

Lambda-CDM model and small-scale-cosmology “crisis”: from astrophysical explanations to new fundamental physics models

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I review briefly three problems where tensions between predictions based on numerical simulations of the Lambda-CDM prototype and observations at small (galactic) scales occur. These include (i) the core-cusp problem on the galactic profiles, (ii) the missing satellite problem, and the (iii) too-big-to-fail problem. I explain what these problems are and present potential resolutions, first through some astrophysical mechanisms, which however, as I argue, fail to alleviate completely the problems, at least currently. Then, I discuss fundamental modifications of the Lambda-CDM model, through the inclusion of self-interacting dark matter (SIDM). I argue that a simple model of SIDM, with (warm) self-interacting right-handed neutrinos (RHN), that exist in minimal extensions of the Standard model of particle physics, appears promising in providing a resolution of the aforementioned “small-scale-Cosmology crisis”, in particular the core-cusp problem, and an observationally consistent description of the core-halo structure in galaxies.

Keywords: Cosmological observations; self-interacting interacting dark matter; right-handed-neutrinos.

1. Introduction

A plethora of precision cosmological observations¹, based on type Ia supernovae, cosmic microwave background (CMB) radiation, large scale structure, baryon acoustic oscillations and strong and weak lensing data, point towards the fact that only $\sim 4\%$ of the current energy budget of the Universe consists of the known form of matter (mainly baryons), while the rest seems to be of unknown nature, specifically $\sim 69\%$ dark energy (DE) and $\sim 27\%$ dark matter (DM). The current universe appears to enter an acceleration phase (for a second time after inflation), and the data point towards an equation of state for the dark energy to be close to that of a de Sitter space-time, $w \simeq -1$, corresponding to a positive cosmological-constant, $\Lambda > 0$. The best fit to the large-scale data seems to be provided by the so-called Λ -Cold-Dark-Matter (Λ CDM) model for the Universe, according to which the DE is provided by a positive cosmological constant $\Lambda > 0$, while the DM content of the Universe is thermal, of “Cold” type, corresponding to a mass range for the (assumed) dominant DM particle species $m_\chi > 100$ keV. Several particle physics models, in particular Supersymmetry, provide candidates for this type of DM, although currently there is no experimental evidence from collider experiments that such extensions of the Standard Model (SM) are realised in nature², thus leaving us only with Cosmological evidence for the existence of DM.

Despite the success of the Λ CDM model at large (cosmological) scales, however, there are tensions at galactic scales which, if not due to observational fluctuations, or admitting astrophysical resolution, may hint new physics beyond the Λ CDM

paradigm. In this talk we mention briefly some of these challenges, and discuss their potential resolution by modifying the Λ CDM model via the inclusion of self interactions in the Dark Matter sector.^a

2. Small Scale Challenges of the Λ CDM Model

There are three challenges to the Λ CDM model at galactic scales, collectively called some times as “*small-scale cosmology crisis*”, which we focus our attention upon in this section:

(i) *The Core-Cusp problem*, that is, a discrepancy between the observed dark matter density profiles of low-mass galaxies and the density profiles predicted by cosmological N-body simulations. Characteristically, all the Λ CDM-based (DM only) simulations form dark matter halos which have “cuspy” dark matter distributions, with the density increasing steeply, i.e. as $\rho \propto r^{-1}$, at small radii. This is, e.g., evidenced in the standard Navarro-Frenk-White (NFW) DM profile.⁵ On the contrary, the rotation curves of most of the observed dwarf galaxies indicate flat central density profiles (“cores”).⁶

(ii) *The “missing satellite or dwarf galaxy problem”*, which pertains to a discrepancy between observations and Λ CDM-based numerical cosmological simulations, which predict the evolution of the distribution of matter in the universe. Such simulations point towards a hierarchical clustering of DM, where smaller halos merge to form larger halos. Although there seem to be enough observed normal-sized galaxies to account for such a numerical distribution, the number of (observed) dwarf galaxies is orders of magnitude lower than that expected from the simulations.⁷

(iii) *The too big to fail problem*, which is a discrepancy arising between the most massive subhaloes predicted in (dissipationless) Λ CDM simulations and the observed dynamics of the brightest dwarf spheroidal (dSph) galaxies in the Milky way. In other words, the Λ CDM simulations predict that the most massive subhaloes of the Milky way are too dense to host any of its bright satellites, with luminosity higher than 10^5 the luminosity of the Sun.⁸

The problems may admit astrophysical explanations, and some of them may be alleviated by improved measurements. For instance, many studies have shown that including baryonic feedback (particularly feedback from supernovae and active galactic nuclei) can “flatten out” the core of a galaxy’s dark matter profile, since feedback-driven gas outflows produce a time-varying gravitational potential that transfers energy to the orbits of the collisionless dark matter particles.⁹ The missing satellite problem also admits standard Astrophysical solutions.¹⁰ One is that the smaller halos do exist but only a few of them end up becoming visible because they have not been able to attract enough baryonic matter to create a visible dwarf

^aWe do not discuss here other recent challenges of Λ CDM, e.g. those associated with the different value of the Hubble parameter H_0 , obtained between CMB/Planck-Coll. measurements¹ and Cepheid-galaxies observations³, which could be alleviated via modified DE models, such as the “running vacuum”.⁴

galaxy. In support of this, Keck observations in 2007 of eight newly discovered ultra-faint Milky Way dwarf satellites showed that six were almost exclusively composed of DM, around 99.9% (with a mass-to-light ratio of about 1000).¹¹ Such ultra-faint dwarfs substantially alleviate the discrepancy between the predicted and observed numbers of satellites around the Milky Way, but there are still discrepancies by a factor of about four too few dwarf galaxies over a significant range of masses. See also the recent work Ref. 12 in this direction. Other solutions may be that dwarf galaxies tend to be merged into or tidally stripped apart by larger galaxies due to complex interactions. This tidal stripping has been part of the problem in identifying dwarf galaxies in the first place, which is an extremely difficult task since these objects have low surface brightness and are highly diffused, so much that they are virtually unnoticeable. Finally, the Too-Big-to-Fail Problem may also be tackled by taking into account baryonic feedback.

From the point of view of the Λ CDM simulations, all three problems have their root in the fact that the cold DM particles, which the Λ CDM simulations rely upon, have too short free streaming length during the epochs of galaxy formation, and therefore they form too clumped and too many structures compared to those observed. The interesting question therefore is whether these problems, especially the core-cusp problem, provide hint towards a paradigm shift of the underlying fundamental physics, implying deviations from Λ CDM framework. One such deviation is to include self interactions among the DM particles.

3. Resolution through Self Interacting Dark Matter Models

Self interactions have been argued to play an important rôle in galactic structure already in Ref. 13. The original idea of Self-Interacting DM (SIDM) was implemented for CDM particles with rest masses above $1 \text{ MeV}/c^2$ (up to $10 \text{ GeV}/c^2$), consistent with the nature of the effective interactions and the mean free paths considered in that work. This way of thinking regarding self-interactions was applied uniquely on DM halo scales with typical densities of $10^{-2} M_{\odot}/\text{pc}^3$, suggesting that normalized total cross-sections of order $\sigma/m \sim 0.1 - 100 \text{ cm}^2/\text{g}$, would imply observational effects in the inner regions of the DM halos. It was also shown that a SIDM regime with these values of σ/m would generate shallower inner DM profiles, with a necessary reduction in the amount of sub-structures, thereby alleviating the core-cusp and the missing satellite problems of collisionless Λ CDM¹⁴. However, contemporaneously, some tension with upper limits in the DM cross sections as obtained from lensing studies on galaxy cluster scales emerged. In a subsequent work,¹⁵ motivated by updated analysis of the Bullet Cluster,¹⁶ new cosmological simulations within CDM were performed, with the aim of further scrutinizing the effects of SIDM on inner halo cores of galaxies and galaxy clusters. The authors of Ref. 15 concluded that $\sigma/m \sim 0.2 \text{ barn GeV}^{-1} = 0.1 \text{ cm}^2 \text{ g}^{-1}$ is consistent with all the observational constraints. In general, SIDM would make no difference from Λ CDM at large scales,

but individual galaxies would appear more cored and spherical, with higher velocity dispersion.

Stringent constraints of the interaction cross section of SIDM have been inferred in Ref. 17 by means of new DM observables in merging galaxies, taking into account *the DM drag*. In particular, the study of 72 mergers using this new technique imposed a more stringent upper bound on the SIDM cross section per unit DM mass, thus defining a new range for the total SIDM cross section^b

$$0.1 \leq \frac{\sigma_{\text{SIDM}}/m}{\text{cm}^2 g^{-1}} \leq 0.47 \quad (1)$$

This might lead to a potential resolution of the three “small scale cosmology problems” of DM, discussed previously. For instance, self interactions may tackle the core-cusp and too big to fail problems by reducing the central density. In this latter respect we mention the works of Ref. 20, according to which self interactions can lead to both deceleration and evaporation of a DM halo when the latter moves through a background of DM particles. This results in a shift of the halo’s centroid relative to the collisionless stars and galaxies.

4. Self-Interacting Right-Handed Neutrinos as (Warm) DM and Galactic Structure

A concrete model of SIDM, which may have significant effects on the galactic structure, and which plays an important rôle in particle physics, has been studied in Ref. 21. It is based on a class of self-interacting models of massive right-handed neutrinos that exist in minimal extensions of the standard model²². The model constitutes a microscopic (particle physics) realisation of previous ideas that populations of electrically neutral fermionic particles might play an important rôle for the galactic core-halo structure²³. In such a scenario, the DM fermion is played by the right-handed neutrino (RHN), which has a mass of a order a few tens of keV, and implies a universal and novel DM density profile (*compact core - dilute halo*), with important implications for the very central and halo regions of galaxies. The model is in agreement with observations, provided sufficiently strong self-interactions among the RHNs are considered²¹. The Lagrangian of the right-handed neutrino sector, including gravity, reads (in units $\hbar = c = 1$, which we use throughout here):

$$\mathcal{L} = \mathcal{L}_{GR} + \mathcal{L}_{N_{R1}} + \mathcal{L}_V + \mathcal{L}_I \quad (2)$$

^bHowever, some tension with the upper bound of (1) still remains, since the analysis and assumptions of Ref. 18 on their study of the Abell Cluster 3827 have been questioned in Ref. 19, which presented corrected estimates for the SIDM cross section: $\sigma_{\text{SIDM}}/m \simeq 3 \text{ cm}^2 g^{-1}$, when self interactions result in a drag force, and $\sigma_{\text{SIDM}}/m \simeq 1.5 \text{ cm}^2 g^{-1}$ in the case of contact interactions.

where

$$\begin{aligned}\mathcal{L}_{GR} &= -\frac{R}{16\pi G}, \quad \mathcal{L}_{N_{R1}} = i\bar{N}_{R1}\gamma^\mu \nabla_\mu N_{R1} - \frac{1}{2}m\bar{N}_{R1}N_{R1}, \\ \mathcal{L}_V &= -\frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \frac{1}{2}m_V^2 V_\mu V^\mu, \quad \mathcal{L}_I = -g_V V_\mu J_V^\mu = -g_V V_\mu \bar{N}_{R1}\gamma^\mu N_{R1},\end{aligned}\quad (3)$$

with R the Ricci scalar for the static spherically symmetric metric background considered in Ref. 21, ∇_μ the gravitational covariant derivative acting on a Majorana spinor, m the mass of the sterile RHN neutrino, and m_V the mass of the dark vector boson providing the carrier of the self-interactions among RHN. When the Lagrangian (3) is expressed in terms of the Majorana field $N^c = N$, the massive-vector-mesons V_μ couple to the axial current $\bar{N}\gamma^\mu\gamma^5 N$.

In general one may add to (2) a Yukawa term, coupling RHNs to the active neutrino sector (see, e.g., the case of ν MSSM in Ref. 22)

$$\mathcal{L}_{\text{Yuk}} = \sum_{\alpha, I} F_{\alpha I} \bar{\ell}_\alpha N_{RI} \phi^c + \text{h.c.}, \quad I = 1, 2, 3, \quad \alpha = e, \mu, \tau, \quad (4)$$

where ℓ_α are the lepton doublets of the SM, $\alpha = e, \mu, \tau$, $F_{\alpha I}$ are appropriate Yukawa couplings, and ϕ^c is the SM conjugate Higgs field, *i.e.* $\phi^c = i\tau_2\phi^*$, with τ_2 the 2×2 Pauli matrix. If one ignores self interactions among the RHN, then, upon considering such a coupling, one obtains stringent X-ray and Big Bang Nucleosynthesis (BBN) constraints²⁴ of the mixing angle and mass of N_{R1} , given that (4) implies decays of the heavy neutrinos $N_I \rightarrow \nu H$, where H is the Higgs excitation field, $\phi = v + H$, with v the vacuum expectation value. In the context of ν MSSM²², the decay time of the lightest of the RHN neutrinos can be longer than the age of the universe, hence the latter can be considered as stable for all practical purposes, thus playing the rôle of the entirety of DM. This assumption requires sufficiently low mixing with the standard model sector, with the corresponding mixing angle given by $\sin^2\theta = \sum_{\alpha=e,\mu,\tau} \frac{\langle\phi\rangle^2 F_{\alpha I}}{m^2}$. On the other hand, the requirement that the sterile neutrino constitutes the entirety of DM, requires it to be produced in sufficient amounts in the early Universe, and hence, if no self-interactions are assumed, there are lower bounds for θ , depending on the RHN mass m . The best fit to date²⁴ points to a narrow allowed region in the RHN mass $m \sim 10$ keV and mixing angle $\sin^2(2\theta) \sim 10^{-12}$ (slightly wider regions for m are allowed²⁵ if one uses the galactic density profile of Ref. 23).

In the RHN SIDM model of Ref. 21 the mixing with the SM sector was ignored, $F_{\alpha 1} = 0$, in which case the lightest neutrino is absolutely stable. The important feature are the self-interactions of the right-handed neutrino, which ensured phenomenologically correct values for the radius and mass of the galactic core within certain mass density profiles²³. Such interactions among RHNs also lead to production mechanisms of the RHN DM that no longer require lower bounds on the mixing angle θ with the standard model sector, hence the stringent X-ray constraints of Ref. 24 can be evaded. The analysis of Ref. 21 showed that, under the assumption of the absence of a central black hole in the galaxy²³, the allowed range for RHN

DM mass that fit the Milky Way observables is: $47 \text{ keV}/c^2 < m < 350 \text{ keV}/c^2$, the upper bound being set by reaching the critical core mass for gravitational collapse, $M_c^{cr} \propto M_{\text{Pl}}^3/m^2 \approx 4.4 \times 10^6 M_\odot$. One also concludes that the inclusion of sufficiently strong interactions in the dark sector among the sterile neutrinos can lead to significantly more compact cores and higher central degeneracies than in the free case^{23c}.

The total RHN-RHN scattering cross-section in the quantum core of the Galaxy has been calculated in Ref. 21 in a perturbative regime $g < 1$ for the dimensionless interaction coupling, $\sigma_{core}^{tot} \approx 29 \frac{(g_V/m_V)^4}{4^3 \pi} m^2$ ($p^2/m^2 \ll 1$). This allows for a comparison of these results with the generic ones existing in the literature and mentioned in the previous section, on the required range of the total cross section per unit DM mass, σ/m , in order to resolve the small-scale Cosmology ‘‘crisis’’ (Eq. (1)). To put things in perspective, one can normalize the interaction field strength in terms of the visible sector (SM) weak interaction dimensionfull coupling, the Fermi ‘‘constant’’ G_F . Thus, if, e.g. one constrains the total cross-section to the N-body simulation value $\sigma^{tot}/m = 0.1 \text{ cm}^2/\text{g}$ (or, in general, to lie in the region (1) discussed above), the coupling constant \overline{C}_V would be constrained to the value $\overline{C}_V \in (2.6 \times 10^8, 7 \times 10^8)$, for RHN-DM masses in the range $m \in (47, 350) \text{ keV}$. It worths noticing that for $C_V \sim 10^8 G_F$, the mass of the massive-vector meson would be constrained to values $m_V \lesssim 3 \times 10^4 \text{ keV}$, in order to satisfy $g_V \lesssim 1$ as requested by the self-consistency of the perturbation scheme applied to the computation of the cross-section²¹.

As stressed in Ref. 21, the inclusion of baryonic matter is not expected to change the basic conclusions that the introduction of WDM fermion self interactions affects the core/halo structure and in particular induces higher central degeneracies and higher compactness of the inner quantum core of galaxies. Moreover, the model of self-interacting sterile neutrinos provides a natural resolution to the core-cusp problem of DM at small scales, because the density profiles based on fermionic phase-space distributions develop always an extended plateau on halo scales, in a way that resemble Burkert or cored Einasto profiles. Another important feature is the fact that the right-handed neutrino DM mass is ‘colder’ by a few keV as compared to most of the Warm Dark Matter (WDM) models available in the literature, which implies that the model does not suffer from such standard WDM problems.

It goes without saying, that the presence of self-interacting right-handed neutrinos with a mass range of a few tens of keV may co-exist harmonically with other types of DM, given that the latter may consist (like ordinary matter) of more than one dominant species. In this way, at large scales, the predictions of the Lambda-CDM model might still be valid, while at galactic scales the r ole of self-interacting

^cIf the assumption that the RHN DM co-exists with a central black hole is made, then one may accommodate the narrow window of Ref. 24 in the analysis of Ref. 21, although, as already mentioned, the existence of self interactions of RHN evades the constraints of Ref. 24.

RHN WDM may play a crucial rôle in providing agreement of the predicted galactic structure and the pertinent DM distribution with observations.

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