



Building Qudit-based Quantum Computing Architecture using SRF Cavities

This manuscript has been authored by Fermi Research Alliance, LLC under Contract No. DE-AC02-07CH11359 with the U.S. Department of Energy, Office of Science, Office of High Energy Physics.

Tanay Roy

SQMS division, Fermilab

15 December 2023

Report number: FERMILAB-SLIDES-23-0409-SQMS

Why Quantum Computing?

Frontier

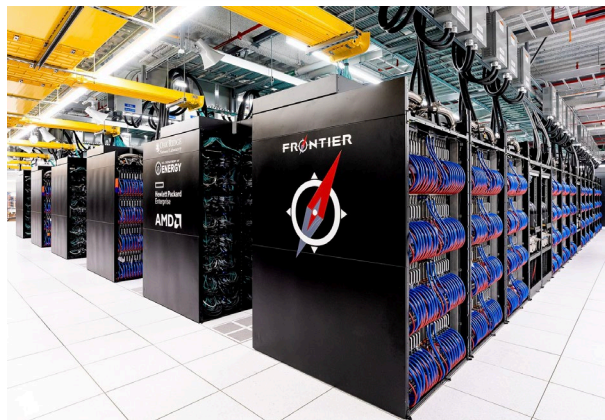


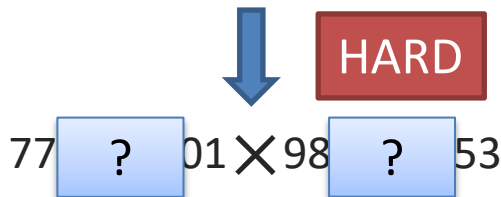
Image: Wikipedia

1.2×10^{18} calculations / sec

Not efficient for
all problems

1. Prime Factorization

762904558518855853



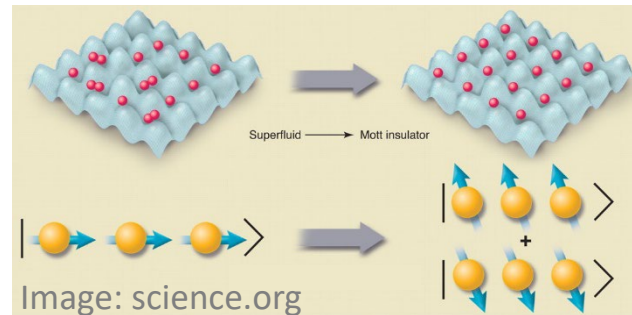
Shor's factoring
algorithm 1994



Image: mit.edu

Build a Quantum
Computer

2. Quantum Simulation



Simulate one QM
system with another

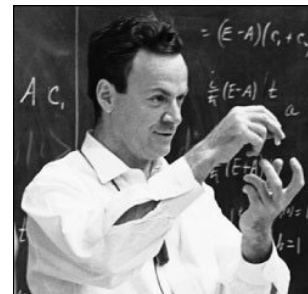
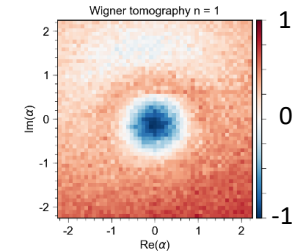
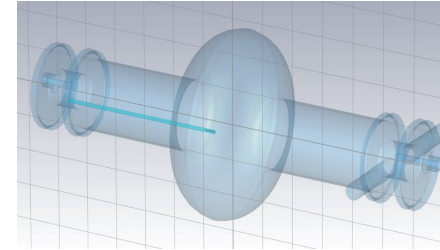
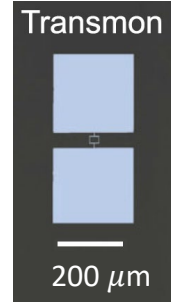
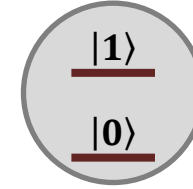


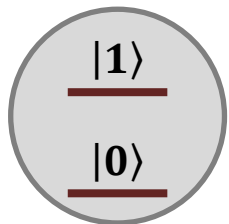
Image: needull.com

Outline

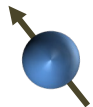
- ❖ Basic requirements and challenges
- ❖ Introduction to superconducting qubits
- ❖ Benefits of 3D SRF cavities
- ❖ Gate schemes and measurements
- ❖ Current achievements and outlook



Basic Requirements for a Quantum Computer

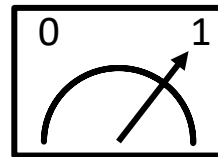


Quantum two
level systems

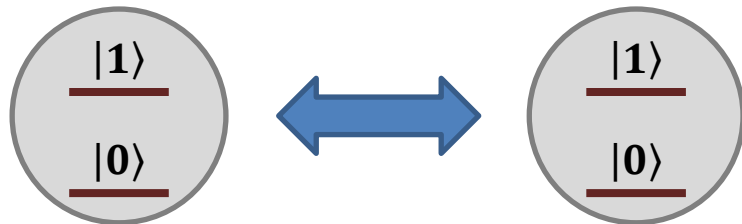


$$\alpha|0\rangle + \beta|1\rangle$$

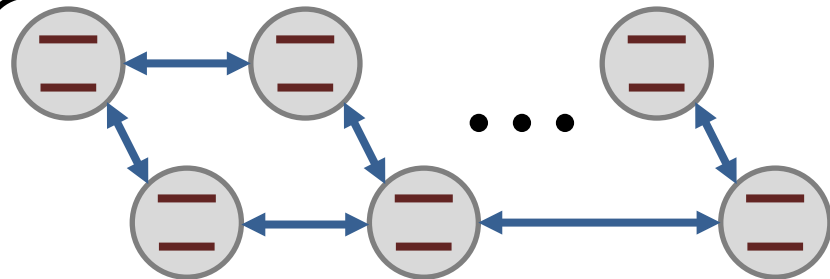
Create arbitrary
states



Measure
quantum states

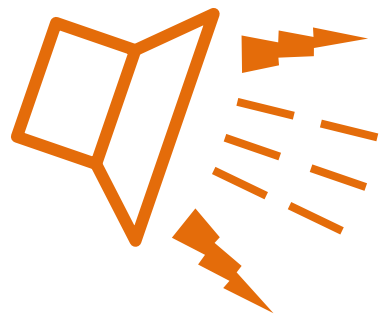


Couple multiple qubits



Scalable architecture

Challenges: Decoherence

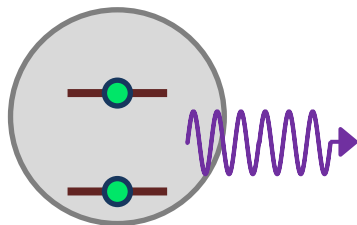


Decoherence

Relaxation (T_1)

Dephasing (T_ϕ)

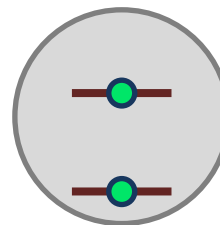
Noise



$$\alpha|0\rangle + \beta|1\rangle$$



$$|0\rangle$$

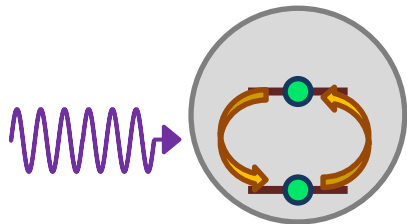


$$|0\rangle + e^{i\phi}|1\rangle$$



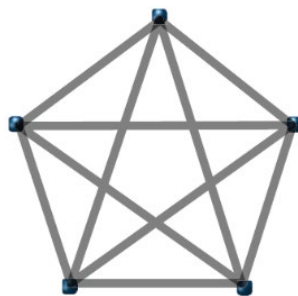
Incoherent mix of $|0\rangle$ and $|1\rangle$

Challenges: Gates and Connectivity



Fast & high-fidelity

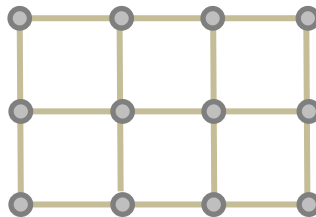
$\frac{\text{Coherence time}}{\text{Gate time}}$



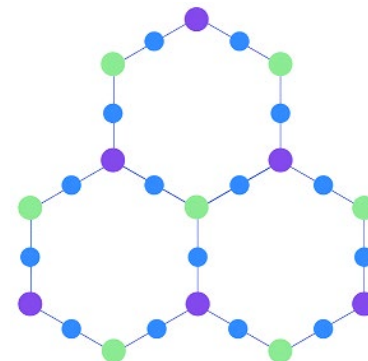
All-to-all



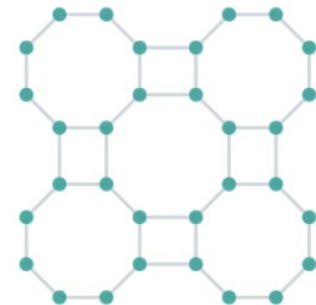
Linear chain



Square lattice



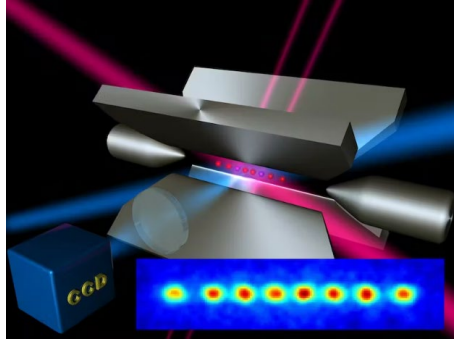
Heavy hexagon



Octagonal

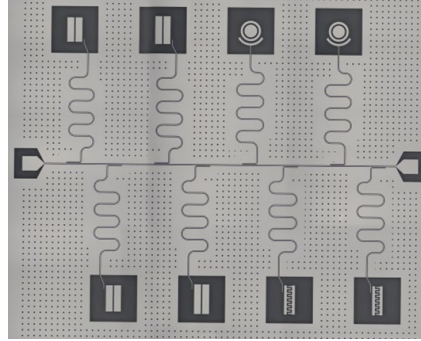
Different Platforms

Trapped ions



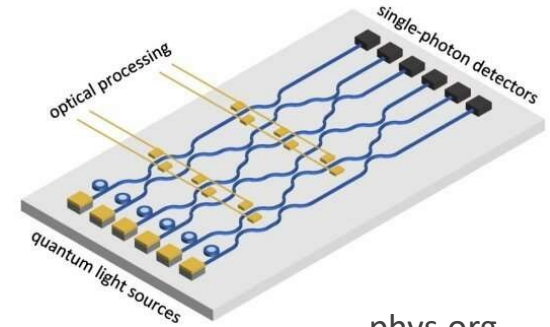
laserfocusworld.com

Superconducting circuits



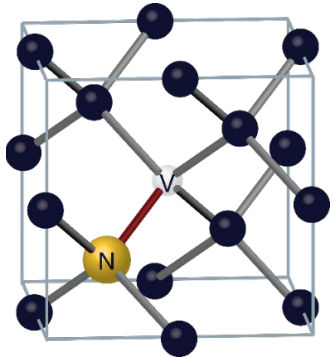
SQMS

Photonic crystals



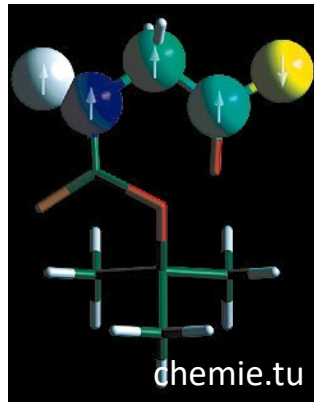
phys.org

NV centers



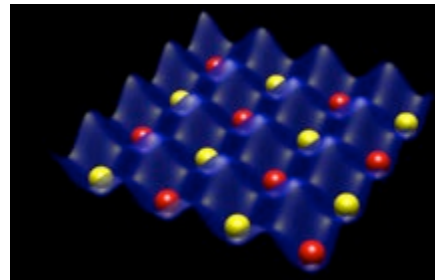
phys.org

NMR



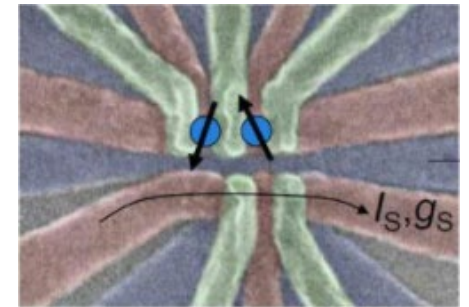
chemie.tu

Neutral atoms



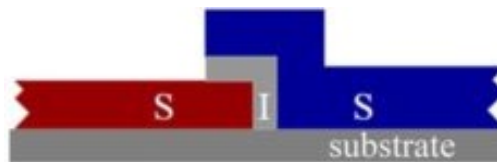
NIST

Quantum dots



sciencemag.org

Superconducting Circuits



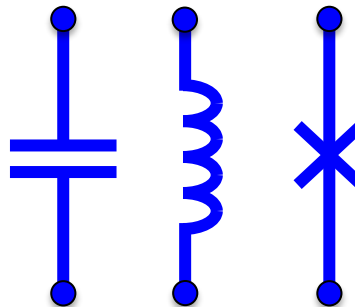
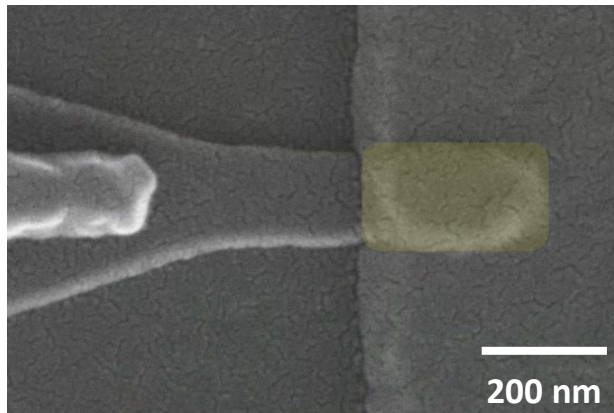
Josephson Junction

$$I(t) = I_0 \sin \delta(t)$$

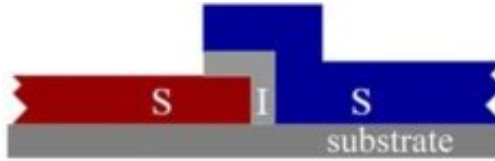
$$V(t) = \varphi_0 \dot{\delta}(t)$$

Lossless nonlinear inductor

$$L_J(I) = \frac{\varphi_0}{(I_0^2 - I^2)^{1/2}}$$



Transmon Circuit



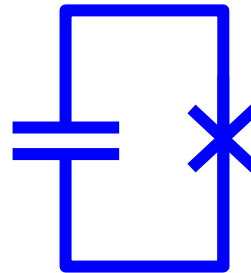
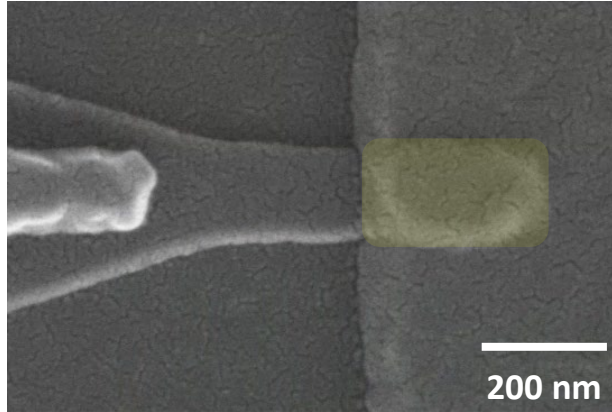
Josephson Junction

$$I(t) = I_0 \sin \delta(t)$$

$$V(t) = \varphi_0 \dot{\delta}(t)$$

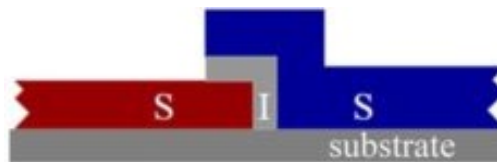
Lossless nonlinear inductor

$$L_J(I) = \frac{\varphi_0}{(I_0^2 - I^2)^{1/2}}$$



Transmon

Transmon Circuit



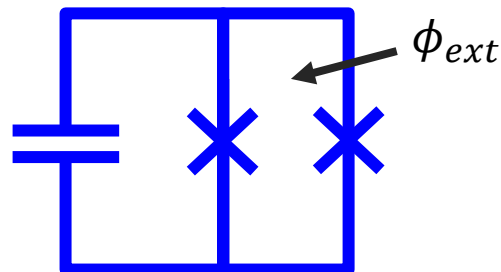
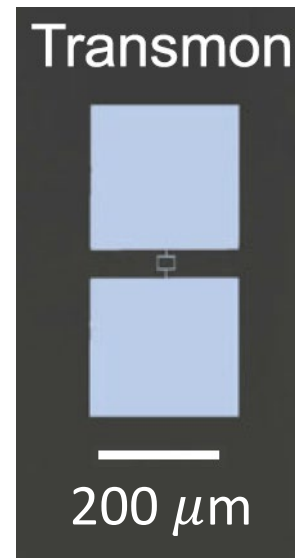
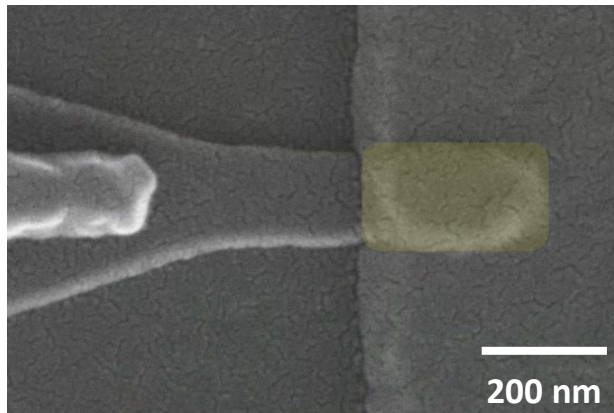
Josephson Junction

$$I(t) = I_0 \sin \delta(t)$$

$$V(t) = \varphi_0 \dot{\delta}(t)$$

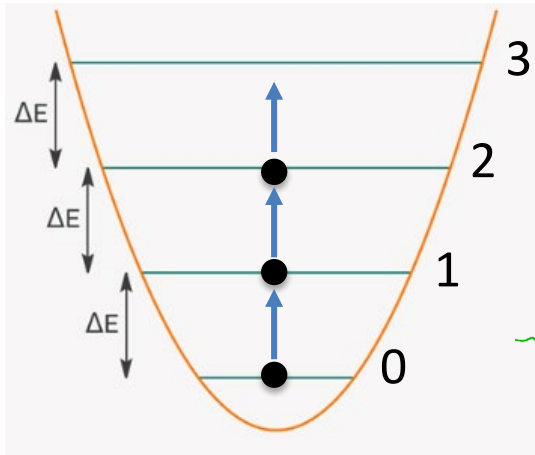
Lossless nonlinear inductor

$$L_J(I) = \frac{\varphi_0}{(I_0^2 - I^2)^{1/2}} \frac{1}{\cos \phi_{ext}}$$

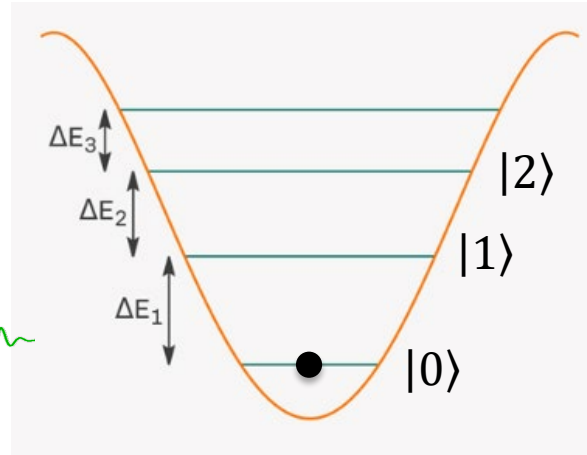
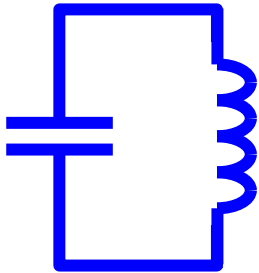


Tunable Transmon

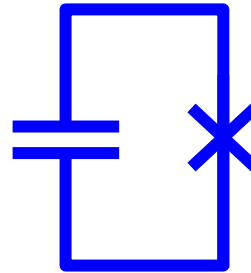
Transmon: Anharmonic Oscillator



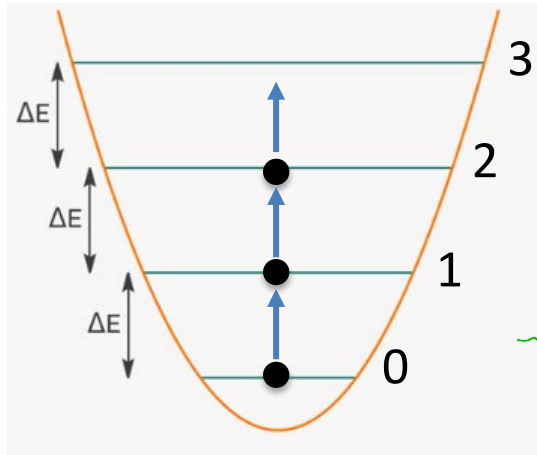
Harmonic Oscillator



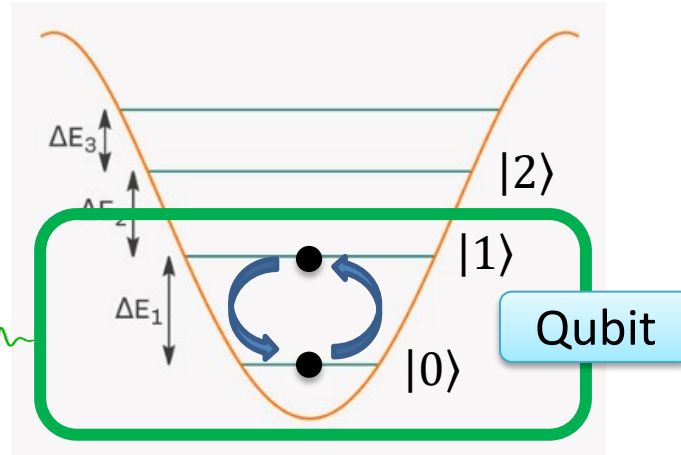
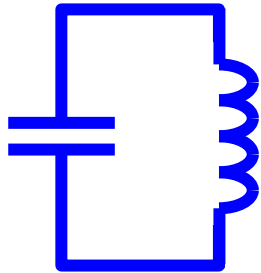
Anharmonic Oscillator



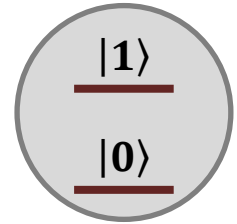
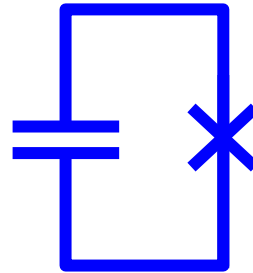
Transmon: Anharmonic Oscillator



Harmonic Oscillator



Anharmonic Oscillator



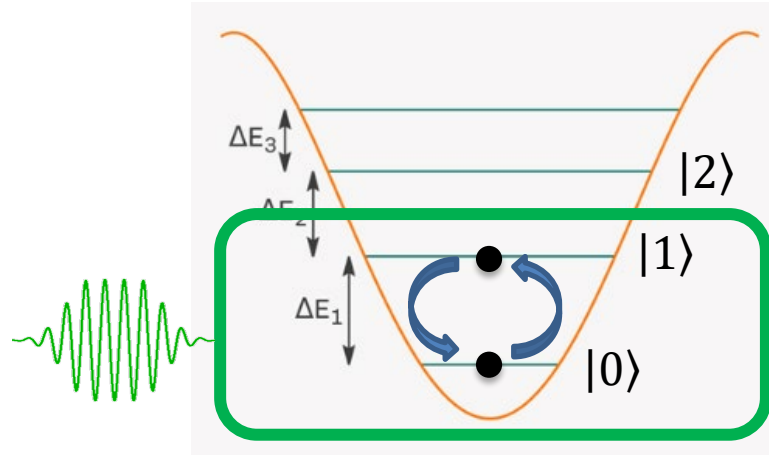
Operating Temperature

$$f_{01} \approx \frac{1}{2\pi\sqrt{L_J C}} \\ \sim 5 \text{ GHz}$$

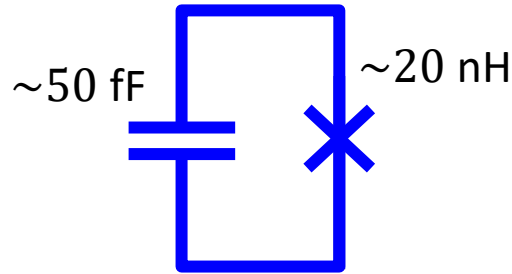
$$k_B T \ll h f_{01}$$

20 mK

$\sim 240 \text{ mK}$



Anharmonic Oscillator



Operating Temperature

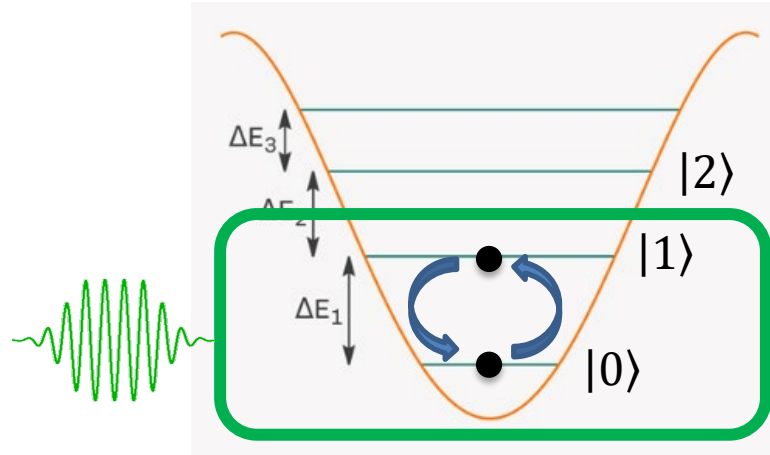
$$f_{01} \approx \frac{1}{2\pi\sqrt{L_J C}}$$

$\sim 5 \text{ GHz}$

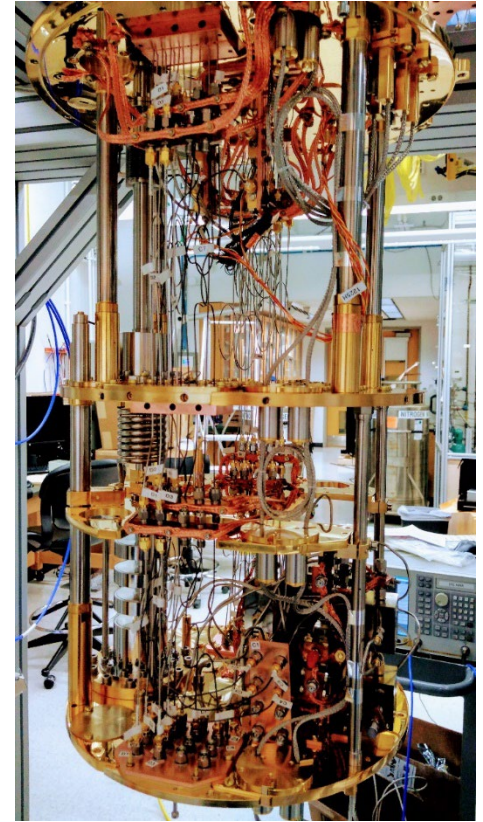
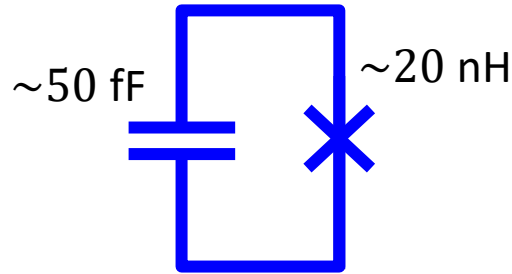
$$k_B T \ll h f_{01}$$

20 mK

$\sim 240 \text{ mK}$

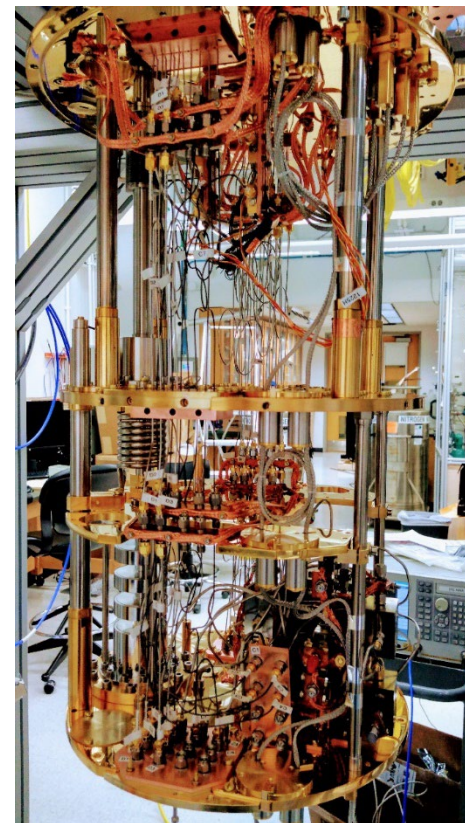
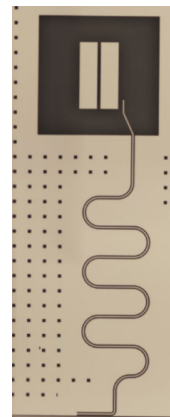
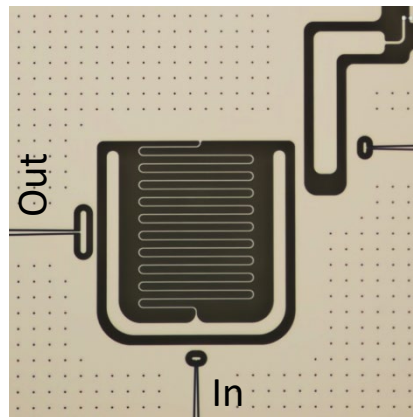
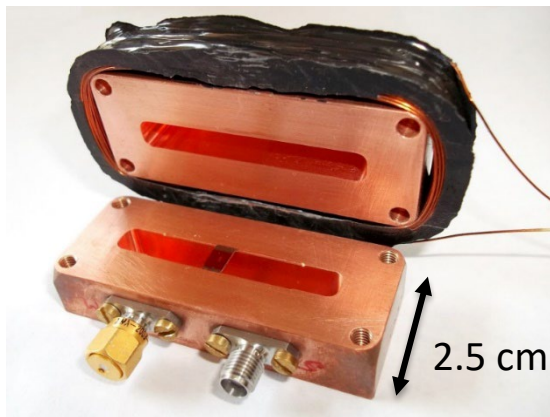
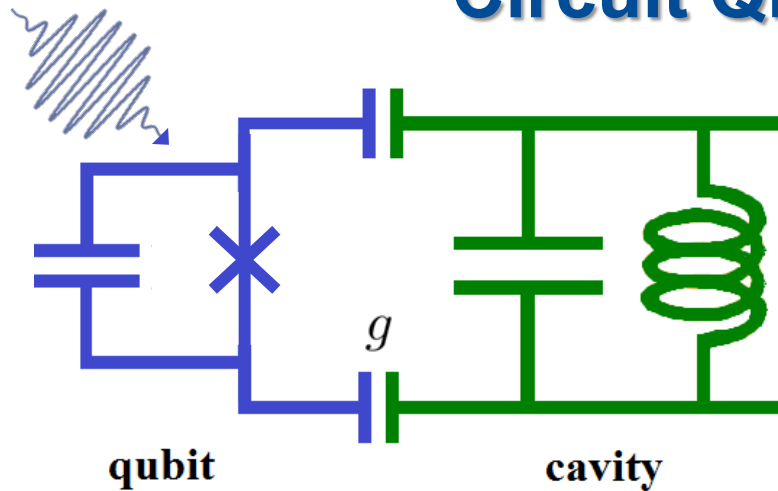


Anharmonic Oscillator



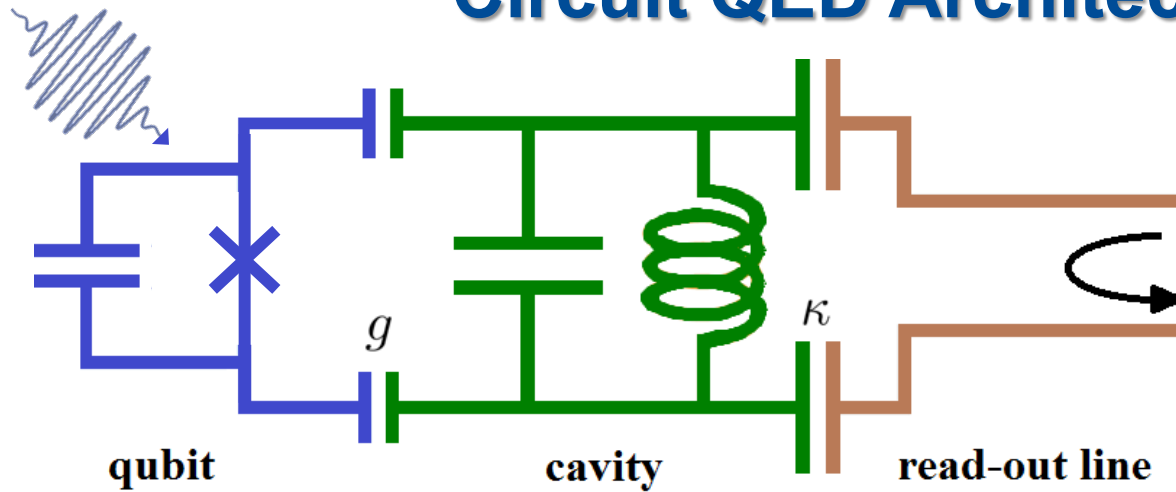
Dilution fridge $\sim 10 \text{ mK}$

Circuit QED Architecture



Dilution fridge ~ 10 mK

Circuit QED Architecture



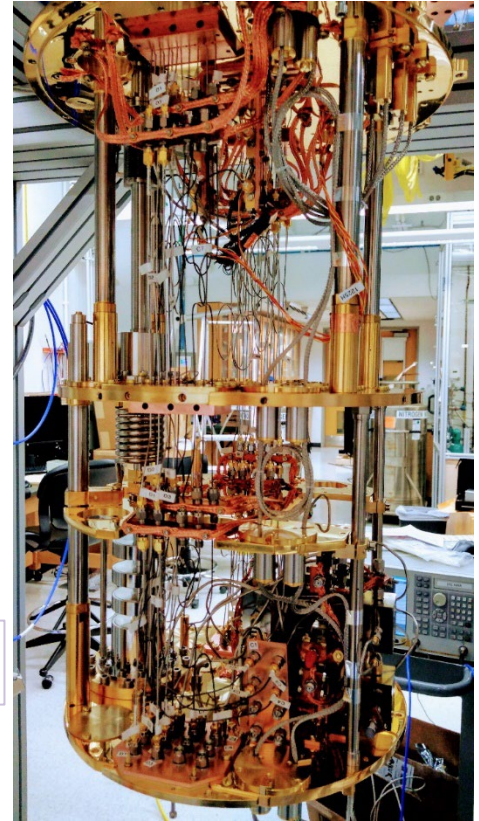
$$H = \frac{\omega_q}{2} \sigma_z + \omega_c a^\dagger a + g(a^\dagger \sigma_- + a \sigma_+)$$

$$\Delta = \omega_q - \omega_c$$

$$\approx \frac{\omega_q}{2} \sigma_z + \omega_c a^\dagger a + \frac{\chi}{2} (a^\dagger a) \sigma_z$$

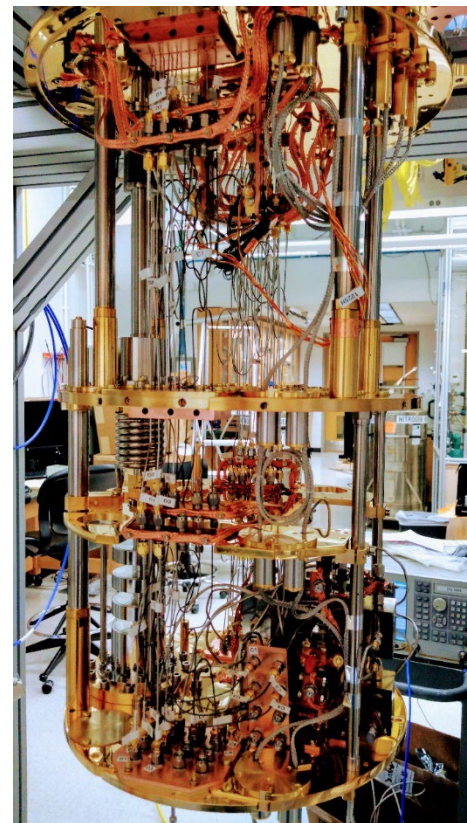
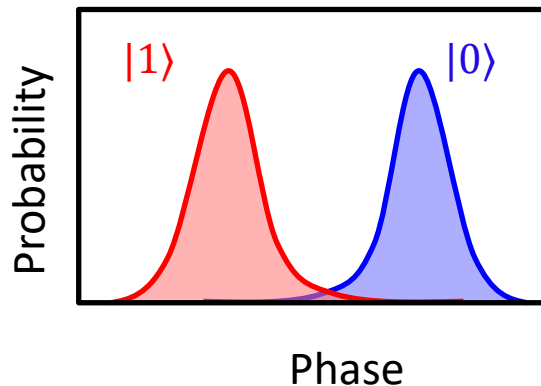
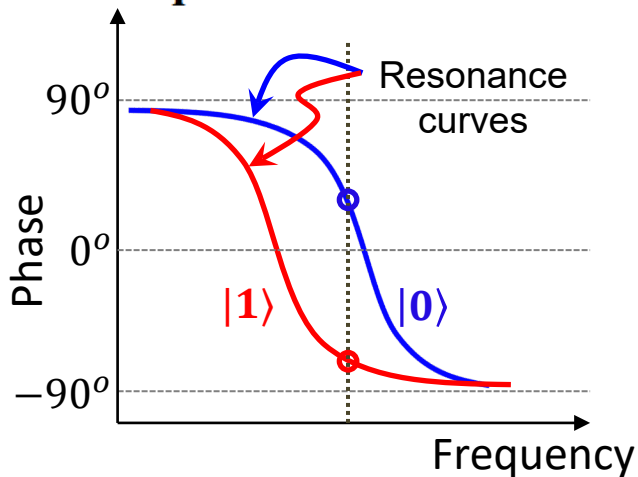
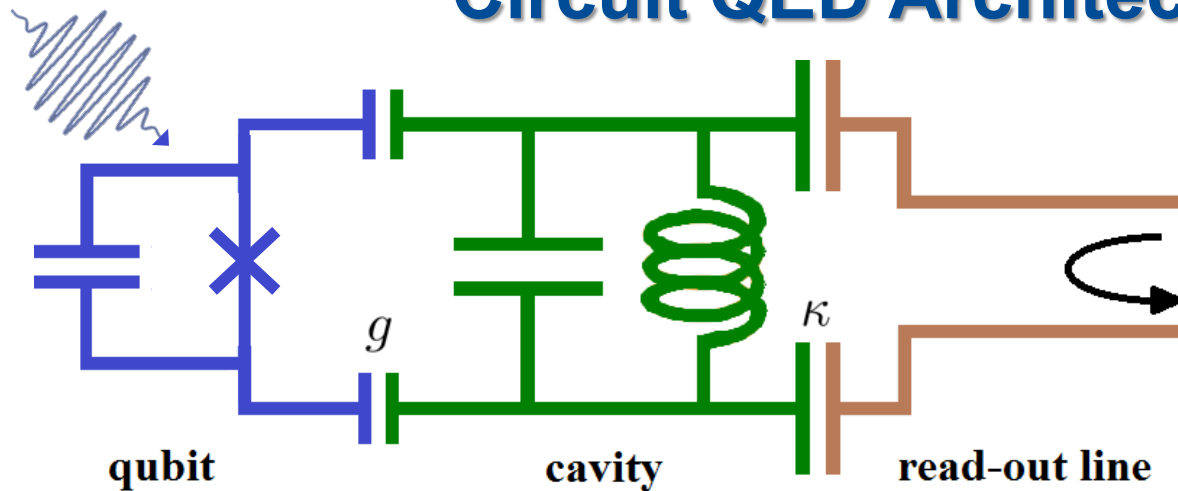
$$g \ll \Delta, \chi = 2g^2/\Delta$$

$$= \frac{\omega_q}{2} \sigma_z + \left(\omega_c + \frac{\chi}{2} \sigma_z \right) a^\dagger a$$



Dilution fridge ~ 10 mK

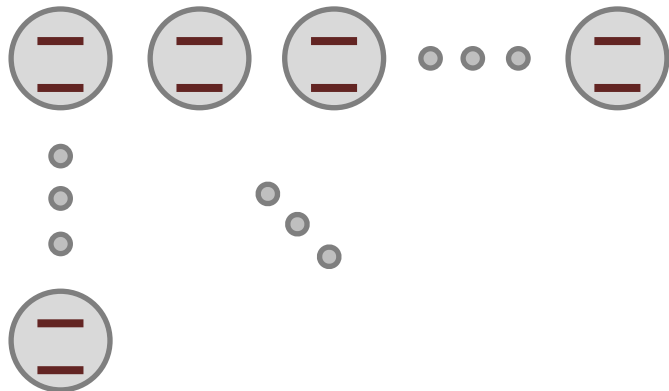
Circuit QED Architecture



Dilution fridge ~ 10 mK

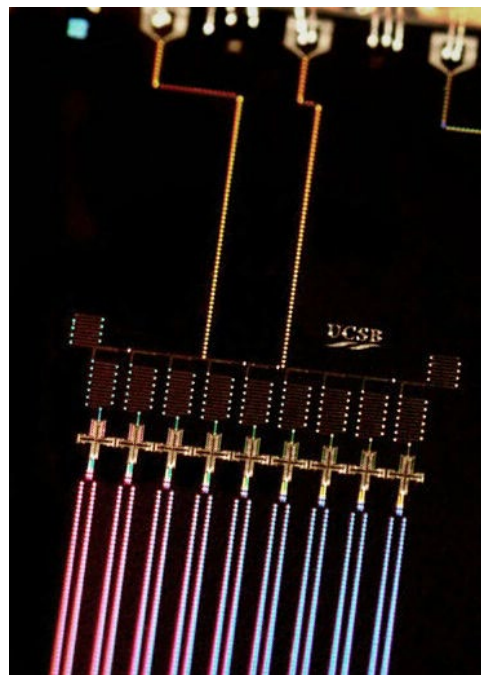
Traditional Multi-qubit Architecture

Linear or planar geometry

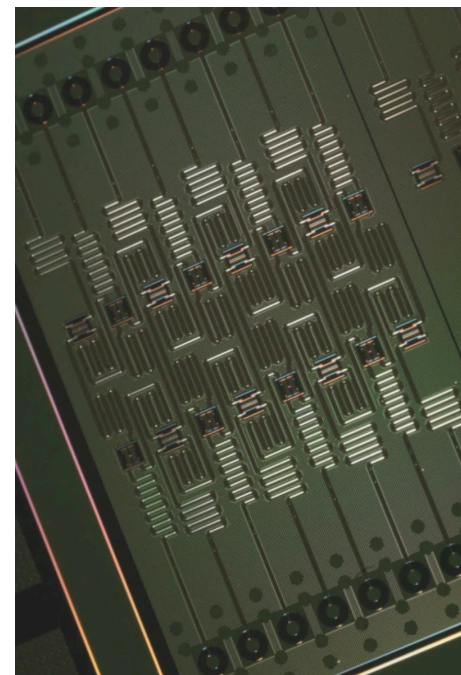


Computational space: 2^N

Can we do **better**?



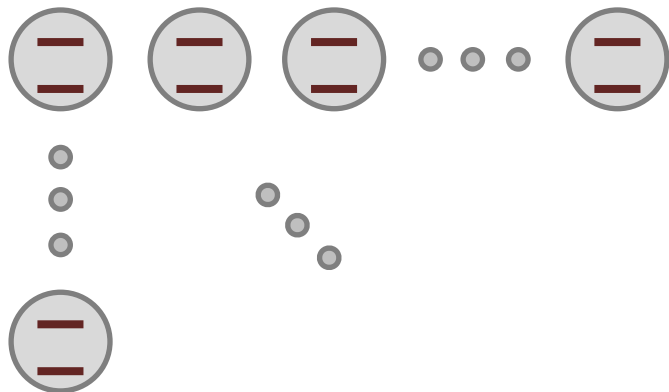
UCSB, Nature 519 (7541)



IBM

Traditional Multi-qubit Architecture

Linear or planar geometry

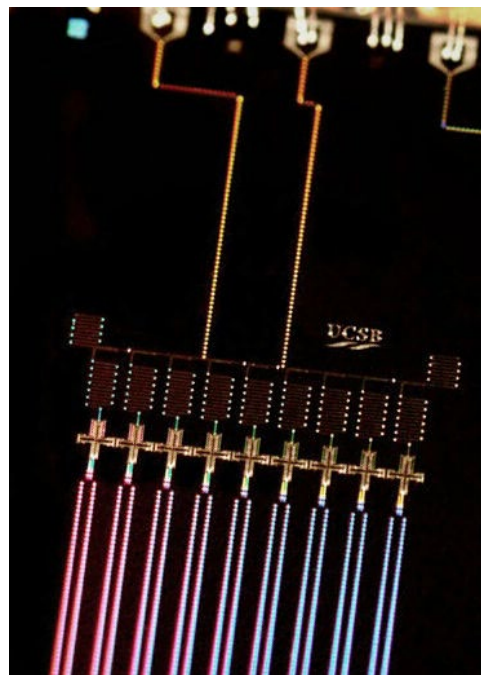


Computational space: 2^N

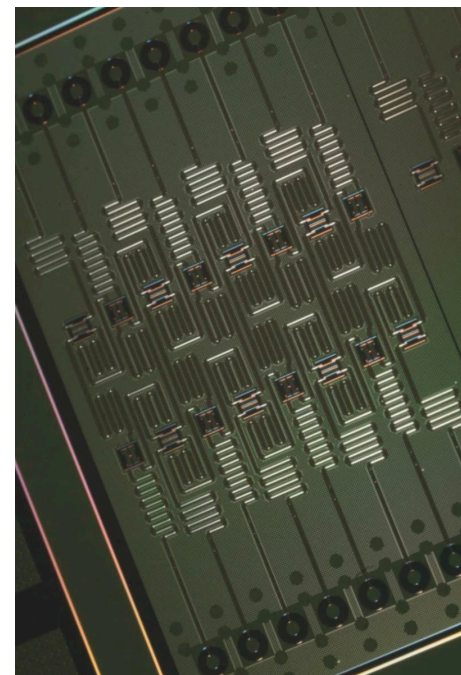
Can we do **better**?

Scaling: d^N , $d > 2$

Qudit



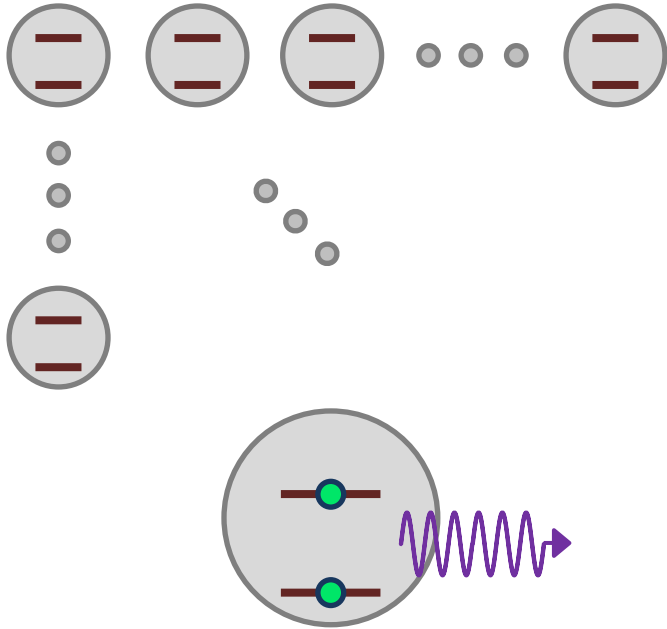
UCSB, Nature 519 (7541)



IBM

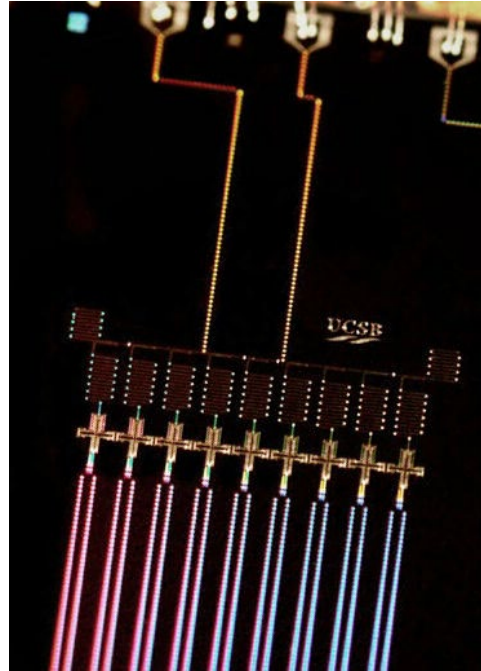
Problem of Relaxation

Linear or planar geometry

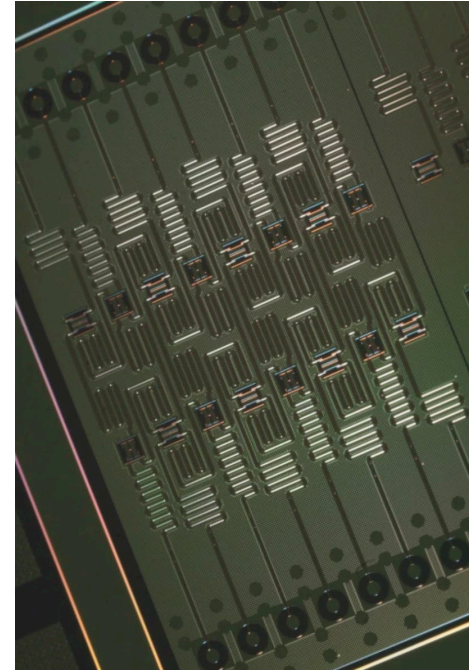


$$T_1 \sim 100 \mu\text{s}$$

Q: a few 10^6



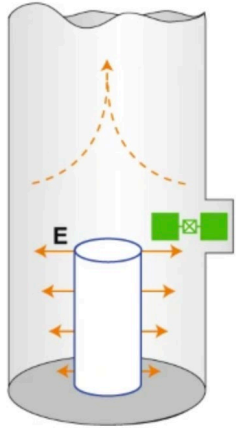
UCSB, Nature 519 (7541)



IBM

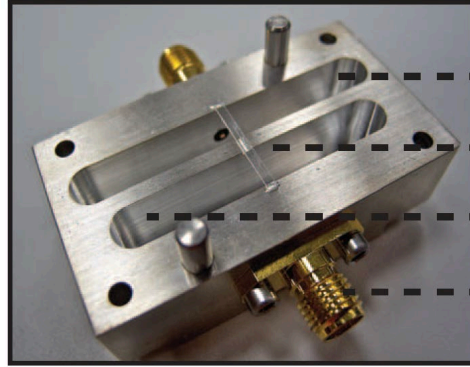
Can we do **better**?

Zoo of Cavities

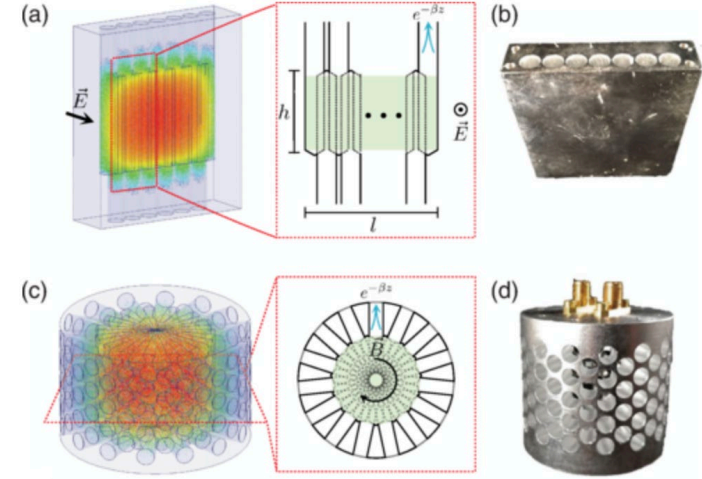


Nat. Phys. 16, 247

Yale, U. Pittsburgh

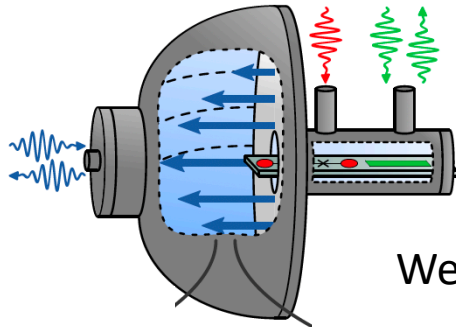


Science 342, 6158



PRL 127, 107701

U. Chicago, Rutgers



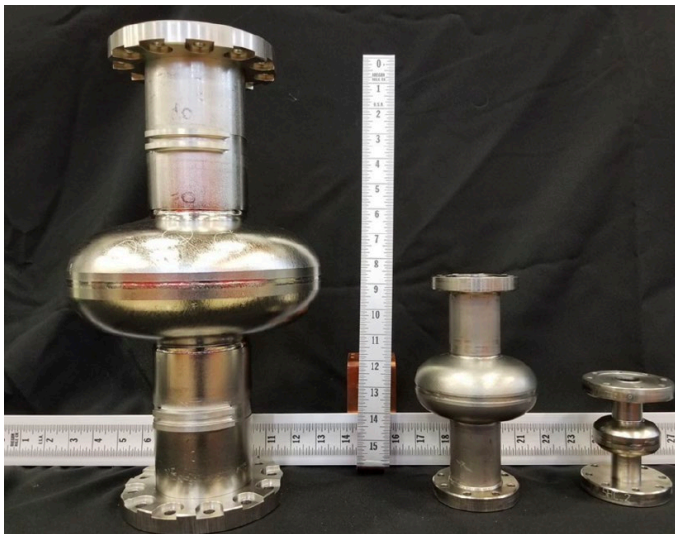
Weizmann

Under
exploration

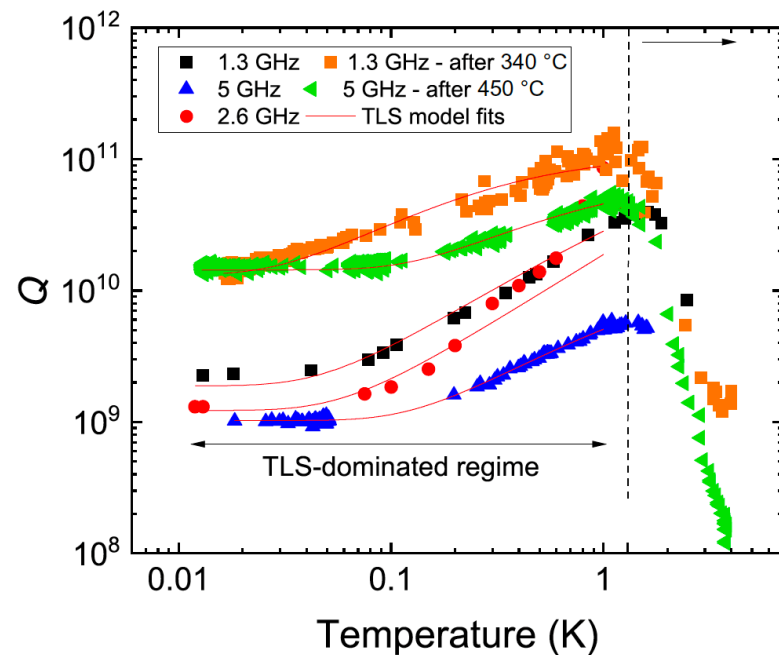
PRX Quant. 4, 030336

Tanay Roy - Fermilab

High-Q 3D SRF Cavities



Romanenko et al. PRApplied 13, 034032



1.3 GHz SRF: $Q > 10^{11}$ at 1 K



$T_1 > 2$ s

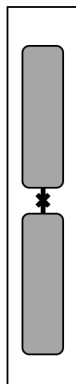
5 GHz SRF: $Q > 10^{10}$ at 10 mK



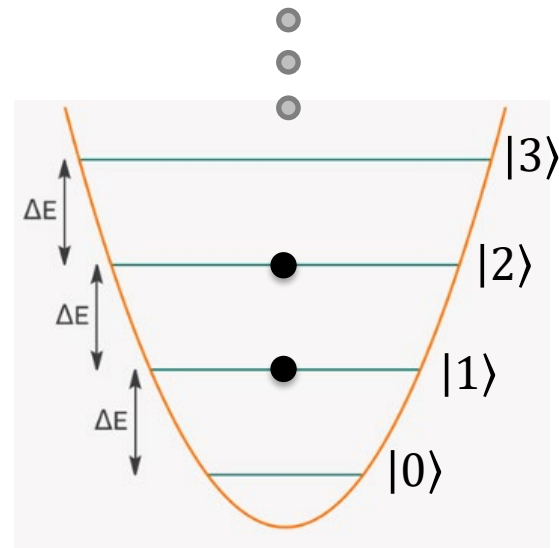
$T_1 > 300$ ms

**>1000 times better than
transmons**

High-Q 3D Cavities as Qudits



Romanenko et al. PRApplied 13, 034032



Qudit

Still much better than transmon qubits

$$T_1^{|1\rangle} > 300 \text{ ms}$$

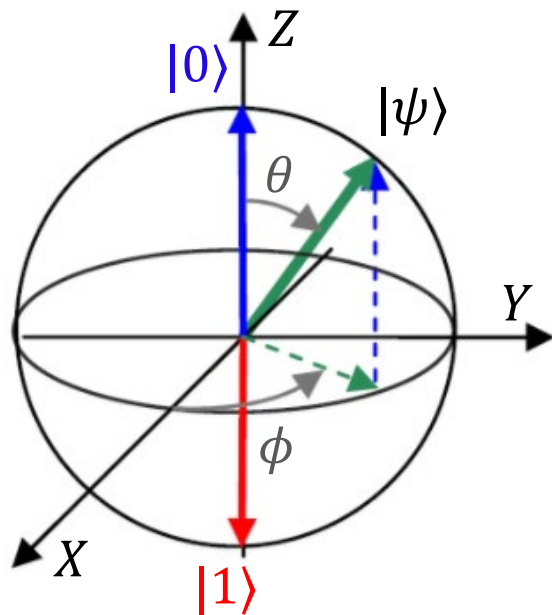
$$T_1^{|2\rangle} > 150 \text{ ms}$$

$$T_1^{|n\rangle} > T_1^{|1\rangle}/n$$

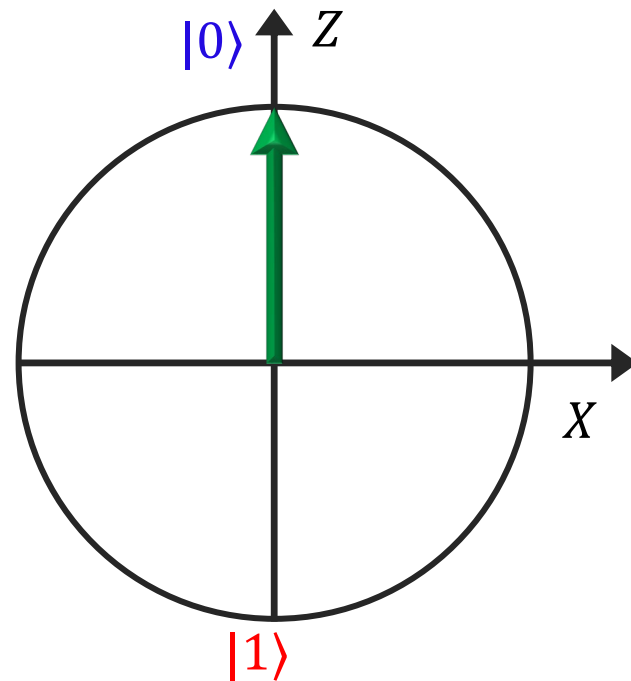
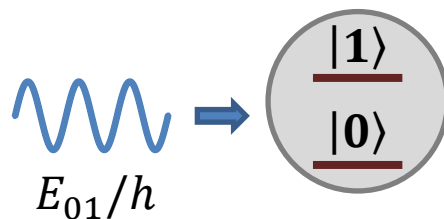
$$T_1^{|10\rangle} > 30 \text{ ms}$$

Qubit Visualization

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$



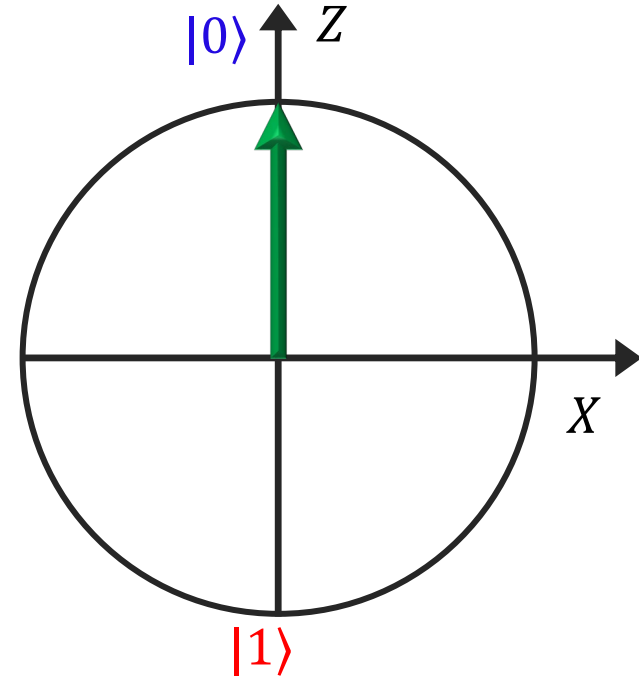
Bloch Sphere



Rabi Oscillation

Single-Qubit Gates

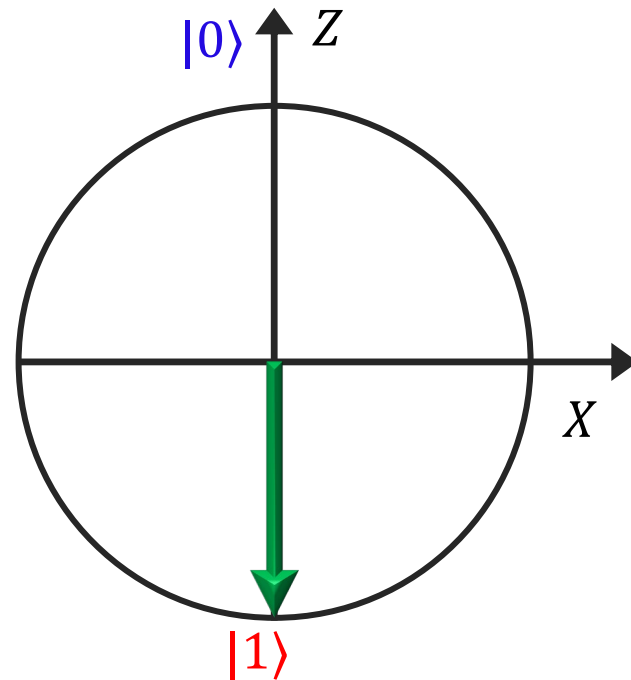
$\pi/2$ pulse: $|0\rangle \rightarrow (|0\rangle + |1\rangle)/\sqrt{2}$



Single-Qubit Gates

$\pi/2$ pulse: $|0\rangle \rightarrow (|0\rangle + |1\rangle)/\sqrt{2}$

$|1\rangle \rightarrow (|0\rangle - |1\rangle)/\sqrt{2}$



Single-Qubit Gates

$\pi/2$ pulse: $|0\rangle \rightarrow (|0\rangle + |1\rangle)/\sqrt{2}$

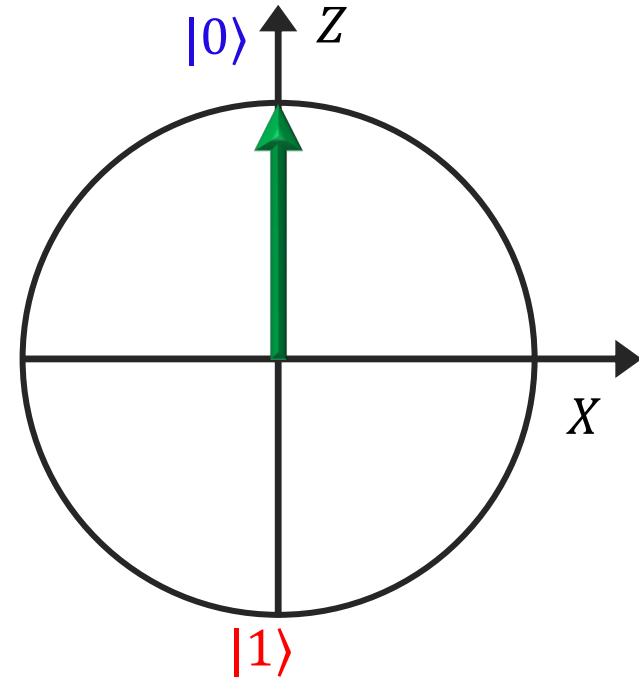
$|1\rangle \rightarrow (|0\rangle - |1\rangle)/\sqrt{2}$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

π pulse: $|0\rangle \rightarrow |1\rangle$

$|1\rangle \rightarrow |0\rangle$

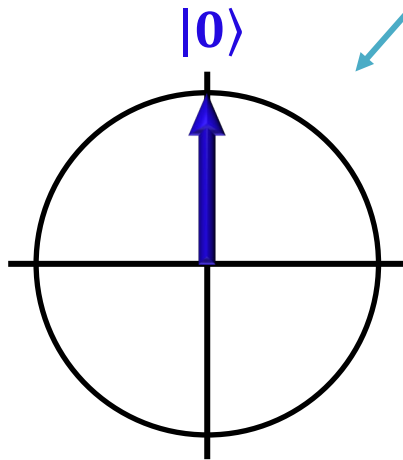
$$X = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$



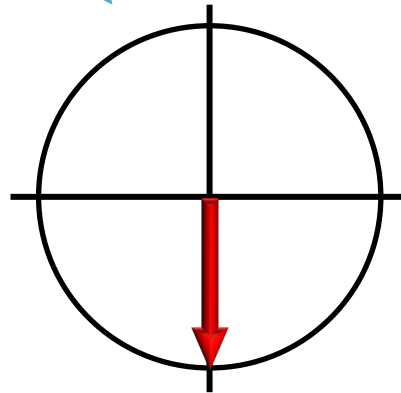
Single-Qubit Measurement

$$\alpha|0\rangle + \beta|1\rangle$$

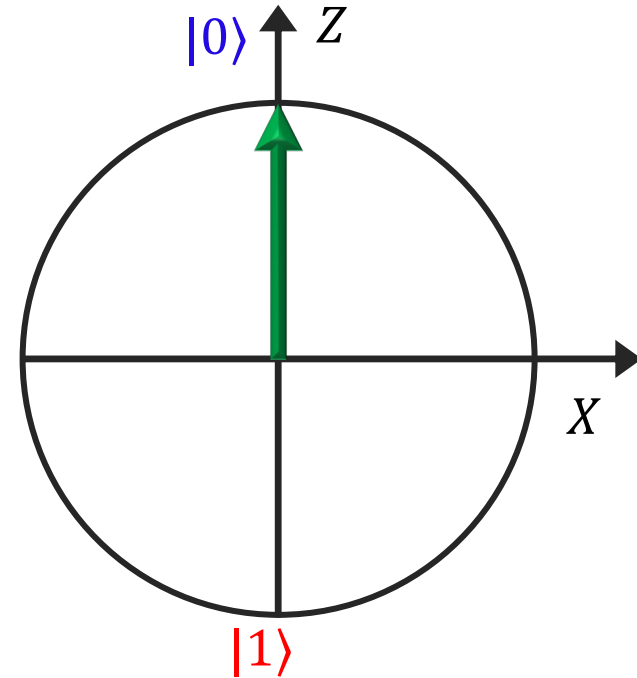
Measure



$$P(0) = |\alpha|^2$$

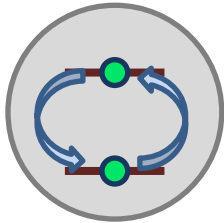
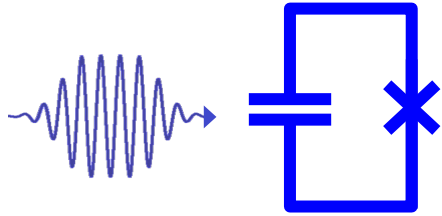


$$P(1) = |\beta|^2$$

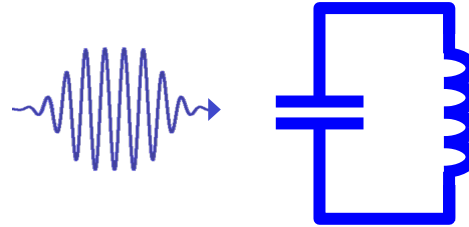


Transmon vs. Cavity Drive

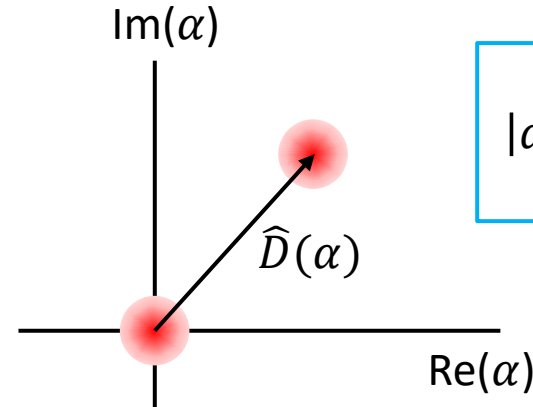
Qubit: $\alpha|0\rangle + \beta|1\rangle$



Qudit: $\alpha_0|0\rangle + \alpha_1|1\rangle + \dots + \alpha_d|d\rangle$



Coherent
state

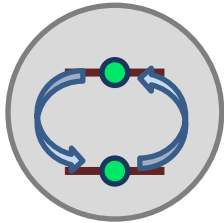
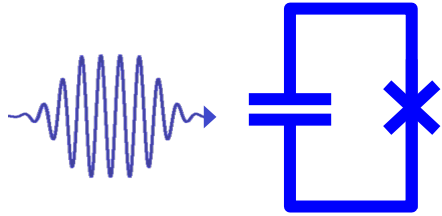


$$|\alpha\rangle = c \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

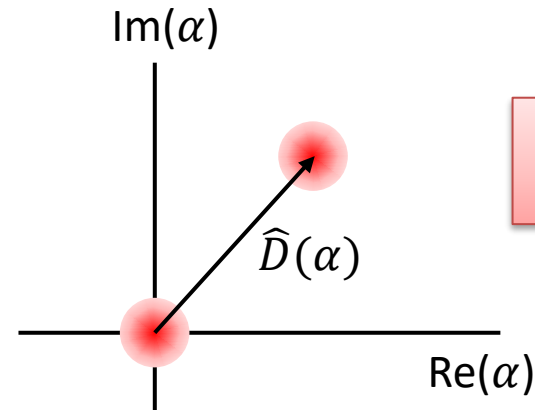
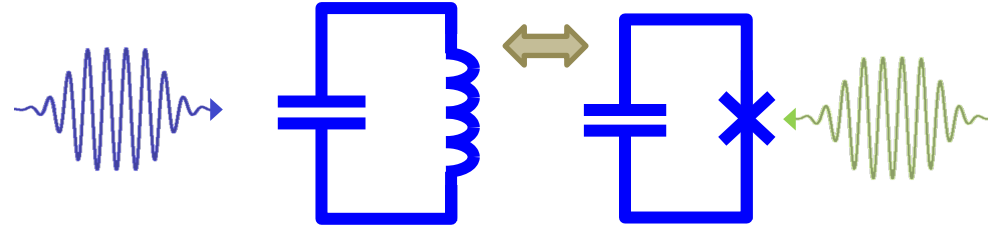
Classical

Transmon vs. Cavity Drive

Qubit: $\alpha|0\rangle + \beta|1\rangle$



Qudit: $\alpha_0|0\rangle + \alpha_1|1\rangle + \dots + \alpha_d|d\rangle$



Quantum states?

Qudit Operation

$$|0\rangle \xrightarrow{\mathcal{D}(\alpha=1)} \alpha_0|0\rangle + \alpha_1|1\rangle + \cdots + \alpha_d|d\rangle$$

$$|1\rangle \rightarrow e^{i\pi}|1\rangle$$

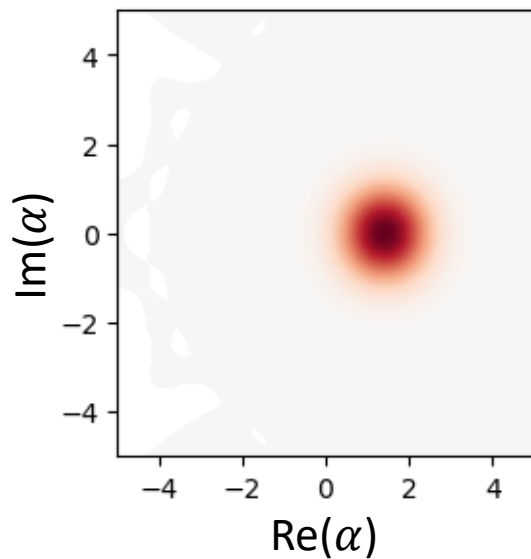
Quantum state

$$\alpha_0|0\rangle - \alpha_1|1\rangle + \cdots + \alpha_d|d\rangle$$

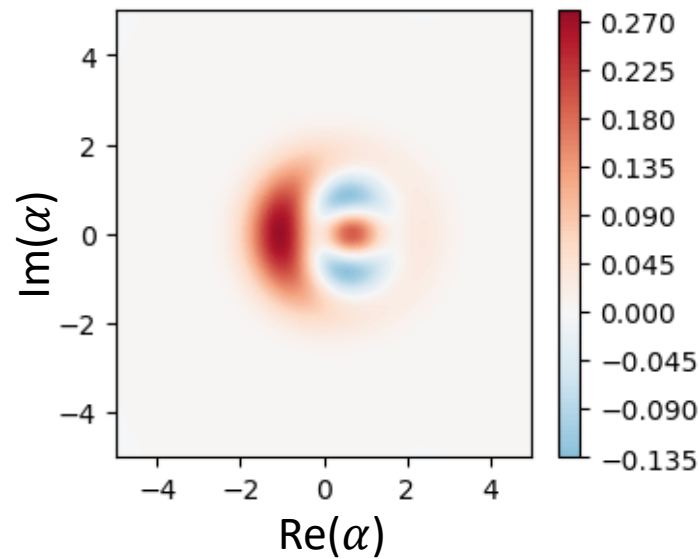
$$|n\rangle \rightarrow e^{i\theta}|n\rangle$$

Selective number-dependent arbitrary phase (SNAP) gate

PRL 115, 137002 (2015)

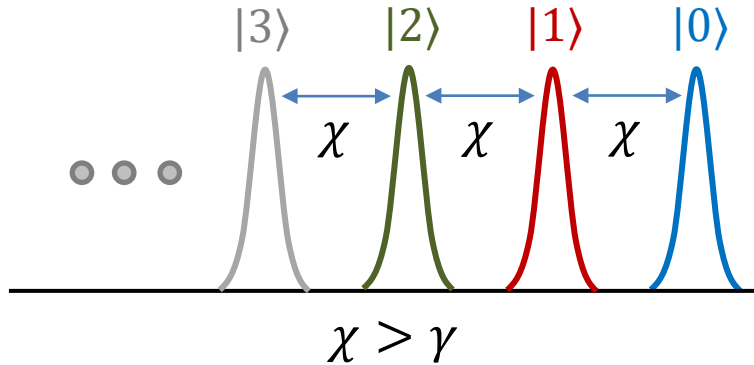


$$|1\rangle \rightarrow e^{i\pi}|1\rangle$$



Qubit Frequency Dependence

$$H = \omega_c a^\dagger a + (\omega_q + \chi a^\dagger a) \frac{\sigma_z}{2}$$

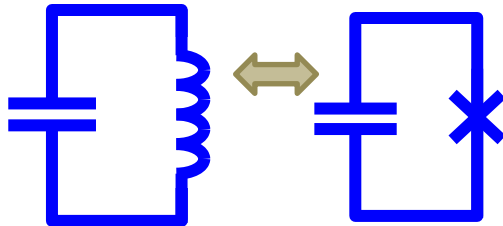


$$\omega'_q(|0\rangle_c) = \omega_q$$

$$\omega'_q(|1\rangle_c) = \omega_q + \chi$$

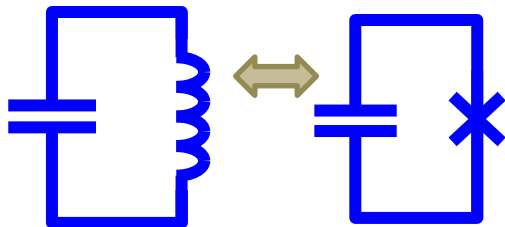
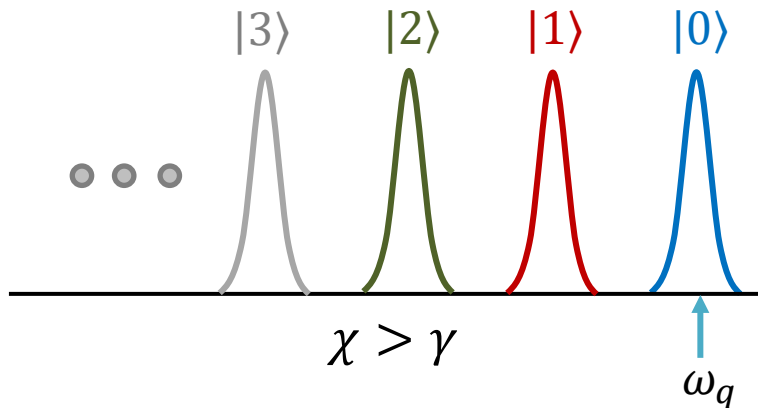
⋮

$$\omega'_q(|n\rangle_c) = \omega_q + n\chi$$



Visualization of SNAP

Selective number-dependent
arbitrary phase pulse



$$(|0\rangle + |1\rangle + |2\rangle + \dots)_c |0\rangle_q$$

$$= |0\rangle|0\rangle + |1\rangle|0\rangle + |2\rangle|0\rangle + \dots$$

$$\downarrow \cos \omega_q t \text{ (}\pi \text{ pulse)}$$

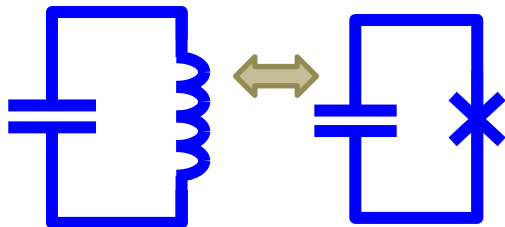
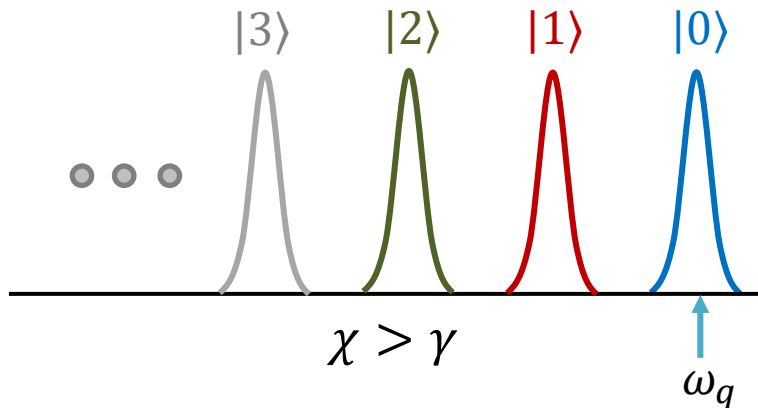
$$|0\rangle|1\rangle + |1\rangle|0\rangle + |2\rangle|0\rangle + \dots$$

$$\downarrow \cos \omega_q t \text{ (}\pi \text{ pulse)}$$

$$-|0\rangle|0\rangle + |1\rangle|0\rangle + |2\rangle|0\rangle + \dots$$

Visualization of SNAP

Selective number-dependent
arbitrary phase pulse



$$(|0\rangle + |1\rangle + |2\rangle + \dots)_c |0\rangle_q$$

$$= |0\rangle|0\rangle + |1\rangle|0\rangle + |2\rangle|0\rangle + \dots$$

$$\downarrow \cos \omega_q t \text{ (\pi pulse)}$$

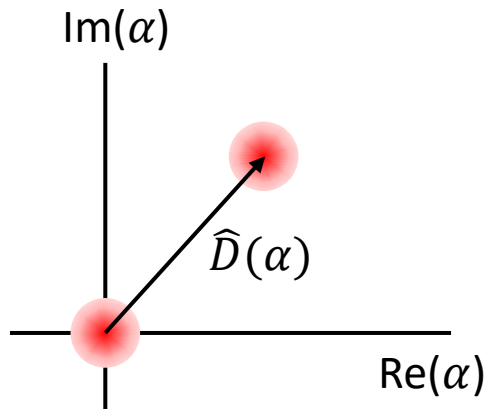
$$|0\rangle|1\rangle + |1\rangle|0\rangle + |2\rangle|0\rangle + \dots$$

$$\downarrow \cos(\omega_q t + \theta')$$

$$e^{i\theta} |0\rangle|0\rangle + |1\rangle|0\rangle + |2\rangle|0\rangle + \dots$$

$$= (e^{i\theta} |0\rangle + |1\rangle + |2\rangle + \dots) |0\rangle$$

Universal Gate Set



$$\text{Qudit: } \alpha_0|0\rangle + \alpha_1|1\rangle + \dots + \alpha_d|d\rangle$$



SNAP gate

$$\text{Qudit: } \alpha_0 e^{i\theta_0}|0\rangle + \alpha_1 e^{i\theta_1}|1\rangle + \dots + \alpha_d e^{i\theta_d}|d\rangle$$

Unconditional
operation on cavity



Cavity
drive



SNAP

Conditional operation on
cavity enabled by a transmon



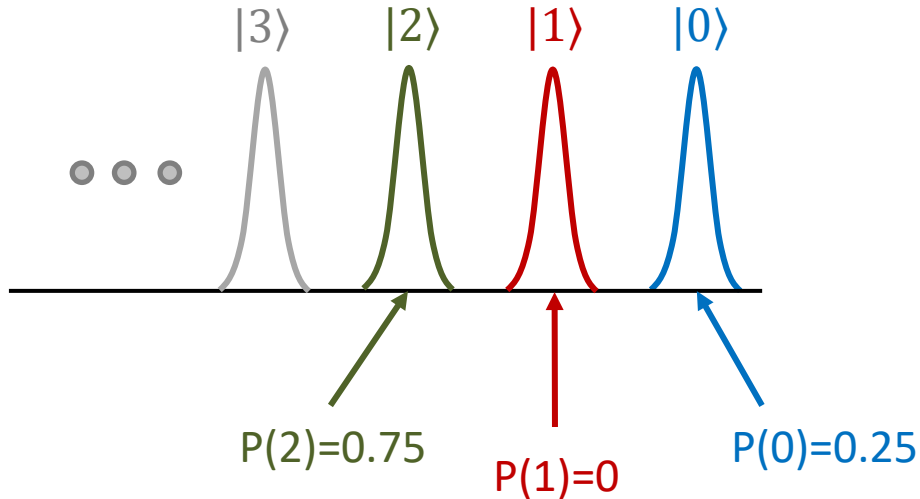
Universal control

Qudit Measurement

$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle + \cdots + \alpha_d|d\rangle$$

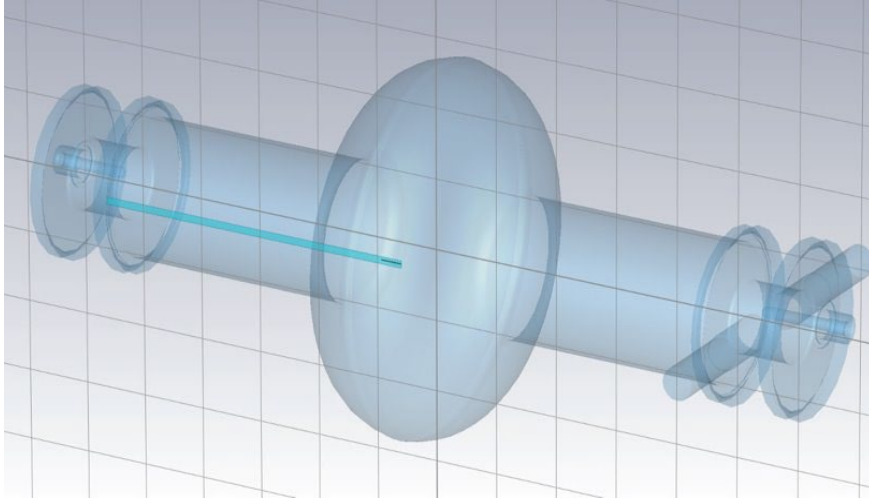
Ask the transmon if
there are n photons

Transmon jumps to
 $|1\rangle$ if yes



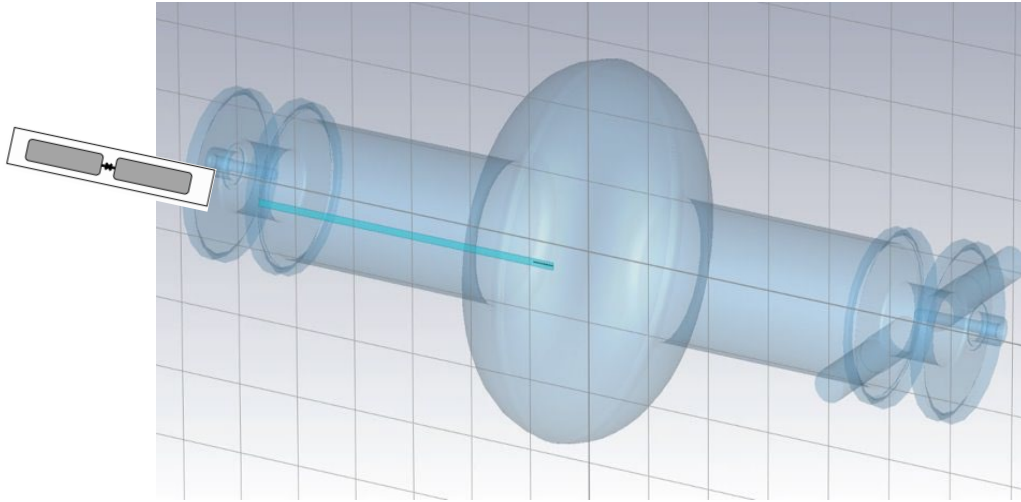
$$|\psi\rangle = \sqrt{\frac{1}{4}}|0\rangle + \sqrt{\frac{3}{4}}|2\rangle$$

First Milestone



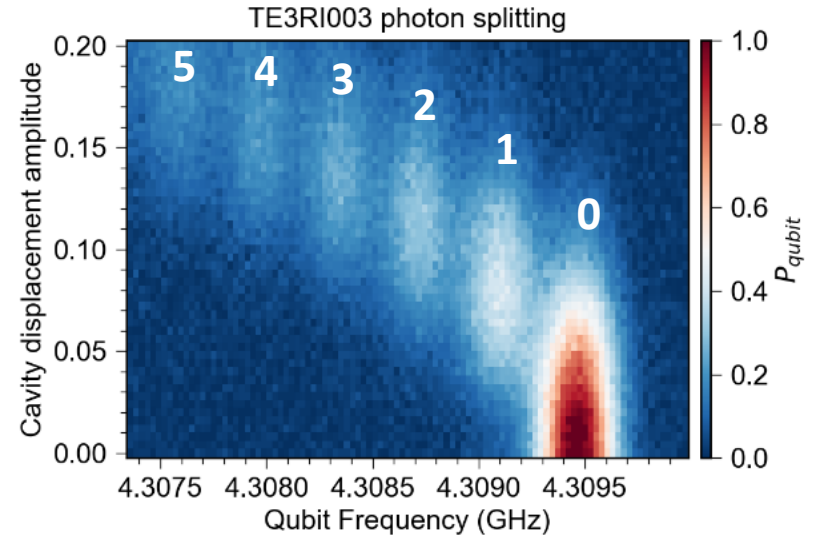
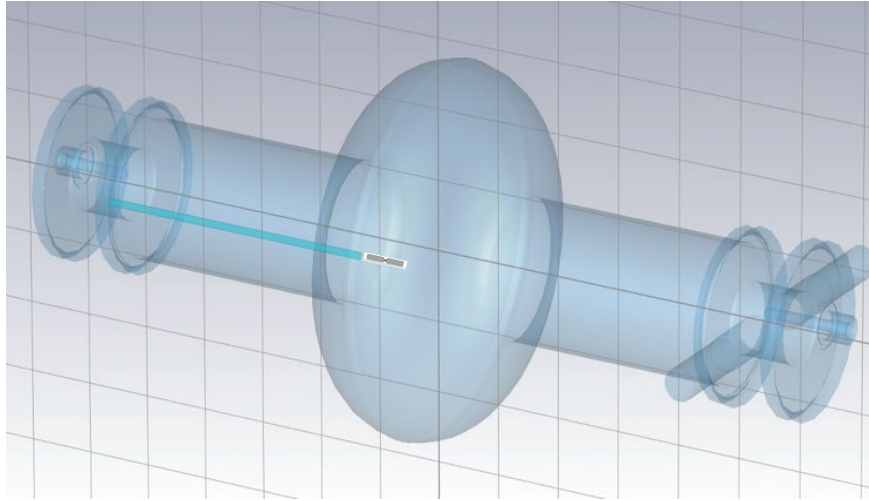
Incorporate Transmon into a
TESLA cavity

First Milestone



Incorporate Transmon into a
TESLA cavity

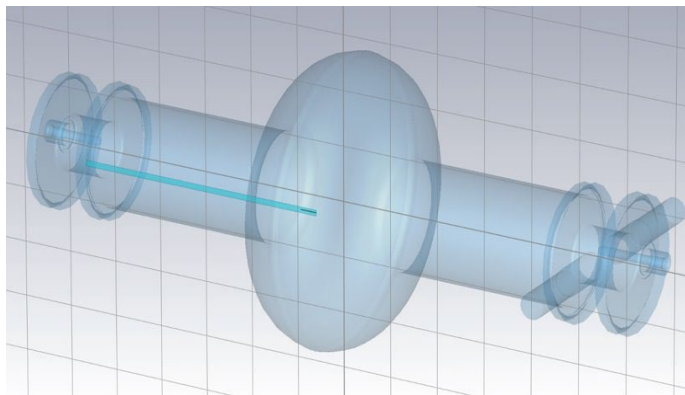
First Milestone



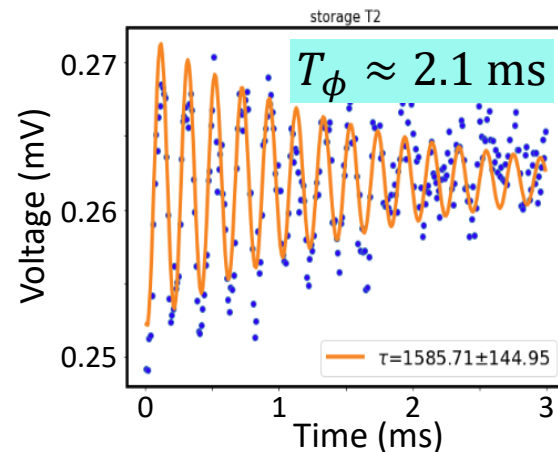
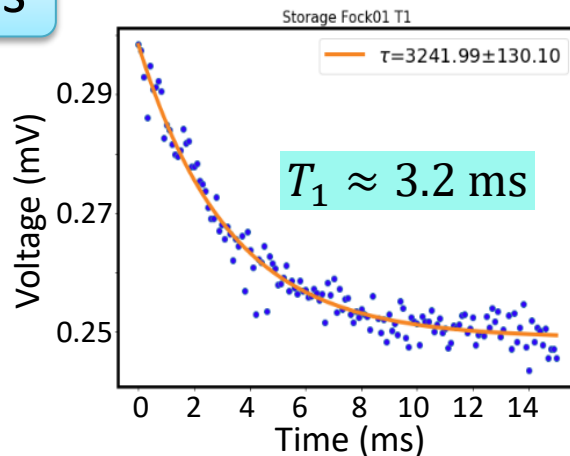
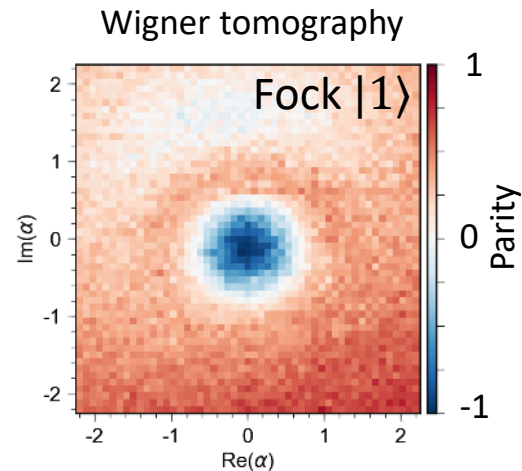
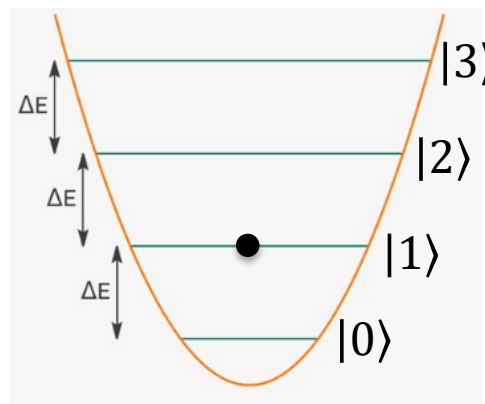
Incorporate Transmon into a
TESLA cavity

Achieved photon counting

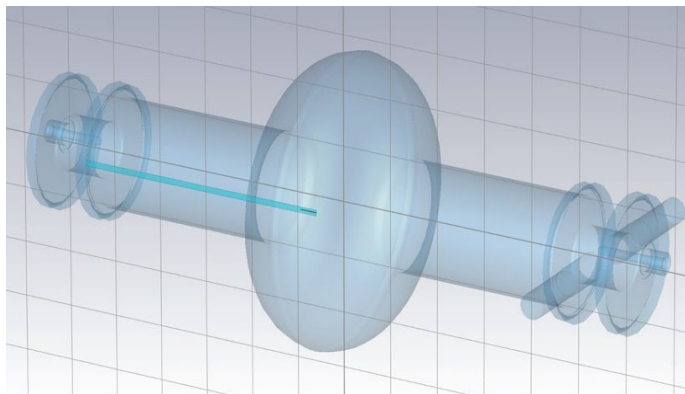
Second Milestone



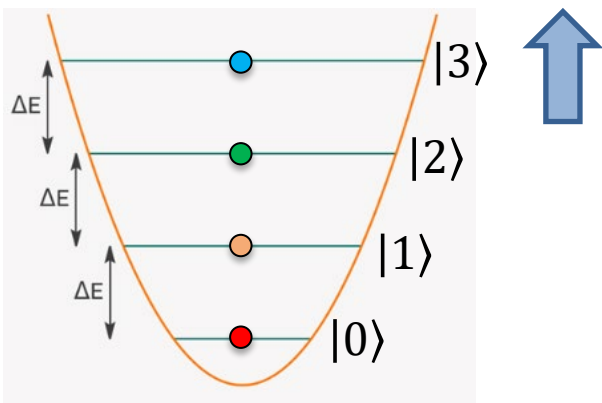
Prepare quantum states



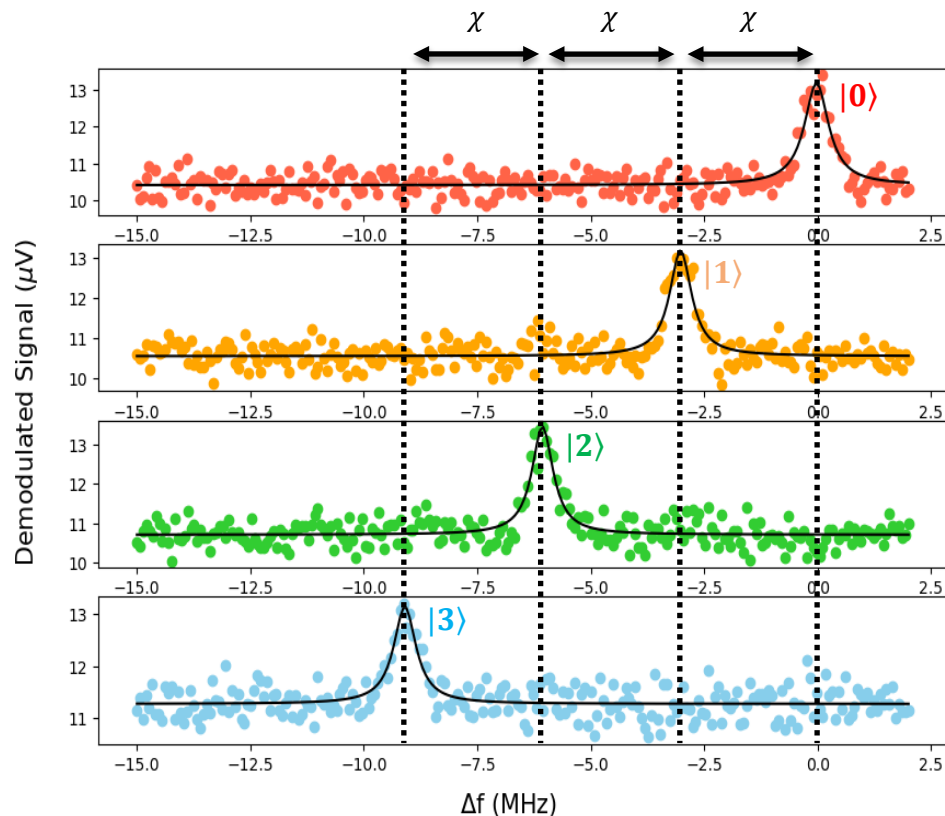
Second Milestone



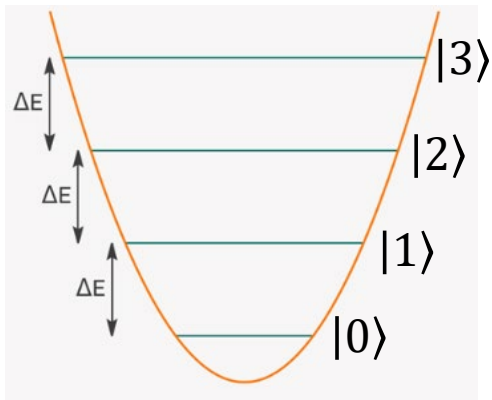
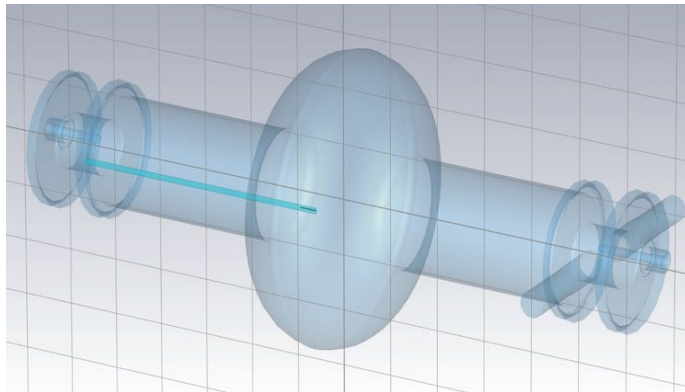
Prepare quantum states



Qubit spectroscopy



Advantages



Longer coherence

- $T_1 = 3.2$ ms vs. ~ 100 μ s
- $T_\phi = 2.0$ ms vs < 100 μ s

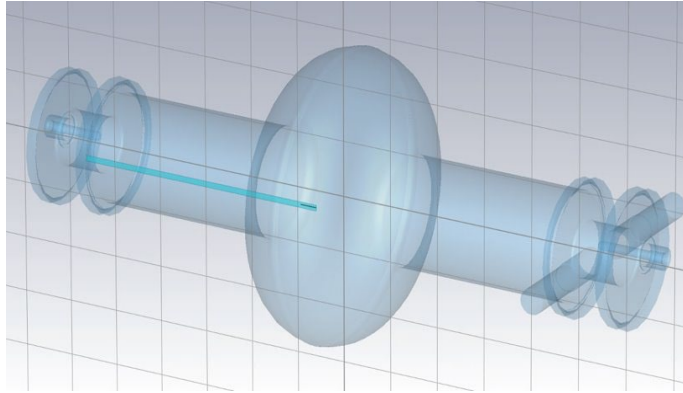
Larger Hilbert Space

- Effectively $\log_2 d$ qubits
- Enables qudit encoding

Efficient control

- One input, one output
- Reduced control hardware

Further Improvements



Substrate

Transmon

Geometry

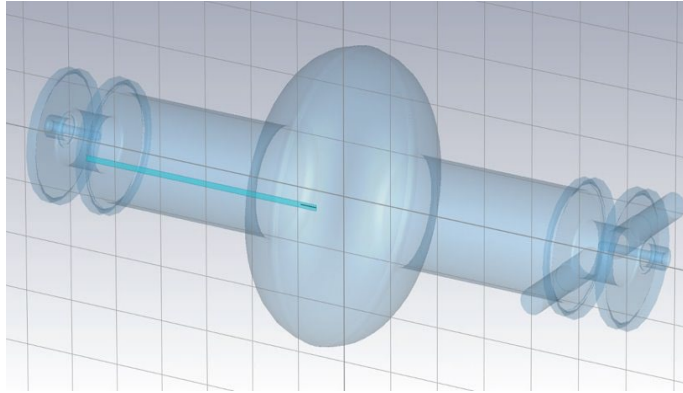
Shielding

$T_1 > 50$ ms when no transmon

Find loss sources
and eliminate

cite

Further Improvements



$T_1 > 50$ ms when no transmon

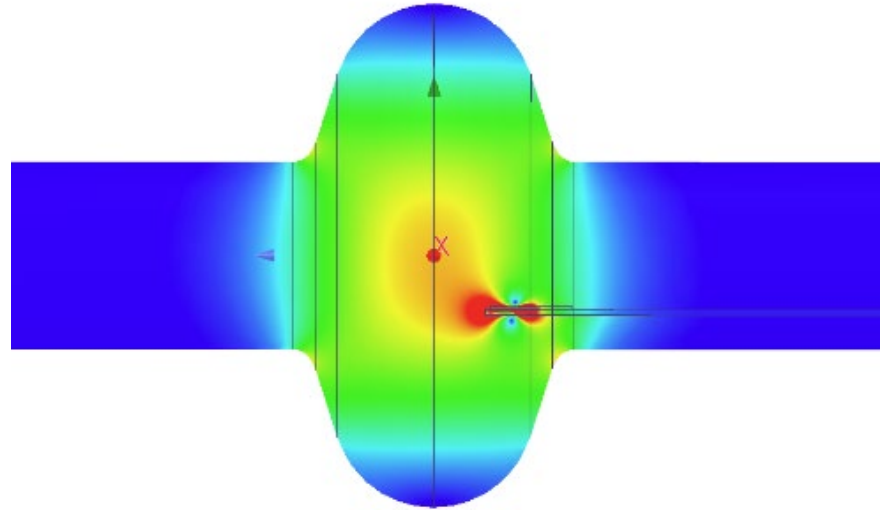
Find loss sources
and eliminate

Substrate

Transmon

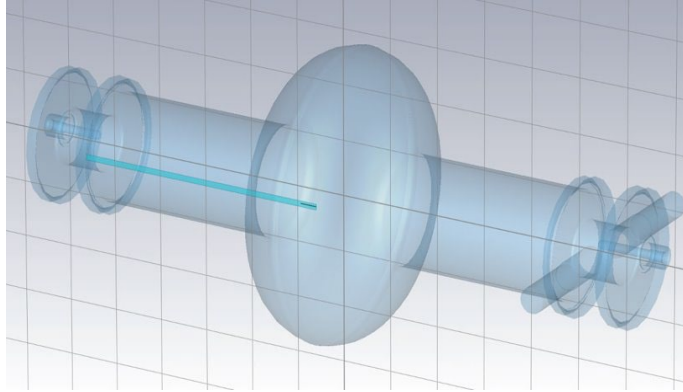
Geometry

Shielding



- Material: silicon or sapphire
- Manufacturer
- Treatment during fabrication

Further Improvements



$T_1 > 50$ ms when no transmon

Find loss sources
and eliminate

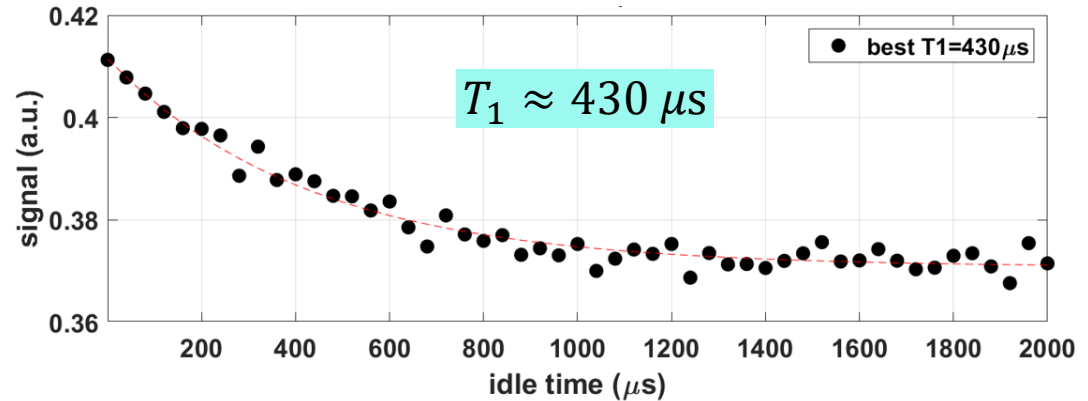
Substrate

Transmon

Geometry

Shielding

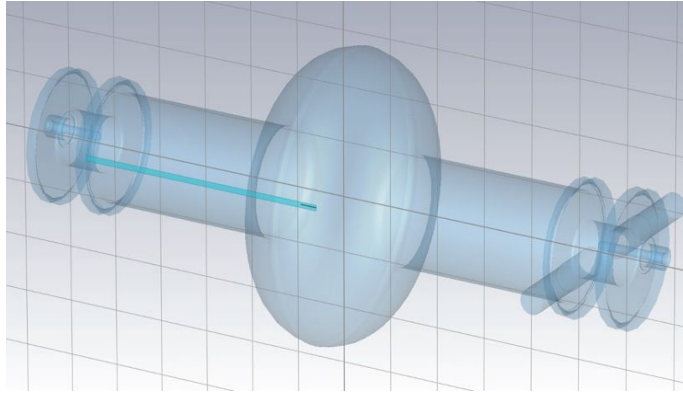
- Qubit sets Purcell limit
- Improve T_1 of transmon



Credit: Shaojiang Zhu

arxiv:2304:13257

Further Improvements



$T_1 > 50$ ms when no transmon

Find loss sources
and eliminate

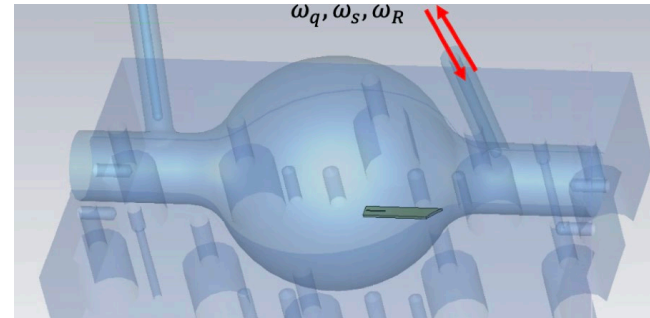
Substrate

Transmon

Geometry

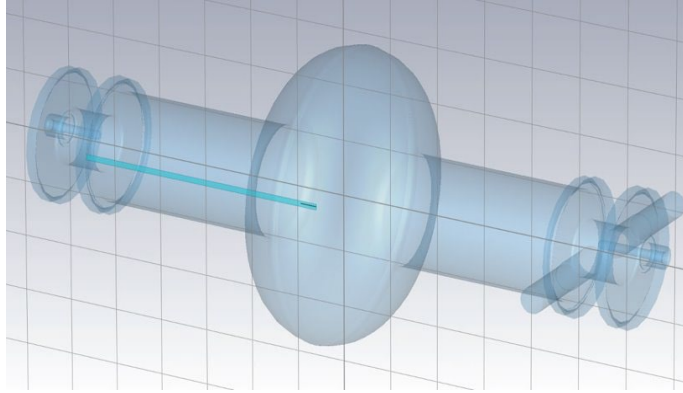
Shielding

- Surface loss
- Participation of lossy components



Credit: Taeyoon Kim

Further Improvements



$T_1 > 50$ ms when no transmon

Find loss sources
and eliminate

Substrate

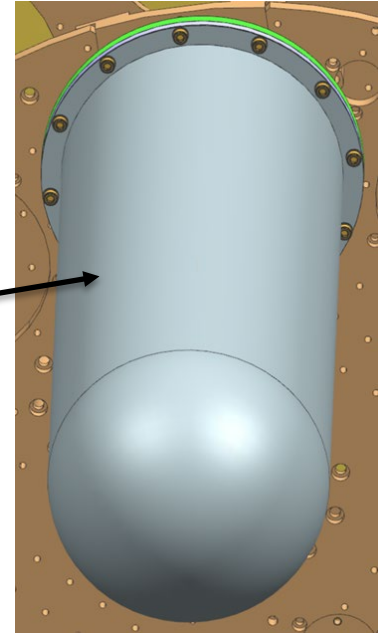
Transmon

Geometry

Shielding

- Infrared shielding
- Magnetic shielding

Multilayered shield



Credit: Oleg Pronitchev

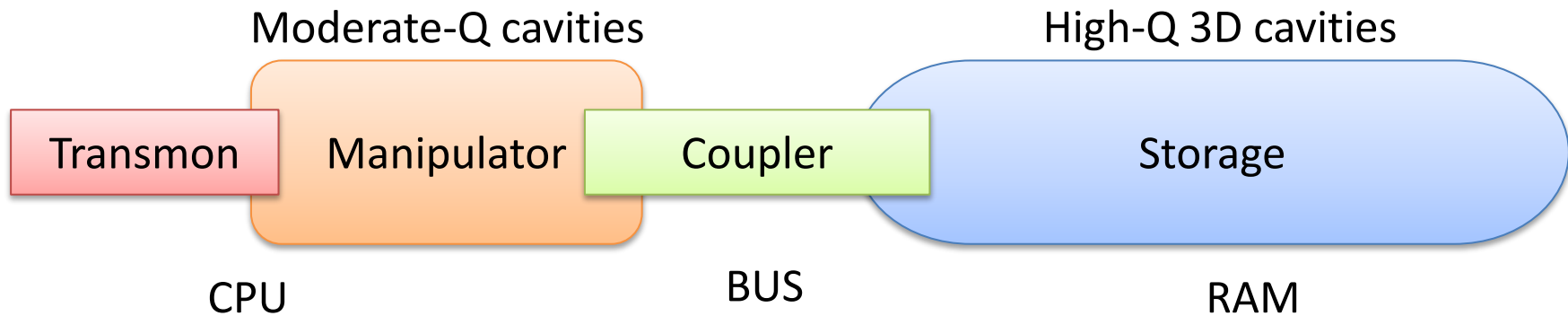
Multi-qudit Architecture

Crosstalk issues



All-to-all coupling

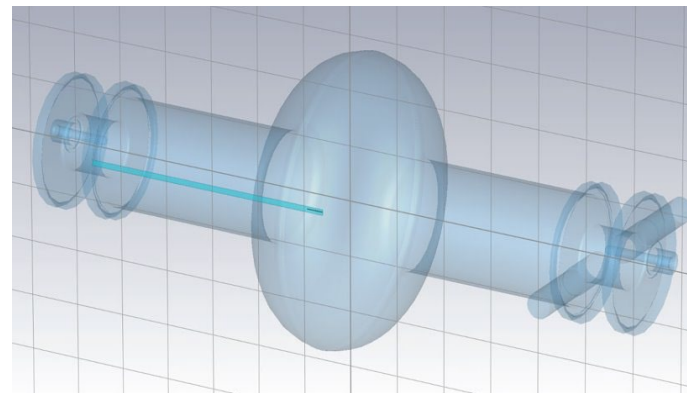
Faster scaling: $d^N > 2^N$



Summary

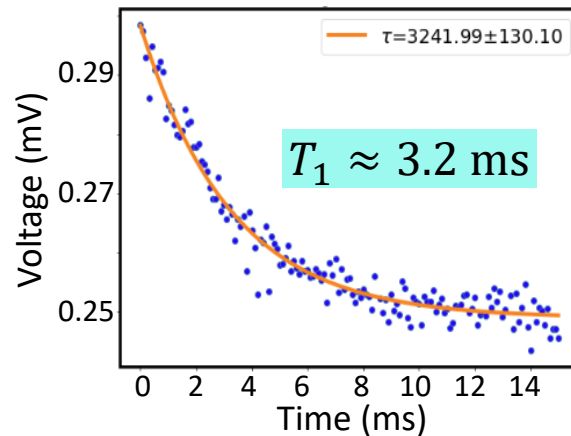
❖ Achievements

- Integrated transmon into TESLA
- Obtained $T_1 \sim 3.2$ ms, $T_\phi \sim 2.1$ ms
- Prepared several quantum states



❖ Future directions

- Implement quantum algorithms
- Keep improving hardware
- Scale up



Brand New SQMS Facility at Fermilab



Quantum Garage

Thank You!

