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A new string model for hadron and muon production in extensive air showers

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Quark-gluon string fragmentation is usually seen as a scalable process independent of the type of colliding system. Existing experimental data from both collider-based and cosmic-ray experiments suggest the need to break this universality. While some improvements can be achieved by including the collective effects, a free string fragmentation is still of interest. In this paper I present the first results of multi-particle production simulations using the ATROPOS quark-gluon string fragmentation model. ATROPOS is founded on the strict first principles of the Nambu-Goto theory with relativistic string being defined by a mathematically derived method based on the Virasoro conditions. To differentiate between various colliding systems, ATROPOS allows consideration of the angular momentum of quark-gluon string that affects particle production. The natural introduction of non-universality in hadronization process is demonstrated. A preliminary estimate of the impact on hadron (and muon) production in extensive air showers is presented.

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1. Introduction

Up to this date, the string model of hadron production is the most consistent and reliable method to describe the hadronization process and is used in numerous hadronic interaction models [1–5]. Cosmic ray simulations require extensive use of hadronic models in the soft QCD regime and, thus, strongly rely on the hadronization model implemented. Some cosmic ray data are challenging to describe even for the state-of-the-art models (e.g., the “muon puzzle” [6, 7]). Solving these issues requires changing the mechanism of hadron production on the one hand and preserving the well-established description of, e.g., $e^+e^- \longrightarrow q\bar{q}$ processes on the other.

A common characteristic of existing string models is their universality in relation to the properties of the colliding system. In order to solve this issue, different ways of adding collective effects were proposed [8]. An interesting alternative way of breaking the universality of string fragmentation is to implement an angular momentum conservation mechanism. In hadronic collisions an impact parameter of the interacting partons may produce a huge angular momentum between the string end-points. This property alone can influence particle production during string fragmentation [9].

To develop a model where the angular momentum of the confined system of partons could be properly calculated and considered during the fragmentation process, an advanced mathematical apparatus is needed. It seems natural to apply the Nambu-Goto string theory for that. Moreover, it is already used in some models, though not to such a degree.

In this paper I will present the basic concepts and first results of particle production simulation in the quark-gluon string fragmentation model ATROPOS. I will demonstrate a possible way of properly defining the rotating relativistic string using the Virasoro conditions. It introduces a new parameter, an eigenvalue of the string oscillation mode, which can be tuned to reproduce the data on the collisions of different systems.

The string fragmentation mechanism defined in ATROPOS is quite unique, too. A brief description is given. Some results that may be important for extensive air shower physics are discussed in the last Section.

2. Basic principles of the classical Nambu-Goto relativistic string theory

The Nambu-Goto action for a relativistic string has the form [10]:

$$S_{\text{string}} = -\kappa \int_{\sigma_1}^{\sigma_2} d\sigma \int_{\tau_1(\sigma)}^{\tau_2(\sigma)} d\tau \sqrt{(\dot{x}x')^2 - \dot{x}^2 x'^2}, \tag{1}$$

where

$$\dot{x}_\mu(\tau,\sigma) \equiv \frac{\partial x_\mu(\tau,\sigma)}{\partial \tau}, \quad x'_\mu(\tau,\sigma) \equiv \frac{\partial x_\mu(\tau,\sigma)}{\partial \sigma}.$$

Here, $x_\mu(\tau,\sigma)$ ($\mu = 0, \ldots, 3$) is a 4-dimensional vector of string coordinates, τ is an evolutionary parameter of the theory, σ is a parameter that numerates the points of the string, κ is a dimensional parameter of the theory usually identified with the string tension, $[\kappa] = \text{GeV}^2$ (the system of units where $\hbar = c = 1$ is used).

By the standard procedure of equating the variation of the action (1) to zero, one can obtain the equations of motion and boundary conditions. However, in their original form, the equations of motion are too complex to attempt to solve them directly. To simplify them, a specific parameterization of τ, σ (an orthonormal gauge) is chosen:

$$\dot{x}^2 + x'^2 = 0, \quad \dot{x}x' = 0. \tag{2}$$

In the orthonormal gauge one can obtain the following equations of motion and boundary conditions:

$$\ddot{x}_\mu - x''_\mu = 0, \quad x'_\mu(\tau, 0) = x'_\mu(\tau, \pi) = 0. \tag{3}$$

The solution to Eqs. (3) has the following form:

$$x_\mu(\tau, \sigma) = Q_\mu + \frac{P_\mu \tau}{\kappa\pi} + \frac{i}{\sqrt{\kappa\pi}} \sum_{n\neq 0} e^{-in\tau} \frac{\alpha_{n\mu}}{n} \cos(n\sigma), \quad \mu = 0, \ldots, 3, \tag{4}$$

where

$$P_\mu \equiv \kappa \int_0^\pi \dot{x}_\mu(0, \sigma) d\sigma = \kappa \int_0^\pi v_\mu(\sigma) d\sigma, \quad Q_\mu \equiv \frac{1}{\pi} \int_0^\pi x_\mu(0, \sigma) d\sigma = \frac{1}{\pi} \int_0^\pi \rho_\mu(\sigma) d\sigma$$

is the string total momentum 4-vector and the coordinates of the center of mass of the string at $\tau = 0$. The Fourier amplitudes $\alpha_{n\mu}$ are calculated in the following way:

$$\alpha_{n\mu} = \sqrt{\frac{\kappa}{\pi}} \int_0^\pi \left[v_\mu(\sigma) - in\rho_\mu(\sigma)\right] \cos(n\sigma) d\sigma, \quad n \neq 0, \quad \alpha_{0\mu} = \frac{P_\mu}{\sqrt{\kappa\pi}}. \tag{5}$$

The functions $\rho_\mu(\sigma)$, $v_\mu(\sigma)$ are called the initial conditions of the problem for string motion and define the shape and velocity of the string at the initial moment in time.

The function (4) can indeed describe the motion of the relativistic string only when it satisfies the orthonormal gauge. The conditions that arise after the substitution of formula (4) in Eq. (2) can be expressed in terms of Fourier amplitudes:

$$\sum_{m=-\infty}^{+\infty} \alpha_{n-m} \alpha_m = 0, \quad n = 0, \pm 1, \pm 2, \ldots \tag{6}$$

The relations (6) are called Virasoro conditions for the relativistic string. They impose restrictions on the functions of the initial data $\rho_\mu(\sigma)$, $v_\mu(\sigma)$, meaning that their choice is not arbitrary.

3. Quark-gluon string in the ATROPOS model

3.1 A “folded” string with spin

The initial conditions for the relativistic string in ATROPOS are set (in the center-of-mass frame) in the following way:

$$v_\mu(\sigma) = (\kappa\pi)^{-1} M \left[\delta_{0\mu} + \xi \delta_{3\mu} \cos(\nu\sigma)\right], \quad \rho_\mu(\sigma) = (\nu\kappa\pi)^{-1} M \delta_{1_\mu} \cos(\nu\sigma). \tag{7}$$

Here, M is the mass of the string, ξ is the rotation signature of the string, and $\delta_{\mu\nu}$ is the Kronecker delta. The parameter ν defines the order of the eigenharmonic of the string oscillation, $\nu = 1, 2, \ldots$.

It is straightforward to check that the initial data (7) indeed satisfy the Virasoro conditions (6). It is remarkable that no other solutions have yet been found for a string with non-zero invariant mass (which is needed for hadron production) [11]. To describe the arbitrary motion of the string, one has to rotate the plane of the string and perform the Lorentz boost.

The eigenvalue ν of the string oscillation is an essential parameter of the ATROPOS model. The classical value of the spin of the string (7) is

$$J = \frac{M^2}{2\kappa\pi\nu}. \tag{8}$$

For a given value of the string mass, it is possible to select its spin by adjusting the value of ν. This procedure may be seen as "folding" of the string with $\nu = 1$ around the "joint" points $(\nu - 1)$ times (see Fig. 1).

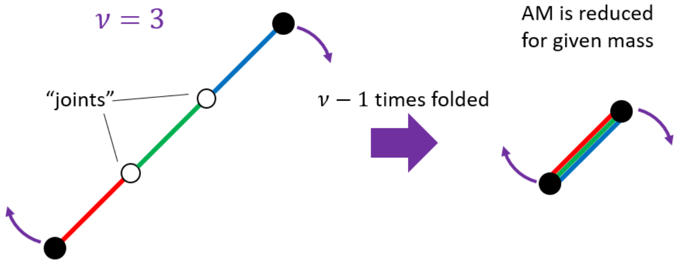


Figure 1: The "folding" of the relativistic string in ATROPOS model. The colours of the string segments are chosen arbitrarily for demonstration purpose only.

The position of the "joints" is described by the formula

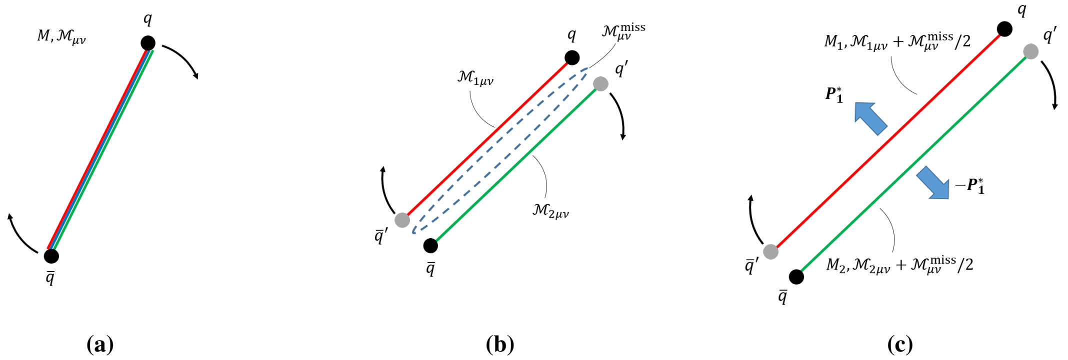
$$\sigma_r^{\text{joint}} = \frac{\pi r}{\nu}, \quad r = 1, \ldots, \nu - 1. \tag{9}$$

3.2 String fragmentation and transition to hadrons

The hadronization is described as a consecutive process of quark-gluon string fragmentation via parton-antiparton pair production at the break point with light string states associated with final-state hadrons. The daughter strings (fragments) must also satisfy the Virasoro conditions. This restriction yields a remarkable consequence: the string can only break in a *countable* set of points [11]. It turns out that these are the same "joints" (9) the string is "folded" around.

However, as one can see, an arbitrary segment between any of the two "joint" points has zero momentum in the rest frame of the string. Fragmenting the string by breaking it up in a single point would result in daughters production at rest. This contradicts the picture of, say, e^+e^- annihilation where particles are produced with significant momentum, while the initial string state can be assumed to be at rest. To overcome this issue, a new mechanism of string fragmentation was proposed for ATROPOS. The string is fragmented by deleting a chunk of the string between *two* break points, allowing the release of energy that turns into a repulsive momentum between the fragments of the string, Fig. 2.

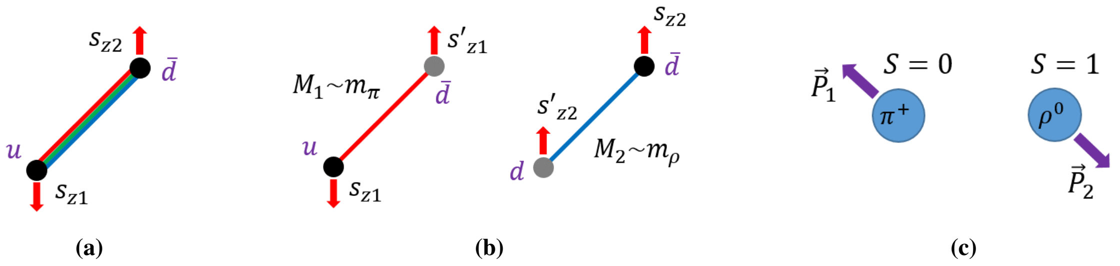
To conserve the total angular momentum of the system, the missing part of the angular momentum tensor $\mathcal{M}_{\mu\nu}^{\text{miss}}$, that was carried by the deleted chunk, is redistributed between the



(a) **(b)** **(c)**

Figure 2: A schematic view of the string fragmentation. A "folded" string rotates as a rigid rod (Fig. 2a). Then the string fragments by having the inner section disappear (Fig. 2b). The missing angular momentum is redistributed between the daughters, their mass is increased, and the released energy is transferred to them by the momentum boost (Fig. 2c).

daughter strings. The Virasoro conditions once again impose unavoidable restrictions, permitting the only way of changing daughters angular momentum: by adjusting their length (i.e. masses), Fig. 2. This is also an interesting feature of the ATROPOS model, as it means that angular momentum conservation affects the mass selection of the string fragments. If the string has a large spin, the fragmentation via the production of light fragments with high repulsive momentum is suppressed due to angular momentum conservation. Thus, angular momentum can influence the multiplicity of hadronic interactions.



(a) **(b)** **(c)**

Figure 3: An example of string-to-hadron transition in ATROPOS model. Fig. 3a shows a $u\bar{d}$ "folded" string with spin projections $s_{z1} = -1/2$ and $s_{z2} = 1/2$ assigned to its ends. The string is fragmented and the new $d\bar{d}$ pair with spin projections s'_{z1}, s'_{z2} is sampled at the break point (Fig. 3b). As the daughter strings have close-to-hadron mass and their spin projections sum up to hadron spins, they are identified with the corresponding hadrons (with momentum correction), Fig. 3c.

The transition to hadrons is modeled in ATROPOS in the following way. Suppose that we have the $u\bar{d}$ string (Fig. 3a). Partons at the string end points are also assigned spin projection values to take into account the spin states of hadrons of the same flavor content. Then the string is fragmented and the new parton-antiparton pair is sampled at the break point with corresponding probability (in this case, a $d\bar{d}$ pair), Fig. 3b. New partons are also assigned random spin projections. The algorithm checks if the daughter string has a mass that is close to a mass of a hadron of the same quark composition (a parameter is used that defines the relative mass difference for the string-to-hadron transition). It also checks if the absolute sum of the spin projections of the string end-points is equal to the spin of a potential hadron (for resonances with spin states $J > 3/2$ the co-directional configuration is accepted). If all the transition criteria are met, the hadron is produced on-shell, and the momentum is re-shuffled in the end to conserve energy (Fig. 3c).

4. Results

It is important to note that ATROPOS is a string fragmentation framework, so it requires a parton-level input for operation. The following results were obtained with the Pythia 8.313 [1] (default tune) parton level generator.

The scaled momentum spectra for some selected particles in e^+e^- interactions at $\sqrt{s} = 91.2$ GeV are shown in Fig. 4. The good agreement with the experimental data [12] demonstrates the correctness of the new fragmentation algorithm. The yields of particles are also well-reproduced with minor underestimation of vector states (to be improved). Another important result is the natural asymmetrical production of ρ-mesons: $n_{\rho^0} : n_{\rho^+} : n_{\rho^-} \approx 1.08 : 1 : 1$.

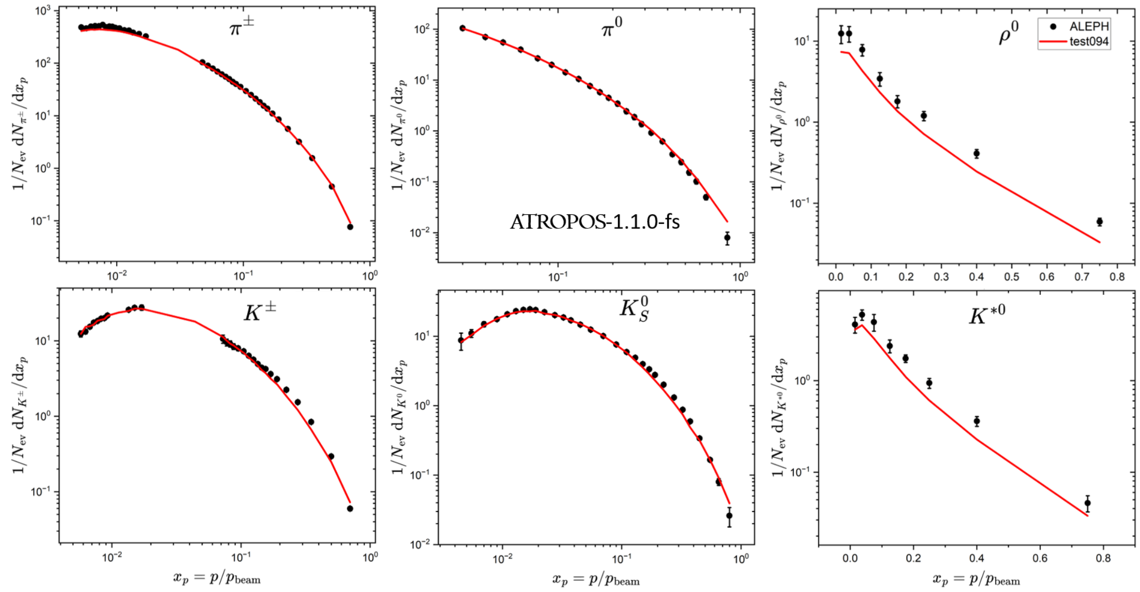


Figure 4: Scaled momentum $x_p = p/p_{\mathrm{beam}}$ spectra of selected particles in e^+e^- interactions at $\sqrt{s} = 91.2$ GeV. The dots indicate the ALEPH data [12], curves represent simulation with ATROPOS-1.1.0-fs.

The comparison of the charged particles multiplicity in pp collisions at $\sqrt{s} = 7$ TeV with the ALICE [13] and ATLAS [14] data is shown in Fig. 5. It should be noted that while good agreement can be achieved in e^+e^- interactions with any value of $\nu > 70$, for pp the best fit is obtained with the eigenharmonic of the string $\nu \approx 90$. Smaller values of ν lead to a multiplicity underestimate, and larger values lead to an overestimate. This demonstrates the importance of the orbital angular momentum of interacting partons in hadronic interactions.

Some preliminary simulations of π^-C collisions were performed. Fig. 6 shows the momentum spectra of π^+ and K^+ at the beam momentum of 158 GeV. The model needs further improvements for a better description of the data [15]. However, the essential point is that the best fit is achieved

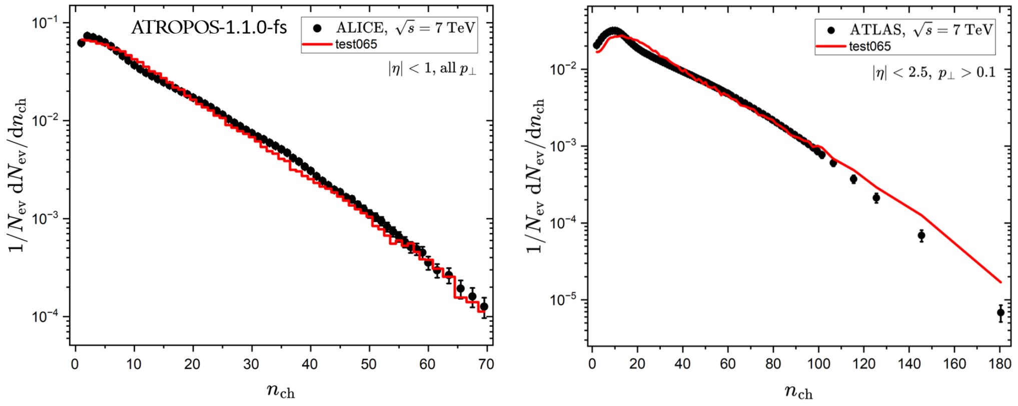


Figure 5: Multiplicity distributions of pp collisions at $\sqrt{s}$ = 7 TeV. The data points are from ALICE [13] and ATLAS [14] detectors, simulation is done with ATROPOS-1.1.0-fs (curves).

with $\nu \approx 20$, which is noticeably lower than in pp collisions. This further proves the possibility to distinguish between different types of colliding systems when considering string angular momentum. To consider the possible consequence for extensive air shower physics, let us compare the average

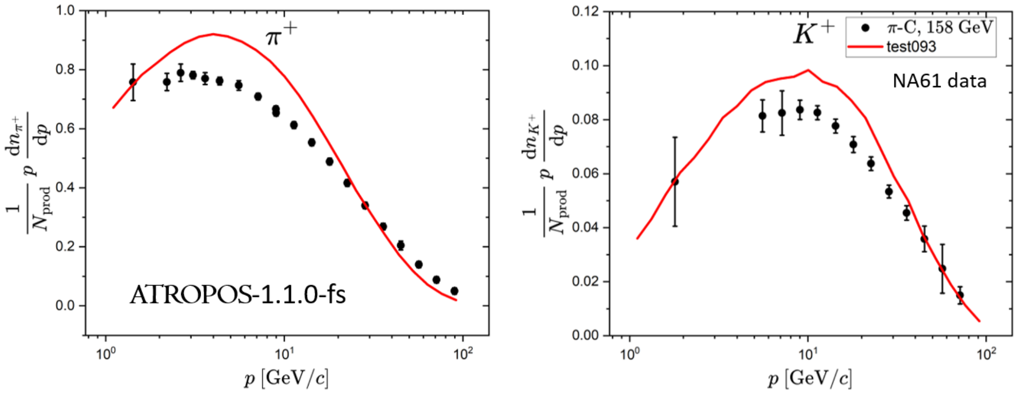


Figure 6: Momentum spectra of positive pions and kaons in π^-C collisions at $p_{\rm beam}$ = 158 GeV. The data is taken from NA61/SHINE Collaboration [15] (dots), simulation is done with ATROPOS-1.1.0-fs (curves). Pythia 8.313 with the Angantyr model is used for parton level input.

fraction $R_{\rm em}$ of electromagnetic energy in pp interactions in different modern hadronic models, Fig. 7. One can clearly see that ATROPOS is the only model demonstrating the clear trend of decreasing $R_{\rm em}$ with increasing $\sqrt{s} > 1$ TeV. The smaller $R_{\rm em}$ can result in a more extensive muon production in air showers due to the greater portion of energy remaining in the hadronic component.

5. Summary

ATROPOS is the first model in which the conservation of the angular momentum of quark-gluon strings during the fragmentation process is considered. This feature provides the opportunity to introduce the natural breaking of the universality principle of the independent string fragmentation

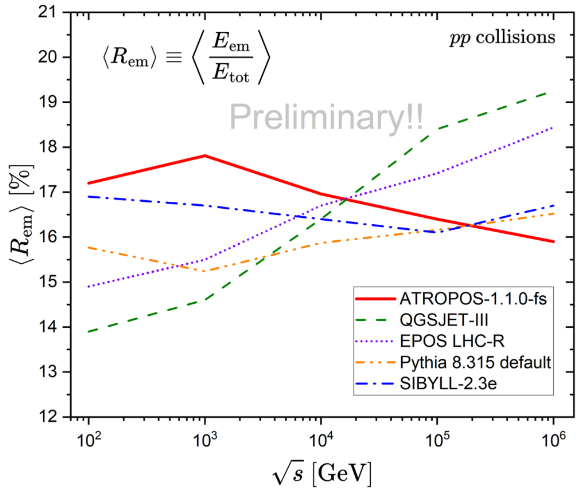


Figure 7: A comparison of electromagnetic energy ratio as a function of center-of-mass energy of the collision for modern hadronic interaction models. Pythia 8.313 is used for the ATROPOS parton level input.

and could contribute to improving of our understanding of the nature of ultra-high energy cosmic ray interactions.

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