that the behavior of the spin datum is determined by the product of the energy (E) and the time duration (Δt), rather than either of these parameters $(E,\Delta t)$ individually. Remarkably, we observe that the probability of tunneling is significantly influenced by $(E\Delta t)$ more than by $C^{+/-}(0)$. This observation underscores a dramatic distinction between quantum erasure and its classical counterpart.

In dissipative dynamics, the process of erasing a bit of information necessitates a concentration of probability in phase space, consequently adhering to Landauer's bound. Conversely, within Hamiltonian dynamics, it is conceivable to move a particle, for example, from the left well to the right well at negligible cost (or approaching zero cost) [19]. However, the challenge in a Hamiltonian memory lies in the fact that simultaneously, the particle in the right well may transition to the left well (or another location—essentially not remaining in the same well). Fortunately, the tunneling observed in the spin–spin magnetic interaction experiment [27] is irreversible, as energy is input through the application of a magnetic field that exclusively favors and flips a spin in the opposite direction. As previously mentioned, a comparable phenomenon (where spins can tunnel to the opposite side of the potential barrier, effectively reducing the activation energy for spin reversal) was also noted in a collective $S_z = \pm 10(20\mu_B)$ giant spin with magnetic hysteresis [13, 14]. Consequently, the erasure of a single-spin or a giant-spin qubit does not adhere strictly to pure Hamiltonian dynamics, and there is still observable probability concentration in phase space.

6 Using Heisenberg's principle to define information

To further illustrate the exchange of information with energy over time, we employed Heisenberg's time-energy uncertainty relation (TEUR) to define information, as depicted in Fig. 6b. Heisenberg initially presented his uncertainty principle in 1927 as $\Delta x \Delta p \geq h$, utilizing the full Planck constant h [17]. In 1928, this inequality was modified to: $\sigma_x \sigma_p \geq \frac{\hbar}{2}$, where $\hbar = \frac{h}{2\pi}$ represents the reduced Planck constant [6, 30]. In 1945, a non-relativistic time–energy uncertainty relation was established as $\Delta E \Delta t \geq \frac{\hbar}{2}$ [6, 17].

In 1990, Anandan and Aharonov considered an arbitrary quantum evolution from the Fubini-Study metric with a new geometric meaning to time and gave a new quantum limit: $\langle \Delta E \rangle \Delta t \geq \frac{1}{4} h$, where $\langle \Delta E \rangle$ is the time-averaged uncertainty in energy during the time interval Δt [2]. The above inequality is more stringent than the conventional time-energy uncertainty relations having $h/2$ or $\hbar/2$ on the right-hand side [2]. Here, we define "1 bit" based on the inequality $\frac{4}{h} \Delta E \Delta t \geq 1$.

From a measuring perspective, one bit of information corresponds to the smallest error when quantifying the product of the measured energy uncertainty (ΔE) and the measured time duration (Δt). A bit of information is quantitatively defined as follows:

$$1(bit) = \frac{4}{h} \Delta E \Delta t. \tag{14}$$

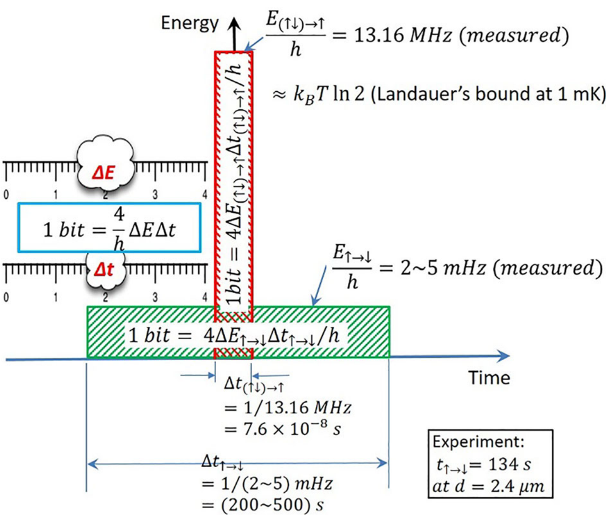


Fig. 6 Heisenberg's time-energy uncertainty relation (TEUR) [2, 17, 30] is utilized to provide a quantitative definition of information from a measuring perspective: the smallest error in quantifying the product of the measured energy uncertainty (ΔE) and the measured time duration (Δt) corresponds to 1 bit of information. The two measured energy gaps (13.16MHz and $2 \sim$ 5mHz) in the spin–spin experiment [27] were utilized as two extreme examples here

In our approach, we adopted either a modern interpretation of the TEUR or a quantum interpretation of the TEUR. In the modern interpretation, measuring the energy of a system with an uncertainty of ΔE necessitates the measurement duration to be at least Δt. Conversely, in the quantum interpretation, a quantum state with a spread in energy ΔE requires a time of at least Δt to evolve from one orthogonal (distinguishable) state to another orthogonal state [2, 30]. In both interpretations, Δt represents the time required to write/erase a bit of information. This time duration is physically equivalent to the duration of the energy measurement in terms of the energy being consumed throughout the Landauer erasure protocol.

The role and interpretation of the TEUR continue to be topics of debate [6]. Heisenberg's initial perspective, which remains the most widely accepted, asserts that measuring the energy of a system with an uncertainty of ΔE necessitates the measurement duration to be at least Δt. However, it's worth noting that Heisenberg's original derivation of the TEUR [17] is currently considered flawed, primarily due to the fact that time is not a quantum variable, but rather a classical parameter. Consequently, there is no "time operator," and thus, no direct analogy to other uncertainty relations, particularly the position and momentum uncertainty relation. In our approach, we do not adhere to this classical interpretation, as we consider Δt as a "duration" rather than an "uncertainty of time measurement itself."

Notably, the higher the input energy, the shorter the time needed to write/erase a bit of information, and vice versa. This new information definition holds significance in our theory, particularly in the context of the energy–time product being a constant, as vividly illustrated in Fig. 6. In essence, the energy required to (irreversibly) erase a spin qubit closely approaching $k_B T$ is theoretically plausible and has been experimentally verified due to this constant product (as denoted by the shaded areas). Our novel information definition based on Heisenberg's principle enables us to ascertain the trade-off between energy and the speed of manipulating a spin qubit.

Also note that the earlier mention of "one bit of information as the smallest error in quantifying the product of the measured energy uncertainty (ΔE) and the measured time duration (Δt)" should not be construed as "the smallest error resulting in one bit when converting the measured analog value to a discrete sequence of digits." Here, one bit represents a quantum in the physical sense—an indivisible unit of a conjugate pair of observables (energy/time) involved in an interaction. According to Heisenberg's TEUR, this unit corresponds to quarter of Planck's constant ($h = 6.626 \times 10^{-34} \mathrm{J} \cdot \mathrm{s}$), which defines the quantum nature of energy and establishes a relationship between the energy of an information carrier and its frequency. Consequently, this new information definition encapsulates the essence of quantum physics, wherein a physical property's magnitude can only assume discrete values composed of integer multiples of one quantum (quarter of Planck's constant). Also important to note is that $4\Delta E \Delta t/h$ is dimensionless, maintaining consistency with the definition of information in terms of units.

This energy–time product stands as the ultimate measure for a unit of information, irrespective of the type of information carrier employed (bead, atom, ion, nanomagnet, giant spin, single spin, or photon) and the encoding/manipulation mechanism utilized—whether rooted in classical physics (electrical, magnetic, optical, chemical, or mechanical) or in quantum physics.

If we employ Landauer's bound at room temperature, the time required to write/erase one bit of information—equivalent, from a physical perspective, to the duration of energy measurement in the TEUR [2, 30], considering energy consumption throughout the write/erase protocol—is given by the relationship:

$$\Delta t = \frac{h}{4\Delta E} = \frac{6.626 \times 10^{-34}\mathrm{J} \cdot \mathrm{s}}{4 \times 3 \times 10^{-21}\mathrm{J}} = 5.5 \times 10^{-14}\mathrm{s} \tag{15}$$

This calculated result aligns reasonably with Brillouin's principle [18].

Historically, multiple definitions of information have existed [16, 38], suggesting that information can be studied from various perspectives, and its definition may not be singular.

The detailed analysis presented above clearly demonstrates that Landauer's bound can be approached quantitatively within the context of a single spin. This approach aligns with the bound's definition as the minimal energy required to erase a unit of information.

In Figs. 5 and 6, we conducted calculations to determine the energy bound during the evolution ($\rightarrow |\uparrow\rangle$) from to $|\uparrow\rangle$. We assumed that once the spin state has successfully tunneled to the other side of the Landauer wall, the information is effectively erased. It is important to emphasize that (reversible) quantum beats such as ($|\uparrow\ \downarrow\rangle \leftrightarrow |\downarrow\ \uparrow\rangle$) or ($|\uparrow\rangle \leftrightarrow |\downarrow\rangle$) via spin tunneling should not be considered as (irreversible) Landauer erasure. In these cases, tunneling can occur in both directions, potentially reintroducing previously seemingly erased information, thus not fulfilling the irreversible nature required for a Landauer erasure. However, (reversible) quantum beats such as ($|\uparrow\ \downarrow\rangle \leftrightarrow |\downarrow\ \uparrow\rangle$) or ($|\uparrow\rangle \leftrightarrow |\downarrow\rangle$) are still governed by Heisenberg's principle [2, 17, 30].

7 Conclusion and discussions

We illustrate an optically manipulated, spin-encoded quantum computer (Fig. 1) that demonstrates an approach to both Landauer's bound and Heisenberg's limit. This research may serve as a significant step toward completing the puzzle of the quantum Landauer erasure and the Heisenberg limit on a single spin qubit.

Present-day few-qubit quantum computers necessitate extensive external cooling systems alongside the actual quantum processors. However, the energy fundamental to quantum computing, as outlined in Eq. 5, presently constitutes a minor portion of the overall energy usage. As quantum technology advances, the cooling energy's scaling might become less than linear concerning the number of qubits, reducing its dominance in the energy consumption [11]. Nonetheless, a spin-encoded quantum computer, while operating at the ultimate energy limit for computation as set by physics (as discussed in Section VI), might exhibit slower performance.

Landauer's bound is widely acknowledged as a fundamental limit in computer science and physics. However, it has faced challenges and debates. Shenker, in 2000, argued that Landauer's dissipation thesis, linking logical irreversibility to dissipation, is incorrect [39]. Bennett, in 2003, proposed an extension of Landauer's principle to counter Shenker's argument and emphasized its pedagogic and explanatory power, despite being a straightforward consequence of the second law of thermodynamics [4]. Norton, in 2005, criticized Bennett's extension, highlighting illicit formations in addressing the no-erasure demon [31]. In 2007, Ladyman et al. defended the qualitative form of Landauer's Principle and clarified its quantitative consequences based on the second law of thermodynamics [28].

Sagawa and Ueda, in 2008, demonstrated that Landauer's principle is a consequence of the second law of thermodynamics with discrete quantum feedback control [36]. Cao and Feito, in 2009, illustrated consequences by computing entropy reduction in feedback-controlled systems [8]. Norton, in 2011, argued that previous proofs selectively neglect thermal fluctuations that may disrupt intended operations [32]. Jordan and Manikandan, in 2019, disagreed with Norton and found the principle easily derivable from basic principles of thermodynamics and statistical physics [21]. Norton countered, asserting that dissipation is unavoidable due to the existence of thermal fluctuations and the high thermodynamic cost of suppressing them [33].

Based on the aforementioned research, we intend to delve deeper into both direct and indirect proofs of Landauer's principle [32] to ascertain if it is merely a direct consequence or a restatement of the second law of thermodynamics (where information erasure leads to reduced entropy). This inquiry is crucial and warranted regardless of whether we maintain Landauer's principle's status as fundamental akin to the second law of thermodynamics. Additionally, we will explore the possibility of realizing an erasable bit without incurring thermodynamic costs by utilizing dissipative dynamics to compress phase space [3, 19, 40].

Despite the many enigmas surrounding Landauer's bound, we might need to consider the possibility of its demise, given the concerns raised by various researchers regarding unsound, incoherent foundations, principles, methods, and/or frameworks present in the above literature. Understanding these fundamental limits is not only of substantial practical significance but also crucial for comprehending the boundaries of what can be achieved with our computing machines [30]. This understanding is tantamount to grasping the limits of the world we inhabit and preparing for transformative shifts, such as energy-efficient quantum computing paradigms nearing both the Landauer bound [41] and the Heisenberg limit.

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Author contributions Frank Wang conceived the research idea, analyzed all the experiments, developed the new theory to explain the experimental verifications, and wrote the manuscript.

Data availability All data generated and analyzed during this study are included in this published article.

References

1. Alexei, O., et al.: Experimental test of Landauer's principle at the sub- kBT level. Japn. J. Appl. Phys. **51**(6), 06FE10 (2012)
2. Anandan, J., Aharonov, Y.: Geometry of quantum evolution. Phys. Rev. Lett. **65**(14), 1697–1700 (1990)
3. Bais, F., Farmer, J.: *The Physics of Information.* 2007–37 ed. s.l.:ITFA (2007)
4. Bennett, C.: Notes on Landauer's principle, reversible computation, and Maxwell's Demon. Stud. Hist. Philos. Mod. Phys. **34**, 501–510 (2003)
5. Bérut, A., et al.: Experimental verification of Landauer's principle linking information and thermodynamics. Nature **483**, 187–189 (2012)
6. Briggs, J.: A derivation of the time-energy uncertainty relation. J. Phys. Conf. Ser. **99**(1), 012002 (2008)
7. Brown, W.: Thermal fluctuations of fine ferromagnetic particles. IEEE Trans. Magnet. **15**(5), 1196–1208 (1979)
8. Cao, F., Feito, M.: Thermodynamics of feedback controlled systems. Phys. Rev. E **79**(4), 041118 (2009)
9. Çetiner, U., Raz, O., Sukharev, S.: *Dissipation During the Gating Cycle of the Bacterial Mechanosensitive Ion Channel Approaches the Landauer's Limit (2020).* https://doi.org/10.1101/2020.06.26.174649. Accessed 29 Feb 2024.
10. Chang, R.: Three-dimensional laser cooling at the doppler limit. Phys. Rev. A (2014). https://doi.org/10.1103/PhysRevA.90.063407
11. Chiribella, G., Yang, Y., Renner, R.: Fundamental energy requirement of reversible quantum operations. Phys. Rev. X (2021). https://doi.org/10.1103/PhysRevX.11.021014
12. Esposito, M.: Landauer principle stands up to quantum test. Physics (2018). https://doi.org/10.1103/Physics.11.49
13. Gaudenzi, R., et al.: Quantum Landauer erasure with a molecular nanomagnet. Nat. Phys. **14**(6), 565–568 (2018)
14. Gaudenzi, R., et al.: Quantum Landauer erasure with amolecular nanomagnet. Nat. Phys. **14**(6), 565568 (2018)
15. Georgescu, I.: 60 years of Landauer's principle. Nat. Rev. Phys. (2021). https://doi.org/10.1038/s42254-021-00400-8
16. Hartley, R.: Transmission of information. Bell Syst. Tech. J. **7**(3), 535–563 (1928)
17. Heisenberg, W.: Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik. Z. Phys. **43**(3–4), 172–198 (1927)
18. Herrera, L.: The mass of a bit of information and the Brillouin's principle. Fluctuat. Noise Lett. **13**(1), 1450002 (2014)
19. Holtzman, R., Arwas, G., Raz, O.: Hamiltonian memory: an erasable classical bit rev. Res. Phys. (2021). https://doi.org/10.1103/PhysRevResearch.3.013232
20. Hong, J., Lambson, B., Dhuey, S., Bokor, J.: Experimental test of Landauer's principle in single-bit operations on nanomagnetic memory bits. Sci. Adv. (2016). https://doi.org/10.1126/sciadv.1501492
21. Jordan, A., Manikandan, S.: *Some Like It Hot. Letter to the Editor in reply to Norton's article* (2019). https://inference-review.com/letter/some-like-it-hot. Accessed 29 Feb 2024
22. Jun, Y., Gavrilov, M., Bechhoefer, J.: High-precision test of Landauer's principle in a feedback trap. Phys. Rev. Lett. **113**(19), 190601 (2014)
23. Koomey, J.: *Koomey's Law Explained: Everything Explained Today* (2020) http://everything.explained.today/Koomey%27s_law/. Accessed 27 Feb 2024
24. Koomey, J., Berard, S., Sanchez, M., Wong, H.: Implications of historical trends in the electrical efficiency of computing. IEEE Ann. Hist. Comput. **33**(3), 46–54 (2010)

25. Kotler, S.: *A Single Ion Perspective on the Environment*. PhD Thesis (the Hebrew University of Jerusalem) (2013)
26. Kotler, S., et al.: Single-ion quantum lock-in amplifier. Nature **473**, 61–65 (2011)
27. Kotler, S., et al.: Measurement of the Magnetic interaction between two bound electrons of two separate ions. Nature **510**(7505), 376–380 (2014)
28. Ladyman, J., Presnell, S., Short, A., Groisman, B.: The connection between logical and thermodynamic irreversibility. Stud. History Philos. Mod. Phy. **38**(1), 58–79 (2007)
29. Landauer, R.: Irreversibility and heat generation in the computing process. IBM J. Res. Develop. **5**, 183–191 (1961)
30. Lloyd, S.: Ultimate physical limits to computation. Nature **406**, 1047–1054 (2000)
31. Norton, J.: Eaters of the Lotus: Landauer's principle and the return of Maxwell's Demon. Stud. History Philos. Mod. Phys. **36**(2), 375–411 (2005)
32. Norton, J.: Waiting for Landauer. Stud. Hist. Philos. Mod. Phys. **42**, 184–198 (2011)
33. Norton, J.: *A Hot Mess: Author's Response* (2019). https://sites.pitt.edu/~jdnorton/papers/Hot_mess_Response.pdf. Accessed 29 Feb 2024
34. Paschotta, R.: *Optical Pumping* (2024). https://doi.org/10.61835/wkk. Accessed 03 Mar 2024
35. Pfeiffer, H.: Influence of Thermal fluctuations on the magnetic properties of particle assemblies. Phys. Stat. Sol. **122**, 371–389 (1990)
36. Sagawa, T., Ueda, M.: Second law of thermodynamics with discrete quantum feedback control. Phys. Rev. Lett. **100**(8), 080403 (2008)
37. Saira, O., et al.: Nonequilibrium thermodynamics of erasure with superconducting flux logic. Phys. Rev. Res. **2**, 013249 (2020)
38. Shannon, C.: A mathematical theory of communication. Bell Syst. Techn. J. **27**, 379–423 (1948)
39. Shenker, O.: *Logic and Entropy* (2000). http://philsci-archive.pitt.edu. Accessed 29 Feb 2024
40. Vaccaro, J.A., Barnett, S.M.: Information erasure without an energy cost. Proc. R. Soc. A: Math. Phys. Eng. Sci. **467**(2130), 1770–1778 (2011)
41. Wang, F.: Near-Landauer-bound quantum computing using single spins. IEEE Trans. Quant. Eng. **4**, 1–13 (2023)
42. Yan, L., et al.: Single-atom demonstration of the quantum landauer principle. Phys. Rev. Lett. **120**, 210601 (2018)