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Interaction of the single-particle and collective degrees of freedom in non-magic nuclei: The role of phonon tadpole terms

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Abstract. A method of consistent treatment of phonon contributions to the self-energy and gap terms in non-magic nuclei is developed in so-called g^2 approximation, where g is the creation amplitude of a low-lying phonon. The method simultaneously takes into account both usual non-local and local phonon tadpole terms. Relations that allow the tadpoles to be calculated without introduction of new parameters are derived. As an application of the method, the effect of the phonon tadpoles on the single-particle strength distribution, single-particle energies and gap values is considered. Hypothesis of the surface nature of pairing correlations is discussed in the light of the tadpole effect.

PACS. 21.10.-k Properties of nuclei; nuclear energy levels – 21.10.Jx Spectroscopic factors and asymptotic normalization coefficients – 21.10.Re Collective levels – 21.60.-n Nuclear structure models and methods

1 Introduction

In the last two decades, certain progress was achieved in the many-body nuclear theory [1,2] in going beyond the standard Random Phase Approximation (RPA) or the Theory of Finite Fermi Systems (TFFS). It was achieved by means of taking into account coupling of single-particle degrees of freedom with low-lying collective excitations ("phonons"). For magic nuclei refs. [3–6] could be cited, and for nuclei with pairing correlations these are refs. [7–11]. A characteristic feature of early studies in this field was the use of a phenomenological Saxon-Woods mean field. In that case, a double set of the phenomenological parameters was necessary, the first one for the effective force and the second one for the mean field. Note that in ref. [3] the main terms of the mean-field potential well, *i.e.* those without phonon corrections, were found in a self-consistent way within the self-consistent version of the TFFS. However, the phonon characteristics were calculated based on the Saxon-Woods potential, thereby a double set of the parameters was used, too.

Modern developments in this field completely abandon the phenomenological mean field when both the field and the phonon characteristics are calculated self-consistently based on a single set of parameters for forces. The Hartree-Fock-Bogolyubov (HFB) calculations with Skyrme forces [12,13] and those within the relativistic mean-field theory (for magic nuclei only) [6,14] should be cited here. Article [6] is a characteristic example of such an approach. There, in the framework of the old dynamic scheme of ref. [15], the Saxon-Woods mean field was changed by the relativistic mean field. In both studies [6] and [15] the authors solved the Dyson equation for the self-energy in the so-called g^2 approximation with g being a phonon-particle coupling amplitude. Properties of odd nuclei nearby ^{208}Pb, such as the single-particle strength distribution and characteristics of the single-particle levels, were considered.

A consistent generalization of the g^2 approximation for the treatment of nuclei with pairing correlations was performed in ref. [10]. This generalization is based on Eliashberg's approach [16], originally developed for superconductivity in solid-state physics. Realistic calculations were performed for ^{121}Sn and ^{123}Sn nuclei. In refs. [10] and [8] the problem of the evaluation of the phonon contribution to single-particle energies and gap values was consistently formulated and the obtained equations were solved.

The problem of the phonon contribution to the nuclear gap value is, probably, of the prime interest at present because it is closely related to the problem of nature of the nuclear superfluidity. Note that it is also of great interest for the astrophysics, see, *e.g.*, refs. [17] and [18]. As for atomic nuclei themselves, the long-standing problem persists: whether the nuclear superfluidity has a volume or surface nature. In the latter case a question remains to which extent the superfluidity results from a special

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form of the initial pairing force and what is the effect of an additional contribution induced by exchange with the collective low-energy surface phonons. The calculations of ref. [8] demonstrated that we deal with an intermediate case: the phonon contribution to the gap value Δ for the tin region is about 30% of the experimental Δ, the rest of about 70% is due to the "main" pairing force.

Simultaneously and independently, the problem of nuclear pairing was attacked by the Milan group. They combined an *ab initio* calculation of the gap, proceeding from the Argonne v_{14} NN-force, refs. [19] and [9], with evaluation of the phonon contribution to Δ, refs. [7] and [9]. Their conclusion was that about 50% of the gap value for the tin region originates due to the initial NN-force and the other 50%, due to the phonon contribution. Note that an alternative *ab initio* calculation of the gap was recently carried out in refs. [20] (Paris NN-force) and [21] (Argonne v_{18} NN-force) based on the method developed in ref. [22]. Skipping discussion of these two methods of solving the *ab initio* gap equation, we only note that a close agreement with the experimental value of Δ was obtained in refs. [20] and [21], leaving a room of not more than 20% for the phonon corrections. It qualitatively agrees with results of refs. [10] and [8], but significantly contradicts to those of refs. [7] and [9]. It is worth mentioning that in the cited papers by the Milan group the phonon contribution to the gap of Ca and Ti isotopes reaches almost 100%. Thus, a controversial situation persists in the problem under discussion and new studies are required to resolve this problem.

However, in all the above-mentioned papers and in many other studies dealing with the evaluation of the low-lying phonon contribution to nuclear characteristics, one term was lost, which is of the same g^2 order as the usual non-local pole term of the self-energy being taken into account. We mean a so-called tadpole term. For the first time it was evaluated in the pioneering paper by V.A. Khodel [23] and was called the local term. Now we call it the tadpole diagram in accordance with the particle physics terminology. This approach is based on the interpretation of the low-lying surface phonons as members of the Goldstone branch of excitations, which originates due to spontaneous breaking of the translation invariance in nuclei [24]. The ghost 1^--state with the frequency $\omega_1 = 0$ is the head of this branch, being an exact solution of the self-consistency equation of the TFFS [25]. Therefore, for the simplest version of the TFFS effective force, the corresponding eigenfunction is $g_1 = \mathrm{d}U/\mathrm{d}r$, where $U(r)$ is the mean-field potential. The TFFS equations for the natural parity excitations result in small excitation energies ω_L and eigenfunctions g_L being close to g_1, provided the self-consistency relation is met. The main achievement of ref. [23] is a scheme for evaluating the surface phonon corrections to nuclear characteristics in such a way that they automatically vanish for the ghost phonon. As it turned out, such a scheme is impossible without taking into account the tadpole term.

For magic nuclei the tadpole term was considered in detail in ref. [3], see also references therein. As it turned out, the tadpole contribution to the single-particle energies, splitting of the particle-vibration multiplets and other properties of magic nuclei and their odd neighbors are, as a rule, important and are often of the opposite sign as compared with the usual non-local terms. The development of an analogous approach for non-magic nuclei is a problem of great interest.

The present paper is the first step in this direction. In sect. 2 we derive a closed set of equations for the tadpole terms in nuclei with pairing correlations, which involve no new parameters in addition to those used for calculating the mean-field self-energies. In sect. 3 we consider possible approximations that make the treatment more feasible. Closed and transparent relations for the particle-hole (ph) and particle-particle (pp) tadpole terms are obtained. As an application, we study modifications of the results of refs. [10] and [8] for the single-particle strength distribution, single-particle energies and gap values of superfluid nuclei due to the inclusion of the tadpoles. The relevance of the tadpole terms to the problem of the nature of nuclear pairing is discussed.

2 Phonon corrections to the self-energy and gap operators

In this paper, we deal with nuclei with weak phonon-particle coupling when the phonon admixture to the one-particle degrees of freedom could be accounted for within a perturbation theory scheme. More specifically, the quantity

$$\alpha = \frac{\bar{g}_L^2}{(2j+1)\omega_L^2} \tag{1}$$

plays the role of the perturbation parameter, and it should be small. Here $\bar{g}_L$ is the average value of the matrix element of the L-phonon creation amplitude at the Fermi surface, ω_L is its excitation energy, and j is a typical value of the single-particle angular momentum ($j \propto A^{1/3}$). In other words, the coupling strength should be not too high and the excitation energy, not too low. We limit ourselves with the first-order terms in α and call this the g^2 approximation for the self-energy.

Such a situation takes place in magic and semi-magic nuclei. There is no pairing at all in the first case and partially, in the magic subsystem, in the second one. For the magic nuclei, the problem under consideration was consistently solved in [3], see also references therein. Our aim is to develop a similar approach for semi-magic nuclei with pairing in the non-magic subsystem taken into account.

In the general case of nuclei with pairing it is necessary to consider four one-particle generalized Green functions, *i.e.* two Green functions, G and G^h, and two Gor'kov functions $F^{(1)}$ and $F^{(2)}$. In addition to two normal self-energies, Σ and Σ^h, their two anomalous counterparts appear. In the textbook [26] they are denoted as Σ_{02} and Σ_{20}, in refs. [8] and [10] as $\Sigma^{(1)}$ and $\Sigma^{(2)}$. Here we use the notation [1], where they are denoted as $\Delta^{(1)}$ and $\Delta^{(2)}$. Therefore we will name them the gap operators. We also use the term "mass operators" for all the four quantities under discussion.

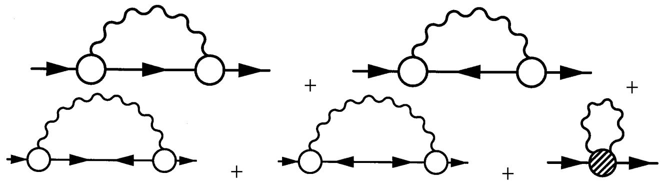

Fig. 1. Phonon g^2 corrections to the self-energy $\Sigma(\varepsilon)$ in non-magic nuclei (general case).

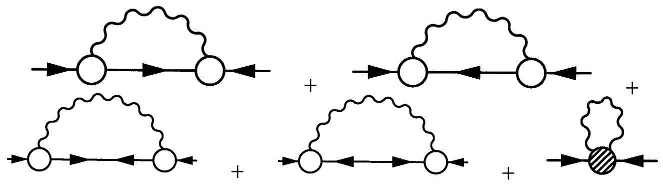

Fig. 2. Phonon g^2 corrections to the gap operator $\Delta^{(1)}(\varepsilon)$ in non-magic nuclei (general case).

The main part of the mass operators is determined by the mean-field contribution. In the g^2 approximation, for single-particle energies ε_λ and average gap values Δ_λ we have:

$$\begin{aligned} \varepsilon_\lambda &= \Sigma_{\lambda\lambda} = \varepsilon_\lambda^{(0)} + \delta^{(2)}\Sigma_{\lambda\lambda}(\varepsilon_\lambda), \\ \Delta_\lambda &= \Delta_{\lambda\lambda} = \Delta_{\lambda\lambda}^{(0)} + \delta^{(2)}\Delta_{\lambda\lambda}(\varepsilon_\lambda), \end{aligned} \tag{2}$$

with the obvious notation. In the approach discussed, the mean-field parts of Σ and Δ are supposed to be calculated within a self-consistent approach, say, the HFB method or the self-consistent TFFS. However, in practice the phenomenological Woods-Saxon potential and a phenomenological pairing force are often used. It should be emphasized that in the latter case it is necessary to use the so-called "refinement" procedure [5,10] in order to avoid a double counting of phonon contributions into the terms $\varepsilon_\lambda^{(0)}$ and $\Delta_\lambda^{(0)}$ and to extract the "refined" values from experimental ones. In principle, analogous precautions should be made in the case of the self-consistent calculation with the use of phenomenological forces, too. Indeed, the force parameters should be chosen in such a way that the total expressions (2) (not the "zero" ones) would reproduce the experimental values. Note that this very idea was utilized in [3].

In the g^2 approximation, one can write down the phonon corrections, *e.g.*, to the operators Σ and $\Delta^{(1)}$, see fig. 1 and fig. 2, as follows:

$$\delta^{(2)}\Sigma(\varepsilon) = M(\varepsilon) + K^{ph}\,, \tag{3}$$

$$\delta^{(2)}\Delta^{(1)}(\varepsilon) = M^{(1)}(\varepsilon) + K^{pp}\,. \tag{4}$$

The set of diagrams and the relation for $\delta^{(2)}\Delta^{(2)}$ are absolutely similar to fig. 2 and eq. (4). In fig. 1 and fig. 2 empty circles denote the phonon creation amplitudes (vertexes). To distinguish between the g- and d-vertexes it is necessary just to look at the direction of ingoing and outgoing arrows. In the case of one ingoing and one outgoing arrow we deal with the g- (or g^h-) vertex. If there are two ingoing arrows, we deal with the $d^{(1)}$-vertex, whereas two outgoing arrows mean the $d^{(2)}$-vertex. Terms with two phonon creation amplitudes are energy-dependent non-local operators, the sums of them being denoted as M and $M^{(1)}$. The last terms are the corresponding *ph*- and *pp*-phonon tadpoles. An essential property of these tadpoles is that they do not depend on the energy ε.

Explicit expressions for the phonon tadpoles K^{ph} and K^{pp} are given by

$$K^{ph} = \int \frac{\mathrm{d}\omega}{2\pi i} D(\omega)\delta g(\omega), \tag{5}$$

$$K^{pp} = \int \frac{\mathrm{d}\omega}{2\pi i} D(\omega)\delta d^{(1)}(\omega), \tag{6}$$

where D is the phonon Green function and δg and $\delta d^{(1)}$ are the changes of the *ph*- and *pp*-phonon creation amplitudes in the external field of an another phonon with the same quantum numbers as the one whose contribution we analyze.

2.1 Magic nuclei

To begin with, let us first outline briefly the method for magic nuclei following to [3]. In this case there is no pairing and only the first and the last terms in fig. 1 for the corrections to the self-energy Σ survive. The first term is the usual pole diagram, where the Green function G, of course, does not contain pairing effects. The last term means the sum of all the irreducible diagrams. In the problem under consideration, as it was mentioned in the introduction, this sum was evaluated firstly in ref. [23]. In accordance with the particle physics terminology, we name it the tadpole diagram. The second order in the g_L correction to the self-energy Σ reads

$$\delta_{LL}^{(2)}\Sigma(\varepsilon) = \int \frac{\mathrm{d}\omega}{2\pi i} S_{LL}(\varepsilon,\omega)D_L(\omega), \tag{7}$$

where D_L stands for the L-phonon D-function and S_{LL} is the phonon-particle scattering amplitude. As usual, the symbolic multiplication means the integration over intermediate coordinates and summation over the spin variables. In accordance with fig. 1, S_{LL} is the sum

$$S_{LL} = g_L G g_L + \delta_L g_L\,, \tag{8}$$

where $\delta_L g_L$ is the tadpole term. Obviously, it is of the second-order correction to Σ, as far as the first-order correction to Σ is the particle-phonon interaction amplitude, $g_L = \delta_L \Sigma$. According to the recipe of [3], it can be found by direct variation of the equation for the vertex g_L,

$$g_L = \mathcal{F} A g_L\,, \tag{9}$$

where $\mathcal{F}$ is the Landau-Migdal effective NN-interaction amplitude [1] and $A = GG$ is the particle-hole propagator. After variation of eq. (9) over the phonon creation amplitude one obtains:

$$\delta_L g_L = (\delta_L \mathcal{F})A g_L + \mathcal{F}(\delta_L A) g_L + \mathcal{F} A\,\delta_L g_L\,. \tag{10}$$

This is an integral equation for the quantity $\delta_L g_L$ with the inhomogeneous term

$$m_{LL} = (\delta_L \mathcal{F}) A g_L + \mathcal{F}(\delta_L A) g_L \,. \tag{11}$$

The procedure of finding the second term of the inhomogeneous term is quite obvious. The direct variation of the particle-hole propagator yields

$$(\delta_L A) = 2G(\delta_L G) = 2GGg_L G. \tag{12}$$

Up to now, we have used a symbolic notation. To obtain the explicit relations, let us, for simplicity, suppose that the self-energy Σ is momentum independent. In this case, for the first-order correction we have

$$\delta^{(1)}_{LM} \Sigma(\mathbf{r}) = g_{LM}(\mathbf{r}) = g_L(r) Y_{LM}(\mathbf{n}). \tag{13}$$

Note that within the Bohr-Mottelson (BM) liquid-drop model [27] one has

$$g_L^{\mathrm{BM}}(r) = \alpha_L \frac{\mathrm{d}U(r)}{\mathrm{d}r} \,, \quad L > 1, \tag{14}$$

where $U(r)$ stands for the nuclear mean-field potential and α_L is a constant which determines the amplitude of the surface vibration. As it was demonstrated in [3], the direct solution of the RPA-like equation (9) for a low-lying excitation in an even-even nucleus is very close to the BM model prescription (14). As to the ghost 1^--state, this relation is exact, due to the TFFS self-consistency relation [3]. Therefore, for qualitative estimations, one can use this simplified form of the particle-phonon vertex.

To obtain the second term in (11) we could fold the quantity (12) with g_L. Let us introduce the notation $T = (\delta_L A) g_L$. In the explicit form, with the help of (12), we obtain

$$\begin{aligned} T_{LM_1LM_2}(\mathbf{r}, \omega) = \int \frac{\mathrm{d}\varepsilon}{2\pi i} \mathrm{d}\mathbf{r}_1 \mathrm{d}\mathbf{r}_2 G(\mathbf{r}, \mathbf{r}_1; \varepsilon) g_{LM_1}(\mathbf{r}_1) \\ \times [G(\mathbf{r}_1, \mathbf{r}_2; \varepsilon - \omega) + G(\mathbf{r}_1, \mathbf{r}_2; \varepsilon + \omega)] \\ \times g_{LM_2}(\mathbf{r}_2) G(\mathbf{r}_2, \mathbf{r}; \varepsilon). \end{aligned} \tag{15}$$

The problem of finding the first term of (11) looks less obvious. To find it, an ansatz was used in [3] based on the density dependence of the Landau-Migdal amplitude $\mathcal{F}$,

$$\delta_L \mathcal{F} = \frac{\delta \mathcal{F}}{\delta \rho} \delta_L \rho, \tag{16}$$

where $\delta_L \rho$ is the transition density associated with the L-phonon excitation. It obeys the relation

$$\delta_L \rho = A g_L \,. \tag{17}$$

In the approximation similar to (14), we have

$$(\delta_L \rho)^{\mathrm{BM}}(r) = \alpha_L \frac{\mathrm{d}\rho(r)}{\mathrm{d}r} \,, \quad L > 1. \tag{18}$$

Thus, the equation for the *ph*-tadpole in magic nuclei reads

$$K^{ph} = \delta_L \mathcal{F} A g D + \mathcal{F}(\delta_L A) g_L D + \mathcal{F} A K^{ph} \,. \tag{19}$$

In the graphic form this equation is shown in fig. 3.

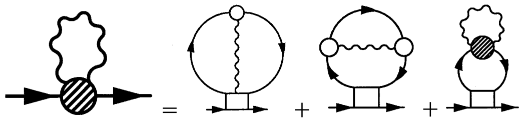

Fig. 3. Equation for the tadpole in magic nuclei.

2.2 Non-magic nuclei

As it was discussed above, in the general case of nuclei with pairing, in addition to the one-particle Green function G, two Gor'kov functions $F^{(1,2)}$ enter the TFFS relations, and two gap functions $\Delta^{(1,2)}$ appear in addition to the self-energy Σ. They are related to each others,

$$\Delta^{(1,2)} = \mathcal{F}^{\xi} F^{(1,2)} \,, \tag{20}$$

in terms of the interaction amplitude $\mathcal{F}^{\xi}$ irreducible in the particle-particle channel. It should be noted that hereafter we mean that the Green function G takes the superfluidity effects into account.

In the systems with pairing, eq. (9) for the phonon-particle vertex is generalized and two new amplitudes appear [1]:

$$d^{(1,2)}_{LM}(\mathbf{r}) = \delta_{LM} \Delta^{(1,2)}(\mathbf{r}) = d^{(1,2)}_L(r) Y_{LM}(\mathbf{n}). \tag{21}$$

Let us omit for a while the subscripts and upper indices. As far as the amplitude $\mathcal{F}^{\xi}$ in the TFFS is considered to be density dependent [28–30], two terms in (21) appear:

$$d = \mathcal{F}^{\xi} \delta F + \delta \mathcal{F}^{\xi} F. \tag{22}$$

It is worth pointing out that usually the first term in (22) is only taken into account. V.A. Khodel [24] was, evidently, the first who turned attention to the second one. It is natural to use for it the ansatz analogous to (16):

$$\delta_L \mathcal{F}^{\xi} = \frac{\delta \mathcal{F}^{\xi}}{\delta \rho} \delta_L \rho, \tag{23}$$

but now, due to pairing effects, the transition density obeys the equation which is more complicated than eq. (17). It will be written down below.

To introduce the standard TFFS notation, let us omit for a while the second term in eq. (22). As far as we deal with the low-lying excitations of natural parity, contributions of spin-dependent forces could be neglected in the equations for g and g^h [1]. As the result, the relation $g^h = g$ is valid. In this case, the quantities g_L and $d^{(1,2)}$ obey the set of equations [1], which could be written in the form similar to (9),

$$\hat{g} = \hat{\mathcal{F}} \hat{A} \hat{g}, \tag{24}$$

but now all the ingredients of (24) are matrices:

$$\hat{g} = \begin{pmatrix} g \\ d^{(1)} \\ d^{(2)} \end{pmatrix}, \tag{25}$$

$$\hat{\mathcal{F}} = \begin{pmatrix} \mathcal{F} & 0 & 0 \\ 0 & \mathcal{F}^{\xi} & 0 \\ 0 & 0 & \mathcal{F}^{\xi} \end{pmatrix}, \tag{26}$$

$$\hat{A} = \begin{pmatrix} \mathcal{L} & \mathcal{M}^{(1)} & \mathcal{M}^{(2)} \\ \mathcal{O} & \mathcal{N}^{(1)} & \mathcal{N}^{(2)} \\ \tilde{\mathcal{O}} & \tilde{\mathcal{N}}^{(1)} & \tilde{\mathcal{N}}^{(2)} \end{pmatrix}. \tag{27}$$

Here $\mathcal{L}$, $\mathcal{M}^{(1)}$, and so on, denote integrals over ε of different double products of the Green function $G(\varepsilon)$ and Gor'kov functions, $F^{(1)}(\varepsilon)$ and $F^{(2)}(\varepsilon)$. They could be found in [1] and we write down now explicitly only two of them,

$$\mathcal{L}(\omega) = \int \frac{\mathrm{d}\varepsilon}{2\pi i} \left[G(\varepsilon)G(\varepsilon+\omega) - F^{(1)}(\varepsilon)F^{(2)}(\varepsilon+\omega)\right], \tag{28}$$

and

$$\mathcal{O}(\omega) = -\int \frac{\mathrm{d}\varepsilon}{2\pi i} \left[G(\varepsilon)F(\varepsilon+\omega) + F(\varepsilon)G(-\varepsilon-\omega)\right]. \tag{29}$$

Let us come back to the second term of (23). With the help of the above short notation the transition density could be written in a compact form similar to (17):

$$\delta\rho = \sum_i A_{1i} g_i \,. \tag{30}$$

Using this relation, we may express the term under consideration in terms of the "generalized vertex function" $\hat{g}$:

$$\delta\mathcal{F}^\xi = \frac{\delta\mathcal{F}^\xi}{\delta\rho} \sum_i A_{1i} g_i \,. \tag{31}$$

By substituting this relation to (23) we find that the general structure of eq. (24) remains valid, but now the "interaction matrix" $\hat{\mathcal{F}}$ becomes more complicated, in particular, non-diagonal. To be more exact, the first line of (26) remains unchanged, but new non-diagonal terms appear in two other lines. To simplify their explicit form, let us use the approximation for the effective pairing interaction amplitude $\mathcal{F}^\xi$ which is usually utilized in the TFFS (*e.g.*, see [30]). Namely, it is considered as an energy-independent delta-force with a density-dependent strength $\mathcal{F}^\xi(\rho(\mathbf{r}))$. In this case, the second term of (22) is reduced to

$$\delta\mathcal{F}^\xi F = \frac{\mathrm{d}\mathcal{F}^\xi}{\mathrm{d}\rho}\, \delta\rho(\mathbf{r})\, \chi(\mathbf{r}), \tag{32}$$

where

$$\chi(\mathbf{r}) = \int \frac{\mathrm{d}\varepsilon}{2\pi i} F(\varepsilon, \mathbf{r}, \mathbf{r}) \tag{33}$$

is the anomalous density. Combining the above relations, one can readily find two new non-diagonal terms of the matrix $\hat{\mathcal{F}}$,

$$\mathcal{F}_{21} = \mathcal{F}_{31} = \frac{\mathrm{d}\mathcal{F}^\xi}{\mathrm{d}\rho}\, \chi(\mathbf{r}). \tag{34}$$

Thus, we obtain the matrix equation (24) for $\hat{g}$ in the general case where both terms of eq. (22) are taken into account. After the variation of this equation over the field of the surface L-phonon under consideration we find the matrix equation for the tadpole term in a superfluid nucleus:

$$\delta_L \hat{g}_L = (\delta_L \hat{\mathcal{F}}) \hat{A} \hat{g}_L + \hat{\mathcal{F}} (\delta_L \hat{A}) \hat{g}_L + \hat{\mathcal{F}} \hat{A}\, \delta_L \hat{g}_L \,. \tag{35}$$

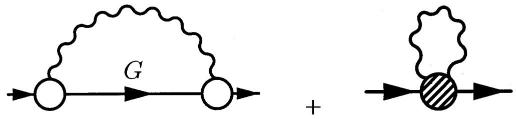

Fig. 4. Phonon g^2 corrections to the self-energy $\Sigma(\varepsilon)$ in non-magic nuclei in the small-d approximation (the Green function G contains pairing effects).

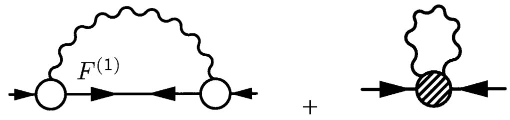

Fig. 5. Phonon g^2 corrections to the gap operator $\Delta^{(1)}(\varepsilon)$ in non-magic nuclei in the small-d approximation.

In principle, this set of equations solves the problem of finding the tadpole terms under discussion. In accordance with eqs. (3), (4), they should be obtained by folding the solutions of eq. (35) with the phonon D-function. However, the explicit form of this equation is quite cumbersome. The second term on the r.h.s. of eq. (35) is the most complicated. To find $\delta\mathcal{L}$ and variations of other components of the $\hat{A}$-matrix, one can use the well-known expressions for δG, δG^h, $\delta F^{(1,2)}$ [1]. In the result, one obtains a lot of integrals of triple combinations of the Green and Gor'kov functions of the type of eq. (15). To obtain more feasible relations, some approximations should be made.

3 Small-d approximation

As it was discussed in the introduction, the collectivity of the low-lying *ph*-phonons, *i.e.* the surface vibrations, exceeds significantly that of the *pp*-phonons, *i.e.* the pairing vibrations. Therefore, we concentrate here on the contributions to the self-energy and gap operators of the phonons of the first type. In this case, the g_L component of the generalized vertex $\hat{g}_L$ dominates in eqs. (24) and (35) and the inequality $g \gg d^{(1,2)}$ is valid. For this reason, we can omit the terms with the pairing phonon creation amplitudes $d^{(1,2)}$ in these equations and, correspondingly, in figs. 1 and 2. In this approximation, the diagrams depicted in figs. 4 and 5 should only be taken into account. In addition, we assume that we deal with the "developed pairing" case when pairing properties of neighboring even-even nuclei should be considered identical. In this case, we have $\Delta^{(1)} = \Delta^{(2)} = \Delta$ and $d^{(1)}(\omega) = \pm d^{(2)}(-\omega) = d(\omega)$ [1]. The sign "+" takes place for the states of natural parity we consider. The appropriate explicit expressions for $M(\varepsilon) = M^h(-\varepsilon)$ and $M^{(1)}(\varepsilon) = M^{(2)}(\varepsilon)$ are given in [10].

In the approximation under consideration, instead of the set (24), we obtain the closed equation for g,

$$g = \mathcal{F}\mathcal{L}g, \tag{36}$$

and the closed expression for the d-vertex in terms of g:

$$d(\omega) = \left(\mathcal{F}_{21}\mathcal{L}(\omega) + \mathcal{F}^{\xi}\mathcal{O}(\omega)\right) g. \tag{37}$$

Sometimes the initial equation (22) for the d-vertex is more convenient. In the small-d approximation, it reads:

$$d(\omega) = \mathcal{F}^{\xi}\mathcal{O}(\omega)g + \delta\mathcal{F}^{\xi}F\,. \tag{38}$$

The set of eqs. (35) for the tadpole terms is also simplified. It could be obtained either by omitting terms containing $d^{(1)}$, $d^{(2)}$ in (35) or by the direct variation of eqs. (36) and (37). In the result, one receives:

$$\delta g = \delta\mathcal{F}\mathcal{L}g + \mathcal{F}\delta\mathcal{L}g + \mathcal{F}\mathcal{L}\delta g, \tag{39}$$

$$\begin{aligned}\delta d = {}& \delta\mathcal{F}_{21}\mathcal{L}g + \mathcal{F}_{21}\delta\mathcal{L}g + \delta\mathcal{F}^{\xi}\mathcal{O}g \\ &+\mathcal{F}^{\xi}\delta\mathcal{O}g + \left(\mathcal{F}_{21}\mathcal{L} + \mathcal{F}^{\xi}\mathcal{O}\right)\delta g.\end{aligned} \tag{40}$$

Thus, we obtained the integral equation (39) for δg with the inhomogeneous term, which is similar to the m_{LL} term (11) for magic nuclei, and expression (40) for δd. The latter contains δg and four terms, which are analogous to the inhomogeneous terms of the equation for the δg. Let us consider them in detail.

An alternative relation for δd could be found by variation of eq. (38). It is as follows:

$$\delta d = \delta\mathcal{F}^{\xi}\mathcal{O}g + \mathcal{F}^{\xi}\delta\mathcal{O}g + \delta\mathcal{F}^{\xi}\delta F + (\delta^2\mathcal{F}^{\xi})F + \mathcal{F}^{\xi}\mathcal{O}\delta g. \tag{41}$$

To obtain the quantities $\delta\mathcal{L}$ and $\delta\mathcal{O}$ in the equations discussed above in the explicit form, one should variate the propagators $\mathcal{L}$ and $\mathcal{O}$:

$$\delta\mathcal{L} = \delta(GG - F^{(1)}F^{(2)}), \tag{42}$$

$$\delta\mathcal{O} = \delta(GF^{(1)} + F^{(1)}G^h), \tag{43}$$

and use the small-d approximation, omitting the terms with $d^{(1)}$, $d^{(2)}$ in the general expressions for the variation of the Green functions [1]. One obtains

$$\begin{aligned}\delta G &= GgG - F^{(1)}gF^{(2)}\,,\\ \delta G^h &= G^h g G^h - F^{(2)}gF^{(1)}\,,\end{aligned} \tag{44}$$

$$\begin{aligned}\delta F^{(1)} &= GgF^{(1)} + F^{(1)}g^hG^h\,,\\ \delta F^{(2)} &= F^{(2)}gG + G^hg^hF^{(2)}\,.\end{aligned} \tag{45}$$

For a time, we come back to the notation $F^{(1)}$, $F^{(2)}$, G^h to avoid an explicit specification of the energy variables in the integrals similar to that in eq. (15), which appear after folding expressions (42) and (43) with g. They could be obtained by combining eqs. (42)–(45) and, in a symbolic form, are as follows:

$$\begin{aligned}\delta\mathcal{L}g = {}& g(GGG - F^{(1)}F^{(2)}G + GGG - GF^{(1)}F^{(2)} \\ &-GF^{(1)}F^{(2)} - F^{(1)}GF^{(2)} \\ &-F^{(1)}F^{(2)}G - F^{(1)}G^hF^{(2)})g,\end{aligned} \tag{46}$$

and

$$\begin{aligned}-\delta\mathcal{O}g = {}& g(GGF^{(1)} - F^{(1)}F^{(2)}F^{(1)} + GGF^{(1)} + GF^{(1)}G^h \\ &+GF^{(1)}G^h + F^{(1)}G^hG^h \\ &+F^{(1)}G^hG^h - F^{(1)}F^{(2)}F^{(1)})g.\end{aligned} \tag{47}$$

Thus, even for the simplified case under consideration, we obtained eight terms for $\delta\mathcal{L}g$ instead of the one in eq. (10). In addition to them, eight new terms for $\delta\mathcal{O}g$ appear in the expression for δd. The explicit form of each integral entering eqs. (46) and (47) is similar to that of eq. (15).

3.1 Final relationships for the tadpoles

For magic nuclei, eq. (19) for the tadpole term was solved in the coordinate representation [3]. Even in this case the procedure turned out to be quite cumbersome. In principle, this method could be generalized to the systems with pairing, using the coordinate representation for the Green and Gor'kov functions [31]. However, as it is clear from the above formulas, in this case it will be much more complicated. For this reason, we prefer to use the representation of the single-particle wave functions, the so-called λ-representation. To make the final equations for the tadpoles more transparent, we also use the diagonal in λ approximation. The matter is that the set $\{\lambda\}$ is chosen in such a way that the mean-field self-energy $\Sigma^{(0)}$ and the corresponding Green function $G^{(0)}$ are diagonal in λ. We use the approximation supposing that the mean-field gap function $\Delta^{(0)}$, the Green function G with pairing and Gor'kov functions $F^{(1,2)}$ are also diagonal in λ [1]. In this approximation, $\mathcal{L}$, $\mathcal{O}$ and other two-particle propagators contain two λ-subscripts, $\mathcal{L}_{\lambda_1\lambda_2}$ and so on, whereas in the general case we have $\mathcal{L}_{\lambda_1\lambda_2} \to \mathcal{L}_{\lambda_1\lambda_2\lambda_3\lambda_4}$, etc. The corresponding generalization of the equations written down below is quite obvious.

The equations for the tadpoles are obtained by substitution of eqs. (39) and (40) (or (41)) into eqs. (5) and (6). The final equation for the K^{ph} tadpole, in the obvious short notation, has the form:

$$\begin{aligned}K^{ph}_{12} = {}& \sum_{3,4}\int\frac{\mathrm{d}\varepsilon}{2\pi i}\frac{\mathrm{d}\omega}{2\pi i}\,\delta\mathcal{F}_{1234}(\omega)\mathcal{L}_{34}(\varepsilon,\omega)g_{34}D_L(\omega) \\ &+\sum_{3,4}\mathcal{F}_{1234}\int\frac{\mathrm{d}\varepsilon}{2\pi i}\frac{\mathrm{d}\omega}{2\pi i}(\delta\mathcal{L}g)_{34}(\varepsilon,\omega)D_L(\omega) \\ &+\sum_{3,4}\mathcal{F}_{1234}\int\frac{\mathrm{d}\varepsilon}{2\pi i}\mathcal{L}_{34}(\varepsilon,\omega_L)K^{ph}_{34}\,.\end{aligned} \tag{48}$$

For the K^{pp} tadpole, let us first use eq. (41) for δd. We find:

$$\begin{aligned}K^{pp}_{12} = {}& 2\sum_{3,4}\int\frac{\mathrm{d}\varepsilon}{2\pi i}\frac{\mathrm{d}\omega}{2\pi i}\delta\mathcal{F}^{\xi}{}_{1234}(\omega)\mathcal{O}_{34}(\varepsilon,\omega)g_{34}D_L(\omega) \\ &+\sum_{3,4}\mathcal{F}^{\xi}_{1234}\int\frac{\mathrm{d}\varepsilon}{2\pi i}\frac{\mathrm{d}\omega}{2\pi i}(\delta\mathcal{O}g)_{34}(\varepsilon,\omega)D_L(\omega) \\ &+\sum_{3}\int\frac{\mathrm{d}\varepsilon}{2\pi i}\frac{\mathrm{d}\omega}{2\pi i}\delta^{(2)}\mathcal{F}^{\xi}{}_{1233}(\omega)F_3(\varepsilon)D_L(\omega) \\ &+\sum_{3,4}\mathcal{F}^{\xi}{}_{1234}\int\frac{\mathrm{d}\varepsilon}{2\pi i}\mathcal{O}_{34}(\varepsilon,\omega_L)K^{ph}_{34}\,.\end{aligned} \tag{49}$$

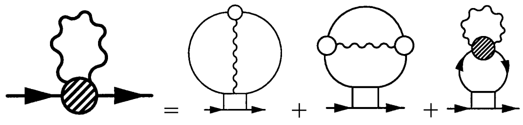

Fig. 6. Equation for the tadpole K^{ph} in non-magic nuclei in the small-d approximation.

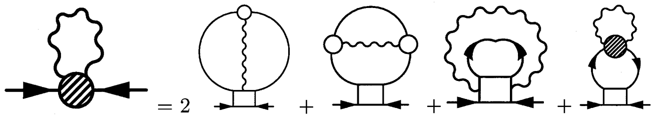

Fig. 7. Expression for the tadpole K^{pp} in non-magic nuclei in the small-d approximation.

The factor 2 in the first term in eq. (49) appears due to the fact that, in the small-d approximation, the terms $\delta\mathcal{F}^{\xi}\mathcal{O}g$ and $\delta\mathcal{F}^{\xi}\delta F^{(1)}$ in eq. (41) are equal to each other.

In the graphic form, eqs. (48) and (49) are illustrated in fig. 6 and fig. 7, respectively. For the sake of simplicity, contrary to fig. 3, here we do not draw all the internal Green functions and omit arrows for those which are drawn. The arrows are only conserved in the cases where it is necessary for understanding. In particular, in the last diagrams of both figures the arrows show that we deal with the tadpole K^{ph}. Remember that the second term on the r.h.s. of eq. (48) in the detailed presentation includes 8 particular diagrams, in accordance with eq. (46). In the case of K^{pp} tadpole, the number of diagrams is even larger. Therefore the detailed diagram representation of eqs. (48) and (49) is rather complicated.

Equation (49) is convenient for the graphical representation, but it possesses one drawback: it contains the term with $\delta^{(2)}\mathcal{F}^{\xi}$ with the "hidden" tadpole K^{ph}. To separate the latter one explicitly it is necessary to use eq. (40) instead of (41). In the result, we find:

$$\begin{aligned}
K^{pp}_{12} &= \sum_{3,4}\int \frac{\mathrm{d}\varepsilon}{2\pi i}\frac{\mathrm{d}\omega}{2\pi i}(\delta_L\mathcal{F}_{21})_{1234}(\omega)\mathcal{L}_{34}(\varepsilon,\omega)g_{34}D_L(\omega)\\
&+\sum_{3,4}\int \frac{\mathrm{d}\varepsilon}{2\pi i}\frac{\mathrm{d}\omega}{2\pi i}\delta\mathcal{F}^{\xi}{}_{1234}(\omega)\mathcal{O}_{34}(\varepsilon,\omega)g_{34}D_L(\omega)\\
&+\sum_{3,4}(\mathcal{F}_{12})_{1234}\int \frac{\mathrm{d}\varepsilon}{2\pi i}\frac{\mathrm{d}\omega}{2\pi i}(\delta\mathcal{L}g)_{34}(\varepsilon,\omega)D_L(\omega)\\
&+\sum_{3,4}\mathcal{F}^{\xi}_{1234}\int \frac{\mathrm{d}\varepsilon}{2\pi i}\frac{\mathrm{d}\omega}{2\pi i}(\delta\mathcal{O}g)_{34}(\varepsilon,\omega)D_L(\omega)\\
&+\sum_{3,4}\Bigg[(\mathcal{F}_{21})_{1234}\int \frac{\mathrm{d}\varepsilon}{2\pi i}\mathcal{L}_{34}(\varepsilon,\omega_L)\\
&+\mathcal{F}^{\xi}{}_{1234}\int \frac{\mathrm{d}\varepsilon}{2\pi i}\mathcal{O}_{34}(\varepsilon,\omega_L)\Bigg]K^{ph}_{34}\,. \qquad (50)
\end{aligned}$$

One remark should be made concerning the integrals over ω in the above equations for the tadpoles. The poles of the D-function should be taken into account only because they lead to the terms which strongly depend on the low-lying phonon frequency ω_L and other phonon characteristics. They could change significantly from one nucleus to another. On the other hand, the terms appearing due to poles of $\mathcal{L}$, $\mathcal{O}$ and other two-particle propagators do not practically depend on ω_L. They are smooth functions of all the variables under consideration and should be included into the corresponding mean-field quantities.

The solution of the integral equations (48) and (49) (or (50)) yields the tadpole values which, according to eqs. (2) and (3), should be added to the usual non-local terms. Note that in the approach discussed there is no need for any new parameters. Below we consider some applications of the results obtained.

3.2 Applications to description of the single-particle characteristics of non-magic nuclei

In refs. [10] and [8] the approach to describe the single-particle strength distribution for non-magic odd nuclei and to take into account the phonon contributions to the single-particle energies and gap values has been developed on the basis of a generalization of the Eliashberg theory [16] to nuclei, with the first application of the Eliashberg theory to nuclei made in [32]. The general set of equations of refs. [10] and [8] for the energy and gap operators, with account for the dynamic spread of a single-particle level (the "dynamic" case), has been derived in the diagonal approximation for the self-energy and gap operators. Arguments in favor of such an approximation could be found in [8,10]. The equations are as follows:

$$\begin{aligned}
\varepsilon_{\lambda\eta} &= \frac{\varepsilon^{(0)}_{\lambda} + M^{\mathrm{even}}_{\lambda}(E_{\lambda\eta})}{1+q_{\lambda\eta}(E_{\lambda\eta})}\,,\\
\Delta_{\lambda\eta} &= \frac{\Delta^{(0)}_{\lambda} + M^{(1)}_{\lambda}(E_{\lambda\eta})}{1+q_{\lambda\eta}(E_{\lambda\eta})}\,,\\
E_{\lambda\eta} &= \sqrt{\varepsilon^2_{\lambda\eta}+\Delta^2_{\lambda\eta}}\,, \qquad (51)
\end{aligned}$$

where

$$q_{\lambda\eta} = -\frac{M^{\mathrm{odd}}_{\lambda}(E_{\lambda\eta})}{E_{\lambda\eta}}\,. \qquad (52)$$

Here M^{even} and M^{odd} are even and odd in the energy components of the non-local term of the self-energy M ($M = M^{\mathrm{even}} + M^{\mathrm{odd}}$) which enters the r.h.s. of eq. (3), and $M^{(1)}$ is the same for the gap operator, see eq. (4). The subscript η numerates solutions of the set of eqs. (51) and (52). This yields the distribution of the single-particle strength in non-magic nuclei.

In order to obtain the single-particle energies and gap values (the "static" case) from eqs. (51) and (52), it is necessary, for each λ, to separate the dominant solution η from the set $\{\lambda\eta\}$. For this aim, the spectroscopic factors should be analyzed. They are given [10] with

$$S^{\pm}_{\lambda\eta} = \frac{(1+q_{\lambda\eta})(E_{\lambda\eta}\pm\varepsilon_{\lambda\eta})}{\dot{\Theta}_{\lambda}(E_{\lambda\eta})}\,, \qquad (53)$$

where

$$\begin{aligned}
\Theta_{\lambda}(\varepsilon) &= (\varepsilon-\varepsilon^{(0)}_{\lambda}-M_{\lambda}(\varepsilon))(\varepsilon+\varepsilon^{(0)}_{\lambda}+M^{h}_{\lambda}(\varepsilon))\\
&\quad-(\Delta^{(0)}_{\lambda}+M^{(1)}_{\lambda}(\varepsilon))^2\,. \qquad (54)
\end{aligned}$$

The η-component with the maximal spectroscopic factor should be associated with the experimental single-particle level, the details see in [10]. Let us denote the observed single-particle energies and gap values as ε_λ and Δ_λ and the corresponding mean-field values as $\varepsilon_\lambda^{(0)}$ and $\Delta_\lambda^{(0)}$. They are related to each other by eqs. (51) and (52) with η equal to the dominant value. Let us rewrite them explicitly omitting the subscript η:

$$\begin{aligned} \varepsilon_\lambda &= \frac{\varepsilon_\lambda^{(0)} + M_\lambda^{\mathrm{even}}(E_\lambda)}{1+q_\lambda(E_\lambda)}\,, \\ \Delta_\lambda &= \frac{\Delta_\lambda^{(0)} + M_\lambda^{(1)}(E_\lambda)}{1+q_\lambda(E_\lambda)}\,, \\ E_\lambda &= \sqrt{\varepsilon_\lambda^2 + \Delta_\lambda^2}\,, \end{aligned} \tag{55}$$

where

$$q_\lambda = -\frac{M_\lambda^{\mathrm{odd}}(E_\lambda)}{E_\lambda}\,. \tag{56}$$

The energies ε_λ and $\varepsilon_\lambda^{(0)}$ are reckoned from the corresponding chemical potentials μ and $\mu^{(0)}$. Note that in refs. [10] and [8] the phenomenological Saxon-Woods potential was utilized as the mean-field one and the phenomenological pairing forces were used as well. As far as both of them are adjusted to the observed values of ε_λ and Δ_λ, a special "refinement" procedure mentioned above is necessary to find $\varepsilon_\lambda^{(0)}$ and $\Delta_\lambda^{(0)}$ values. It is described in detail in the cited articles.

Now it is necessary to modify these results in order to include the tadpoles in accordance with eqs. (3) and (4). In fact, there was no specialization of the mass operators in [8,10] to derive the relations (51) and (55). For this reason, in order to include the tadpoles into consideration we should just change the mass and gap operators of refs. [10] and [8] to the ones from eqs. (3) and (4). Remember that the tadpole terms K^{ph} and K^{pp} do not depend on the energy. Supposing that they, just as the non-local operators M and $M^{(1)}$, are diagonal in λ, we obtain

$$\begin{aligned} \varepsilon_{\lambda\eta} &= \frac{\varepsilon_\lambda^{(0)} + M_\lambda^{\mathrm{even}}(E_{\lambda\eta})}{1+q_{\lambda\eta}(E_{\lambda\eta})} + \frac{K_\lambda^{ph}}{1+q_{\lambda\eta}(E_{\lambda\eta})}\,, \\ \Delta_{\lambda\eta} &= \frac{\Delta_\lambda^{(0)} + M_\lambda^{(1)}(E_{\lambda\eta})}{1+q_{\lambda\eta}(E_{\lambda\eta})} + \frac{K_\lambda^{pp}}{1+q_{\lambda\eta}(E_{\lambda\eta})}\,, \\ E_{\lambda\eta} &= \sqrt{\varepsilon_{\lambda\eta}^2 + \Delta_{\lambda\eta}^2}\,, \end{aligned} \tag{57}$$

with

$$q_{\lambda\eta} = -\frac{M_{\lambda\eta}^{\mathrm{odd}}(E_{\lambda\eta})}{E_{\lambda\eta}}\,, \tag{58}$$

instead of eqs. (51) and (52). In the same way, instead of eqs. (55) and (56) for the single-particle and gap values, we find:

$$\begin{aligned} \varepsilon_\lambda &= \frac{\varepsilon_\lambda^{(0)} + M_\lambda^{\mathrm{even}}(E_\lambda)}{1+q_\lambda(E_\lambda)} + \frac{K_\lambda^{ph}}{1+q_\lambda(E_\lambda)}\,, \\ \Delta_\lambda &= \frac{\Delta_\lambda^{(0)} + M_\lambda^{(1)}(E_\lambda)}{1+q_\lambda(E_\lambda)} + \frac{K_\lambda^{pp}}{1+q_\lambda(E_\lambda)}\,, \\ E_\lambda &= \sqrt{\varepsilon_\lambda^2 + \Delta_\lambda^2}\,, \end{aligned} \tag{59}$$

with

$$q_\lambda = -\frac{M_\lambda^{\mathrm{odd}}(E_\lambda)}{E_\lambda}\,. \tag{60}$$

We see that both the single-particle energy and gap values are changed due to inclusion of the tadpoles in the dynamic and static cases. In the latter case, the solution of the set of eqs. (59) should answer the question about the total phonon contribution, including the tadpole terms, to the pairing gap, as compared with the mean-field, or "refined", value $\Delta_\lambda^{(0)}$. Up to now, all calculations of the phonon corrections to the gap have ignored the tadpole contributions.

Let us briefly discuss the situation in nuclei without pairing. In this case, the equations of sect. 2.1 for the phonon corrections to the single-particle energies were solved in the coordinate representation in [3] (see references therein, in particular, [33]). It turned out that the tadpole contribution to ε_λ was, as a rule, significant and comparable with that of the non-local term of the self-energy. For a qualitative analysis, we limit ourselves with the diagonal in λ approximation using eqs. (57) and (59) without any pairing contribution. For the spread of a single-particle state we have:

$$\varepsilon_{\lambda\eta} = \varepsilon_\lambda^{(0)} + M_\lambda(\varepsilon_{\lambda\eta}) + K_\lambda^{ph}\,, \tag{61}$$

and for the single-particle energies:

$$\varepsilon_\lambda = \varepsilon_\lambda^{(0)} + M_\lambda(\varepsilon_\lambda) + K_\lambda^{ph}\,. \tag{62}$$

As far as K^{ph} does not depend on the energy, the shift of the solutions, for a fixed λ, will be the same for all the values of η. For the same reason, that is independence of K^{ph} on energy, the spectroscopic factors which are determined by the residues of the Green function and, therefore, by the energy derivative of the self-energy in eq. (3), are not changed.

This conclusion about the role of the ph-tadpole in magic nuclei agrees with the results of calculations for single-particle level properties in the odd neighbors of ^{208}Pb in ref. [6] cited above, where the tadpole contribution was not taken into account. Indeed, as can be seen from table III of [6], the authors obtained a good agreement with the experimental data for the spectroscopic factors, where there is no tadpole contribution, but the agreement for the single-particle energies is considerably worse due to the fact that the tadpole contribution does exist in this case.

Things are different in nuclei with pairing. In this case, in the absence of the tadpole, the single-particle spectroscopic factors are given with eqs. (53) and (54). If the tadpole terms K^{ph} and K^{pp} are included, as can be easily checked, in addition to a change of $E_{\lambda\eta}$ and $\varepsilon_{\lambda\eta}$, the expression (53) itself is modified. Thus, for non-magic nuclei both the energies and spectroscopic factors should change due to inclusion of the tadpoles.

4 Conclusion

In this work we have developed a consistent scheme of calculating self-energies and gap terms in non-magic nuclei, which takes into account the particle-phonon coupling in the g^2 approximation. In addition to usual non-local terms, this approach explicitly considers tadpole terms. A general set of equations for the phonon corrections under discussion is obtained, which involves no new parameters apart those used in the self-consistent calculation of the "zero" mean field, *i.e.* that without phonon contributions. This set is simplified for the case of corrections induced by low-lying surface phonons in the "small-d approximation" ($d^{(1,2)} \ll g$). This approximation means that the admixture of *pp*-phonons to *ph*-phonons under consideration, *i.e.* the contribution of the $d^{(1,2)}$-vertexes as compared to the g-vertex, can be neglected. A closed integral equation for the *ph*-tadpole K^{ph} as well as an integral relation for the *pp*-tadpole K^{pp} in terms of K^{ph} are obtained. Even in such a simplified case the obtained relations turn out to be much more complicated than those for magic nuclei.

As an application of the relations obtained, the effect of the phonon tadpoles on single-particle strength distribution, the single-particle energies and gap values has been analyzed. The relations of refs. [10] and [8], where only usual non-local self-energies (in the g^2 approximation) were taken into account, have been modified by the explicit contribution of the tadpole terms. The set of obtained equations has been analyzed. Even before numerical calculations, the analysis of the structure of these equations and their comparison with those for magic nuclei has led us to the conclusion that the tadpole terms should significantly modify the nuclear characteristics under consideration. Indeed, on the one hand, this comparison has shown that the *ph*-tadpole K^{ph} in non-magic nuclei should be close to that in magic ones. On the other hand, the expression for the *pp*-tadpole K^{pp} in terms of K^{ph} indicates that the former has no smallness as compared with the second one. Therefore, we could rely on the experience of calculations in ref. [3] for magic nuclei where the contribution of the tadpole term, *e.g.*, to the single-particle energies is significant. Note that, contrary to magic nuclei, in non-magic ones the tadpoles should also modify the spectroscopic factors. A preliminary analysis of the modified gap equation shows that here the tadpole could be significant, too. This is important for the problem of the pairing nature in finite nuclei.

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