Analysis of α + ^{16}O elastic scattering in the semi-microscopical dispersive optical model

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Abstract. We study the elastic scattering cross sections of the $\alpha + ^{16}$O system at 49.5, 69.5, and 80.7 MeV using the optical potentials calculated in the framework of the semi-microscopical dispersive optical model. In this model, the optical potential contains an energy-independent part calculated in the double folding model with a realistic effective nucleon-nucleon interaction, as well as an energy-dependent dynamic polarization potential, the parameters of which are adjusted to the data. The parameters of the real and imaginary parts of the dynamic polarization potential are connected by the dispersion relations.

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1. Introduction

The search for an effective potential that describes the interaction of colliding nuclei in the elastic channel is the important part of heavy-ion reaction studies. This problem is usually solved by using an optical model [1-3], where an effective complex nucleus-nucleus potential or optical potential (OP) is constructed phenomenologically and contains various parameters determined by fit the data. Also the global OPs that have few energy-dependent parameters are deduced to describe the interaction of light projectiles with target nuclei at wide energy range [2, 4].

Alternatively, a semi-microscopic approach involves the effective OP, which is represented as a sum of two components. The first of them, a static mean-field potential, is commonly calculated in the microscopic double-folding model [3,5-13] by using a realistic effective nucleon-nucleon interaction [14]. The second part is usually called a dynamic polarization potential (DPP); it depends on the projectile energy and describes the inelastic interaction between colliding nuclei. The real and imaginary parts of the DPP are constructed phenomenologically. Thus a semi-microscopic approach combines a microscopic calculation of the mean-field potential and empirical adjustment of the DPP. The effective nucleus-nucleus potential must be nonlocal, complex, and depends on the nuclear matter density [3]. The semi-microscopic OP can be written in the following form [10, 11]:

$$U_E = V_F + V_P + iW + V_C \tag{1}$$

where V_F is a mean-field potential, V_P is the real part of the DPP, W is the imaginary part of the DPP, and V_C is the Coulomb potential.

2. Calculation of the double folding potential

The mean-field potential for the interaction of two nuclei is expressed as a sum of the direct V_F^D and exchange V_F^E components,

$$V_F = V_F^D + V_F^E, \quad (2)$$

and is written in terms of folding integral by considering nuclei in their ground states [6],

$$V_F =< \phi_{p0}\, \phi_{t_0}\, |V|\, \phi_{p0}\, \phi_{t_0}\ >, \quad (3)$$

where V denotes the interaction of the projectile p and the target t, which is presented as a sum of two-body interactions

$$V = \sum_{pt} v_{pt}. \quad (4)$$

Involving the exchange of nucleons, an extra term with the exchange operator of all coordinates of the two interacting nucleons is added

$$v_{pt} \rightarrow v_{pt}(1 - \widehat{P}_{pt}).$$

Thus, the direct and exchange terms of Eq. (2) have the following explicit forms [8-12]

$$V_F^D(\mathbf{r}) = \int \rho_p(\mathbf{r}_p)\rho_t(\mathbf{r}_t)v^D(\rho, E, r_{pt})\ d^3r_p d^3r_t, \quad (5)$$

$$V_F^E(\mathbf{r}) = \int \rho_p(\mathbf{r}_p, \mathbf{r}_p + \mathbf{r}_{pt})\rho_t(\mathbf{r}_t, \mathbf{r}_t - \mathbf{r}_{pt})v^E(\rho, E, r_{pt})\mathrm{exp}\left(\frac{i\,\mathbf{K}(r)\mathbf{r}_{pt}}{M}\right)\ d^3r_p d^3r_t \quad (6)$$

in terms of the nuclear matter distribution functions $\rho_{p,t}(r)$ of each of the colliding nuclei, the effective density-dependent nucleon-nucleon interaction potential $v^{D,E}(E, \rho, r_{pt})$, and the local momentum

$$K(r) = \sqrt{\frac{2\mu}{\hbar^2}\,[E - V(r) - V_C(r)]},$$

where $\mathbf{r}_{pt} = \mathbf{r} - \mathbf{r}_p + \mathbf{r}_t$.

The most important part of the calculation of the mean-field potential is a choice of the realistic effective nucleon-nucleon (NN) interaction. We use in our calculations the M3Y-Paris interaction [6, 9, 10, 11, 12] (corresponding to the notation DDM3Y1(Paris)), in which the nucleon knock-on exchange is included and then a density-dependent correction is performed following Refs. [9, 11]:

$$\begin{aligned} v^D(\rho, E, r_{pt}) &= F(\rho, E)v_{NN}^D(r_{pt}), \\ v^E(\rho, E, r_{pt}) &= F(\rho, E)v_{NN}^E(r_{pt}), \end{aligned} \quad (7)$$

where

$$F(\rho, E) = C[1 + Ae^{-B\rho} + G\rho], \quad (8)$$

$$v_{NN}^D(r) = 11061.625\frac{e^{-4r}}{4r} - 2537.5\frac{e^{-2.5r}}{2.5r}, \quad (9)$$

and

$$v^E(r) = -1524.25\frac{e^{-4r}}{4r} - 518.75\frac{e^{-2.5r}}{2.5r} - 7.8474\frac{e^{-0.7072r}}{0.7072r}. \quad (10)$$

Parameters used at the density-dependent function $F(\rho, E)$ are shown in Table 1.

Table 1. Parameters of the density-dependent function $F(\rho, E)$ for the DDM3Y (Paris) potential

Interaction	C	A	B (fm^3)	G (fm^3)
DDM3Y1(Paris)	0.2963	3.7231	3.7384	0.0

The nuclear matter densities of the projectile and target are taken based on the 3-parameter modified Fermi form [15]

$$\rho(r) = \rho_0 \left(1 + \frac{wr^2}{c^2}\right) \left[1 + \exp\left(\frac{r-c}{z}\right)\right].$$

For α particles, we have the following values: $c_p = 1.008$ fm, $z_p = 0.327$ fm, $w_p = 0.445$, likewise, for the target, these parameters are chosen as: $c_t = 2.608$ fm, $z_t = 0.513$ fm, $w_t = -0.051$ [15].

3. The DPP potential

The dynamic polarization potential $(V_P + iW)$ describes virtual excitations to inelastic channels and the flux loss (absorption) in the elastic channel. The DPP is a phenomenological potential and is obtained by comparison with the scattering data.

Following Ref. [11], we present the volume and surface parts of the DPP in the Woods-Saxon form and its derivative

$$V_P(r, E) = \alpha(E) W_0(E) f(x_W) + \beta(E) \times 4W_D(E) \frac{f(x_D)}{dr}, \tag{11}$$

$$W = -W_0(E) f(x_W) + 4W_D(E) \frac{df(x_D)}{dr}, \tag{12}$$

where $f(x) = (e^x + 1)^{-1}$ with $x_{W,(D)} = (r - R_{W,(D)})/a_{W,(D)}$ and $R_{W,(D)} = r_{W,(D)} A^{1/3}$.

Phenomenological parameters $\alpha(E)$, $\beta(E)$, $W_0(E)$, $W_D(E), r_{W\ (D)}$, and $a_{W\ (D)}$ are chosen by fit the data.

4. Results and discussion

In modern studies of nucleus-nucleus elastic scattering, various models of double folding potentials and dynamic polarization potentials are widely used (see, for example, [16-19] and references therein). We study the α + ^{16}O elastic scattering at 49.5, 69.5, and 80.7 MeV. The double folding potentials were calculated with the computer code FOLDEG [11] and the DPPs were calculated with an original code WDPP.

Parameters of the DPP for the α + ^{16}O scattering are shown in Table 2.

Table 2. Parameters of the dynamic polarization potential for the α + ^{16}O scattering

E_{lab}[MeV]	$\alpha(E)$	$\beta(E)$	$-W_0$[MeV]	$-W_D$[MeV]	$-J_V$ [MeV·fm^3]
49.5	−0.0001	−0.34	11.9	5.5	416.8
69.5	−0.27	−0.36	13.1	6.4	397.7
80.7	−0.45	−0.33	15.2	4.5	391.0

The geometrical parameters of the DPP were fixed at the values: $r_W = 0.9$ fm, $a_W = 0.45$ fm, r_D= 1.0 fm, $a_D = 0.55$ fm.

The calculated nucleus-nucleus potentials are shown in Figure 1, where the mean-field potential, the real part of the DPP, V_P, and the absorption potential W are indicated for each energy.

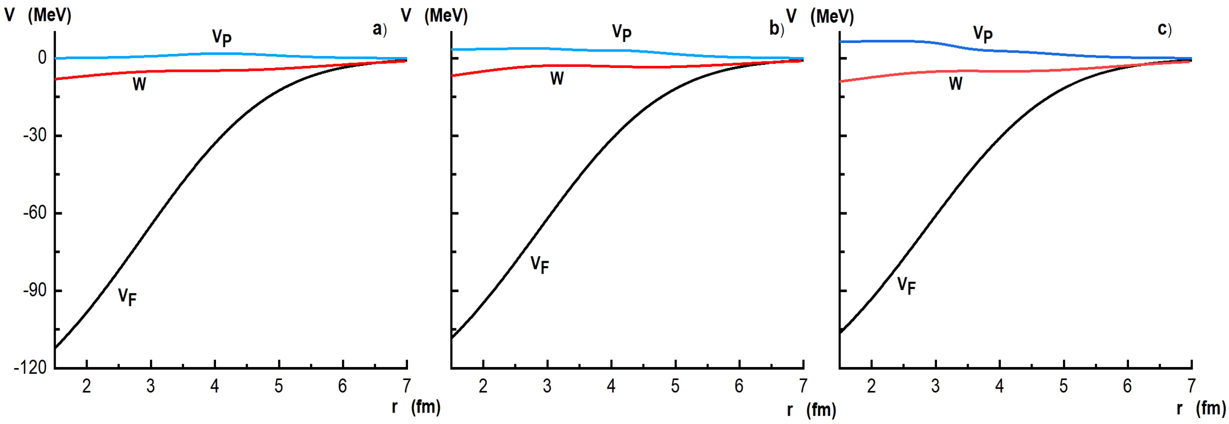

Figure 1. Effective α + ^{16}O potentials resulting from the calculations in the semi-microscopic model at a)$E_{\rm lab}$ = 49.5 MeV, b)$E_{\rm lab}$ = 69.5 MeV, c)$E_{\rm lab}$ = 80.7 MeV

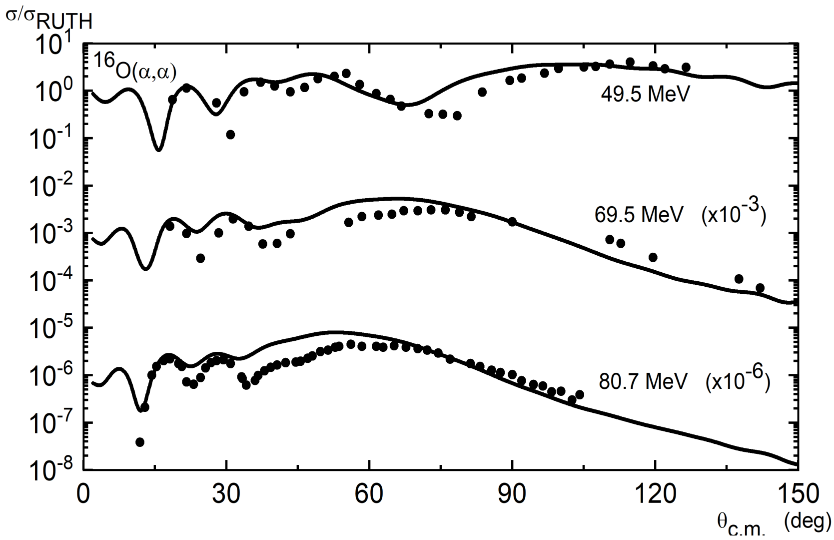

Figure 2. Comparison of the calculated differential cross section of α + ^{16}O elastic scattering with at 49.5, 69.5, and 80.7 MeV

The folding potential V_F is deeper compared to the other parts of the potential, being the dominant part of the effective potential. The real part of the DPP contributes as an energy-dependent correction of the V_F and its influence increases with increase of the projectile energy.

The elastic scattering cross sections calculated using the code FRESCO [20] are compared with the data [4,21] in Fig. 2. The calculated cross sections are in a qualitative agreement with the data and can be improved by applying a tuned version of the model.

5. Conclusion

The semi-microscopic optical model was used for the analysis of the elastic α+ ^{16}O scattering at 49.5, 69.5, and 80.7 MeV. The effective optical potential in this model contains two parts: the first one is a mean-field potential for the interaction of α-particle with ^{16}O calculated within the double-folding model with allowance for exchange effects. The second term is a phenomenological dynamic polarization potential, the real and imaginary parts of the which are related by dispersion relations and are chosen to be proportional adjusted scaling parameters. Their energy independence was controlled by the energy dependence of the DPP volume integrals at fixed geometric parameters.

The mean-field potential was calculated applying the density-dependent version of the M3Y (Paris) effective nucleon–nucleon interactions and a nuclear matter density of the 3-parameter modified Fermi form.

The calculated (using computer code FRESCO) differential cross sections for the elastic $\alpha+$ ^{16}O scattering were compared with the data and a reasonable description of the data was demonstrated. We confirmed that the double folding model with a realistic effective nucleon-nucleon interaction and the exchange term to account antisymmetrization of the system has an advantage to reduce ambiguities inherent to pure phenomenological methods. In general, the applied model involves a smaller number of parameters and makes it possible to control them in accordance with physical conditions.

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