Vol. 29 (1998) ACTA PHYSICA POLONICA B No 5

HARD EXCLUSIVE LEPTOPRODUCTION OF MESONS*

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(Received March 19, 1998)

We present an analysis of leading order QCD amplitudes for hard exclusive leptoproduction of mesons in terms of double/nonforward parton distribution functions. Using a model for quark and gluon double distributions we derive explicit expressions for hard exclusive π^0, η, neutral and charged ρ meson production amplitudes.

PACS numbers: 12.38. Aw, 12.38. Bx

1. Introduction

Our current knowledge of the sub-structure of nucleons is based to a large extent on high-energy scattering experiments which probe its quark and gluon distribution functions. The most prominent processes in this respect are deep-inelastic scattering and Drell–Yan leptoproduction. In addition a large amount of information can be deduced from measurements of electromagnetic and weak form factors.

Recently new observables, namely nonforward parton distribution functions, attract a great amount of interest. Although being discussed already some time ago [1–3], they were introduced in the context of the spin structure of nucleons recently in Ref. [4]. Nonforward parton distributions are a straightforward generalization of ordinary parton distributions [4, 5]. Recall that at the twist-2 level the latter can be represented as normalized Fourier transforms of forward nucleon matrix elements of non-local QCD operators [6]. Nonforward parton distributions are defined by the same non-local QCD operators — just sandwiched between nucleon states with different momenta and eventually spin.

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* Presented at the Cracow Epiphany Conference on Spin Effects in Particle Physics and Tempus Workshop, Cracow, Poland, January 9-11, 1998.

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Nonforward parton distributions are probed in hard processes where the nucleon target recoils elastically. One possible process is deeply-virtual Compton scattering [4]. Another promising class of reactions is hard exclusive leptoproduction of mesons [5, 7, 8]. A factorization theorem, proven recently in Ref. [8], states that at large photon virtualities $Q^2 \gg \Lambda^2_{\rm QCD}$ and moderate momentum transfers $|t| \sim \Lambda^2_{\rm QCD}$, and for incident longitudinally polarized photons the meson production amplitudes can be factorized into a hard, perturbatively calculable part, and matrix elements which contain all information about the long-distance non-perturbative strong interaction dynamics in the produced meson and the nucleon target. The latter are nothing else than the wanted nonforward distribution functions.

In this paper we outline the calculation of hard exclusive leptoproduction amplitudes for a variety of mesons. To obtain insights in the magnitude and behavior of the corresponding cross sections we employ a simple ansatz for double distribution functions which satisfies constraints due to forward parton distributions, form factors and discrete symmetries. As a result, we obtain baseline estimates for the differential cross sections for π^0, η, ρ^0 and $\rho^\pm$ meson production.

2. Double and nonforward distribution functions

In leading twist quark and gluon distribution functions in nucleons are defined by forward matrix elements of non-local light-cone operators sandwiched between nucleon states [6]:

$$\langle N(P,S)|\,\hat{O}(0,z)\,|N(P,S)\rangle_{z^2=0} \sim \int_0^1 dx\,\left[e^{-ix(P\cdot z)}f(x) \pm e^{ix(P\cdot z)}\bar{f}(x)\right]. \quad (1)$$

Here $|N(P,S)\rangle$ represents a nucleon with momentum P and spin S. The operator $\hat{O}(0,z)$ stands for a product of two quark or gluon fields, respectively, separated by a light-like distance $z \sim n$, with $n \cdot a = a^+ = a^0 + a^3$ for an arbitrary vector a. Forward matrix elements as in Eq. (1) are explored in processes where no momentum is transfered to the target, *e.g.* in deep-inelastic scattering.

2.1. Double distribution functions

In exclusive production processes a non-vanishing momentum $r = P - P'$ is transfered to the target. The nonforward matrix elements of quark and gluon light-cone operators can be parametrized through so called "double distribution functions", introduced in Ref. [5]. For the unpolarized quark distribution one has:

$$\begin{aligned}
&\left\langle N(P',S')\right|\bar{\psi}(0)\hat{z}\,[0,z]\,\psi(z)\,|N(P,S)\rangle_{z^2=0}\\
&=\bar{u}(P',S')\,\hat{z}\,u(P,S)\int_0^1 dx\int_0^{\bar{x}} dy\Big[e^{-ix(P\cdot z)-iy(r\cdot z)}F(x,y,t)\\
&-e^{ix(P\cdot z)-i\bar{y}(r\cdot z)}\bar{F}(x,y,t)\,\Big]\\
&+\bar{u}(P',S')\frac{\sigma_{\mu\nu}z^\mu r^\nu}{iM}u(P,S)\int_0^1 dx\int_0^{\bar{x}} dy[\,e^{-ix(P\cdot z)-iy(r\cdot z)}K(x,y,t)\\
&-e^{ix(P\cdot z)-i\bar{y}(r\cdot z)}\bar{K}(x,y,t)\,]\;, \qquad (2)
\end{aligned}$$

where we use the notation $\hat{z}=z_\mu\gamma^\mu$. The operator on the LHS of Eq. (2) is built from quark fields connected by the path-ordered exponential $[0,z]=\mathcal{P}\exp[-igz_\mu\int_0^1 d\lambda A^\mu(z\lambda)]$ which guarantees gauge invariance and reduces to one in axial gauge $n\cdot A=0$.

On the RHS we have the product of the Dirac spinors for the initial and final nucleon. The quark and antiquark double distribution functions F, $\bar{F}$, K and $\bar{K}$ depend on the momentum transfer $t=r^2$, and two light-cone variables x and y ($\bar{x}=1-x$, $\bar{y}=1-y$). The spin-flip distributions K and $\bar{K}$ enters proportional to the momentum transfer r. A comparison of Eqs. (1) and (2) demonstrates that in the forward limit, $r\to 0$, the double distributions reduce to ordinary quark distributions, *e.g.*

$$f(x)=\int_0^{\bar{x}} dy\,F(x,y,t=0). \qquad (3)$$

For the polarized quark and the unpolarized gluon distribution one has [5]:

$$\begin{aligned}
&\left\langle N(P',S')\right|\bar{\psi}(0)\hat{z}\gamma^5\,[0,z]\,\psi(z)\,|N(P,S)\rangle_{z^2=0}\\
&=\bar{u}(P',S')\hat{z}\gamma^5 u(P,S)\int_0^1 dx\int_0^{\bar{x}} dy\Big[e^{-ix(P\cdot z)-iy(r\cdot z)}\Delta F(x,y,t)\\
&+e^{ix(P\cdot z)-i\bar{y}(r\cdot z)}\Delta\bar{F}(x,y,t)\,\Big]+\text{“}K-\text{term''}\,, \qquad (4)\\
&z_\mu z_\nu\left\langle N(P',S')\right|\mathrm{Tr}\,G^{\mu\xi}(0)[\Delta;0]G_\xi{}^{\nu}(z),|N(P,S)\rangle_{z^2=0}\\
&=\bar{u}(P',S')\,\hat{z}\,u(p,s)\frac{\bar{P}\cdot z}{4}\int_0^1 dx\int_0^{\bar{x}} dy[\,e^{-ix(P\cdot z)-iy(r\cdot z)}\\
&+e^{ix(P\cdot z)-i\bar{y}(r\cdot z)}\,]\,G(x,y,t)+\text{“}K-\text{term''}\,, \qquad (5)
\end{aligned}$$

with $\bar{P} = (P' + P)/2$. In Eqs. (4), (5) we have only indicated spin-flip terms which are proportional to the momentum transfer r as in Eq. (2). Of course, relations similar to (3) hold also for the double distributions in Eqs. (4,5).

It is important to realize that the double distributions defined above fulfill a symmetry constraint based on hermiticity. It implies that the double distributions in Eqs. (2), (4), (5) are symmetric with respect to an exchange of

$$y \longleftrightarrow 1 - x - y. \tag{6}$$

This symmetry, besides being important for modeling double distribution functions, is crucial for establishing proper analytical properties of meson production amplitudes.

In the forward limit, $r \to 0$, Eqs. (2), (4), (5) immediately reduce to the definitions of ordinary twist-two parton distributions (1). On the other hand in the limit $z \to 0$ Eqs. (2), (4), (5) define familiar nucleon form factors. One therefore obtains the following sum rules for double parton distributions [3–5]:

$$\int_0^1 dx \int_0^{\bar{x}} dy \left[F(x,y,t) - \bar{F}(x,y,t)\right] = F_1(t), \tag{7}$$

$$\int_0^1 dx \int_0^{\bar{x}} dy \left[K(x,y,t) - \bar{K}(x,y,t)\right] = F_2(t), \tag{8}$$

$$\int_0^1 dx \int_0^{\bar{x}} dy \left[\Delta F(x,y,t) + \Delta\bar{F}(x,y,t)\right] = G_A(t), \tag{9}$$

$$\int_0^1 dx \int_0^{\bar{x}} dy \left[G(x,y,t) + \bar{G}(x,y,t)\right] = \mathcal{G}(t). \tag{10}$$

F_1, F_2 and G_A are the Dirac, Pauli and axial-vector form factors of the nucleon. The gluon form factor $\mathcal{G}(t)$ is experimentally not observable. Its dependence on the momentum transfer has been estimated within QCD sum rules [9], and is determined by a characteristic radius of the order of $0.4 - 0.5$ fm. Similar sum rules hold for the remaining distributions [4].

2.2. Nonforward distribution functions

In the definitions of double distribution functions (2,4,5) the variable y always enters in the combination $x + y\,\zeta \equiv X$ with $\zeta = \frac{r\cdot z}{P\cdot z}$. As a consequence it is possible to define *e.g.*, unpolarized nonforward quark and antiquark distribution functions [5]:

$$F_\zeta(X,t) = \int_0^1 dx \int_0^{\bar{x}} dy\, F(x,y,t)\, \delta\left(X - (x+\zeta y)\right)$$
$$= \Theta(X \geq \zeta) \int_0^{\bar{X}/\bar{\zeta}} dy\, F(X - y\zeta, y, t) + \Theta(X \leq \zeta) \int_0^{X/\zeta} dy\, F(X - y\zeta, y, t),$$
$$\bar{F}_\zeta(X,t) = \int_0^1 dx \int_0^{\bar{x}} dy\, \bar{F}(x,y,t)\, \delta\left(X - (x+\zeta y)\right)$$
$$= \Theta(X \geq \zeta) \int_0^{\bar{X}/\bar{\zeta}} dy \bar{F}(X - y\zeta, y, t) + \Theta(X \leq \zeta) \int_0^{X/\zeta} dy \bar{F}(X - y\zeta, y, t)\,, \quad (11)$$

and similar for the polarized quark and gluon case. Since the scattered nucleon is on its mass shell, *i.e.* $P'^2 = M^2$ with the nucleon mass M, one finds $\zeta \leq 1$. Together with the kinematic constraint $x + y \leq 1$ this gives $0 \leq X \leq 1$. The variable X can be identified with the nucleon light-cone momentum fraction of the parton being removed by the operator $\hat{O}$ from the target, while the light-cone momentum fraction of the returning parton is equal to $X - \zeta$.

In terms of the nonforward distribution functions the light-cone correlators in (2), (4), (5) read:

$$\left\langle N(P',S') \right| \bar{\psi}(0) \hat{z}\, [0,z]\, \psi(z) \left| N(P,S) \right\rangle_{z^2=0}$$
$$= \bar{u}(P',S')\, \hat{z}\, u(P,S) \int_0^1 dX \left[e^{-iX(P\cdot z)} F_\zeta(X,t) - e^{i(X-\zeta)(P\cdot z)} \bar{F}_\zeta(X,t) \right]$$
$$+ \bar{u}(P',S') \frac{\sigma_{\mu\nu} z^\mu r^\nu}{iM} u(P,S) \int_0^1 dX \left[e^{-iX(P\cdot z)} K_\zeta(X,t) \right.$$
$$\left. - \, e^{i(X-\zeta)(P\cdot z)} \bar{K}_\zeta(X,t) \right] \,,$$
$$\left\langle N(P',S') \right| \bar{\psi}(0) \hat{z} \gamma^5\, [0,z]\, \psi(z) \left| N(P,S) \right\rangle_{z^2=0}$$
$$= \bar{u}(P',S') \hat{z} \gamma^5 u(P,S) \int_0^1 dX \left[e^{-iX(P\cdot z)} \Delta F_\zeta(X,t) \right.$$
$$\left. + e^{i(X-\zeta)(P\cdot z)} \Delta \bar{F}_\zeta(X,t)\, \right] + \text{“}K - \text{term''}\,,$$

$$z_\mu z_\nu \left\langle N(P',S')\right| \mathrm{Tr}\, G^{\mu\xi}(0)[\Delta;0]G_\xi{}^\nu(z), |N(P,S)\rangle_{z^2=0}$$
$$= \bar{u}(P',S')\,\hat{z}\,u(p,s)\frac{\bar{P}\cdot z}{4}\int_0^1 dX\,[\,e^{-iX(P\cdot z)}$$
$$+e^{i(X-\zeta)(P\cdot z)}\,]\,G_\zeta(X,t) + \text{“}K - \text{term}'' . \quad (12)$$

Both definitions of nonforward parton distributions are of course equivalent.

3. Model

Nonforward parton distributions have not yet been measured. Therefore one has to rely on models [10] in order to provide estimates for exclusive production cross sections. To guarantee a proper analytic behavior of the involved amplitudes, as given by dispersion relations [11], it is favorable to model double distributions (2), (4), (5) instead of nonforward distributions. We parametrize the former, denoted generically by $F(x,y,t;\mu_0^2)$, such that they fulfill all constraints which we are aware of. As a model ansatz at some low normalization scale μ_0^2 we take [12]:

$$F(x,y,t;\mu_0^2) = \frac{f(x,\mu_0^2)}{(1-x)^3}\,h(x,y)\,f(t), \quad (13)$$

where $f(x,\mu_0^2)$ stands for the corresponding ordinary quark and gluon distribution, respectively. It is clear that one can model in this way only non spin-flip distributions. The "K-terms" in (2), (4), (5) remain unconstrained because the corresponding forward parton distributions are not known. We choose:

$$h(x,y) = 6\,y\,(1-x-y), \quad (14)$$

such that $F(x,y,t;\mu_0^2)$ satisfies the symmetry constraint in Eq. (6). Of course there are many possible choices for $h(x,y)$, *e.g.* $h(x,y) = 12\left[y - \frac{1-x}{2}\right]^2$. The latter however results in a nonforward parton distribution which is not continuously differentiable at $X = \zeta$. Although we are not aware of any principle which forbids such a behavior, we use in the following the parametrization from Eq. (14) leading to nonforward distributions smooth in X.

The form factor $f(t)$ in Eq. (13) is responsible for the t-dependence of double distributions. Motivated by the relationship between double distributions and nucleon form factors in Eqs. (7 – 10) we assume that $f(t)$ is given by a dipole form:

$$f(t) = \left(\frac{1}{1-t/\Lambda^2}\right)^2 . \quad (15)$$

For quark distributions we take Λ from fits to the nucleon vector and axial form factors [13].

4. Exclusive meson production amplitudes

A solid QCD description of hard exclusive meson production is based on the factorization of long- and short-distance dynamics which has been proven in Ref. [8]. The main result is that the amplitudes for the production of mesons from longitudinally polarized photons can be split into three parts: the perturbatively calculable hard photon-parton interaction arises from short distances, while the long-distance dynamics can be factorized in terms of nonperturbative meson distribution amplitudes and nucleon nonforward parton distributions. In the following we discuss production amplitudes for a variety of neutral mesons in terms of these building blocks, staying at leading order in the strong coupling constant α_s.

The kinematics is defined as follows: the initial and scattered nucleon carries a momentum P and $P' = P - r$, respectively, while $t = r^2$ stands for the squared momentum transfer. The momentum of the exchanged virtual photon is denoted as usual by q with $Q^2 = -q^2$. Then $q' = q + r$ is the momentum of the produced meson. Calculating the hard subprocess to leading twist accuracy allows to neglect the momentum transfer t, the invariant mass of the nucleon target $P^2 = P'^2$, and the mass of the produced vector meson ${q'}^2$, as compared with the virtuality of the photon Q^2. Then the Bjorken scaling variable $x_{\rm Bj} = Q^2/2P\cdot q$ coincides with the longitudinal momentum transfer, i.e. $\zeta \equiv r\cdot n/P\cdot n = x_{\rm Bj}$.

4.1. Neutral vector mesons

To leading twist accuracy there are two amplitudes for longitudinally and transversely polarized vector mesons (see [14] for a recent discussion). In the longitudinal case one has:

$$\langle M(q')|\,\bar\psi(x)\hat P\psi(y)\,|0\rangle = q'\cdot P\, f_{V_L}\int_0^1 d\tau\, \Phi_{V_L}(\tau)\, e^{iq'\cdot(\tau x+\bar\tau y)}, \qquad (16)$$

where f_{V_L} and Φ_{V_L} denote the corresponding decay constant and distribution amplitude. According to our previous discussion this implies that the quark contribution to the production of longitudinally polarized vector mesons is determined by the nucleon matrix element $\langle N(P')|\bar\psi(z)\hat q'\psi(y)|N(P)\rangle$. As a consequence the nonforward quark distributions F and K, defined in Eq. (2), enter. The corresponding amplitude reads:

$$\mathcal{A}^q_{V_L} \;=\; \pi\,\alpha_s\,\frac{C_F}{N_c}\frac{1}{Q}\frac{\bar N(P',S')\hat q' N(P,S)}{P\cdot q'} f_{V_L}\int_0^1 d\tau\frac{\Phi_{V_L}(\tau)}{\tau\bar\tau}$$

$$\times \int_0^1 dX \left[F_\zeta(X,t) + \bar{F}_\zeta(X,t)\right] \left(\frac{1}{X - i\epsilon} + \frac{1}{X - \zeta + i\epsilon}\right)$$

$$+ \pi\, \alpha_s \frac{C_F}{N_c} \frac{1}{Q} \frac{\bar{N}(P',S')(\hat{q}'\hat{r} - \hat{r}\hat{q}')N(P,S)}{2\, M\, P \cdot q'} f_{V_L} \int_0^1 d\tau \frac{\Phi_{V_L}(\tau)}{\tau\bar{\tau}}$$

$$\times \int_0^1 dX \left[K_\zeta(X,t) + \bar{K}_\zeta(X,t)\right] \left(\frac{1}{X - i\epsilon} + \frac{1}{X - \zeta + i\epsilon}\right). \quad (17)$$

Since the virtual photon and the final state vector meson carry the same C-parity, vector meson production can take place also in the gluon background field. Its contribution is [5, 7]:

$$\mathcal{A}^g_{V_L} = 2\pi\, \alpha_s \frac{1}{N_c} \frac{1}{Q} \frac{\bar{N}(P',S')\hat{q}' N(P,S)}{P \cdot q'} \left(1 - \frac{\zeta}{2}\right) f_{V_L} \int_0^1 d\tau \frac{\Phi_{V_L}(\tau)}{\tau\bar{\tau}}$$

$$\times \int_0^1 dX \frac{G_\zeta(X,t)}{(X - i\epsilon)(X - \zeta + i\epsilon)} + \text{"}K - \text{term"}, \quad (18)$$

with G being the nonforward gluon distribution (5) of the nucleon.

Transversely polarized vector mesons are described by the amplitude [14]:

$$\langle M(q')|\, \bar{\psi}(x)\sigma_{\mu\nu}\psi(y)\, |0\rangle = -i(\epsilon_\mu q'_\nu - \epsilon_\nu q'_\mu) f_{V_T} \int_0^1 d\tau\, \Phi_{V_T}(\tau)\, e^{iq'\cdot(\tau x + \bar{\tau} y)}. \quad (19)$$

Their production from longitudinally polarized photons involves chiral-odd nucleon nonforward parton distributions. Thus, in general, only the nonforward quark transversity distribution contributes. An explicit calculation shows, however, that in leading order α_s the twist-2 contribution vanishes, and one has to go either to higher orders, or to higher-twists to obtain a non-zero result.

4.2. Charged vector mesons

In the following we will need a generalization of the usual, flavor-diagonal, double distributions (2) to the flavor non-diagonal case:

$$\left\langle p(P',S')\right|\, \bar{\psi}_u(0)\hat{z}\, [0,z]\, \psi_d(z)\, |n(P,S)\rangle_{z^2=0}$$

$$= \bar{u}(P',S')\, \hat{z}\, u(P,S) \int_0^1 dx \int_0^{\bar{x}} dy \Big[e^{-ix(P\cdot z) - iy(r\cdot z)} F^{ud}(x,y,t)$$

$$-e^{ix(P\cdot z)-i\bar{y}(r\cdot z)}\bar{F}^{ud}(x,y,t)\ ]\ \text{“}K-\text{term}'' \tag{20}$$

and

$$\begin{aligned}&\langle n(P',S')|\,\bar{\psi}_u(0)\hat{z}\,[0,z]\,\psi_d(z)\,|p(P,S)\rangle_{z^2=0}\\&=\bar{u}(P',S')\,\hat{z}\,u(P,S)\int_0^1 dx\int_0^{\bar{x}} dy\Big[e^{-ix(P\cdot z)-iy(r\cdot z)}F^{du}(x,y,t)\\&-e^{ix(P\cdot z)-i\bar{y}(r\cdot z)}\bar{F}^{du}(x,y,t)\ ]+\text{“}K-\text{term}''\,.\end{aligned} \tag{21}$$

In above equations ψ_u and ψ_d represent u- and d-quark field, and $|p(P,S)\rangle$ and $|n(P,S)\rangle$ denote the proton and neutron states, respectively.

We will show now that using the isospin symmetry one can relate the flavor non-diagonal double distributions to the flavor-diagonal ones. Because the argument z of the non-local twist-2 string operators is a light-like vector, it is convenient to introduce conserved isospin charges defined on a light-like hypersurface which contains z:

$$\hat{\tau}^i=\frac{1}{2}\int d^2x^{\perp}dx^{-}\bar{\Psi}(x^+=0,\overline{x})\gamma^+\tau^i\Psi(x^+=0,\overline{x}), \tag{22}$$

where $\overline{x}=(x^-,x^{\perp})$, $\Psi=(\psi_u,\psi_d)^{\mathrm{T}}$, and τ^i is a Pauli matrix. As the definition of $\hat{\tau}^i$ employs only the "good" component of the spinor fields, using canonical anticommutation relations on the light-cone it is easy to show that

$$[\hat{\tau}^i,\hat{\tau}^j]=i\epsilon^{ijk}\hat{\tau}^k \tag{23}$$

and therefore that charges (22) are generators of the isospin symmetry. Hence, in the isospin-symmetric world proton and neutron states can be related to each other by the usual ladder operators $\hat{\tau}^+$ and $\hat{\tau}^-$:

$$\begin{aligned}|p(P,S)\rangle &= \hat{\tau}^+\,|n(P,S)\rangle\ ,\\ |n(P,S)\rangle &= \hat{\tau}^-\,|p(P,S)\rangle\ ,\end{aligned} \tag{24}$$

where $\hat{\tau}^{\pm}=\hat{\tau}^x\pm i\hat{\tau}^y$, and

$$\begin{aligned}\hat{\tau}^+\,|p(P,S)\rangle &= 0\,,\\ \hat{\tau}^-\,|n(P,S)\rangle &= 0\,.\end{aligned} \tag{25}$$

Now, consider the matrix elements (20) and (21) which define the flavor non-diagonal double quark distributions, respectively. Introducing the notation

$$\hat{O}^{q\,q'}(z)=\bar{\psi}_q(0)\hat{z}\,[0,z]\,\psi_{q'}(z)_{z^2=0} \tag{26}$$

with q and q' denoting quark flavors, and using (24) and (25) one readily obtains

$$\begin{aligned}&\left\langle p(P',S')\right|\hat{O}^{ud}(z)\left|n(P,S)\right\rangle = \left\langle p(P',S')\right|\hat{O}^{ud}(z)\,\tau^{-}\left|p(P,S)\right\rangle\\&= \left\langle p(P',S')\right|[\hat{O}^{ud}(z),\hat{\tau}^{-}]\left|p(P,S)\right\rangle\\&= \left\langle p(P',S')\right|\hat{O}^{uu}(z)\left|p(P,S)\right\rangle - \left\langle p(P',S')\right|\hat{O}^{dd}(z)\left|p(P,S)\right\rangle \qquad (27)\end{aligned}$$

and, similarly

$$\begin{aligned}&\left\langle p(P',S')\right|\hat{O}^{ud}(z)\left|n(P,S)\right\rangle\\&= \left\langle n(P',S')\right|\hat{O}^{dd}(z)\left|n(P,S)\right\rangle - \left\langle n(P',S')\right|\hat{O}^{uu}(z)\left|n(P,S)\right\rangle\\&\left\langle n(P',S')\right|\hat{O}^{du}(z)\left|p(P,S)\right\rangle\\&= \left\langle p(P',S')\right|\hat{O}^{uu}(z)\left|p(P,S)\right\rangle - \left\langle p(P',S')\right|\hat{O}^{dd}(z)\left|p(P,S)\right\rangle\\&\left\langle n(P',S')\right|\hat{O}^{du}(z)\left|p(P,S)\right\rangle\\&= \left\langle n(P',S')\right|\hat{O}^{dd}(z)\left|n(P,S)\right\rangle - \left\langle n(P',S')\right|\hat{O}^{uu}(z)\left|n(P,S)\right\rangle\,. \qquad (28)\end{aligned}$$

Using results from the previous section it is easy to derive the corresponding amplitudes for charged ρ production. The amplitude for transversally polarized ρ production vanishes in the leading α_s order. In the case of longitudinal polarization the transition of the quark-antiquark pair into the final meson is parametrized by the twist-2 matrix element

$$\langle\rho^{\pm}(q')|\,\bar{\psi}(x)\hat{n}\psi(y)\,|0\rangle = \pm q'\cdot n\, f_\rho^L\int_0^1 d\tau\,\Phi_\rho^L(\tau)\,e^{iq'\cdot(\tau x+\bar{\tau}y)}, \qquad (29)$$

where f_ρ^L and Φ_ρ^L denote the ρ meson decay constant and the distribution amplitude, respectively.

Denoting by e_u and e_d u- and d-quark charges one can obtain the final result for the ρ^+ production amplitude:

$$\begin{aligned}\mathcal{A}^+ =&\ \pi\,\alpha_s\,C_F\frac{1}{Q}\frac{\bar{N}(P',S')\hat{n}N(P,S)}{\bar{P}\cdot n}f_\rho^L\int_0^1 d\tau\frac{\Phi_\rho^L(\tau)}{\tau\bar{\tau}}\\&\times\int_0^1 dx\int_0^{\bar{x}} dy\,\Big[(e_u\,F^{ud}+e_d\,\bar{F}^{ud})\frac{\bar{\omega}}{x+2y+x\bar{\omega}-i\epsilon}\\&-(e_d\,F^{ud}+e_u\,\bar{F}^{ud})\frac{\bar{\omega}}{x+2y-x\bar{\omega}-i\epsilon}\,\Big] + \text{“}K-\text{terms''}\,. \qquad (30)\end{aligned}$$

To obtain the symmetric in $\bar{\omega}$ form of this equation we have made use of the symmetry (6). The amplitude for ρ^- production can be obtained by exchanging everywhere $u \leftrightarrow d$ and applying an overall minus sign, compare with Eq. (29). Because of the charge transfer in the t-channel, gluons do not contribute to the hard exclusive electroproduction of charged mesons.

Applying now the isospin symmetry relations (27) and (28) one can rewrite amplitudes for $\rho^\pm$ production in a form

$$\begin{aligned}
\mathcal{A}^\pm \sim & \int_0^1 dx \int_0^{\bar{x}} dy \left[(F^u - \bar{F}^u) - (F^d - \bar{F}^d) \right] \\
\times & \left(\frac{\bar{\omega}}{x + 2y + x\bar{\omega} - i\epsilon} - \frac{\bar{\omega}}{x + 2y - x\bar{\omega} - i\epsilon} \right) \\
\pm & \frac{1}{3} \int_0^1 dx \int_0^{\bar{x}} dy \left[(F^u + \bar{F}^u) - (F^d + \bar{F}^d) \right] \\
\times & \left(\frac{\bar{\omega}}{x + 2y + x\bar{\omega} - i\epsilon} - \frac{\bar{\omega}}{x + 2y - x\bar{\omega} - i\epsilon} \right) + \text{“}K - \text{terms''}, \qquad (31)
\end{aligned}$$

where we have inserted $e_u + e_d = \frac{1}{3}$, and $e_u - e_d = 1$. The second term in the above equation can be related to the isovector part of the ρ^0 amplitude. Explicitely, one obtains:

$$\mathcal{A}^{(I=1)}_{\rho^0} = \frac{1}{2\sqrt{2}} \left(\mathcal{A}_{\rho^+} - \mathcal{A}_{\rho^-} \right). \qquad (32)$$

4.3. Pseudoscalar mesons

In the case of pseudoscalar mesons one ends up with the amplitude:

$$\langle M(q')| \, \bar{\psi}(x) \gamma_5 \hat{P} \psi(y) \, |0\rangle = i \, q' \cdot P \, f_{PS} \int_0^1 d\tau \, \Phi_{PS}(\tau) \, e^{iq' \cdot (\tau x + \bar{\tau} y)}, \qquad (33)$$

with the meson decay constant f_{PS}, and the distribution amplitude Φ_{PS}. According to the above discussion the production process is sensitive to the nonforward nucleon matrix element $\langle N(P')|\bar{\psi}(z)\gamma_5 \hat{q}' \psi(y)|N(P)\rangle$. Thus the nonforward polarized quark distributions ΔF and ΔK from Eq. (4) enter. Collecting all leading order contributions gives for the pseudoscalar meson production amplitude:

$$\mathcal{A}_{\mathcal{PS}} = i \, \pi \, \alpha_s \, \frac{C_F}{N_c} \frac{1}{Q} \frac{\bar{N}(P', S') \gamma_5 \hat{q}' N(P, S)}{P \cdot q'} f_{PS} \int_0^1 d\tau \frac{\Phi_{PS}(\tau)}{\tau \bar{\tau}}$$

$$\times \int_0^1 dX \left[\Delta F_\zeta(X,t) - \Delta \bar{F}_\zeta(X,t)\right] \left(\frac{1}{X - i\epsilon} + \frac{1}{X - \zeta + i\epsilon}\right) + \text{“}K - \text{term}'', \tag{34}$$

Due to C-parity conservation pseudoscalar meson production receives contributions only from C-odd nonforward distribution functions. As an immediate consequence two gluon exchange is impossible. At least three gluons have to be exchanged, which in the language of twist expansion corresponds to higher-twist.

5. Results

Using the derived production amplitudes together with the model distributions from Section 3 allows to calculate the production of pseudoscalar and vector mesons. In both cases we use the asymptotic meson distribution amplitude [15, 16]:

$$\Phi_{PS}(\tau) = \Phi_{V_L}(\tau) = 6\tau(1-\tau). \tag{35}$$

In numerical calculations we have neglected the practically unconstrained "K-terms". As their contribution enters proportional to r it is bound to be small at small momentum transfers.

5.1. π^0 and η production

To obtain predictions for a specific process the generic amplitudes from Section 4 have to be furnished with flavor charges. Using standard SU(3) wave functions for the pseudoscalar meson octet implies for π^0 and η production[1] an replacement of the nonforward distributions in Eq. (34) through distributions with specific flavor as listed in Table I. For π^0 production we use the standard value for the decay constant $f_\pi = 133$ MeV. In Fig. 1 we present the corresponding differential production cross section for a proton target taken at $t = t_{\rm min} = -x_{\rm Bj}^2 M^2/(1 - x_{\rm Bj})$. We restrict ourselves to the region $|t_{min}| < 1$ GeV^2, which implies $x_{\rm Bj} < 0.6$.

One finds that, up to logarithmic corrections, the production cross section drops as $1/Q^6$. Furthermore, at small values of $x_{\rm Bj}$ the calculated cross section drops as $x_{\rm Bj}^{2\lambda}$ with $\lambda \approx 0.6$. This decrease is controlled by the small-$x_{\rm Bj}$ behavior of the polarized valence quark distributions which enter in Eq. (13). For the used parametrizations from Ref. [17] one has indeed

[1] We restrict our considerations to the pure octet state η_8, neglecting mixing with the singlet state η_0.

TABLE I

Flavor structure for π^0 and η production. Analogous relations hold for the "$K-$ terms".

	$\Delta F_\zeta(X,t) - \Delta\bar{F}_\zeta(X,t)$
π^0	$\sqrt{2}\left[\frac{1}{3}(\Delta u_\zeta - \Delta\bar{u}_\zeta) + \frac{1}{6}(\Delta d_\zeta - \Delta\bar{d}_\zeta)\right]$
η	$\sqrt{6}\left[\frac{1}{9}(\Delta u_\zeta - \Delta\bar{u}_\zeta) - \frac{1}{18}(\Delta d_\zeta - \Delta\bar{d}_\zeta) + \frac{2}{9}(\Delta s_\zeta - \Delta\bar{s}_\zeta)\right]$

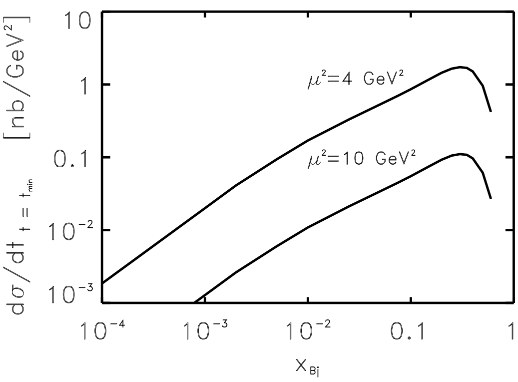

Fig. 1. Differential cross section for exclusive π^0 production from protons at $t = t_{min}$.

$x_{\rm Bj}\Delta u_v(x_{\rm Bj}) \sim x_{\rm Bj}\Delta d_v(x_{\rm Bj}) \sim x_{\rm Bj}^{\lambda}$. The decrease of the production cross section at large $x_{\rm Bj}$ is due to the form factor in Eq. (13). It is important to note that this behavior should be quite general and largely independent of a specific model for double distributions. The reason is that the rise of the production cross section with increasing $x_{\rm Bj}$ is brought to an end through the decrease of the involved nonforward or double distributions at large momentum transfers $|t|$. Since the latter is determined by a typical nucleon scale, $\Lambda \sim 1$ GeV, the maximum of the pseudoscalar meson production cross section at $t = t_{min}$ should occur when t_{min} starts to become sizeable, say $-t_{min}/\Lambda^2 \sim 0.2$. This implies $x_{\rm Bj} \approx 0.3$ in accordance with our result.

Our prediction for the η production cross section turns out to be approximately a factor 2/3 smaller than for π^0, but of similar shape. This is due to the comparable shape of the polarized u and d valence quark distributions from Ref. [17] which enter in Eq. (13).

5.2. Vector meson production

For ρ^0, Φ and ω production the nonforward distributions in Eq. (17), (18) have to be modified according to Table II. In the kinematic domain

TABLE II

Flavor structure for ρ^0, ω and Φ meson production

	$F_\zeta(X,t) + \bar{F}_\zeta(X,t)$	$G_\zeta(X,t)$
ρ^0	$\sqrt{2}\left[\frac{1}{3}(u_\zeta + \bar{u}_\zeta) + \frac{1}{6}(d_\zeta + \bar{d}_\zeta)\right]$	$\frac{1}{\sqrt{2}} G_\zeta$
ω	$\sqrt{2}\left[\frac{1}{3}(u_\zeta + \bar{u}_\zeta) - \frac{1}{6}(d_\zeta + \bar{d}_\zeta)\right]$	$\frac{1}{3\sqrt{2}} G_\zeta$
Φ	$-\frac{1}{3}(s_\zeta + \bar{s}_\zeta)$	$-\frac{1}{3} G_\zeta$

of small $x_{\rm Bj} < 0.01$ the quark part of the production amplitude (17) is negligible. Then the well known SU(3) relation for the cross section ratios $\sigma(\rho) : \sigma(\omega) : \sigma(\Phi) = 9 : 1 : 2$ follows immediately from Table II.

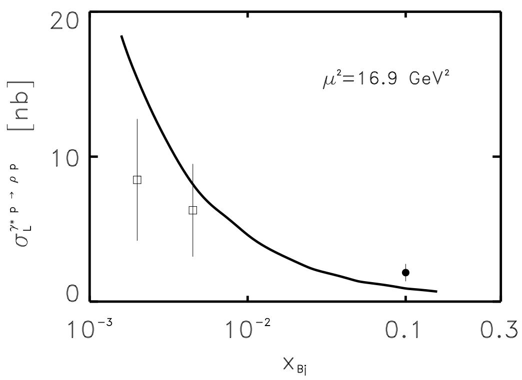

Fig. 2. Cross section for ρ^0 production from a proton through the interaction of longitudinally polarized photons. The data are taken from ZEUS [18] (open squares) and NMC [20] (filled circles).

In Fig. 2 we present the result for ρ^0 production from a proton in the kinematic domain of HERA [18] as obtained from the amplitudes in Eqs. (17), (18), combined with the double distributions from Section 3. For the ρ meson decay constant $f_\rho = 216$ MeV has been used. After multiplying with a suppression factor $T(Q^2 = 16.9\,\mathrm{GeV}^2) \approx 0.1$ from [19] qualitative agreement can be achieved.

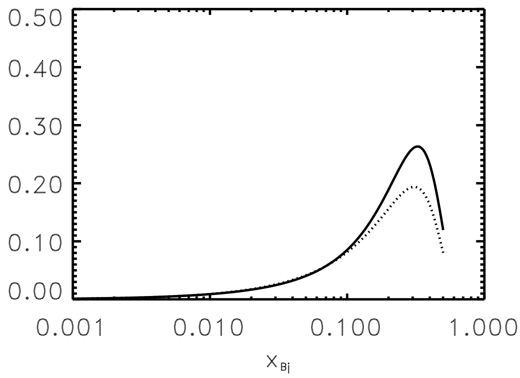

Fig. 3. Cross section for ρ^+ and ρ^- production from a nucleon through the interaction of longitudinally polarized photons.

In Fig. 3 we present the result for ρ^+ and ρ^- production from a proton or neutron respectively in a kinematic domain typical for current and future experiments. At small $x_{\rm Bj}$ the total cross section $\sigma_L^{\gamma^* N \to \rho^\pm N}$ vanishes power-like according to Regge theory. At medium $x_{\rm Bj} \sim 0.3$ we observe a maximum of the cross section which is of the order $\mathcal{O}(0.1\ {\rm nb})$ at $Q^2 = 10\ {\rm GeV}^2$ followed by a fall off at large $x_{\rm Bj}$ due to form factor suppression. Note, that due to isospin violation in electromagnetic processes the production cross sections for ρ^+ and ρ^- mesons differ.

6. Summary

Ordinary parton distributions accessible *e.g.*, in deep-inelastic scattering measure the nucleon response to a process where one parton is removed and subsequently inserted back into the target along a light-like distance, without changing its longitudinal momentum. Generalized parton distributions, so-called nonforward distributions, can be studied in deeply virtual Compton scattering and hard exclusive leptoproduction of mesons. They describe a situation where the removed parton changes its longitudinal momentum before returning to the nucleon. Furthermore, hard exclusive charged meson production provides possibilities to investigate new processes where the removed quark carries different flavor than the returning one.

In this paper we have derived the amplitudes for the hard exclusive production of neutral and charged mesons from longitudinally polarized photons. After suggesting a phenomenological model for double or nonforward distribution functions which obey appropriate symmetries, we have presented results for exclusive π^0, η, ρ^0 and $\rho^\pm$ production.

In the pseudoscalar case nonforward polarized valence quark distributions enter. Several features of the presented results should be independent of our specific model for nonforward distribution functions: at small x_{Bj} pseudoscalar meson production cross sections drop for decreasing x_{Bj}. This is related to a similar behavior of the corresponding forward distributions. Furthermore, the production cross sections peak at moderate x_{Bj}. This is due to the dependence of the involved nonforward distributions on the momentum transfer which should be controlled by a typical nucleon scale.

We also present results for ρ^0 production in the kinematic domain of recent HERA measurements. Here the production process is dominated by contributions from the nonforward gluon distribution. On the other hand, hard exclusive charged meson production provides possibilities to investigate new processes where the removed quark has a different flavor than the returning one.

We gratefully acknowledge discussions with A. Radyushkin. This work was supported in part by BMBF.

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